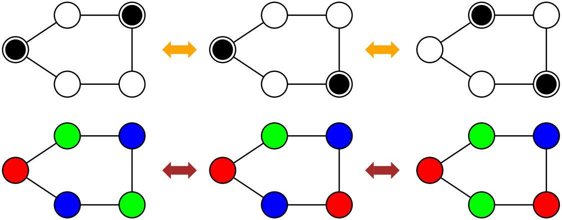
The 3rd Vietnam-Korea Joint Workshop on Selected Topics in Mathematics



Duc A. Hoang

hoanganhduc@hus.edu.vn

VNU University of Science
Hanoi, Vietnam

January 06–09, 2025

The Complexity of Some Reconfiguration Problems on Graphs with Distance Constraints

in collaboration with
Niranka Banerjee (Kyoto University, Japan)
Christian Engels (National Institute of Informatics, Japan)

Contents

Introduction to Reconfiguration

Reconfiguration Setting

- A description of what *states* (≡ *configurations*) are
- One or more *allowed moves* between states (≡ *reconfiguration rule(s)*)

Reconfiguration Process	Reconfiguration Graph	Example RUBIK'S CUBE
state	node	
move *[reconfiguration rule]*	edge	
sequence of moves *[reconfiguration sequence]*	path	

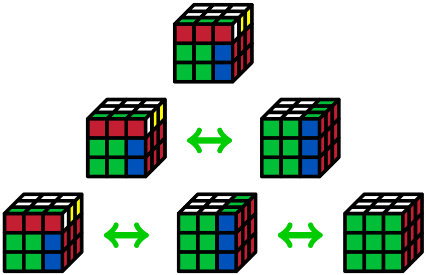


Figure: Reconfiguration Setting

Introduction to Reconfiguration

Reconfiguration vs. Solution Space

For a computational problem $\mathcal{P}$ (e.g., INDEPENDENT SET, DOMINATING SET, VERTEX-COLORING, etc.)

Reconfiguration	Solution Space
States/Configurations	Feasible solutions of $\mathcal{P}$
Allowed Moves	Slight modifications of a solution *without changing its feasibility*

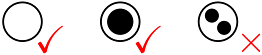
Algorithmic Questions

- REACHABILITY: Given two states S and T, is there a sequence of moves that *transforms S into T*?
- SHORTEST TRANSFORMATION: Given two states S and T and a positive integer ℓ, is there a sequence of moves that *transforms S into T using at most ℓ moves*?
- CONNECTIVITY: Is there a sequence of moves *between any pair of states*?
- and so on

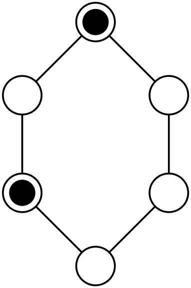
Introduction to Reconfiguration

Token Reconfiguration

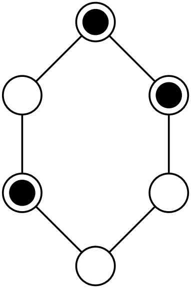
Each vertex has at most one token



Each state is a set of tokens satisfying certain property



Independent Set



Dominating Set

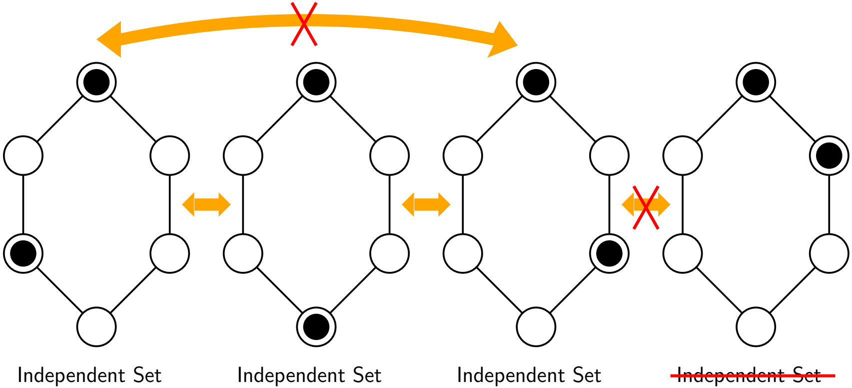
....

Introduction to Reconfiguration

[Hearn and E. D. Demaine 2005]

Token Reconfiguration (Example: Independent Set Reconfiguration)

Token Sliding (TS)

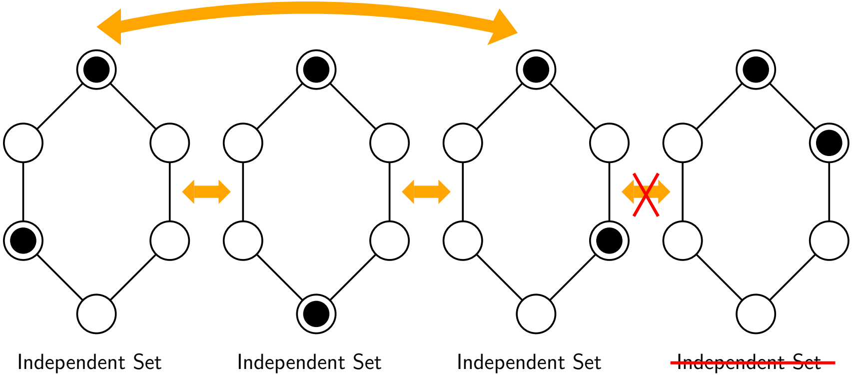


Introduction to Reconfiguration

[Kamiński, Medvedev, and Milanič 2012]

Token Reconfiguration (Example: Independent Set Reconfiguration)

Token Jumping (TJ)

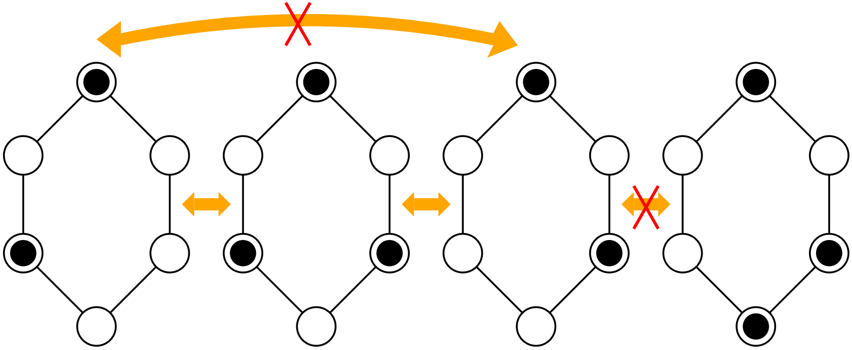


Introduction to Reconfiguration

[Ito et al. 2011]

