ONLINE CLUSTERING: ALGORITHMS, EVALUATION, METRICS, APPLICATIONS AND BENCHMARKING

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KDD 2022, Washington, D.C., United States

https://hoanganhngo610.github.io/river-clustering.kdd.2022/

https://doi.org/10.1145/3534678.3542600

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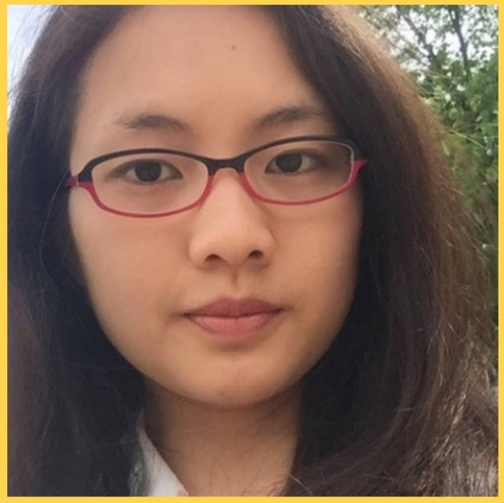

Minh-Huong Le Nguyen

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OUTLINE

- Introduction to ML for data streams
- Introduction to **River**
- Online clustering algorithms
- Online clustering benchmarking with River

https://hoanganhngo610.github.io/river-clustering.kdd.2022/

https://doi.org/10.1145/3534678.3542600

INTRODUCTION TO ML FOR DATA STREAMS

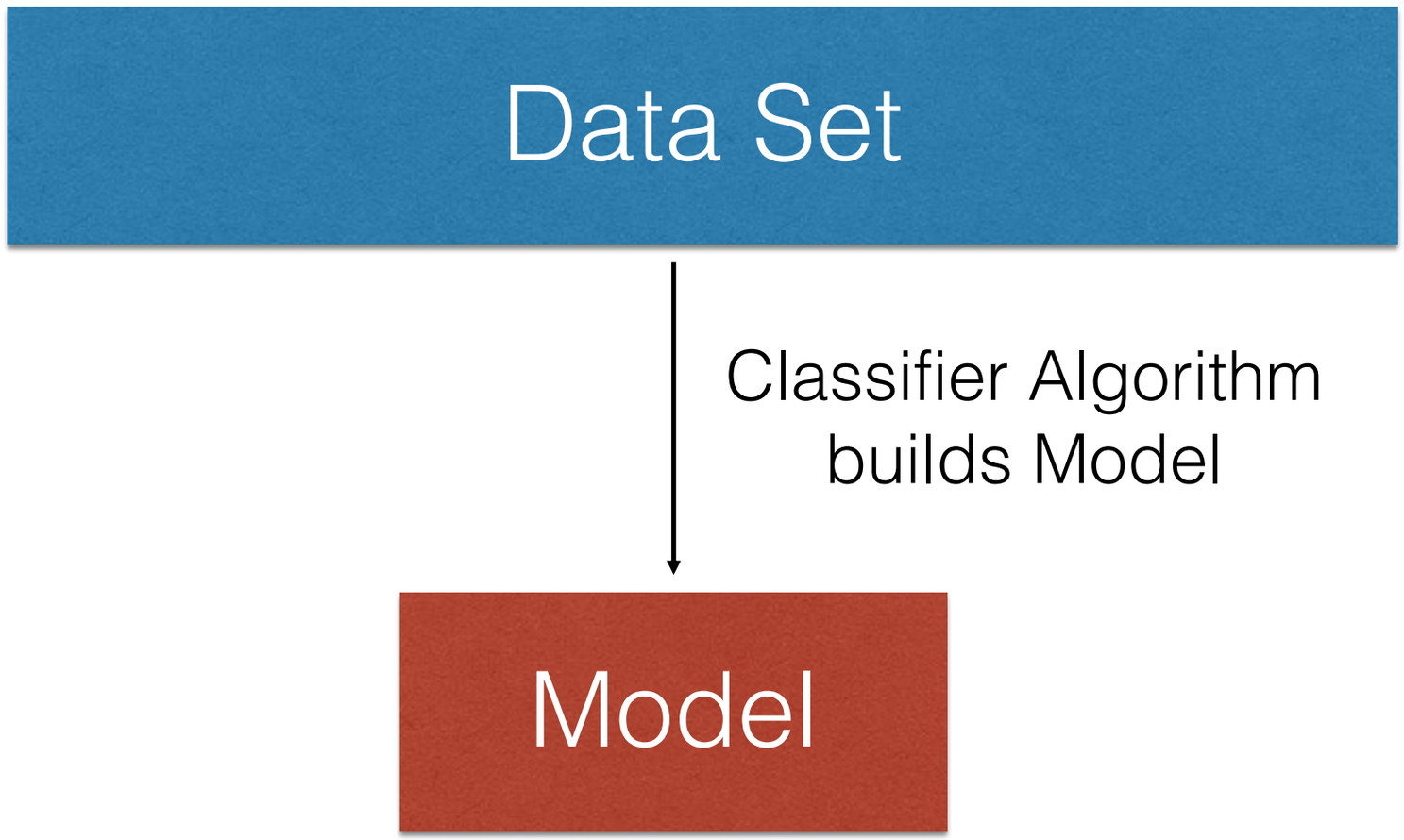
GETTY

Artificial intelligence / Machine learning

Our weird behavior during the pandemic is messing with AI models

Machine-learning models trained on normal behavior are showing cracks —forcing humans to step in to set them straight.

