An Exploration of Optical Correlators
Convolution at the Speed of Light

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Abstract

Convolution is an extremely common computation, but it is rather computationally expensive to perform. We attempt to perform a two dimensional convolution of an image and filter using an optical correlator. While optical correlators have been around for many years, some of the techniques require large amounts of preparatory work for a single operation making them infeasible for regular use. In this paper we explore novel methods of producing an optical correlator that do not require such overhead making them much more feasible to be used for computation. Although the mathematics and theory seem to suggest our filtering method is effective, experimental results could not confirm this to be the case.

1 Introduction

In the age of large data sets, computation has become a nightmare. Disciplines like data searching, image analysis, and artificial intelligence require large numbers of correlation and convolution[1] operations whose time and memory requirements grow quickly with the size of the input. One previously proposed solution to this problem is to change the method of computation from digital to optical.

1.1 Background

According to Fraunhofer diffraction theory, the diffraction pattern produced at infinity is a Fourier transform of the diffracting aperture. Because of the prevalence of the Fourier transform in many areas of science and math, this property makes aperture diffraction considerably useful in optical computation and will form a the backbone of all the computations we perform. While the Fourier transform of the aperture is produced at an infinite distance, we can use a converging lens to focus these parallel beams of light on a point one focal distance from the lens making measurement of the resultant pattern feasible.

According to the convolution theorem, convolution in the spatial domain is equivalent to point-wise multiplication in the frequency domain. This is useful because multiplying light is relatively easy to do as we can simply pass the light through a filter that blocks a certain fraction of the light. This is equivalent to multiplying by a number in the range [0,1] inclusive.

Combining the Fourier transform produced by diffraction from the aperture and multiplication from filters we have enough to perform simple convolutions and correlations. To perform these operations we need only an aperture with the shape of our image and a filter whose pattern is the Fourier transform of the kernel you'd like to perform the correlation operation with. For a given binary image (black, white, no gray), the image itself can be used as the aperture if produced on a transparent medium. Producing a proper filter though is slightly more involved.

In general, the Fourier transform of a function is complex valued. This creates quite a problem as it is difficult to directly interact with the complex components of light. Previously, holographic filters were produced which recorded the entire complex-valued Fourier transform of the desired correlating image [1]. To produce each holographic filter, the aperture diffraction and lens setup described previously was used to produce the Fourier transform of the desired function on a photographic plate while a reference beam from the same light source was also incident on the plate. This obviously required a lot of potentially expensive work because a new holographic filter had to be produced every time you wanted to convolve with a different pattern. Instead of dealing with the hassle of the complex valued filter, we instead avoid the problem al-

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[1] In this paper, correlation and convolution will be referred to interchangeably since they are essentially the same operation. The only difference between them is that the kernel is flipped in convolution and left in the original orientation for correlation.

together by imposing restrictions on the function that we are convolving our image with.

1.2 Filter Restrictions

Being that it is difficult to interact with the non-real component of light we can impose restrictions on our function so that the the imaginary component is approximately zero meaning that our filter must only deal with the real component. The restrictions were formed by inspection of the Fourier integral for an arbitrary function $f(x)$ which is shown below. For our purposes we will only consider the case when $f(x)$ is a real-valued function.

$$\hat{f}(\omega) = \int_{-\infty}^{\infty} f(x)e^{-2\pi ix\omega}dx$$
$$= \int_{-\infty}^{\infty} f(x)\cos(2\pi x\omega)dx$$
$$- i\int_{-\infty}^{\infty} f(x)\sin(2\pi x\omega)dx$$

In order for the imaginary component of our filter to be zero, it is obvious from the the equation shown above that we must require that

$$\int_{-\infty}^{\infty} f(x)\sin(2\pi x\omega)dx = 0 \qquad (1)$$

Since sine is an odd function and zero is an even function, we can deduce that $f(x)$ must be an even function in order for this condition to hold. Although the equations shown above represent the single dimensional Fourier transform the result does extend to the two-dimensional case. Effectively, this requires that the function we wish to convolve our image with must exhibit absolute symmetry in both the vertical and horizontal dimensions.