Token Reconfiguration (Example: Independent Set Reconfiguration)

Token Addition/Removal (TAR)



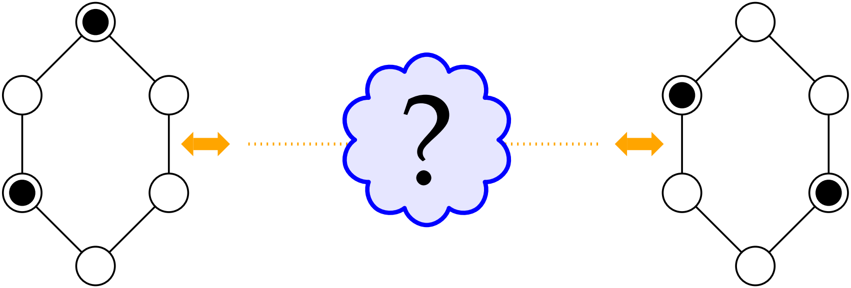
Independent Set | Independent Set | Independent Set | ~~Independent Set~~

Introduction to Reconfiguration

TOKEN RECONFIGURATION (Example: INDEPENDENT SET RECONFIGURATION)

Token Sliding (TS) / Token Jumping (TJ) / Token Addition/Removal (TAR)

PSPACE-complete on general graphs under any rule



Independent Set

Independent Set

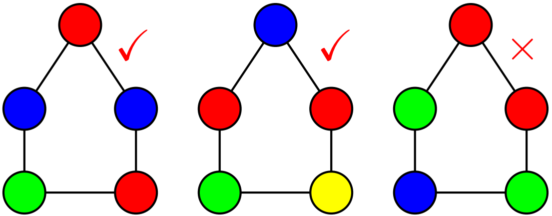
Introduction to Reconfiguration

[Cereceda, van den Heuvel, and Johnson 2008]

Vertex-Coloring Reconfiguration

Each vertex is colored by one of the k given colors

Each state is a k-coloring of all vertices such that
no two adjacent vertices share the same color



3-coloring 4-coloring   ~~3-coloring~~

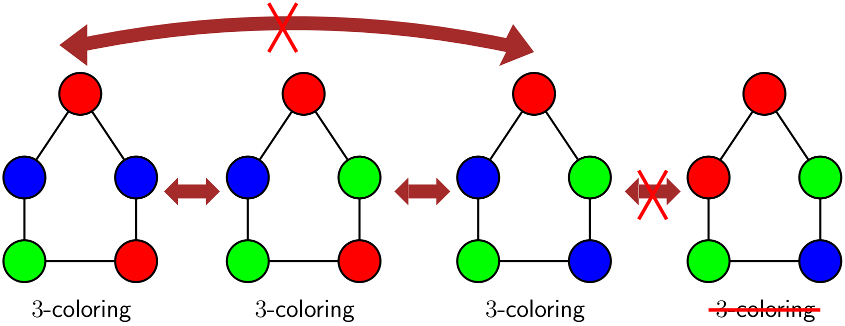
Introduction to Reconfiguration

[Cereceda, van den Heuvel, and Johnson 2008]

Vertex-Coloring Reconfiguration

Recoloring using $\leq k$ colors

Example: $k = 3$



Introduction to Reconfiguration

[Cereceda, van den Heuvel, and Johnson 2008]

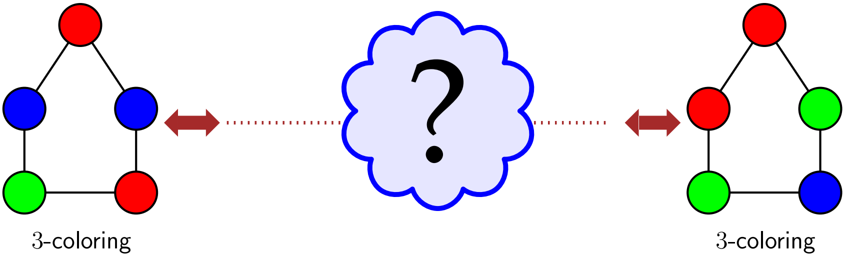
Vertex-Coloring Reconfiguration

Recoloring using $\leq k$ colors

Example: $k = 3$

PSPACE-complete on general graphs for $k \geq 4$

P on general graphs for $k \leq 3$



Introduction to Reconfiguration

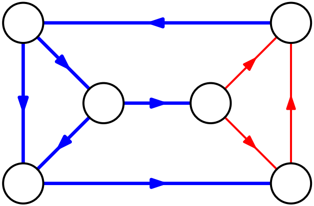
[Hearn and E. D. Demaine 2005]

Nondeterministic Constraint Logic (NCL)

Each state is a weighted, oriented graph

weight 2 weight 1

Total incoming weight at each vertex ≥ 2



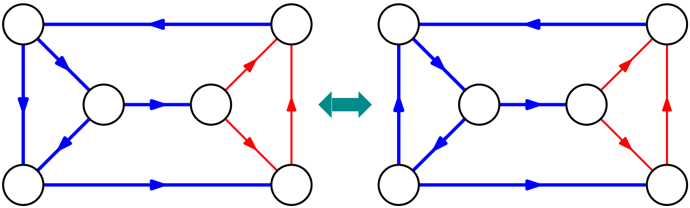
Introduction to Reconfiguration

[Hearn and E. D. Demaine 2005]

NONDETERMINISTIC CONSTRAINT LOGIC (NCL)

Reversing edge direction

Total incoming weight at each vertex ≥ 2



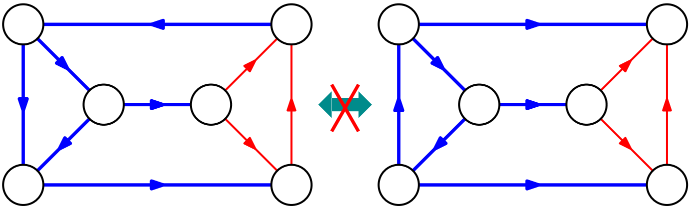
Introduction to Reconfiguration

[Hearn and E. D. Demaine 2005]

NONDETERMINISTIC CONSTRAINT LOGIC (NCL)

Reversing edge direction

Total incoming weight at each vertex ≥ 2



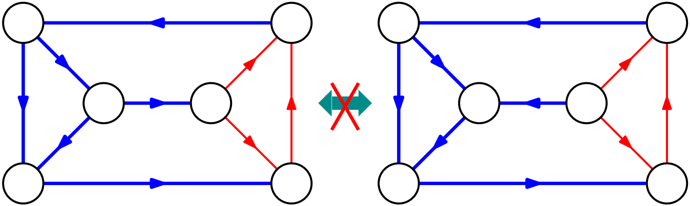
Introduction to Reconfiguration

[Hearn and E. D. Demaine 2005]