by **Will Douglas Heaven** May 11, 2020



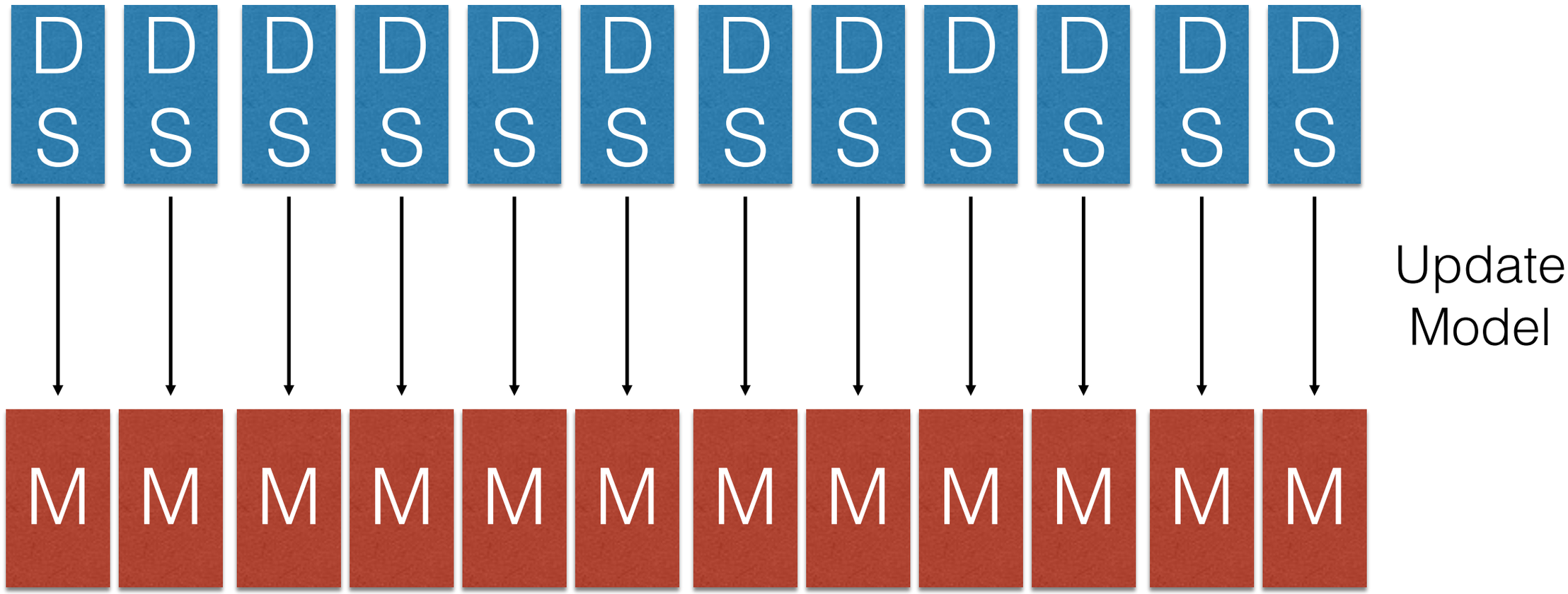
ANALYTIC STANDARD APPROACH

Finite training sets
Static models

Problem: how often and how much data to use to build the model

ML STANDARD APPROACH

Finite training sets
Static models



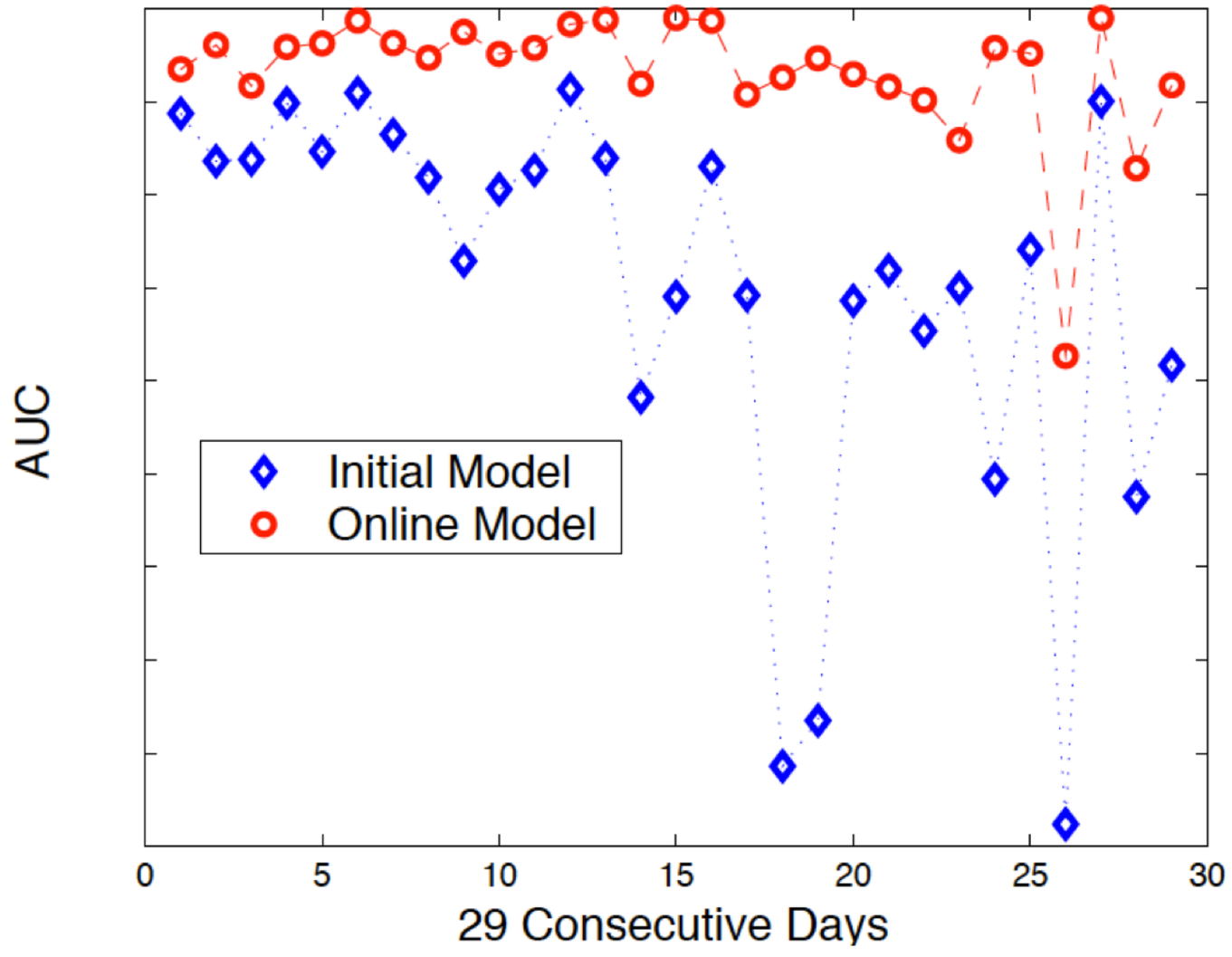
DATA STREAM APPROACH

Infinite training sets
Dynamic models

*DS: Data set, M: Model

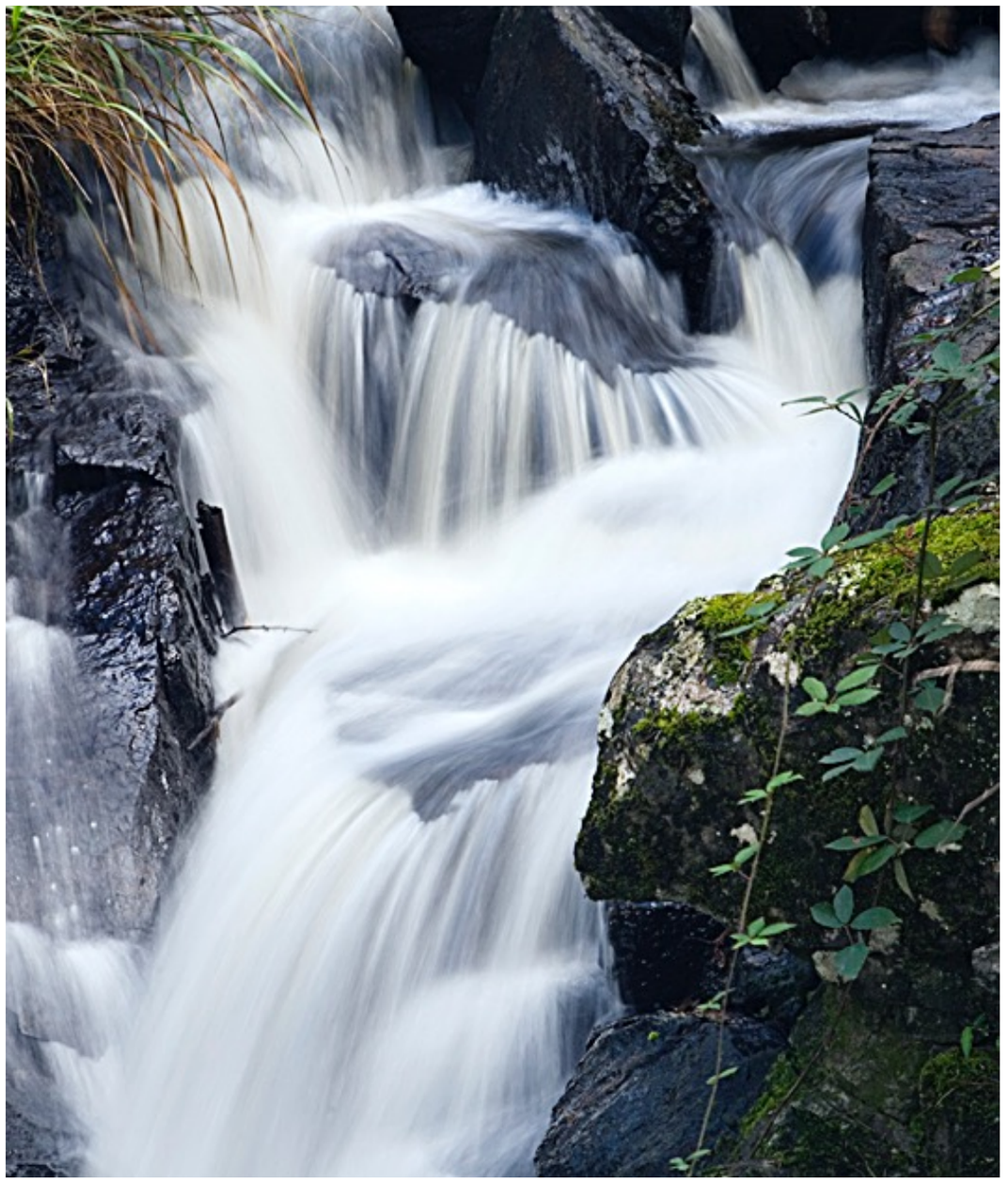
PAIN POINTS

- Need to **retrain!**
  - Things change over time
    - How often?
- Data unused until next update!
  - Value of data wasted



STREAM MINING

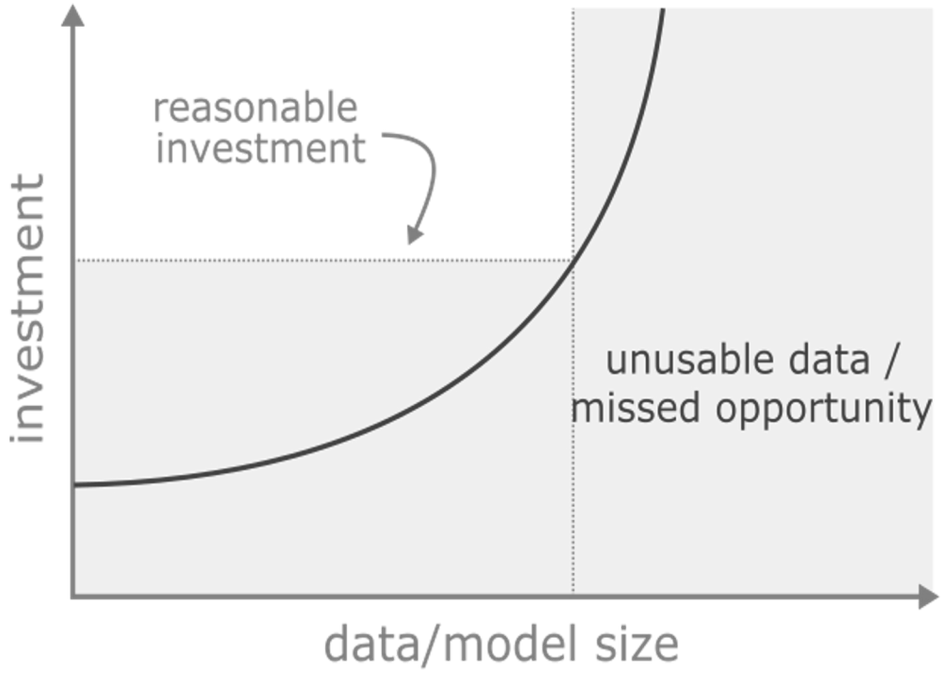
- Maintain models online
  - Incorporate data on the fly
  - Unbounded training sets
  - Resource efficient
  - Detect changes and adapts
  - Dynamic models



