

CHƯƠNG II. TẬP HỢP - ÁNH XẠ

1. Cho $A, B \subset \mathbb{R}$. Hãy viết $\overline{A}, \overline{B}, A \cap B, A \cup B, A \setminus B, B \setminus A$ thành phần hội của các khoảng rời nhau trong \mathbb{R} .

(a)

$$A = (-9, -3) \cup [-1, 2] \cup [4, 5] \cup (7, 11] \text{ và } B = (-\infty, -7] \cup [-4, -2) \cup (0, 3) \cup (6, 8]$$

(b)

$$A = (-\infty, -4) \cup [4, 7] \cup \{-1, 2, 8, 10\} \text{ và } B = (-5, 1] \cup [6, 9) \cup \{-6, 3, 5, 10\}$$

2. Cho $A, B, C, D \subset E$. Hãy rút gọn các biểu thức sau đây

(a)

$$(A \setminus B) \cup (B \setminus A) \cup (A \cap B)$$

(b)

$$(A \cup B) \setminus [(A \setminus B) \cup (A \cap B)]$$

(c)

$$\overline{A} \cup \overline{B} \cup (A \cap B \cap \overline{C})$$

(d)

$$(A \cap B) \cup (A \cap B \cap \overline{C} \cap D) \cup (\overline{A} \cap B)$$

3. Cho $A, B, D \subset E$. Chứng minh

(a)

$$D \setminus (A \cup B) = (D \setminus A) \cap (D \setminus B) = (D \cup B) \setminus (A \cup B)$$

(b)

$$D \setminus (A \cap B) = (D \setminus A) \cup (D \setminus B)$$

(c)

$$(A \cup B) \setminus D = (A \setminus D) \cup (B \setminus D)$$

(d)

$$(A \cap B) \setminus D = (A \setminus D) \cap (B \setminus D)$$

(e)

$$(A \setminus B) \setminus D = A \setminus (B \cup D) = (A \setminus D) \setminus (B \setminus D)$$

4. Xét tính đơn ánh và toàn ánh của các ánh xạ sau

(a)

$$f : \mathbb{R} \setminus \{0\} \longrightarrow \mathbb{R} \\ x \longmapsto \frac{1}{x}$$

(b)

$$f : \mathbb{R} \longrightarrow \mathbb{R} \\ x \longmapsto 3x^2 + 1$$

(c)

$$f : \mathbb{R} \longrightarrow \mathbb{R} \\ x \longmapsto x(x-1)(x+1)$$

(d)

$$f : \mathbb{R} \setminus \{1\} \longrightarrow \mathbb{R} \\ x \longmapsto \frac{2x+1}{x-1}$$

(e)

$$f : \mathbb{R} \setminus \{0\} \longrightarrow \mathbb{R} \\ x \longmapsto (2x-3)x^{-1}$$

(f)

$$f : (-2, +\infty) \longrightarrow (-20, +\infty) \\ x \longmapsto x^2 + 6x - 3$$

(g)

$$f : \mathbb{R} \longrightarrow [-2, 2] \\ x \longmapsto \sin(x) + \sqrt{3}\cos(x)$$

(h)

$$f : (0, +\infty) \longrightarrow \mathbb{R} \\ x \longmapsto 3\ln x - 2$$

5. Kiểm tra các ánh xạ f sau đây có phải ánh xạ song ánh hay không? Nếu f là ánh xạ song ánh, hãy tìm ánh xạ ngược f^{-1} .

(a)

$$f : \mathbb{R} \longrightarrow \mathbb{R} \\ x \longmapsto x + 2$$

(b)

$$f : \mathbb{R} \longrightarrow \mathbb{R} \\ x \longmapsto x^2 + 1$$

(c)

$$f : \mathbb{R} \longrightarrow \mathbb{R} \\ x \longmapsto x^3 - 1$$

(d)

$$f : \mathbb{R} \longrightarrow \mathbb{R} \\ x \longmapsto \frac{x}{x^2 + 1}$$

(e)

$$f : \mathbb{R} \setminus \{0\} \longrightarrow \mathbb{R} \\ x \longmapsto \frac{1}{x}$$

(f)

$$f : \mathbb{R} \setminus \{0\} \longrightarrow \mathbb{R} \\ x \longmapsto \frac{x-3}{x}$$

(g)

$$f : \mathbb{R} \setminus \{2\} \longrightarrow \mathbb{R} \\ x \longmapsto \frac{x+1}{x-2}$$

(h)

$$f : [1, 3] \longrightarrow \mathbb{R} \setminus \{0\} \\ x \longmapsto 3x^2 - 9x + 5$$

(i)

$$f : (2, +\infty) \longrightarrow \mathbb{R} \\ x \longmapsto 2\ln(5x-10) + 3$$

(j)

$$f : \mathbb{R} \setminus \{1\} \longrightarrow \mathbb{R} \setminus \{3\} \\ x \longmapsto (5x-3)(x-1)^{-1}$$

(k)

$$\begin{aligned} f : [1, 2) &\longrightarrow [5, 7) \\ x &\longmapsto 3x + 2x^{-1} \end{aligned}$$

(l)

$$\begin{aligned} f : \mathbb{R} &\longrightarrow (-1, 1) \\ x &\longmapsto x(1 + |x|)^{-1} \end{aligned}$$

(m)

$$\begin{aligned} f : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto 3\cos(2x) - 7x + 8 \end{aligned}$$

(n)

$$\begin{aligned} f : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto e^x - 3e^{-x} + 1 \end{aligned}$$