Nondeterministic Constraint Logic (NCL)

Reversing edge direction

Total incoming weight at each vertex ≥ 2

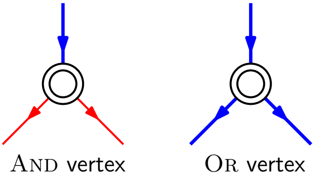


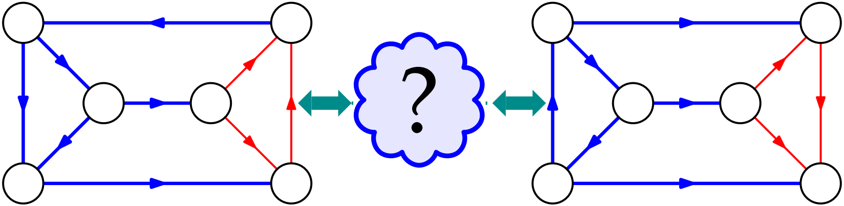
Introduction to Reconfiguration

[van der Zanden 2015]

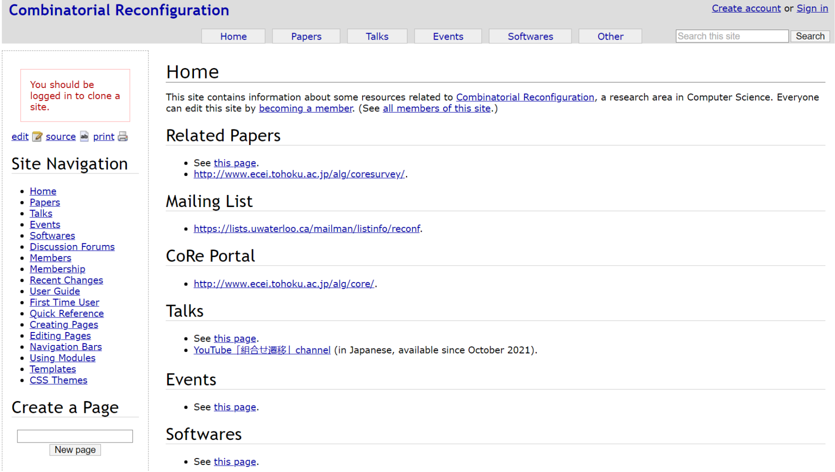
Nondeterministic Constraint Logic (NCL)

PSPACE-complete on
- planar graphs
- having only two types of vertices
- max degree 3
- bounded bandwidth





Introduction to Reconfiguration



Combinatorial Reconfiguration

Create account or Sign in

Home | Papers | Talks | Events | Softwares | Other | Search this site | Search

You should be logged in to clone a site.

edit source print

Site Navigation

- Home
- Papers
- Talks
- Events
- Softwares
- Discussion Forums
- Members
- Membership
- Recent Changes
- User Guide
- First Time User
- Quick Reference
- Creating Pages
- Editing Pages
- Navigation Bars
- Using Modules
- Templates
- CSS Themes

Create a Page

New page

Home

This site contains information about some resources related to Combinatorial Reconfiguration, a research area in Computer Science. Everyone can edit this site by becoming a member. (See all members of this site.)

Related Papers

- See this page.
- http://www.ecei.tohoku.ac.jp/alg/coresurvey/.

Mailing List

- https://lists.uwaterloo.ca/mailman/listinfo/reconf.

CoRe Portal

- http://www.ecei.tohoku.ac.jp/alg/core/.

Talks

- See this page.
- YouTube「組合せ遷移」channel (in Japanese, available since October 2021).

Events

- See this page.

Softwares

- See this page.

Figure: Online Reconfiguration Wiki Page (`https://reconf.wikidot.com/`)

Introduction to Reconfiguration

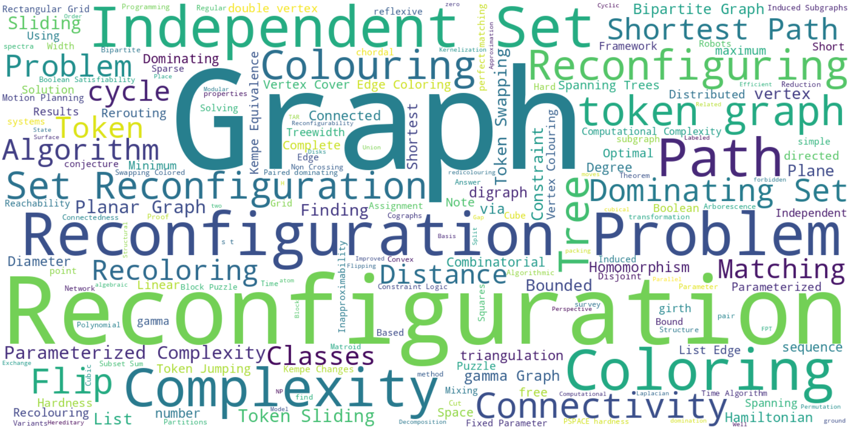


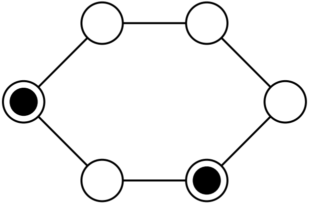
Figure: A word cloud of titles extracted from the current list of papers related to reconfiguration problems available at `https://reconf.wikidot.com/` (Accessed on January 3, 2025)

Distance Token-Set Reconfiguration

Distance constraints for *independent sets*

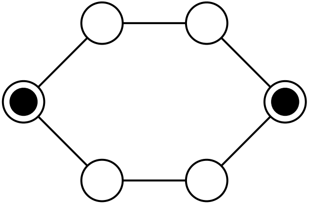
	Independent Set (IS)	Distance-d Independent Set (DdIS)
Distance between any two tokens	≥ 2	$\geq d$

- DdIS was (first?) studied in [Kong and Zhao 1993]
- Dual to the *dispersion problem*, which has numerous applications in *facility location* and in *management decision science* [Agnarsson, Damaschke, and Halldórsson 2003]