INVESTMENT VS DATA SIZE

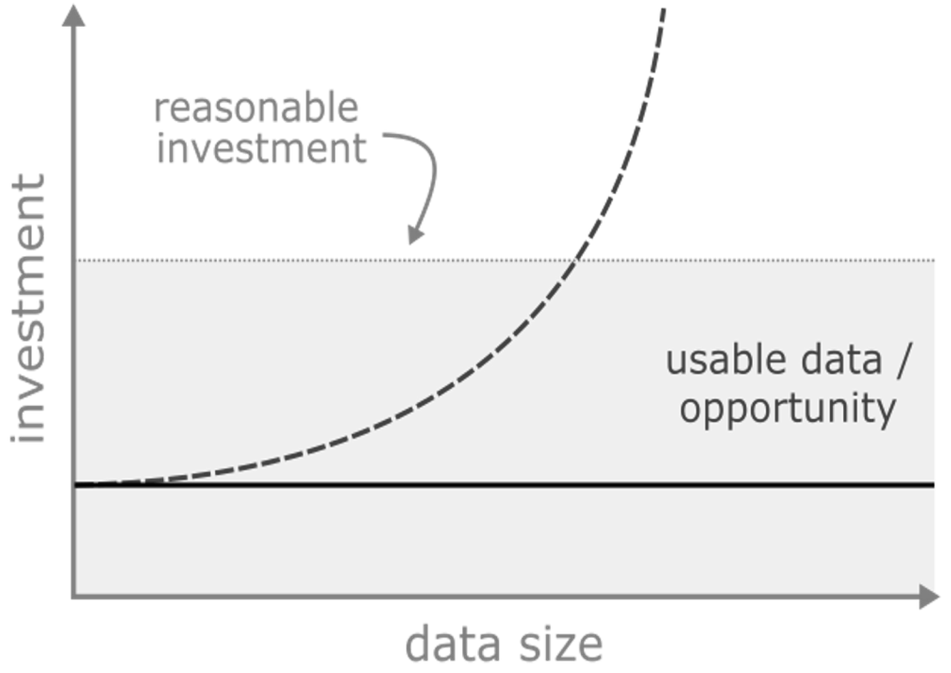
Batch learning

Often struggles to maintain investment
(time, memory, cost) below reasonable level



Stream learning

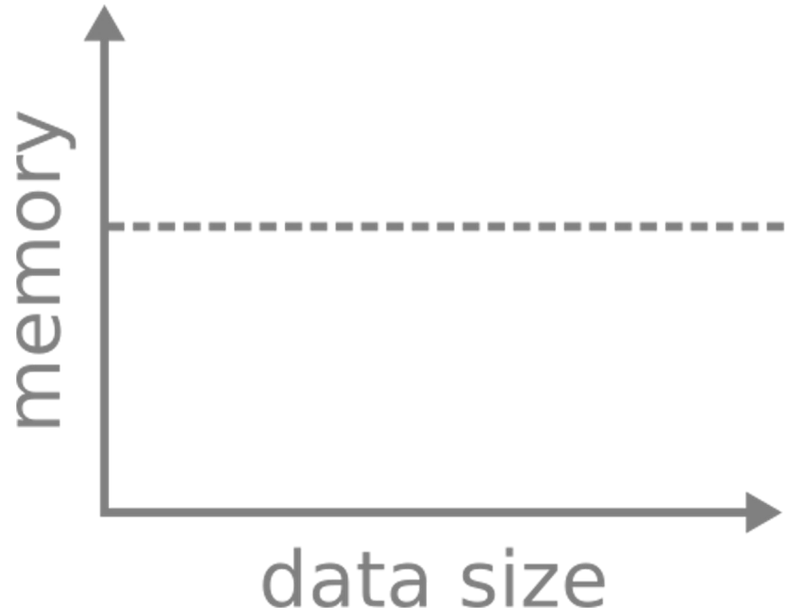
Efficiently generates incremental models
from data streams



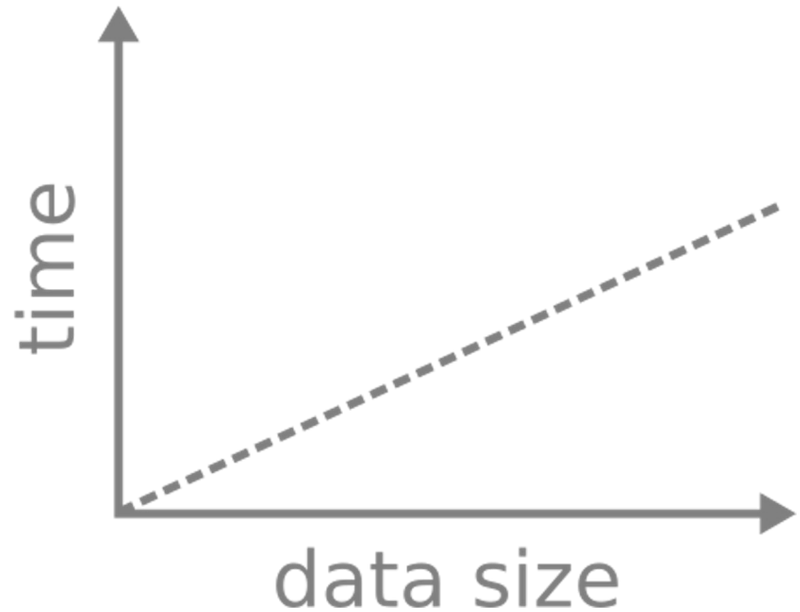
Source: AWS, 2018

RESOURCES

Stream learning upper limits



constant memory



sub-linear processing time (training)

REQUIREMENTS

Process one sample at a time, and inspect it only once

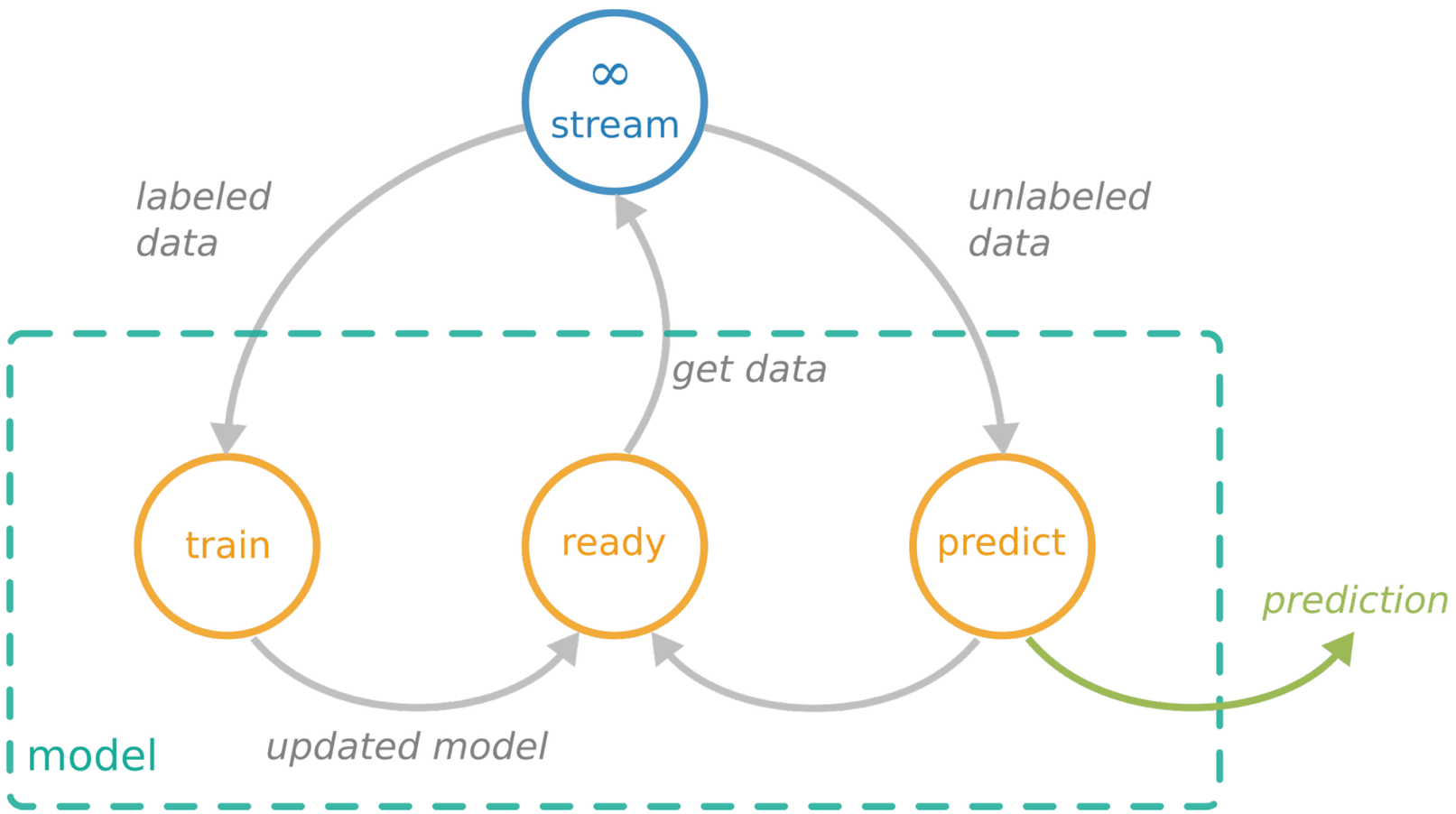
Use a limited amount of memory

Work in a limited amount of time

Always ready to predict

LEARNING FROM DATA STREAMS

Supervised learning



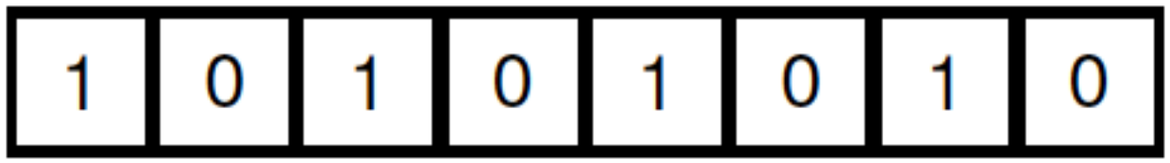
- One sample at a time
- Limited resources
- Predict at any time

APPROXIMATION ALGORITHMS

- General idea, good for streaming algorithms
- Small error ϵ with high probability $1 - \delta$
  - True hypothesis H, and learned hypothesis $\widehat{\mathrm{H}}$
  - $\Pr\left[\left|\mathrm{H} - \widehat{\mathrm{H}}\right| < \epsilon|\mathrm{H}|\right] > 1 - \delta$

APPROXIMATION ALGORITHMS

- What is the largest number that we can store in 8 bits?