This restriction is tight but, luckily, many of the functions commonly used in convolutions exhibit this symmetry. If an asymmetric filter is needed for a specific application this method will not be usable as you must keep track of both the real and imaginary parts of the Fourier transform in order for the resulting convolution to be accurate.

To proceed then, we restrict that the image we wish to produce the filter from be symmetric horizontally and vertically and then take its Fourier transform to produce our filter. When using discretized data like an image and using an implementation of the Fourier transform like the Fast Fourier Transform (FFT), it is usually the case that not all imaginary components of the resulting transform are zero. To remedy this we simply take the real component of the Fourier transform as our filter. Although we are loosing some information by doing this, it is shown in the next section

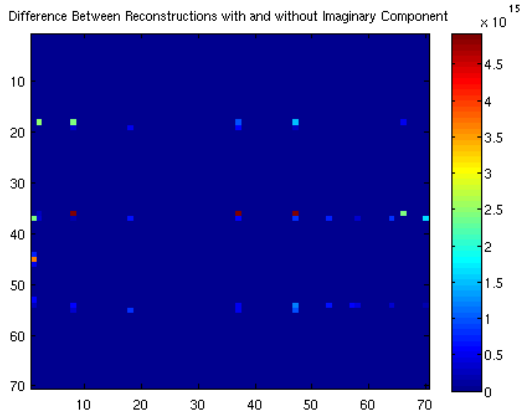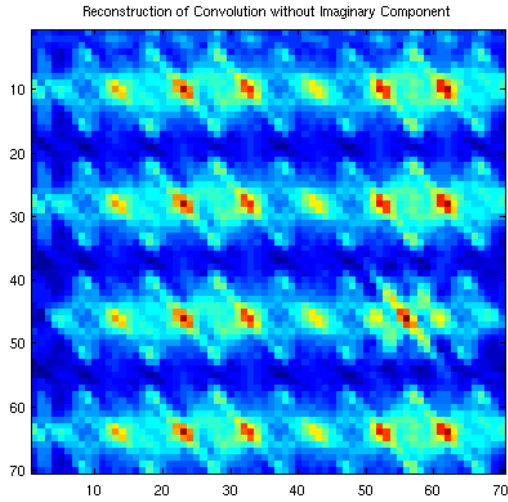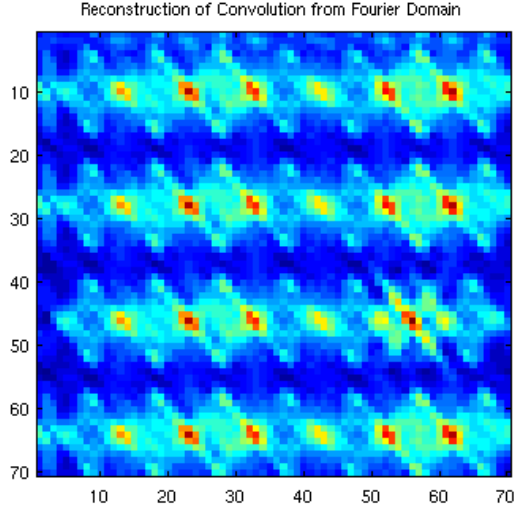that this is typically still a good approximation of even functions.

2 Numerical Simulation

To test whether this method of only using the real component of a Fourier transform functions as we expect, we can run the situation numerically. In this simulation we correlate the two images shown below, making sure that our filtering image is even. Both images are binary so dark pixels have a value of zero while light pixels have a value of one. Our method will be to numerically take the Fourier transform of both images, discard any artifact imaginary values that may arise in the transform of the filter image, multiply the results, and then perform an inverse Fourier transform to return the resulting image to the spatial domain.