(Distance-2) Independent Set

~~Distance-3 Independent Set~~



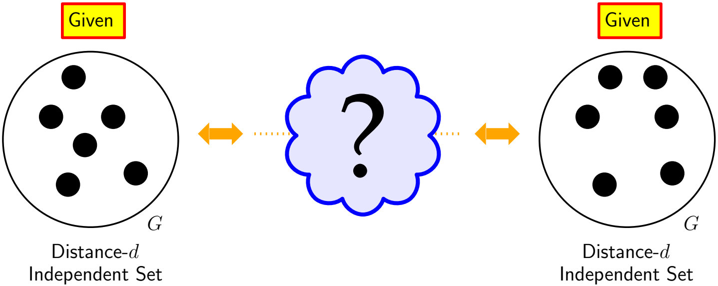
(Distance-2) Independent Set

Distance-3 Independent Set

Distance Token-Set Reconfiguration

DISTANCE-d INDEPENDENT SET RECONFIGURATION (DdISR)

Reconfiguration Rule: Token Sliding (TS) and Token Jumping (TJ)



- The case $d = 2$ has been well-studied [Nishimura 2018]; [Bousquet et al. 2024]
- The case $d \geq 3$ under TAR was first studied from the *parameterized complexity* persepective in [Siebertz 2018] (parameterized by the size of the independent set)
  - FPT on "nowhere dense graphs" for $d \geq 2$
  - W[1]-hard on "somewhere dense graphs" that are closed under taking subgraphs for some value of $d \geq 2$
- We focus on the *classic complexity* for $d \geq 3$

Distance Token-Set Reconfiguration

Graph	d = 2		d ≥ 3	
	TS	TJ	TS	TJ
planar	PSPACE-C [Hearn and E. D. Demaine 2005]		PSPACE-C	
perfect	PSPACE-C [Kamiński, Medvedev, and Milanič 2012]		PSPACE-C	
chordal	PSPACE-C ($\supseteq$ split)	P ($\subseteq$ even-hole-free)	unknown	PSPACE-C if d is odd P if d is even
split	PSPACE-C [Belmonte et al. 2021]	P ($\subseteq$ even-hole-free)	P	PSPACE-C if $d = 3$ P if $d \geq 4$
cograph	P [Kamiński, Medvedev, and Milanič 2012]	P [Bonsma 2016]	P	
tree	P [E. D. Demaine, M. L. Demaine, et al. 2015]	P ($\subseteq$ even-hole-free)	unknown	P
interval	P [Bonamy and Bousquet 2017]	P ($\subseteq$ even-hole-free)	unknown	P

Table: Our Results for $d \geq 3$. We provide the status for $d = 2$ for comparison. Here PSPACE-C stands for PSPACE-complete [**Hoang** 2024]

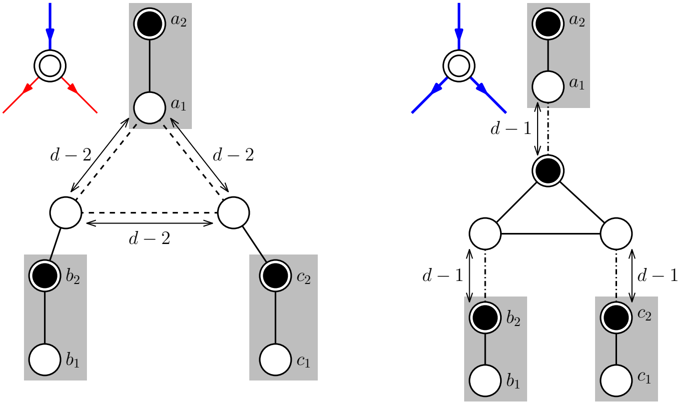
Distance Token-Set Reconfiguration

Theorem (Hoang 2024)

DdISR *is* PSPACE*-complete for* $d \geq 3$ *on planar graphs of maximum degree* 3 *and bounded bandwidth.*

Proof Sketch.

Reduction from NONDETERMINISTIC CONSTRAINT LOGIC



AND gadget

OR gadget

Distance Token-Set Reconfiguration

Theorem (Hoang 2024)

On split graphs,

	TS	TJ
$d = 2$	$\texttt{PSPACE}$-*complete* [Belmonte et al. 2021]	$\texttt{P}$ [Kamiński, Medvedev, and Milanič 2012]
$d = 3$	$\texttt{P}$	$\texttt{PSPACE}$-*complete*
$d \geq 4$		$\texttt{P}$

Proof Sketch.

$d \geq 4$ Trivial. (Any connected split graph has diameter $\leq 3 \Rightarrow$ Any DdIS has size $\leq 1 \Rightarrow$ Reconfiguration is easy!)

$d = 3$
- **Under TS:** Trivial. (If there are more than two tokens then none of them can be moved.)
- **Under TJ:** Reduction from the problem for $d = 1$ on general graphs

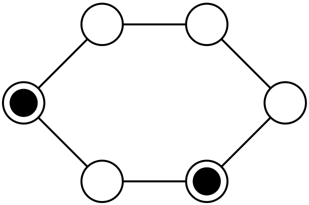
$\square$

Distance Token-Set Reconfiguration

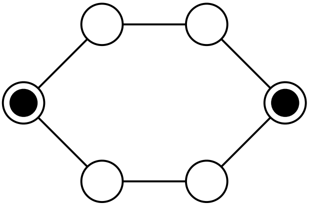
Distance constraints for *dominating sets*

	Dominating Set (DS)	Distance-d Dominating Set (DdDS)
Distance that a token dominates	≤ 1	$\leq d$

- DdDS was first studied in [Meir and Moon 1975]
- Has applications in *facility location* [E. D. Demaine, Fomin, et al. 2005]



~~(Distance-1) Dominating Set~~
Distance-2 Dominating Set

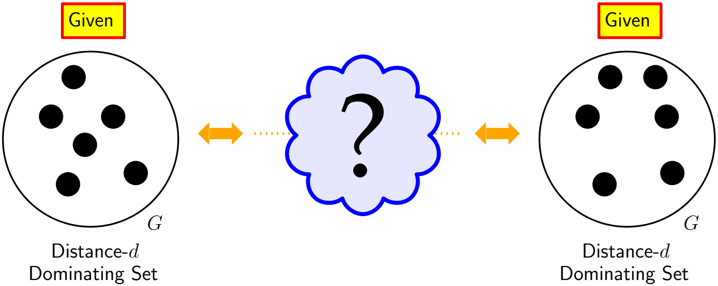


(Distance-1) Dominating Set
Distance-2 Dominating Set

Distance Token-Set Reconfiguration

Distance-d Dominating Set Reconfiguration (DdDSR)

Reconfiguration Rule: Token Sliding (TS) and Token Jumping (TJ)



- The case $d = 1$ (Dominating Set Reconfiguration) has been well-studied [Nishimura 2018]; [Mynhardt and Nasserasr 2019]
- The case $d \geq 2$ under TAR was first studied from the *parameterized complexity* persepective in [Siebertz 2018] (parameterized by the size of the dominating set)
  - `FPT` on "nowhere dense graphs" for $d \geq 2$
  - `W`[2]-hard on "somewhere dense graphs" that are closed under taking subgraphs for some value of $d \geq 2$
- We focus on the *classic complexity* for $d \geq 2$