APPROXIMATION ALGORITHMS

- What is the largest number that we can store in 8 bits?

Robert Morris. 1978. *Counting large number of events in small registers*. Communications of the ACM, 21, 10 (1978), 840-842. https://doi.org/10.1145/359619.359627

Programming Techniques

S.L. Graham, R.L. Rivest Editors

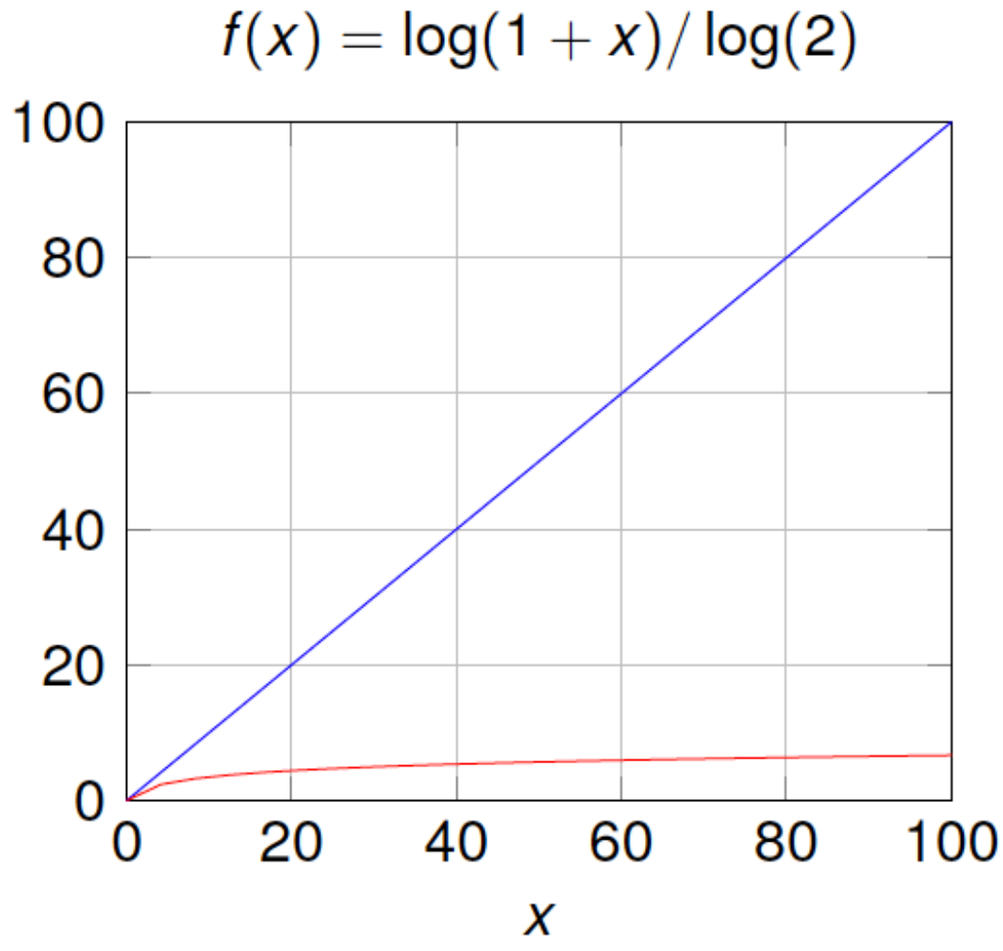
Counting Large Numbers of Events in Small Registers

Robert Morris
Bell Laboratories, Murray Hill, N.J.

It is possible to use a small counter to keep approximate counts of large numbers. The resulting expected error can be rather precisely controlled. An example is given in which 8-bit counters (bytes) are used to keep track of as many as 130,000 events with a relative error which is substantially independent of the number n of events. This relative error can be expected to be 24 percent or less 95 percent of the time (i.e. $\sigma = n/8$). The techniques could be used to advantage in multichannel counting hardware or software used for the monitoring of experiments or processes.

APPROXIMATION ALGORITHMS

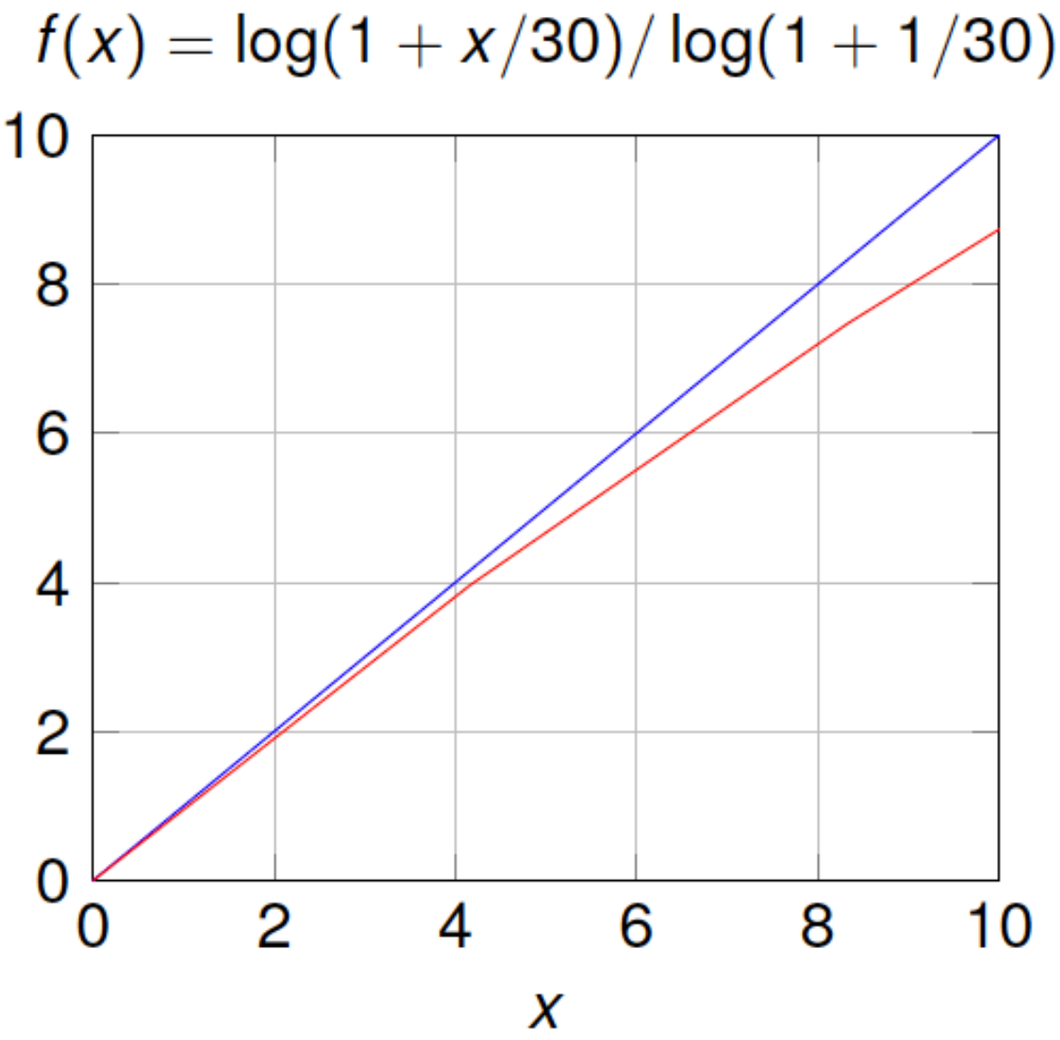
- What is the largest number that we can store in 8 bits?



$$f(0) = 0, f(1) = 1$$

APPROXIMATION ALGORITHMS

- What is the largest number that we can store in 8 bits?