Base Image

$$(2)$$

Even Convolving Image

$$(3)$$
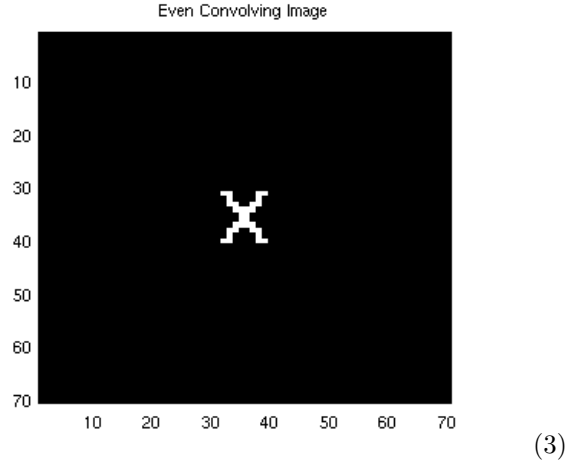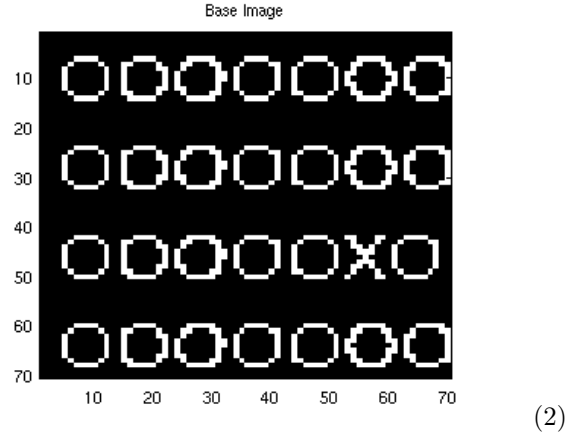
To start we take the Fourier transform of both images using the FFT algorithm. We then create a copy of the transform of the even convolving image and set the imaginary component of each pixel to zero. We then point-wise multiply the two standard transforms and the image transform with the real component of the even image transform and perform the inverse Fourier

transform to reconstruct both results. The two resulting images are shown below along with their difference.

Reconstruction of Convolution from Fourier Domain

(4)

Reconstruction of Convolution without Imaginary Component

(5)

Difference Between Reconstructions with and without Imaginary Component

(6)

As can be seen from the above images, the two results are very similar. By looking at the difference image we can see that the only substantial differences occur in minimums of the pattern and are highly localized (almost always to a single pixel). From these results we expect that using only the real values for a filter will still produce a good approximation of the actual convolution.

Another method of calculating Fourier filters using a down-sampling of the Discrete Cosine Transform (DCT) was also tested but results are not shown as the convolution and difference images are indistinguishable from those produced by ignoring the imaginary components of the Fourier transform which makes sense because these two operations should be equivalent for real-valued, even functions.

3 Experiment

To test whether this method is indeed usable to perform computations, we now construct an optical correlator to test whether this method of using a multiplicative real filter works.

3.1 Experimental Setup

Light from a Helium-Neon Laser passes through a spatial filter (10x magnification). The light then passes through a lens whose focal length is equal to its distance from the spacial filter which focuses the beam at infinity. This beam then travels through the image aperture after which it passes though another converging lens. At exactly one focal length of this lens further on, the light passes through the Fourier filter after which it it reconstructed by a converging lens placed its own focal length beyond the Fourier filter. The light then passes through a diverging lens and is projected onto a screen. The diverging lens is unnecessary but it helped to enlarge the resulting image. A picture of the actual setup used for experiments can be found in the last section of this article.