Distance Token-Set Reconfiguration

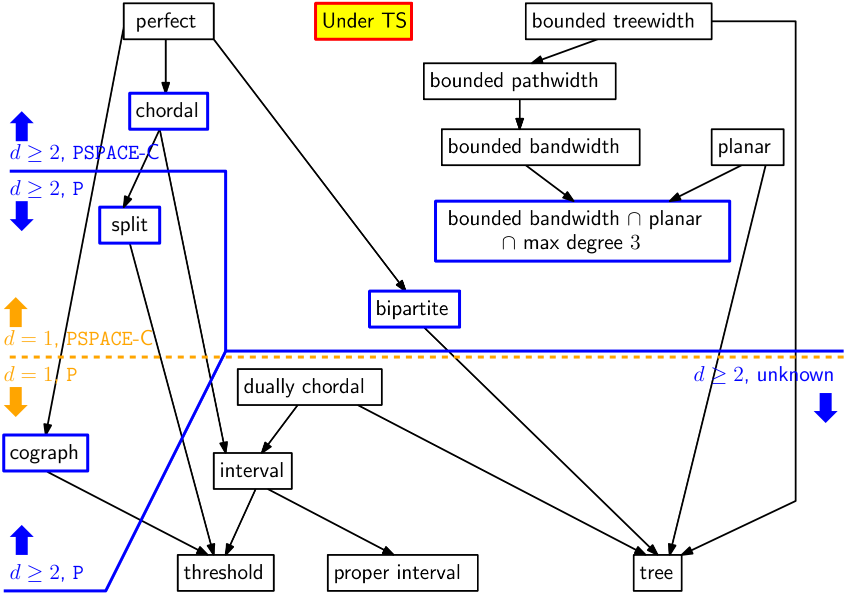


Figure: Our Results for $d \geq 2$. We provide the status for $d = 1$ for comparison. Here PSPACE-C stands for PSPACE-complete [Banerjee and **Hoang** 2024]

Distance Token-Set Reconfiguration

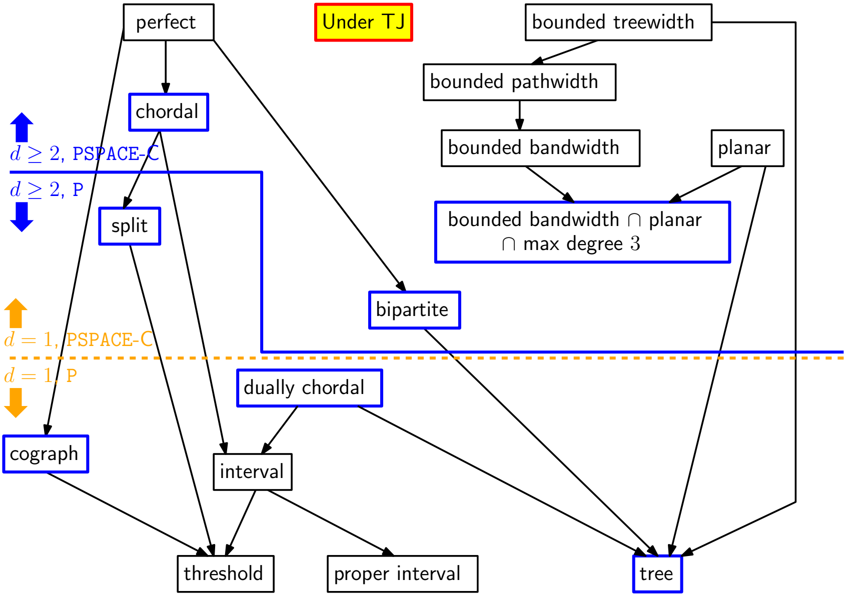


Figure: Our Results for $d \geq 2$. We provide the status for $d = 1$ for comparison. Here PSPACE-C stands for PSPACE-complete [Banerjee and **Hoang** 2024]

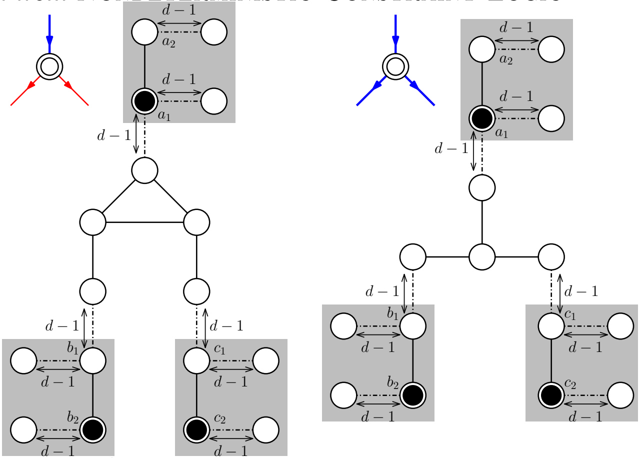
Distance Token-Set Reconfiguration

Theorem (Banerjee and Hoang 2024)

DdDSR *is* PSPACE*-complete for* $d \geq 1$ *on planar graphs of maximum degree* 3 *and bounded bandwidth.*

Proof Sketch.

Reduction from Nondeterministic Constraint Logic



And gadget

Or gadget

Distance Token-Set Reconfiguration

Theorem (Banerjee and Hoang 2024)

On split graphs,

	TS	TJ
$d = 1$	$\mathtt{PSPACE}$-*complete* *[Bonamy, Dorbec, and Ouvrard 2021]*	$\mathtt{PSPACE}$-*complete* *[Haddadan et al. 2016]*
$d \geq 2$	$\mathtt{P}$	$\mathtt{P}$

Proof Sketch.

$d \geq 3$ Trivial. (Any connected split graph has diameter $\leq 3 \Rightarrow$ Any non-empty token-set is a DdDS $\Rightarrow$ Reconfiguration is easy!)