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APPROXIMATION ALGORITHMS

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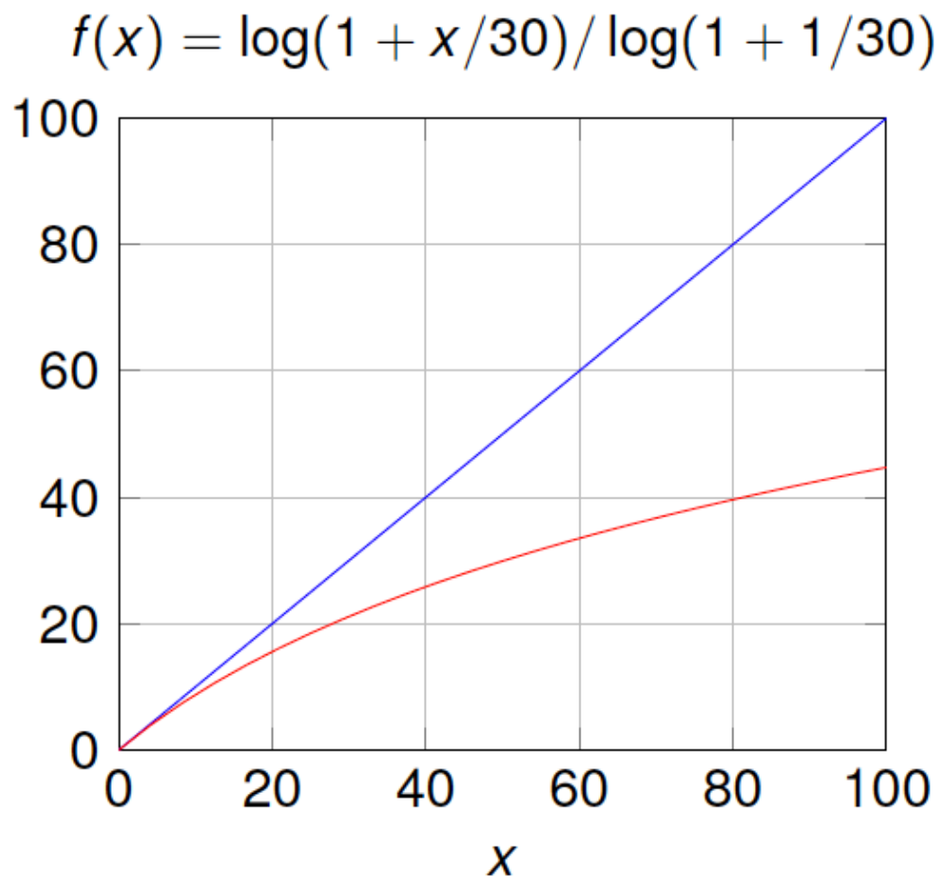


$$f(0) = 0, f(1) = 1$$

APRPOXIMATION ALGORITHMS

MORRIS APPROXIMATE COUNTING ALGORITHM

```
1    Init counter c ← 0
2    for every event in the stream
3         do rand = random number between 0 and 1
4         if rand < p
5              then c ← c + 1
```

- What is the largest number that we can store in 8 bits?

APPROXIMATION ALGORITHMS

101100011110101 0111010

Sliding Window

We can maintain simple statistics over sliding windows, using $O(\frac{1}{\epsilon}\log^2 N)$ space, where

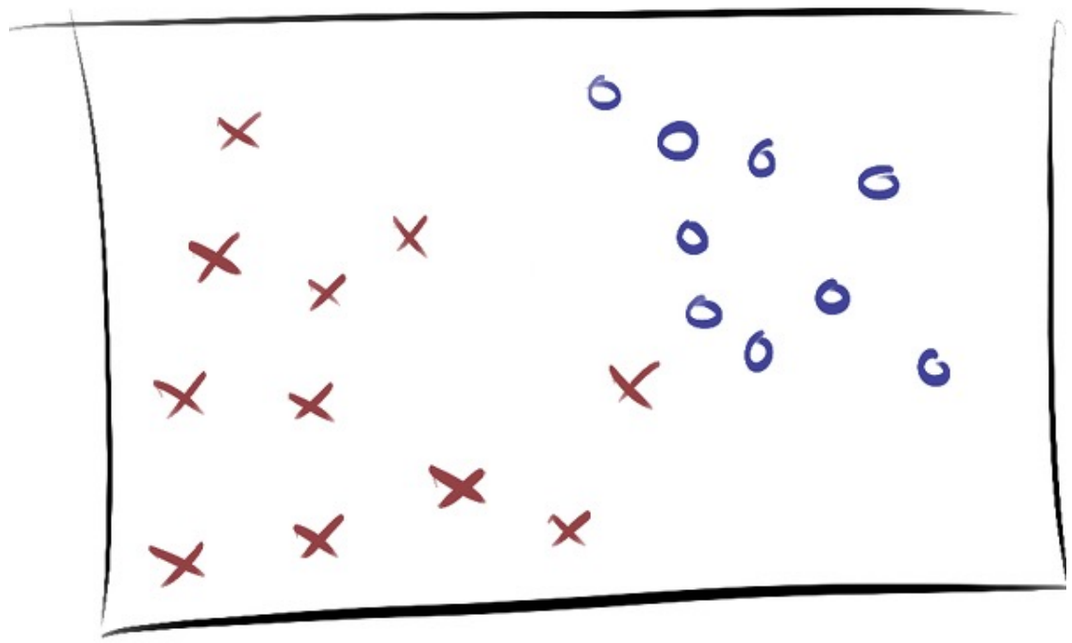
- N is the length of the sliding window
- ϵ is the accuracy parameter

M. Datar, A. Gionis, P. Indyk, and R. Motwani.
Maintaining stream statistics over sliding windows. 2002

CLASSIFICATION

DEFINITION

Given a set of training examples belonging to $\mathbf{n_c}$ different classes, a classifier algorithm builds a model that predicts for every unlabeled instance x the class C to which it belongs



EXAMPLES

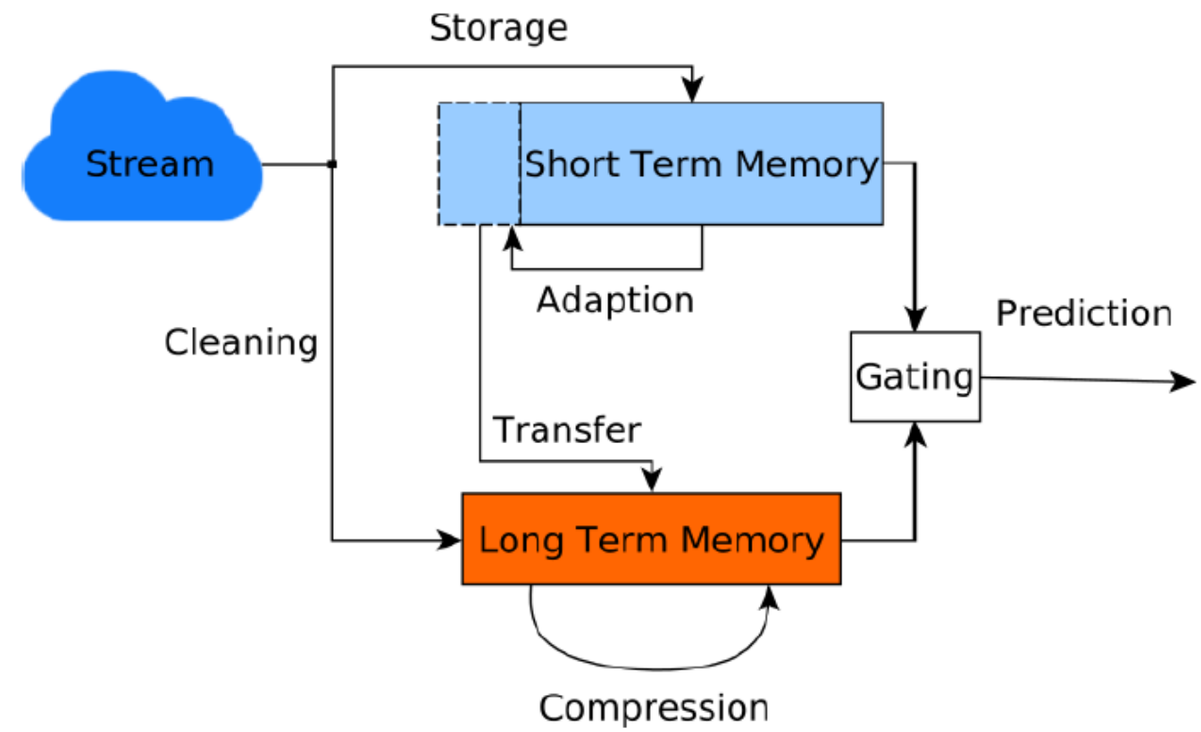
- Email spam filter
- Twitter sentiment analyzer

Photo: Stephen Merity http://smerity.com

SAM-kNN

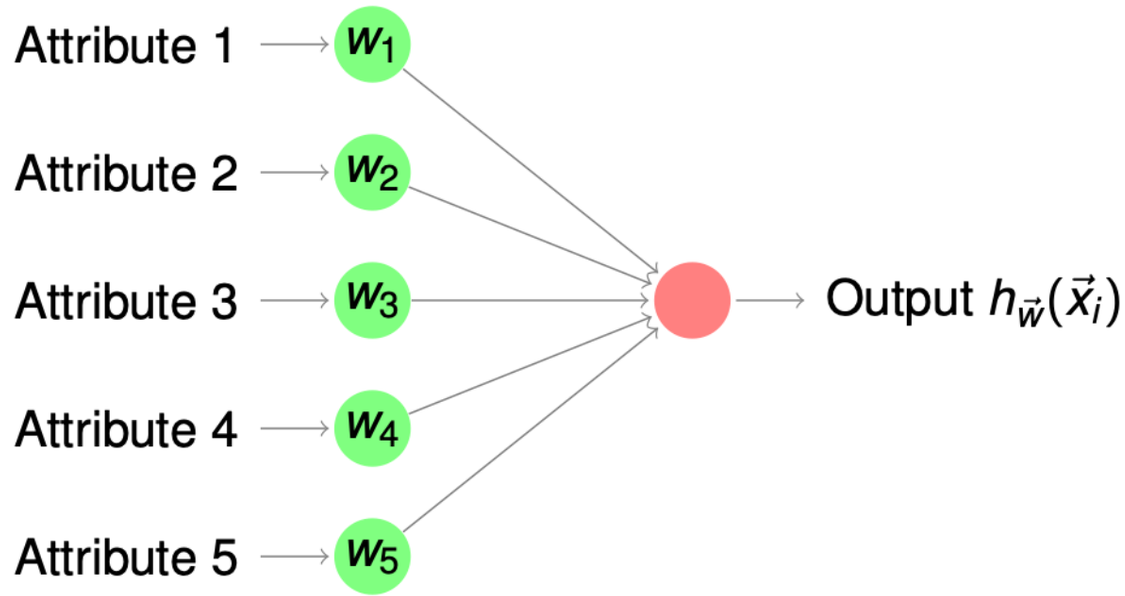
Viktor Losing, Barbara Hammer, and Heiko Wersing. 2016. **KNN Classifier with Self Adjusting Memory for Heteogenerous Concept Drift.** In: *Proceedings of the 16th International Conference on Data Mining (ICDM)*, (Barcelona, Spain), 291-300. https://doi.org/10.1109/ICDM.2016.0040

Received **Best Paper Awards**.