(7)
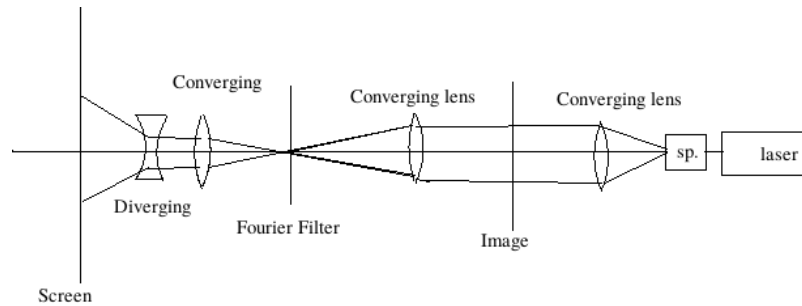
To block extraneous light, dark plate with a hole was placed after the first converging lens and before the

image. This blocked everything but the central maximum produced by the spatial filter in order to achieve a more uniform beam.

3.2 Testing

A transparency of the Base Image (2) was used as the image for all experiments. A total of two different Fourier filters were tested, both filters were calculated using the Even convolving image (3) from the numerical simulation. The first filter tested was the result of taking the real part of the FFT of the even, lone X image. It is shown below. Using a filter in this way, we can only multiply light by a value between zero and one so the negative values produced by the transform are simply set to zero. While this does cause us to lose some accuracy, we saw with the numerical simulation that the reconstructed pattern still looks mostly like those of our original numerical simulation. An image of this numerical result is shown next to the product of our experiments for comparison.
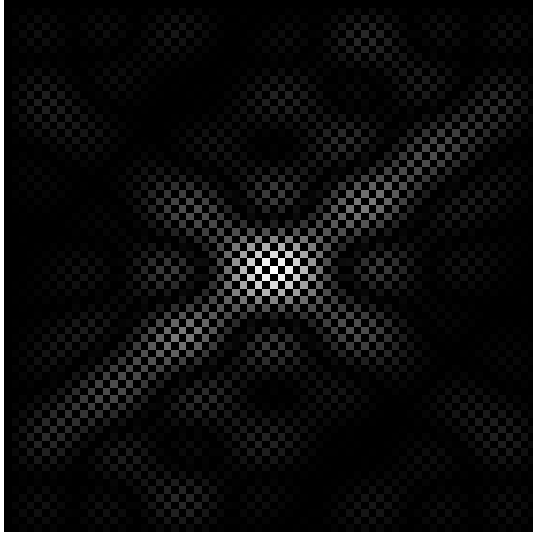
(8)

This image was placed on the Fourier plane and adjusted to get the central light maximum to fall directly on the center of the image. Aligning the filter precisely was difficult so the lens after the Fourier plane was moved slightly further from the Fourier plane so that a real, magnified image of the Fourier filter with a bright spot where the light was hitting it was produced. Using this method finer adjustments could be made by hand to filter placement. An image of the alignment that was used to produce our data is shown on the last page of this article and a zoomed and colorized picture of just the central maximum is shown below with the

maxima on the pattern marked.
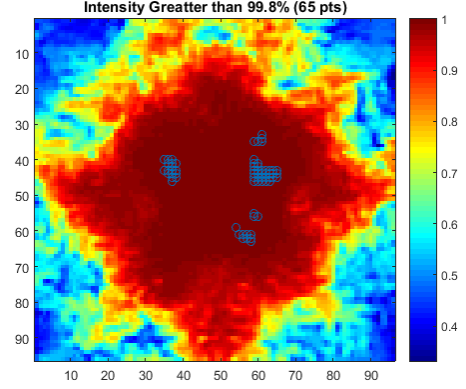
(9)

As can be seen from the location of maxima in the above image (the blue circles), the highest maxima do not occur at the center of the pattern implying that the alignment of the filter was slightly off. From the distribution of maxima it looks like the central part of the filter was low and left of the beam.