$d = 2$ When doing reconfiguration, *always keep at least one token in the clique side of the split graph*

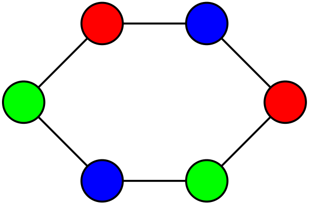
□

Distance Vertex-Coloring Reconfiguration

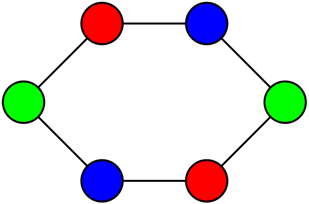
Distance constraints for *vertex colorings*

	k-Coloring	(d, k)-Coloring
Distance between two vertices having the same color	≥ 2	$\geq d + 1$

- (d, k)-coloring was first studied in [F. Kramer and H. Kramer 1969]
- Has applications in *frequency assignment problem* (or radio channel assignment) [F. Kramer and H. Kramer 2008]



3-coloring
~~$(2, 3)$-coloring~~

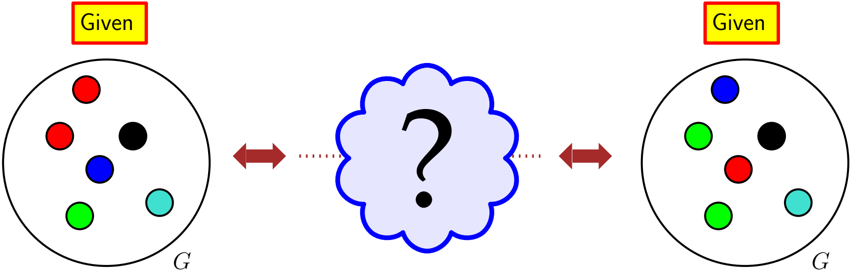


3-coloring
$(2, 3)$-coloring

Distance Vertex-Coloring Reconfiguration

(d, k)-COLORING RECONFIGURATION ((d, k)-CR)

Reconfiguration Rule: Recoloring a vertex



(d, k)-Coloring $\qquad$ (d, k)-Coloring

- The case $d = 1$ (k-COLORING RECONFIGURATION (k-CR)) has been well-studied [Mynhardt and Nasserasr 2019]; [Heuvel 2013]
- We focus on the case $d \geq 2$

Distance Vertex-Coloring Reconfiguration

Graph	k-CR	(d,k)-CR $(d \geq 2)$
general	PSPACE-C $(k \geq 3)$ [Cereceda, Heuvel, and Johnson 2011]	PSPACE-C $(k = \Omega(d^2))$
planar	PSPACE-C $(4 \leq k \leq 6)$ P $(k \geq 7)$ [Bonsma and Cereceda 2009]	
bipartite	PSPACE-C $(k \geq 4)$ [Bonsma and Cereceda 2009]	
planar $\cap$ bipartite	PSPACE-C $(k = 4)$ P $(k \geq 5)$ [Bonsma and Cereceda 2009]	
2-degenerate	P [Hatanaka, Ito, and Zhou 2019]	
planar $\cap$ bipartite $\cap$ 2-degenerate	P ($\subseteq$ 2-degenerate)	
path	P ($\subseteq$ planar $\cap$ bipartite $\cap$ 2-degenerate)	P $(k \geq d+1)$
split	P [Hatanaka, Ito, and Zhou 2019]	PSPACE-C $(d = 2$, large $k)$ P $(d \geq 3)$

Table: Our Results for $d \geq 2$. We provide the status for $d = 1$ for comparison. Here PSPACE-C stands for PSPACE-complete [Banerjee, Engels, and **Hoang** 2024]

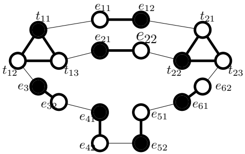
Distance Vertex-Coloring Reconfiguration

Theorem (Banerjee, Engels, and Hoang 2024)

(d, k)-CR is PSPACE-complete for $d \geq 2$ and $k = \Omega(d^2)$ on graphs which are planar, bipartite, and 2-degenerate.

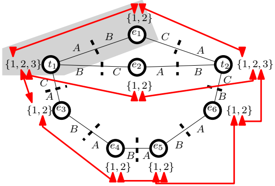
Proof Sketch.

Restricted PSPACE-Complete Variant of ISR under TS
(Restricted Sliding Tokens)



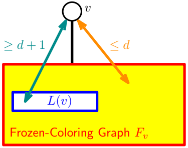
↓

List $(d, \Omega(d))$-Coloring Reconfiguration



↓

$(d, \Omega(d^2))$-Coloring Reconfiguration



Distance Vertex-Coloring Reconfiguration

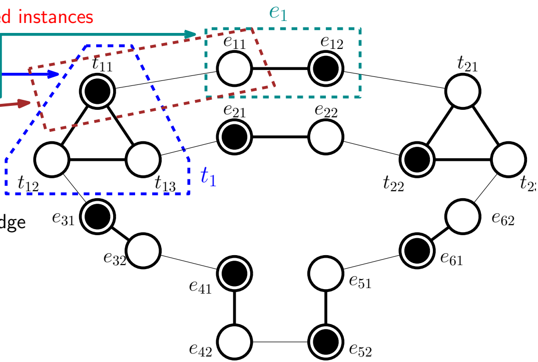
First Phase

RESTRICTED SLIDING TOKENS $\Rightarrow$ LIST $(d, \Omega(d))$-COLORING RECONFIGURATION

RESTRICTED SLIDING TOKENS

PSPACE-complete on very restricted instances

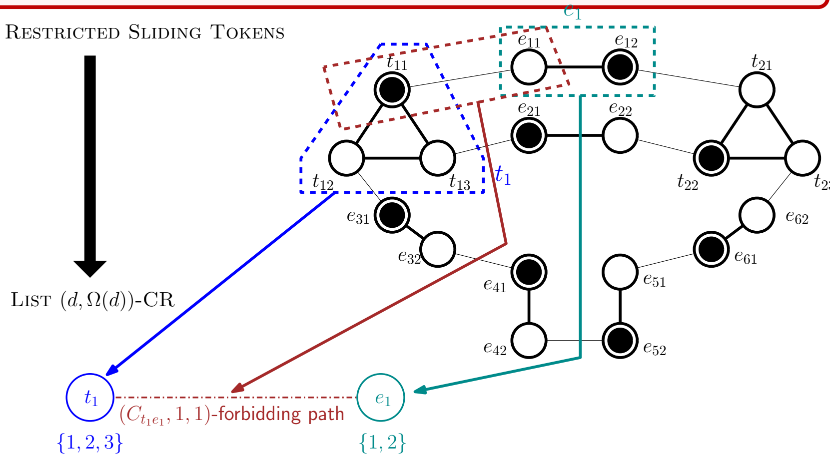
- Three types of gadgets
  - Token triangle
  - Token edge
  - Link edge
- Planar
- Max degree 3, min degree 2
- No two token triangles are directly joined by a link edge
- Each token triangle/token edge has exactly one token