PERCEPTRON

- Linear classifier
- Data stream: $\langle \overrightarrow{x_i}, y_i \rangle$
- $\widetilde{y_i} = h_{\overrightarrow{w}}(\overrightarrow{x_i}) = \sigma(\overrightarrow{w_i}^T \overrightarrow{x_i})$
- $\sigma(x) = \frac{1}{1+e^{-x}}, \sigma' = \sigma(x)(1-\sigma(x))$
- Minimize MSE $J(\overrightarrow{w}) = \frac{1}{2} \times \sum (y_i - \widetilde{y_i})^2$
- SGD $\overrightarrow{w_{i+1}} = \overrightarrow{w_i} - \eta \nabla J \overrightarrow{x_i}$
- $\nabla J = (y_i - \widetilde{y_i})\widetilde{y_i}(1-\widetilde{y_i})$
- $\overrightarrow{w_{i+1}} = \overrightarrow{w_i} + \eta(y_i - \widetilde{y_i})\widetilde{y_i}(1-\widetilde{y_i})\widetilde{x_i}$



PERCEPTRON LEARNING

PERCEPTRON LEARNING(*Stream*, η)

1 **for** each class
2 **do** PERCEPTRON LEARNING(*Stream*, *class*, η)

PERCEPTRON LEARNING(*Stream*, *class*, η)

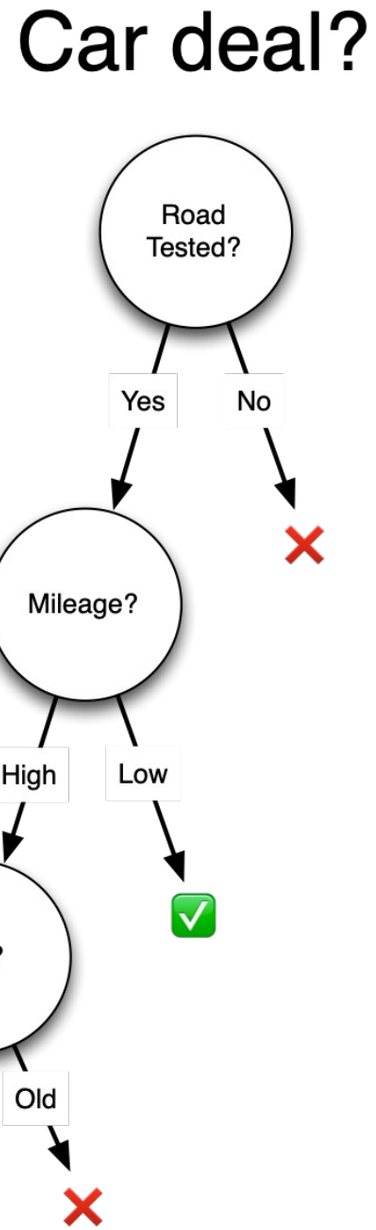
1 ▷ Let w_0 and $\vec{w}$ be randomly initialized
2 **for** each example $(\vec{x}, y)$ in Stream
3 **do if** $class = y$
4 **then** $\delta = (1 - h_{\vec{w}}(\vec{x})) \cdot h_{\vec{w}}(\vec{x}) \cdot (1 - h_{\vec{w}}(\vec{x}))$
5 **else** $\delta = (0 - h_{\vec{w}}(\vec{x})) \cdot h_{\vec{w}}(\vec{x}) \cdot (1 - h_{\vec{w}}(\vec{x}))$
6 $\vec{w} = \vec{w} + \eta \cdot \delta \cdot \vec{x}$

PERCEPTRON PREDICTION($\vec{x}$)

1 **return** $\arg\max_{class} h_{\vec{w}_{class}}(\vec{x})$

DECISION TREE

- Each node tests a features
- Each branch represents a value
- Each leaf assigns a class
- Greedy recursive induction
  - Sort all examples through tree
  - X_i = most discriminative attribute
  - New node for x_i, new branch for each value, leaf assigns majority class
  - Stop if no error | limit on number of instances



HOEFFDING TREE

Pedro Domingos and Geoff Hulten. 2000. **Mining high-speed data stream.** In: *Proceedings of the sixth ACM SIGKDD International Conference on Knowledge Discovery and Data Mining* (Boston, Massachusetts, USA), Association for Computing Machinery, New York, NY, USA, 71-80. https://doi.org/10.1145/347090.347107

- Sample of stream enough for near optimal decision
- Estimate merit of alternatives from prefix of stream
- Choose sample size based on statistical principles
- When to expand a leaf?
  - Let x_1 be the most informative attribute, x_2 the second most informative one
  - **Hoeffding bound:** split if $\mathrm{G}(\mathrm{x}_1) - \mathrm{G}(\mathrm{x}_2) > \epsilon = 2 \times \sqrt{\frac{R^2 \ln\left(\frac{1}{\delta}\right)}{2\mathrm{n}}}$ (∗)

(∗) This bound has been corrected in a recent paper: Pawel Matuszyk, Georg Krempl, and Myra Spiliopoulou. **Correcting the Usage of the Hoeffding Inequality in Stream Mining**. In: *IDA 2013: Advances in Intelligent Data Analysis*, 298-309. https://doi.org/10.1007/978-3-642-41398-8_26

ADAPTIVE RANDOM FOREST

- Why Random Forests?
  - Off-the-shelf learner
  - Good learning performance

Heitor M. Gomes, Albert Bifet, Jesse Read, Jean Paul Barddal, Fabrício Enembreck, Bernhard Pfharinger, Geoff Holmes & Talel Abdessalem. 2017. **Adaptive random forests for evolving data streams classification.** *Machine Learning* 106 (2017), 1469-1495. https://doi.org/10.1007/s10994-017-5642-8

- Based on the original Random Forest by Leo Breiman

Leo Breiman. 2001. **Random Forests**. *Machine Learning* 45 (2001), 5-32. https://doi.org/10.1023/A:1010933404324

ADAPTIVE RANDOM FOREST

1. Leveraging bagging
2. Random Hoeffding Trees
    - Random subsets of features for splits
    - Independent (can train in parallel)
3. Adaptive Learning
    - 1 drift and 1 warning detector per tree
    - Train trees in the background before adding them

STREAMING RANDOM BATCHES

x_1	x_2	x_3	x_4	$x_{...}$	x_m
$v_{1,1}$	$v_{1,2}$	$v_{1,3}$	$v_{1,4}$	$v_{1,5}$	$v_{1,6}$
$v_{2,1}$	$v_{2,2}$	$v_{2,3}$	$v_{2,4}$	$v_{2,5}$	$v_{2,6}$
$v_{3,1}$	$v_{3,2}$	$v_{3,3}$	$v_{3,4}$	$v_{3,5}$	$v_{3,6}$
$v_{4,1}$	$v_{4,2}$	$v_{4,3}$	$v_{4,4}$	$v_{4,5}$	$v_{4,6}$
$v_{5,1}$	$v_{5,2}$	$v_{5,3}$	$v_{5,4}$	$v_{5,5}$	$v_{5,6}$
$v_{6,1}$	$v_{6,2}$	$v_{6,3}$	$v_{6,4}$	$v_{6,5}$	$v_{6,6}$
$v_{...,1}$	$v_{...,2}$	$v_{...,3}$	$v_{...,4}$	$v_{...,5}$	$v_{...,6}$

Heitor Murilo Gomes, Albert Bifet, and Jesse Read. **Streaming Random Patches for Evolving Data Stream Classification**. In: *Proceedings of the 2019 IEEE International Conference on Data Mining (ICDM)*, 240-249. https://doi.org/10.1109/ICDM.2019.00034

STREAMING RANDOM BATCHES

TABLE II: Test-then-train accuracy (%) using $n = 100$ base models. Underlined results means the performance increased in comparison to $n = 10$ version. BAG and LB did not finish execution for SPAM dataset.

