Norm of $d\,f'\,(\rho)$

when 3 divides m

In[1]:=
```
ClearAll[x, y, z, m, a]
poly = (3 x^2 - m / 3) (3 y^2 - m / 3) (3 z^2 - m / 3);
polySym = SymmetricReduction[poly, {x, y, z}][[1]];
polySym
```

Out[4]= $-\dfrac{m^3}{27} + 27\,x^2\,y^2\,z^2 + 6\,m\,x\,y\,z\,(x + y + z) + \dfrac{1}{3}\,m^2\,(x + y + z)^2 - \dfrac{2}{3}\,m^2\,(x\,y + x\,z + y\,z) - 3\,m\,(x\,y + x\,z + y\,z)^2$

In[5]:=
```
s1 = x + y + z;
s2 = x y + y z + z x;
s3 = x y z;
subs = {s1 → 0, s2 → -m / 3, s3 → (a m) / 27};
finalExpr = Simplify[polySym /. subs]
```

Out[9]= $-\dfrac{4\,m^3}{27} + 27\,x^2\,y^2\,z^2 + \dfrac{2}{9}\,a\,m^2\,(x + y + z)$

when 3 does not divide m

In[10]:=
```
ClearAll[x, y, z, m, a]
poly = (3 x^2 - 2 x + (1 - m) / 3) (3 y^2 - 2 y + (1 - m) / 3) (3 z^2 - 2 z + (1 - m) / 3);
polySym = SymmetricReduction[poly, {x, y, z}][[1]];
polySym
```

Out[13]=

$\dfrac{1}{27} - \dfrac{m}{9} + \dfrac{m^2}{9} - \dfrac{m^3}{27} + (-8 + 3\,(2 - 2\,m))\,x\,y\,z + 27\,x^2\,y^2\,z^2 +$

$\left(-\dfrac{2}{9} + \dfrac{4\,m}{9} - \dfrac{2\,m^2}{9}\right)(x + y + z) + (12 + 2\,(-3 + 3\,m))\,x\,y\,z\,(x + y + z) +$

$\left(\dfrac{1}{3} - \dfrac{2\,m}{3} + \dfrac{m^2}{3}\right)(x + y + z)^2 + \left(\dfrac{4}{3} - \dfrac{4\,m}{3} + 2\left(-\dfrac{1}{3} + \dfrac{2\,m}{3} - \dfrac{m^2}{3}\right)\right)(x\,y + x\,z + y\,z) -$

$18\,x\,y\,z\,(x\,y + x\,z + y\,z) + (-2 + 2\,m)\,(x + y + z)\,(x\,y + x\,z + y\,z) + (3 - 3\,m)\,(x\,y + x\,z + y\,z)^2$

In[14]:= `s1 = x + y + z;`
`s2 = x y + y z + z x;`
`s3 = x y z;`
`subs = {s1 → 1, s2 → (1 - m) / 3, s3 → (m (a - 3) + 1) / 27};`
`finalExpr = Simplify[polySym /. subs]`
`SymmetricReduction[finalExpr, {x, y, z}]⟦1⟧`

Out[18]=

$$\frac{1}{27} \left(-1 - 4 m^3 + 6 y + 6 z - 18 y z + 729 x^2 y^2 z^2 - \right.$$
$$6 x (-1 + 3 y + 3 z) + 3 m^2 (9 - 6 x - 6 y - 6 z + 2 a (-1 + x + y + z)) -$$
$$\left. 2 m (3 + 6 y + 6 z - 27 y z + a (-1 + 3 y) (-1 + 3 z) + a x (-3 + 9 y + 9 z) - 3 x (-2 + 9 y + 9 z)) \right)$$

Out[19]=

$$-\frac{1}{27} - \frac{2 m}{9} - \frac{2 a m}{27} + m^2 - \frac{2 a m^2}{9} - \frac{4 m^3}{27} + 27 x^2 y^2 z^2 +$$
$$\left(\frac{2}{9} - \frac{4 m}{9} + \frac{2 a m}{9} - \frac{2 m^2}{3} + \frac{2 a m^2}{9} \right) (x + y + z) + \left(-\frac{2}{3} + 2 m - \frac{2 a m}{3} \right) (x y + x z + y z)$$

In[•]:= $-\frac{1}{27} - \frac{2 m}{9} - \frac{2 a m}{27} + m^2 - \frac{2 a m^2}{9} - \frac{4 m^3}{27} + 27 ((m (a - 3) + 1) / 27) ^ 2 +$
$\left(\frac{2}{9} - \frac{4 m}{9} + \frac{2 a m}{9} - \frac{2 m^2}{3} + \frac{2 a m^2}{9} \right) 1 + \left(-\frac{2}{3} + 2 m - \frac{2 a m}{3} \right) (1 - m) / 3$

In[20]:= `ClearAll[a, m]`
`expr = 5 / 27 - (2 m) / 3 + (4 a m) / 27 + m^2 / 3 - (4 m^3) / 27 +`
`    1 / 27 (1 + (-3 + a) m) ^2 + 1 / 3 (1 - m) (-2 / 3 + 2 m - (2 a m) / 3);`
`exprSub = expr /. m → (a^2 + 27) / 4;`
`Simplify[exprSub]`

Out[23]=

$$-\frac{1}{16} \left(27 + a^2 \right)^2$$