Distance Vertex-Coloring Reconfiguration

First Phase

RESTRICTED SLIDING TOKENS ⇒ LIST $(d, \Omega(d))$-COLORING RECONFIGURATION

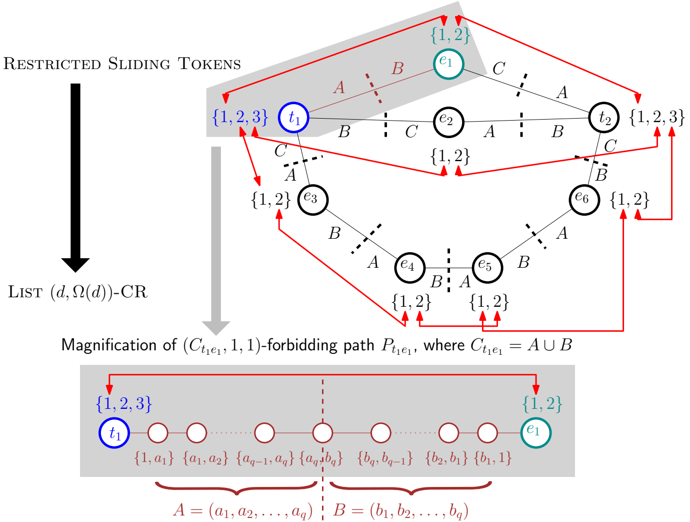


Reconfiguration under Distance Constraints — Duc A. Hoang

Distance Vertex-Coloring Reconfiguration

First Phase

RESTRICTED SLIDING TOKENS $\Rightarrow$ LIST $(d, \Omega(d))$-COLORING RECONFIGURATION



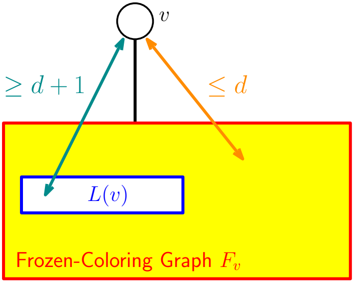
Distance Vertex-Coloring Reconfiguration

Second Phase

LIST $(d, \Omega(d))$-COLORING RECONFIGURATION $\Rightarrow$ $(d, \Omega(d^2))$-COLORING RECONFIGURATION

Key Ideas

1. List Coloring $\equiv$ Coloring with constraints on which colors can be used for each vertex
2. *Frozen-Coloring Graphs:* Pre-colored graphs where *no vertex can be recolored*



Vertices are pre-colored

Containing all possible colors

No vertex can be recolored

Distance Vertex-Coloring Reconfiguration

Second Phase

List $(d, \Omega(d))$-Coloring Reconfiguration $\Rightarrow$ $(d, \Omega(d^2))$-Coloring Reconfiguration

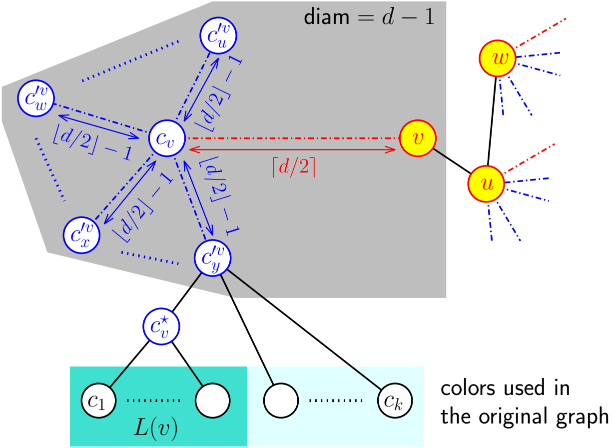


Figure: Construction of the frozen-coloring subgraph F_v for a vertex v. The colors used in this figure are just for illustration of paths.

Distance Vertex-Coloring Reconfiguration

Theorem (Banerjee, Engels, and Hoang 2024)

On split graphs,

	(d, k)-*Coloring Reconfiguration*
$d = 1$	P *for any* k *[Hatanaka, Ito, and Zhou 2019]*
$d = 2$	PSPACE-*complete for large* k
$d \geq 3$	P *for any* k

Proof Sketch.

$d \geq 3$ Trivial. (Any connected split graph has diameter $\leq 3 \Rightarrow$ Reconfiguration is easy!)

$d = 2$ Reduction from the problem for $d = 1$ and $k \geq 4$ on general graphs (which is known to be PSPACE-complete [Bonsma and Cereceda 2009])

□

Concluding Remarks

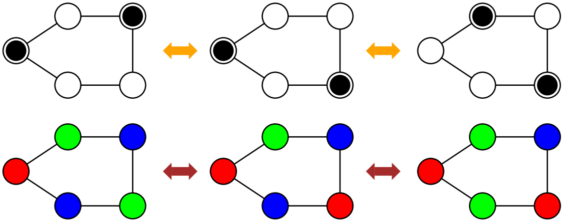
Take-Home Messages

1. *Reconfiguration* studies the "solution space" of a problem
   - Moving from one solution to another *without violating feasibility*
2. Under certain *distance constraints*,
   - *Reconfiguration problems* can be *hard* for very restricted graph classes
   - *Problems on graphs whose diameters are bounded by some constant c* (e.g., split graphs) are interesting when *restricted to distances close to c*
3. *Nondeterministic Constraint Logic* is a powerful tool for *hardness reductions*

Open Problems

The complexities of the following problems remain *open* for *trees*:

- DdISR ($d \geq 3$) under Token Sliding (TS)
- DdDSR ($d \geq 2$) under Token Sliding (TS)
- (d, k)-CR ($d \geq 2$)



Partially supported by

AFSA

VIASM
VIETNAM INSTITUTE FOR ADVANCED STUDY IN MATHEMATICS

ĐẠI HỌC KHOA HỌC TỰ NHIÊN

Came to my talk, you did.
Thank you, I must!

References I

Banerjee, Niranka, Christian Engels, and **Duc A. Hoang** (2024). "Distance Recoloring". In: *arXiv preprint*. arXiv: 2402.12705.

Banerjee, Niranka and **Duc A. Hoang** (2024). "The Complexity of Distance-r Dominating Set Reconfiguration". In: *Proceedings of COCOON 2024*. LNCS. arXiv: 2310.00241.

Bousquet, Nicolas, Amer E. Mouawad, Naomi Nishimura, and Sebastian Siebertz (2024). "A survey on the parameterized complexity of reconfiguration problems". In: *Computer Science Review* 53. (article 100663). DOI: 10.1016/j.cosrev.2024.100663.

Hoang, **Duc A.** (2024). "On the Complexity of Distance-d Independent Set Reconfiguration". In: *Theoretical Computer Science* 1009. (article 114682). DOI: 10.1016/j.tcs.2024.114682.