Data set	*LB*	*OAUE*	*DWM*	*OB*	*ARF*	**SRP**	*SRS*	*BAG*
LED(A)	73.953	73.393	73.958	72.475	73.96	74.027	**74.04**	73.975
LED(G)	73.225	72.582	73.031	72.117	73.094	**73.233**	73.179	73.215
AGR(A)	88.717	90.164	88.299	90.374	87.929	**92.869**	92.807	86.663
AGR(G)	83.713	85.244	79.437	87.834	82.288	89.651	**90.259**	82.52
RBF(M)	84.338	84.262	60.977	74.514	**86.958**	86.039	84.821	86.671
RBF(F)	76.771	57.147	54.531	48.698	76.291	76.375	61.622	**77.686**
AIRLINES	62.82	65.229	64.025	64.556	66.417	**68.564**	68.303	62.093
ELEC	89.508	87.407	87.754	89.515	89.672	89.859	**90.267**	89.822
COVTYPE	95.104	92.857	88.519	92.695	94.967	**95.348**	93.461	95.288
KDD99	99.965	2.445	99.951	99.936	99.972	**99.981**	99.973	99.974
ADS	99.634	15.401	97.499	90.393	99.695	**99.726**	98.353	99.634
NOMAO	97.072	58.233	95.462	96.393	97.197	**97.383**	96.57	97.226
SPAM	NA	80.781	89.361	86.519	97.319	**97.437**	95.924	NA
Avg Rank	4.46	6.33	6.67	6.17	4	**1.58**	3.17	3.63
Avg Rank Synt.	4.17	5.67	6.67	6.17	4.67	**2**	2.83	3.83
Avg Rank Real	4.75	7	6.67	6.17	3.33	**1.17**	3.5	3.42

Heitor Murilo Gomes, Albert Bifet, and Jesse Read. **Streaming Random Patches for Evoving Data Stream Classification**. In: *Proceedings of the 2019 IEEE International Conference on Data Mining (ICDM)*, 240-249. https://doi.org/10.1109/ICDM.2019.00034

River

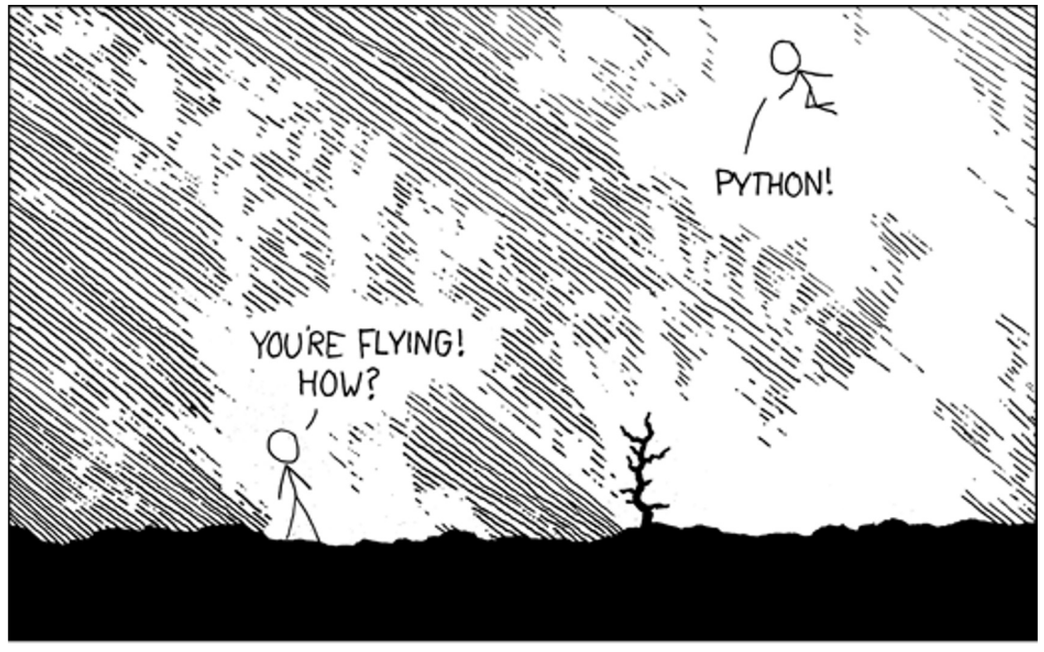
Jacob Montiel, Max Halford, Saulo Martiello Mastelini, Geoffrey Bolmier, Raphael Sourty, Robin Vaysse, Adil Zouitine, Heitor Murilo Gomes, Jesse Read, Talel Abdessalem, and Albert Bifet. 2021. **River: machine learning for streaming data in Python**. *Journal of Machine Learning Research* 22 (April 2021), 1–8. http://jmlr.org/papers/v22/20-1380.html

A Python library for stream learning

- Incremental + adaptive methods
  - *Supervised learning*
    Classification, Regression
  - *Unsupervised learning*
    Anomaly detection*, Clustering
- Drift detection
- Pipelines / Data transformers
- Evaluation / Metrics
- etc.

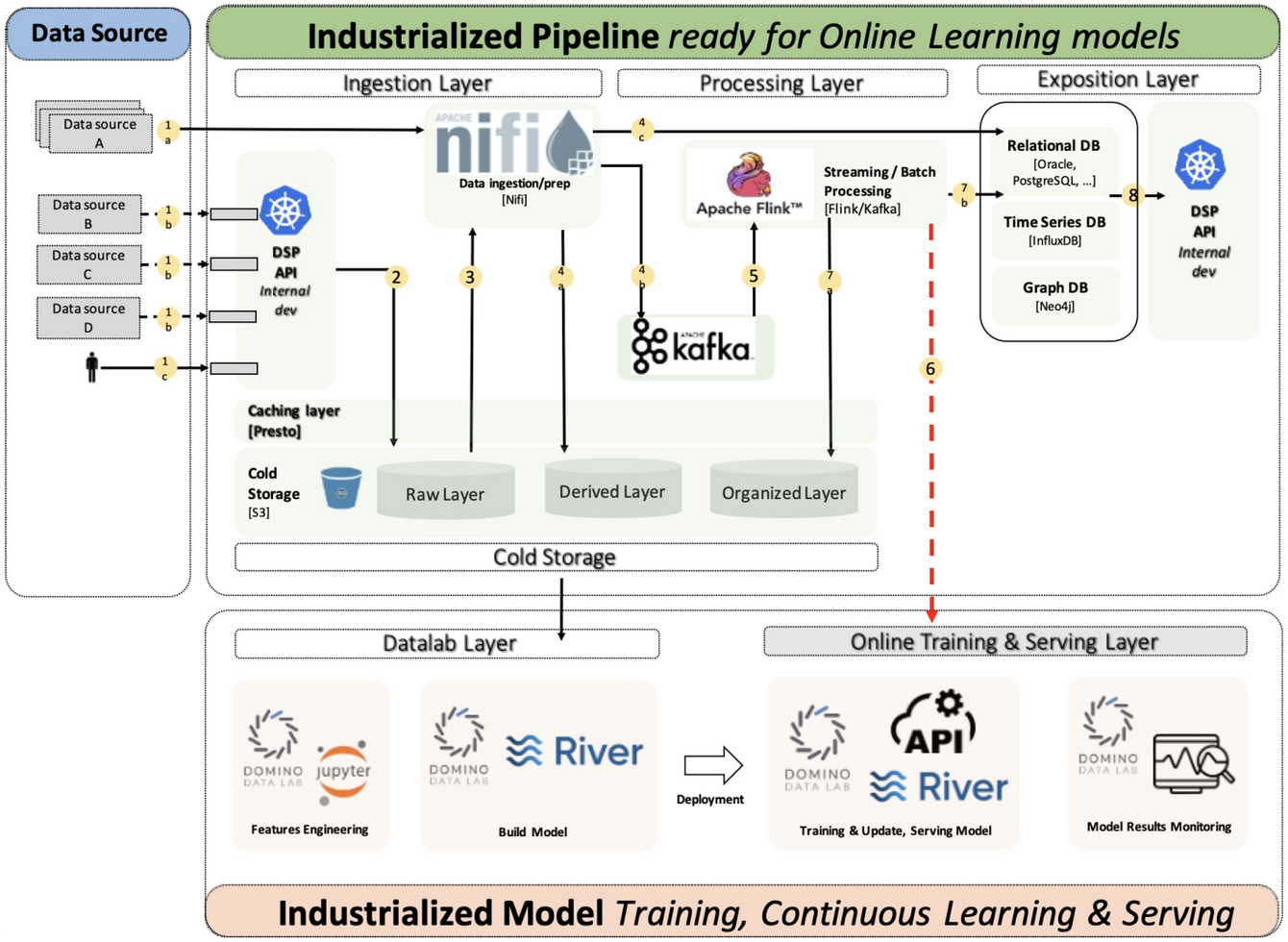
DESIGN PRINCIPLES

- Pythonic
- Easy to use (any expertise level)
- Easy to extend
- Intended to work with other tools in the Python ecosystem
- Users: researchers *and* practitioners



USE CASE: BNP PARIBAS

Marriam Barry and Albert Bifet and Raja Chiky and Jacob Montiel and Vinh-Thuy Tran. **Challenges of Machine Learning for Data Streams in Banking Industry**. In: *BDA 2021: Big Data Analytics*. Springer, 106-118. https://doi.org/10.1007/978-3-030-93620-4_9



Proof of concept.

