Norm of $f'(\rho)$

when 3 divides m

In[1]:=
```
ClearAll[x, y, z, m, a]
poly = (3 x^2 - m / 3) (3 y^2 - m / 3) (3 z^2 - m / 3);
polySym = SymmetricReduction[poly, {x, y, z}][[1]];
polySym
```

Out[4]=
$$-\frac{m^3}{27} + 27\,x^2\,y^2\,z^2 + 6\,m\,x\,y\,z\,(x+y+z) + \frac{1}{3}\,m^2\,(x+y+z)^2 - \frac{2}{3}\,m^2\,(x\,y+x\,z+y\,z) - 3\,m\,(x\,y+x\,z+y\,z)^2$$

In[5]:=
```
s1 = x + y + z;
s2 = x y + y z + z x;
s3 = x y z;
subs = {s1 → 0, s2 → -m / 3, s3 → (a m) / 27};
finalExpr = Simplify[polySym /. subs]
```

Out[9]=
$$-\frac{4\,m^3}{27} + 27\,x^2\,y^2\,z^2 + \frac{2}{9}\,a\,m^2\,(x+y+z)$$

when 3 does not divide m

In[10]:=
```
ClearAll[x, y, z, m, a]
poly = (3 x^2 - 2 x + (1 - m) / 3) (3 y^2 - 2 y + (1 - m) / 3) (3 z^2 - 2 z + (1 - m) / 3);
polySym = SymmetricReduction[poly, {x, y, z}][[1]];
polySym
```

Out[13]=
$$\frac{1}{27} - \frac{m}{9} + \frac{m^2}{9} - \frac{m^3}{27} + (-8 + 3\,(2 - 2\,m))\,x\,y\,z + 27\,x^2\,y^2\,z^2 + \left(-\frac{2}{9} + \frac{4\,m}{9} - \frac{2\,m^2}{9}\right)(x+y+z) + (12 + 2\,(-3 + 3\,m))\,x\,y\,z\,(x+y+z) + \left(\frac{1}{3} - \frac{2\,m}{3} + \frac{m^2}{3}\right)(x+y+z)^2 + \left(\frac{4}{3} - \frac{4\,m}{3} + 2\left(-\frac{1}{3} + \frac{2\,m}{3} - \frac{m^2}{3}\right)\right)(x\,y+x\,z+y\,z) - 18\,x\,y\,z\,(x\,y+x\,z+y\,z) + (-2 + 2\,m)\,(x+y+z)\,(x\,y+x\,z+y\,z) + (3 - 3\,m)\,(x\,y+x\,z+y\,z)^2$$

In[14]:=
```
s1 = x + y + z;
s2 = x y + y z + z x;
s3 = x y z;
subs = {s1 → 1, s2 → (1 - m) / 3, s3 → (m (a - 3) + 1) / 27};
finalExpr = Simplify[polySym /. subs]
SymmetricReduction[finalExpr, {x, y, z}][[1]]
```

Out[18]=

$$\frac{1}{27}\left(-1-4\,m^3+6\,y+6\,z-18\,y\,z+729\,x^2\,y^2\,z^2-6\,x\,(-1+3\,y+3\,z)+3\,m^2\,(9-6\,x-6\,y-6\,z+2\,a\,(-1+x+y+z))-2\,m\,(3+6\,y+6\,z-27\,y\,z+a\,(-1+3\,y)\,(-1+3\,z)+a\,x\,(-3+9\,y+9\,z)-3\,x\,(-2+9\,y+9\,z))\right)$$

Out[19]=

$$-\frac{1}{27}-\frac{2\,m}{9}-\frac{2\,a\,m}{27}+m^2-\frac{2\,a\,m^2}{9}-\frac{4\,m^3}{27}+27\,x^2\,y^2\,z^2+\left(\frac{2}{9}-\frac{4\,m}{9}+\frac{2\,a\,m}{9}-\frac{2\,m^2}{3}+\frac{2\,a\,m^2}{9}\right)(x+y+z)+\left(-\frac{2}{3}+2\,m-\frac{2\,a\,m}{3}\right)(x\,y+x\,z+y\,z)$$

In[20]:=
```
-1/27 - 2 m/9 - 2 a m/27 + m^2 - 2 a m^2/9 - 4 m^3/27 + 27 ((m (a - 3) + 1) / 27) ^ 2 +
 (2/9 - 4 m/9 + 2 a m/9 - 2 m^2/3 + 2 a m^2/9) 1 + (-2/3 + 2 m - 2 a m/3) (1 - m) / 3
```

Out[20]=

$$\frac{5}{27}-\frac{2\,m}{3}+\frac{4\,a\,m}{27}+\frac{m^2}{3}-\frac{4\,m^3}{27}+\frac{1}{27}\,(1+(-3+a)\,m)^2+\frac{1}{3}\,(1-m)\left(-\frac{2}{3}+2\,m-\frac{2\,a\,m}{3}\right)$$

In[21]:=
```
ClearAll[a, m]
expr = 5 / 27 - (2 m) / 3 + (4 a m) / 27 + m^2 / 3 - (4 m^3) / 27 +
    1 / 27 (1 + (-3 + a) m) ^ 2 + 1 / 3 (1 - m) (-2 / 3 + 2 m - (2 a m) / 3);
exprSub = expr /. m → (a^2 + 27) / 4;
Simplify[exprSub]
```

Out[24]=

$$-\frac{1}{16}\left(27+a^2\right)^2$$