Belmonte, Rémy, Eun Jung Kim, Michael Lampis, Valia Mitsou, Yota Otachi, and Florian Sikora (2021). "Token sliding on split graphs". In: *Theory of Computing Systems* 65.4, pp. 662–686. DOI: 10.1007/s00224-020-09967-8.

References II

Bonamy, Marthe, Paul Dorbec, and Paul Ouvrard (2021). "Dominating sets reconfiguration under token sliding". In: *Discrete Applied Mathematics* 301, pp. 6–18. DOI: 10.1016/j.dam.2021.05.014. arXiv: 1912.03127.

Hatanaka, Tatsuhiko, Takehiro Ito, and Xiao Zhou (2019). "The Coloring Reconfiguration Problem on Specific Graph Classes". In: *IEICE Transactions on Information and Systems* E102.D.3, pp. 423–429. DOI: 10.1587/transinf.2018FCP0005.

Mynhardt, C.M. and S. Nasserasr (2019). "Reconfiguration of Colourings and Dominating Sets in Graphs". In: *50 years of Combinatorics, Graph Theory, and Computing*. Ed. by Fan Chung, Ron Graham, Frederick Hoffman, Ronald C. Mullin, Leslie Hogben, and Douglas B. West. 1st. CRC Press, pp. 171–191. DOI: 10.1201/9780429280092-10.

Nishimura, Naomi (2018). "Introduction to Reconfiguration". In: *Algorithms* 11.4, p. 52. DOI: 10.3390/a11040052.

Siebertz, Sebastian (2018). "Reconfiguration on Nowhere Dense Graph Classes". In: *The Electronic Journal of Combinatorics* 25.3. (article P3.24). DOI: 10.37236/7458. arXiv: 1707.06775.

References III

Bonamy, Marthe and Nicolas Bousquet (2017). “Token sliding on chordal graphs”. In: *Proceedings of WG 2017*. Vol. 10520. LNCS. Springer, pp. 127–139. DOI: 10.1007/978-3-319-68705-6_10.

Bonsma, Paul S. (2016). “Independent set reconfiguration in cographs and their generalizations”. In: *Journal of Graph Theory* 83.2, pp. 164–195. DOI: 10.1002/jgt.21992.

Haddadan, Arash, Takehiro Ito, Amer E. Mouawad, Naomi Nishimura, Hirotaka Ono, Akira Suzuki, and Youcef Tebbal (2016). “The Complexity of Dominating Set Reconfiguration”. In: *Theoretical Computer Science* 651, pp. 37–49. DOI: 10.1016/j.tcs.2016.08.016.

Demaine, Erik D., Martin L. Demaine, Eli Fox-Epstein, **Duc A. Hoang**, Takehiro Ito, Hirotaka Ono, Yota Otachi, Ryuhei Uehara, and Takeshi Yamada (2015). “Linear-time algorithm for sliding tokens on trees”. In: *Theoretical Computer Science* 600, pp. 132–142. DOI: 10.1016/j.tcs.2015.07.037.

References IV

van der Zanden, Tom C. (2015). "Parameterized complexity of graph constraint logic". In: *Proceedings of IPEC 2015*. Vol. 43. LIPIcs. Schloss Dagstuhl – Leibniz-Zentrum für Informatik, pp. 282–293. DOI: 10.4230/LIPIcs.IPEC.2015.282.

Heuvel, Jan van den (2013). "The Complexity of Change". In: *Surveys in Combinatorics*. Vol. 409. London Mathematical Society Lecture Note Series. Cambridge University Press, pp. 127–160. DOI: 10.1017/cbo9781139506748.005.

Kamiński, Marcin, Paul Medvedev, and Martin Milanič (2012). "Complexity of independent set reconfigurability problems". In: *Theoretical Computer Science* 439, pp. 9–15. DOI: 10.1016/j.tcs.2012.03.004.

Cereceda, Luis, Jan van den Heuvel, and Matthew Johnson (2011). "Finding Paths Between 3-Colorings". In: *Journal of Graph Theory* 67.1, pp. 69–82. DOI: 10.1002/jgt.20514.

References V

- Ito, Takehiro, Erik D. Demaine, Nicholas J. A. Harvey, Christos H. Papadimitriou, Martha Sideri, Ryuhei Uehara, and Yushi Uno (2011). "On the complexity of reconfiguration problems". In: *Theoretical Computer Science* 412.12-14, pp. 1054–1065. DOI: 10.1016/j.tcs.2010.12.005.
- Bonsma, Paul S. and Luis Cereceda (2009). "Finding Paths Between Graph Colourings: PSPACE-Completeness and Superpolynomial Distances". In: *Theoretical Computer Science* 410.50, pp. 5215–5226. DOI: 10.1016/j.tcs.2009.08.023.
- Cereceda, Luis, Jan van den Heuvel, and Matthew Johnson (2008). "Connectedness of the Graph of Vertex-Colourings". In: *Discrete Mathematics* 308.5-6, pp. 913–919. DOI: 10.1016/j.disc.2007.07.028.
- Kramer, Florica and Horst Kramer (2008). "A Survey on the Distance-Colouring of Graphs". In: *Discrete mathematics* 308.2-3, pp. 422–426. DOI: 10.1016/j.disc.2006.11.059.

References VI

Demaine, Erik D., Fedor V. Fomin, MohammadTaghi Hajiaghayi, and Dimitrios M. Thilikos (2005). "Fixed-parameter algorithms for (k, r)-center in planar graphs and map graphs". In: *ACM Transactions on Algorithms (TALG)* 1.1, pp. 33–47. DOI: 10.1145/1077464.1077468.

Hearn, Robert A. and Erik D. Demaine (2005). "PSPACE-completeness of sliding-block puzzles and other problems through the nondeterministic constraint logic model of computation". In: *Theoretical Computer Science* 343.1-2, pp. 72–96. DOI: 10.1016/j.tcs.2005.05.008.

Agnarsson, Geir, Peter Damaschke, and Magnús M. Halldórsson (2003). "Powers of geometric intersection graphs and dispersion algorithms". In: *Discrete Applied Mathematics* 132.1-3, pp. 3–16. DOI: 10.1016/S0166-218X(03)00386-X.

Kong, M. and Y. Zhao (1993). "On computing maximum k-independent sets". In: *Congressus Numerantium* 95, pp. 47–47.

Meir, A. and John Moon (1975). "Relations between packing and covering numbers of a tree". In: *Pacific Journal of Mathematics* 61.1, pp. 225–233. DOI: 10.2140/pjm.1975.61.225.

References VII

Kramer, Florica and Horst Kramer (1969). "Ein Färbungsproblem der Knotenpunkte eines Graphen bezüglich der Distanz p". In: *Rev. Roumaine Math. Pures Appl* 14.2, pp. 1031–1038.