After alignment, all that was left was to record data. Below is an image of the patten produced when no filter is placed on the Fourier plane so the original image is able to be perfectly reconstructed for comparison purposes. Notice that because of the use of the converging lens, the resulting image is inverted both horizontally and vertically with respect to the original (2) although it only looks to be inverted vertically as these images were taken from the opposite side of the screen so that the camera could be directly in line with the laser.
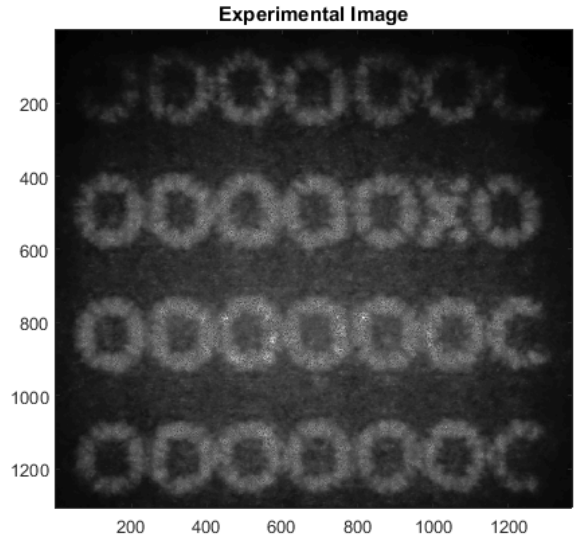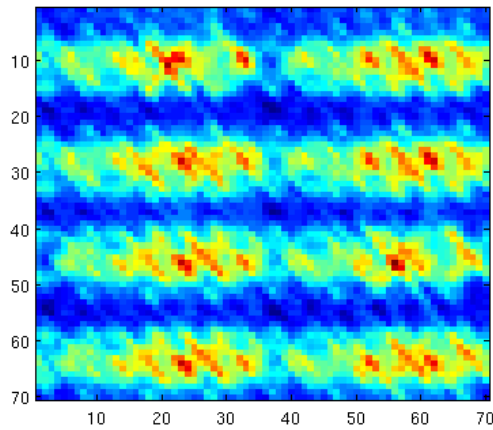

(10)

With the alignment procedure completed we moved the lenses back to their proper positions and recorded the resulting image, for which a colorized version is shown below.
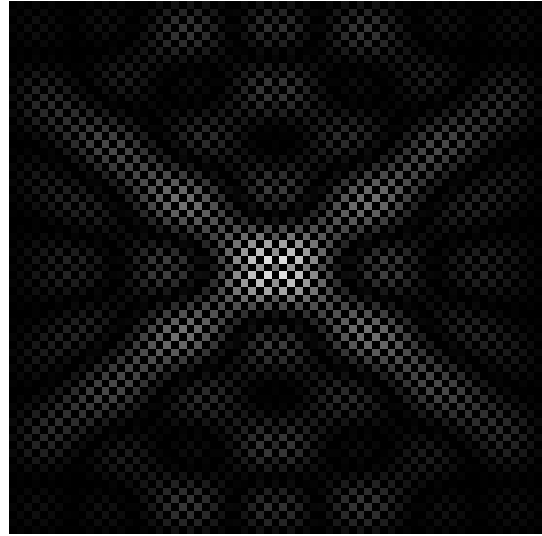
4

Intensity DFT Filter Result

(11)

(13)
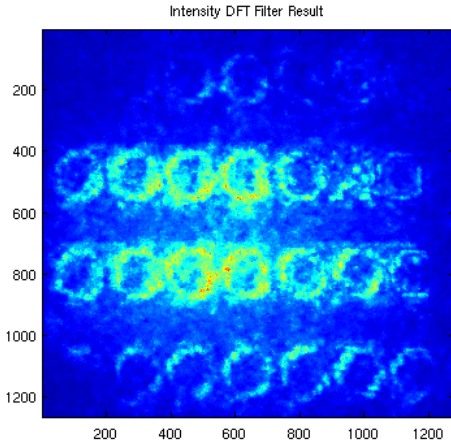
Running a numerical simulation with this same filter gives us a theoretical result with which we can compare. This result is shown below. When comparing, remember that one should be flipped vertically relative to the other. As you can see, this experimental result does not match the pattern of our numerical solutions very well at all. The letters of the original image can still be seen very clearly whereas we would expect not to be able to make much out in the way of letters had a convolution actually occurred.

