Æfing 1.4 **BLS 1**

1. Reiknaðu hnit einingarvigurs sem er samsíða a

$e = \frac{1}{|a|} \cdot a$

a) $a = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$|a| = \sqrt{1^2 + 1^2}$

$|a| = \sqrt{2}$

$e = \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$e = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$

b) $a = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$|a| = \sqrt{4^2 + 3^2}$

$|a| = \sqrt{25}$

$|a| = 5$

$e = \frac{1}{5} \cdot \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$e = \begin{pmatrix} \frac{4}{5} \\ \frac{3}{5} \end{pmatrix}$

c) $a = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$|a| = \sqrt{(-1)^2 + 2^2}$

$|a| = \sqrt{1 + 4}$

$|a| = \sqrt{5}$

$e = \frac{1}{\sqrt{5}} \cdot \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$e = \pm \begin{pmatrix} -\frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix}$

d) $a = \begin{pmatrix} -8 \\ -6 \end{pmatrix}$

$|a| = \sqrt{(-8)^2 + (-6)^2}$

$|a| = \sqrt{64 + 36}$

$|a| = \sqrt{100}$

$|a| = 10$

$e = \frac{1}{10} \cdot \begin{pmatrix} -8 \\ -6 \end{pmatrix}$

$\frac{1}{10} \cdot (-8) = -\frac{8}{10}/2 = -\frac{4}{5}$

$\frac{1}{10} \cdot (-6) = -\frac{6}{10}/2 = -\frac{3}{5}$

$e = \pm \begin{pmatrix} -\frac{4}{5} \\ -\frac{3}{5} \end{pmatrix}$

2. – II – gagnstefna a

a) $a = \begin{pmatrix} -5 \\ 12 \end{pmatrix}$

$|a| = \sqrt{(-5)^2 + 12^2}$

$|a| = \sqrt{25 + 144}$

$|a| = \sqrt{169}$

$|a| = 13$

$-e = \frac{1}{13} \cdot \begin{pmatrix} -5 \\ 12 \end{pmatrix}$

$e = \begin{pmatrix} -\frac{5}{13} \\ \frac{12}{13} \end{pmatrix}$

b) $a = \begin{pmatrix} 24 \\ -7 \end{pmatrix}$

$|a| = \sqrt{24^2 + (-7)^2}$

$|a| = \sqrt{576 + 49}$

$|a| = \sqrt{625}$

$|a| = 25$

$-e = \frac{1}{25} \cdot \begin{pmatrix} 24 \\ -7 \end{pmatrix}$

$-e = \begin{pmatrix} \frac{24}{25} \\ -\frac{7}{25} \end{pmatrix} = \begin{pmatrix} -\frac{24}{25} \\ \frac{7}{25} \end{pmatrix}$

c) $a = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

$|a| = \sqrt{(-3)^2 + 2^2}$

$|a| = \sqrt{9 + 4}$

$|a| = \sqrt{13}$

$-e = \frac{1}{\sqrt{13}} \cdot \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

$e = \begin{pmatrix} \frac{3}{\sqrt{13}} \\ -\frac{2}{\sqrt{13}} \end{pmatrix}$

d) $a = \begin{pmatrix} 2\sqrt{2} \\ 2 \end{pmatrix}$

$|a| = \sqrt{2\sqrt{2}^2 + 2^2}$

$|a| = \sqrt{8 + 4}$

$|a| = \sqrt{12}$

$-e = \frac{1}{\sqrt{12}} \cdot \begin{pmatrix} 2\sqrt{2} \\ 2 \end{pmatrix}$

$e = \begin{pmatrix} -\frac{\sqrt{6}}{3} \\ -\frac{\sqrt{3}}{3} \end{pmatrix}$

3. Reiknaðu hvort eftirfarandi sem er samsíða línunni

a) $y=2x+1$ b) $y=0,5x+2$ c) $y=-x+3$ d) $y=\frac{3}{4}x+4$

Línu $h_a=\frac{2}{1}=1,2$ $h_a=\frac{0,5}{1}=1,0,5$ $h_{\bar{a}}=-1$ $h_a=\frac{a_2}{a_1}=\frac{3}{4}$

