IMPA | Medida e integração | 2/2025

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Lista 6

1. Let μ be a σ-finite measure and λ a complex measure. Prove that Randon-Nikodym theorem still holds.

2. Let $\mu(X) < \infty$, $f \in L^1(\mu)$, S a closed set in the complex plane, and the averages

$$\frac{1}{\mu(E)} \int_E f d\mu$$

lie in S for every $E \in \mathcal{M}$ with $\mu(E) > 0$. Then $f(x) \in S$, for almost all x μ-a.e.

3. Extend Theorem 3 from last class (about duality of L^p spaces) to σ-finite μ.

4. Suppose that (g_n) is a sequence of positive continuous functions on $I = [0, 1]$, μ is a positive Borel measure on I, and m be the Lebesgue measure and that

   (a) $\lim_{n\to\infty} g_n(x) = 0$, m-a.e

   (b) $\int_I g_n dm = 1$

   (c) $\lim_{n\to\infty} f g_n dm = \int_I f\mu$ for all $f \in C(I)$.

   Does it follow that $\mu \perp m$?

5. Let μ be a complex measure on a σ-algebra $\mathcal{M}$. If $E \in \mathcal{M}$, define

$$\lambda(E) = \sup \sum |\mu(E_i)|,$$

the supremum being taken over all finite partitions $\{E_i\}$ of E. Does it follow that $\lambda = |\mu|$?