After aligning this filter on the Fourier plane as we described for the previous filter, data was collected and a colorized image of the result is shown below.

Reconstruction of Convolution without Imaginary Component or Negatives

Intensity DCT Filter Result

(14)

Again we find that this does not agree with the result predicted by numerical simulation shown on the left.
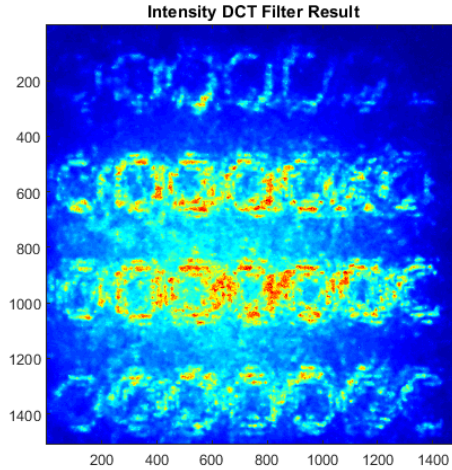
4   Conclusion

(12)

The next experiment used a filter that was a down sampling of a DCT of the even, lone X image. It was thought that even though this filter performed exactly the same as the real component of the Fourier filter in the numerical solution, it might be different in experimental trials as the filter pattern is obviously different. The filter pattern is shown below.

Overall, the experimental results did not agree well with numerical predictions but this is not to say that this method does not perform as expected. The strengths of our proposed method is that we greatly simplify the process of filter creation and should still achieve a decent approximation of the convolution according to numerical simulation. Part of the beauty of this method is its simplicity in that because we are

only multiplying the positive, real components of the light by a number between zero and one this should correspond to simply blocking a certain fraction of the light using the filter. The real question then is why our experimental results did not agree with our numerical simulation if the method is so robust. It is our belief that most of the reason our experimental results were so different than expected was that precise filter alignment was extremely difficult. As we saw in the image showing the alignment of our filters, the filters were sightly misaligned when data was taken. Because of this difference, we cannot expect our results to match as we effectively convolved with the wrong filter as any misplaced light or dark region of the filter would let through or block undesired frequencies. It was also noted that even the darkest areas of the transparancy did not fully block the beam and so we could not actually multiply by zero in the convolution.

4.1 Future Works

In the future, we would like to find a better way of aligning filters either by using a machined channel for the light so the filters are held in the precise alignment or by being able to measure our deviation from the proper alignment using a method similar to the method shown in this document and make fine adjustments to filter position in order to compensate. It would also be beneficial to find a way of printing tranparancies that could actually block 100% of the light when needed.

In commercially produced optical correlators, a spatial light modulator (SLM) is used to create image apertures and filters. Using an SLM the alignment might become easier as the SLM itself would not have to me moved and very fine adjustments to the image position on the SLM could be made. Using an SLM, there is also possibly a way to create a complex-valued filter like the holographic filters mentioned earlier because SLMs are sometimes able to modify the phase of light passing through them [2].