$\bar{a}=\begin{pmatrix}1\\-1\end{pmatrix}$

$a=\begin{pmatrix}1\\2\end{pmatrix}$ $|a|=\sqrt{1^2+0,5^2}$ $|a|=\sqrt{1^2+(-1)^2}$ $|\bar{a}|=\sqrt{4^2+3^2}$

$|a|=\sqrt{1^2+2^2}$ $|a|=\sqrt{1+0,25}$ $|a|=\sqrt{2}$ $|\bar{a}|=\sqrt{16+9}$

$|a|=\sqrt{5}$ $|a|=\sqrt{1,25}$ $\bar{e}=\frac{1}{\sqrt{2}}\cdot\begin{pmatrix}1\\-1\end{pmatrix}$ $|a|=\sqrt{25}$

$|a|=\sqrt{5}$ $|a|=5$

$e=\pm\frac{1}{\sqrt{5}}\cdot\begin{pmatrix}1\\2\end{pmatrix}$ $e=\frac{\frac{\sqrt{5}}{2}}{\frac{\sqrt{5}}{4}}$ $\bar{e}=\begin{pmatrix}\frac{1}{\sqrt{2}}\\-\frac{1}{\sqrt{2}}\end{pmatrix}$ $\bar{e}=\frac{1}{5}\cdot\begin{pmatrix}4\\3\end{pmatrix}=\begin{pmatrix}\frac{4}{5}\\\frac{3}{5}\end{pmatrix}$

$e=\pm\begin{pmatrix}\frac{1}{\sqrt{5}}\\\frac{2}{\sqrt{5}}\end{pmatrix}$

4.-7. Reiknaðu hvort eftirfarandi sem hafa lengdina og eru samsíða línunni

4. Lengd $=10$ samsíða línunni $y=\frac{3}{4}x$

$h_{\bar{a}}=\frac{a_2}{a_1}=\frac{3}{4}$ $a=4,3$

$|a|=\sqrt{4^2+3^2}$

$|a|=\sqrt{25}$

$|a|=5$

$e=\frac{10}{5}\cdot\begin{pmatrix}4\\3\end{pmatrix}$

$\bar{e}=\pm\begin{pmatrix}8\\6\end{pmatrix}$

7. Lengd 10 línunni $y=\frac{3}{5}x$

$h_{\bar{a}}=\frac{3}{5}=(a=3,2)$

$|a|=\sqrt{3^2+2^2}$

$|a|=\sqrt{13}$

$\bar{e}=\frac{10}{\sqrt{13}}\cdot\begin{pmatrix}3\\2\end{pmatrix}=\pm\begin{pmatrix}\frac{30}{\sqrt{13}}\\\frac{20}{\sqrt{13}}\end{pmatrix}$

8. Lengd 2 línu $y=x-5$

$h_{\bar{a}}=\frac{a_2}{a_1}=\frac{1}{1}=(a=1,1)$

$|a|=\sqrt{1^2+1^2}$

$|a|=\sqrt{2}$

$\bar{e}=\frac{2}{\sqrt{2}}\cdot\begin{pmatrix}1\\1\end{pmatrix}=\pm\begin{pmatrix}\sqrt{2}\\\sqrt{2}\end{pmatrix}$

Æfing 1.5 Æfing 1.5

B/S 1

1 a)
$a \cdot b = 3 \cdot (-2) + 5 \cdot 3$
$a \cdot b = -6 + 15$
$a \cdot b = 9$

b)
$a \cdot b = (-12) \cdot 8 + (-5) \cdot (-6)$
$a \cdot b = -96 + 30$
$a \cdot b = -66$

c)
$a \cdot b = (-4) \cdot 4 + 3 \cdot 5$
$a \cdot b = -16 + 15$
$a \cdot b = -1$

2. a)
$a \cdot b = 3 \cdot (-2) + 5 \cdot 3$
$a \cdot b = 9$
Hvasst

b)
$a \cdot b = 4 \cdot 1 + 1 \cdot (-4)$
$a \cdot b = 0$
Hornrétt

c)
$a \cdot b = 1 \cdot (-1) + 2 \cdot (-2)$
$a \cdot b = -1 + (-4)$
$a \cdot b = -6$
gleitt