The last suggestion for further work is to try to create and use a complex valued filter. While they can be produced on photographic plates as a standard hologram, it might also be possible to calculate the hologram pattern and then print it on to transparencies as some sources claim is possible [3].
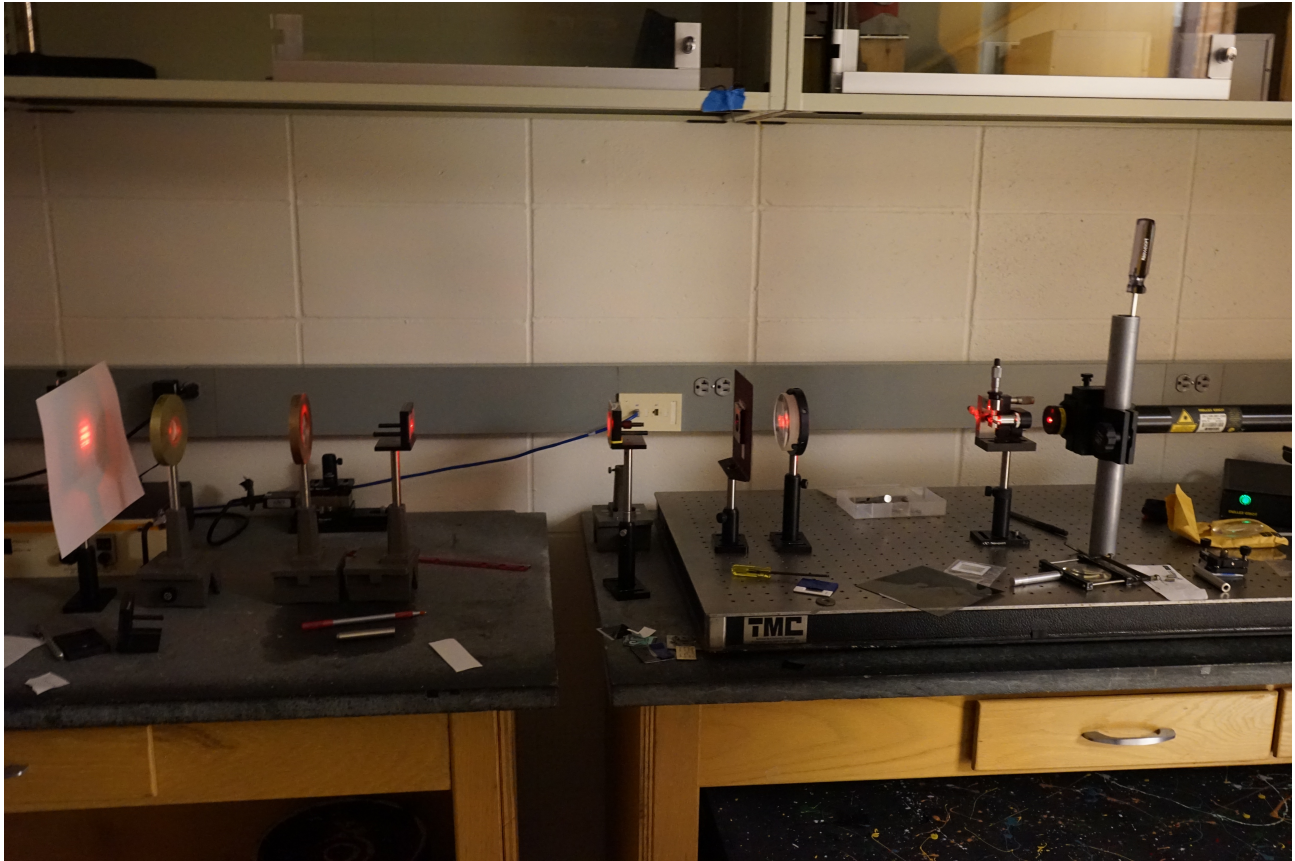
5 Special Thanks

- A special thanks to Edward D. Dudak and Keisuke Hasegawa for equipment acquisition and theory help respectively.

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References

[1] H. Stark, *Application of Optical Fourier Transforms*, (Academic Press, New York, 1982)

[2] Wikipedia: Spatial light modulator
https://en.wikipedia.org/wiki/Spatial_light_modulator

[3] The CorticalCafe Computer Generated Hologram (CGH) Construction Kit
http://corticalcafe.com/prog_CGHmaker.htm

6 Pictures

6.1 Setup

6.2 Allignment