4. $a = \begin{pmatrix} 2 \\ t-2 \end{pmatrix}$ $b = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ (t) leitað að eru "hornrétt" þá er miðað við 0; jöfnuna
—"— er að hvasst þá er gert > 0
gleitt þá er gert < 0

$a \cdot b = 2 \cdot 3 + (t-2)(2) = 0$

$6 + 2t - 4 = 0$
$2t + 2 = 0$
$2t = -2$
$t = -2/2 = -1$

8. $a = \begin{pmatrix} 2 \\ t - 2,5 \end{pmatrix}$ $b = \begin{pmatrix} -3 \\ t \end{pmatrix}$ gildi T hornrétt

$a \cdot b = 2 \cdot (-3) + (t - 2,5)(t) = 0$

$-6 + t^2 - 2,5t = 0$
$t^2 - 2,5t - 6 = 0$

$$t = \frac{-(-2,5) \pm \sqrt{(-2,5)^2 - 4 \cdot (1) \cdot (-6)}}{2 \cdot 1}$$

$$t = \frac{2,5 \pm \sqrt{6,25 + 24}}{2}$$

$$t = 2,5 \pm \sqrt{30,25}$$

$$t = \frac{2,5 \pm 5,5}{2}$$

$$t = \frac{2,5 + 5,5}{2} \qquad t = \frac{2,5 - 5,5}{2}$$

$$t = \frac{8}{2} = 4 \qquad t = \frac{-3}{2} = -1,5$$

$$t = \begin{cases} -1,5 \\ 4 \end{cases}$$

A Bls 1.5 bls 2

11) Reiknaðu x þannig að gefnir vigrarnir $a=\begin{pmatrix} x \\ 4 \end{pmatrix}$ og $b=\begin{pmatrix} x+1 \\ -3 \end{pmatrix}$

a) $a \parallel b \rightarrow a_1/b_1 = a_2/b_2$

$\frac{x}{x+1} = \frac{4}{-3}$ krossa

$-3x = 4(x+1)$

$-3x = 4x+4$

$-7x = 4$

$x = 4/-7$

$x = -\frac{4}{7}$

b) $|a| = |b|$ lengd vigranna: $|a| = \sqrt{a_1^2+a_2^2}$

$\sqrt{x^2+4^2} = \sqrt{(x+1)^2+(-3)^2}$

$x^2+16 = x^2+2x+1+9$

$16-10 = 2x$

$6 = 2x$

$2x = 6$

$x = 6/2 = 3$

c) $a \perp b$

$a \cdot b = x(x+1)+4(-3) = 0$

$x^2+x-12 = 0$

$(x+4)(x-3) = 0$

$x+4 = 0 \quad x-3 = 0$

$x = -4 \quad x = 3$

$x \begin{cases} -4 \\ 3 \end{cases}$

14) $a=\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ lengd 5 sömuleiðis

$|a| = \sqrt{4^2+3^2}$

$|a| = \sqrt{16+9}$

$|a| = \sqrt{25}$

$|a| = 5$

$e = \pm \frac{5}{5}\begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$e = \pm \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

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15) $a=\begin{pmatrix} 6 \\ 8 \end{pmatrix}$ lengd = 10 (hornrétt vigur $b=(-y,x)$

$b = (-8,6)$ $\begin{pmatrix} -8 \\ 6 \end{pmatrix}$

$|b| = \sqrt{(-8)^2+6^2}$

$|b| = \sqrt{64+36}$

$|b| = \sqrt{100}$

$|b| = 10$

$e = \pm \frac{10}{10}\begin{pmatrix} -8 \\ 6 \end{pmatrix} = \pm\begin{pmatrix} -8 \\ 6 \end{pmatrix}$ Hornréttur vigur $a=\begin{pmatrix} 6 \\ 8 \end{pmatrix}$

$b=\begin{pmatrix} -8 \\ 6 \end{pmatrix}$ eða

$b=(-y,x)$