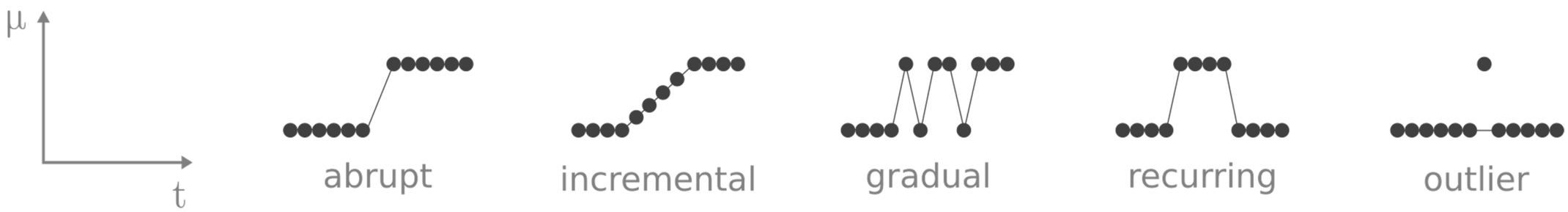
"*...a platform architecture used to deploy online learning models in a production environment and at a large scale.*"

CONCEPT DRIFT

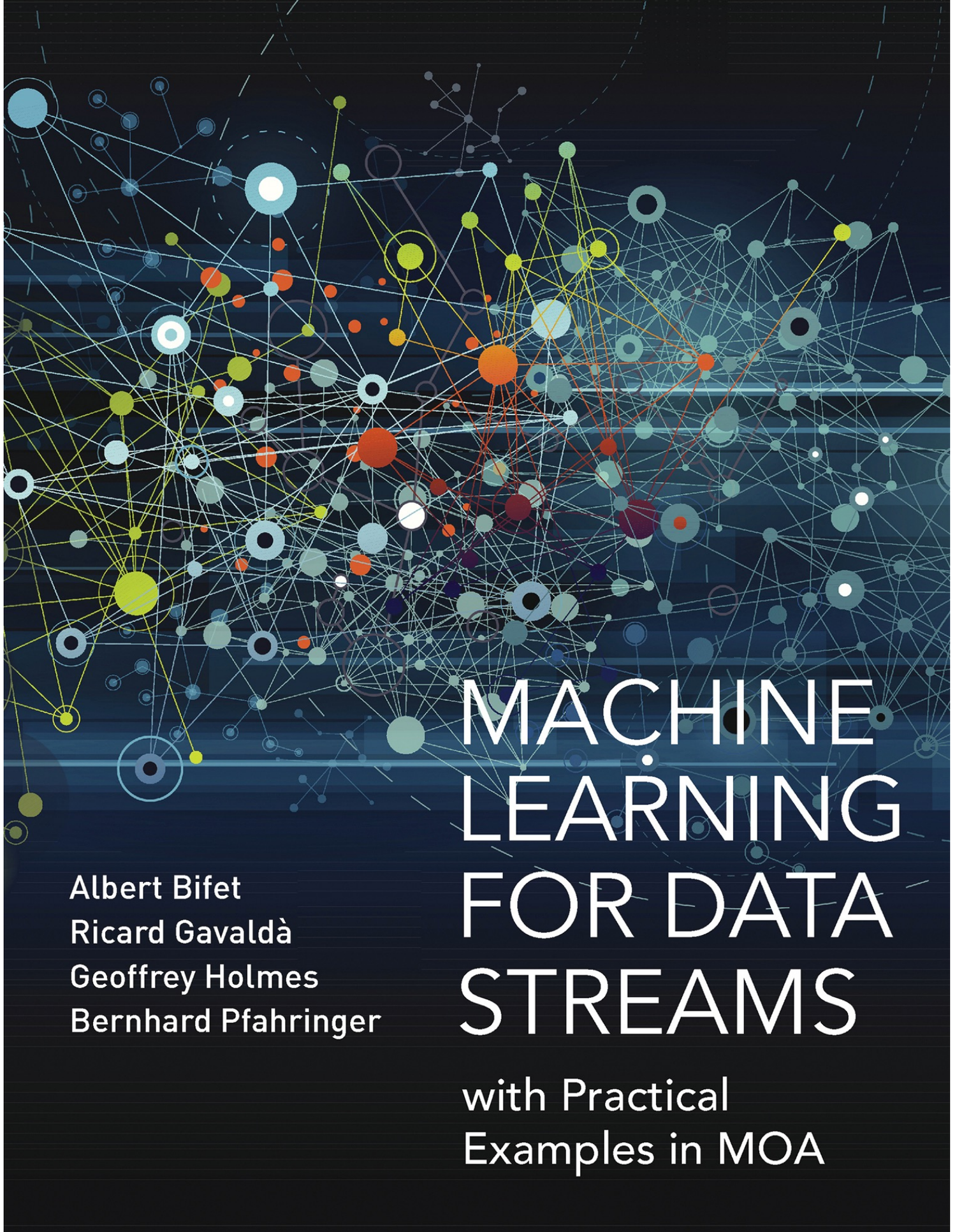
Joao Gama and Indre Zliobaite, Albert Bifet, Mikola Pechenizkiy and Abdelhamid Bouchachia. **A survey on Concept Drift Detection**. *ACM Computing Surveys* 46, 4 (2014), 1-37. https://doi.org/10.1145/2523813

In dynamic and non-stationary environments, the data distribution can change over time.

- **Change detector:** Given an input sequence $\langle x_1, x_2, \ldots, x_t, \ldots \rangle$ raise an alarm signal at instant t if there is a change
- **Application:** Detect changes in model performance



ONLINE CLUSTERING ALGORITHMS



The MIT Press

DEFINITION

Given a set of unlabeled instances, distribute them into homogeneous groups according to some common relations or affinities

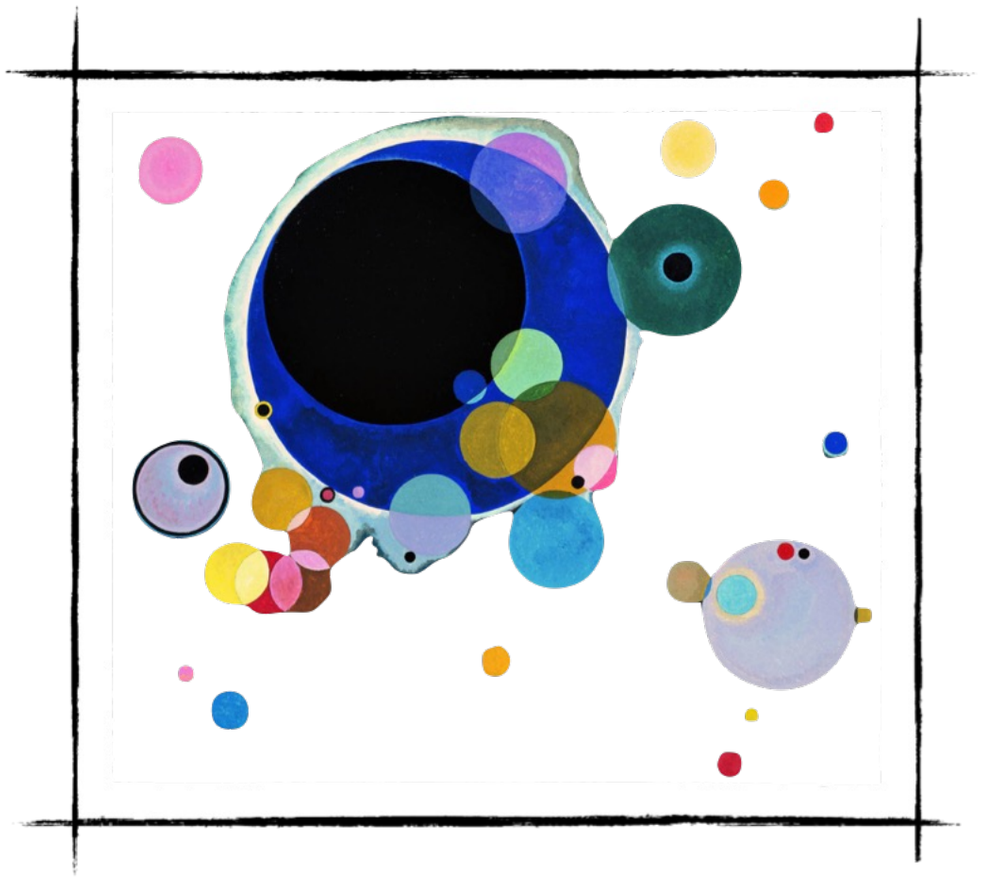


Photo source: W. Kandinsky - Several Circles (edited)

EXAMPLES

- Market segmentation
- Social network communities

CLUSTERING

Given

- a set of instances I
- a number of clusters K
- an objective function $cost(I)$

a clustering algorithm computes an assignment of a cluster for each instance

$$f : I \rightarrow \{1, \ldots, K\}$$

that minimizes the objective function $cost(I)$

CLUSTERING

Given

- a set of instances I
- a number of clusters K
- an objective function $cost(C, I)$

a clustering algorithm computes a set C of instances with $|C| = K$ that minimizes the objective function

$$cost(C, I) = \sum_{x \in I} d^2(x, C)$$

where

- $d(x, c)$: distance function between x and c
- $d^2(x, C) = min_{c \in C} d^2(x, c)$: distance from x to the nearest point in C

AVAILABLE SOFTWARES

Very few implementations (only most prominent ones) and unified frameworks with multiple algorithms co-existing:

- **Massive Online Analysis (MOA):** Most popular framework, written by Bifet et al. (2010) in Java, including the most number (7) of clustering algorithms. However, one major **disadvantage**: only works well when information of data streams are previously known.
- **stream package**: Written in R by Hahsler et al. (2018), with newer algorithms including D-Stream, DBSTREAM and evoStream.

AVAILABLE SOFTWARES

Very few implementations (only most prominent ones) and unified frameworks with multiple algorithms co-existing:

- **Subspace MOA framework:** An extension of MOA from Java into R, written by Hassani et al. (2016), with extra algorithms including HDDStream and PreDeConStream.
- **streamDM:** Written by Huawei Noah's Ark Lab (2015) with Spark Streaming, an extension of Spark engine. Including CluStream and StreamKM++, but no plans for any further implementation

SOLUTION – RIVER

→ River comes into play, with a neat implementation that allows:

- Works with any arbitrary numerical data stream;
- Well-maintained, documented and includes various algorithms of different types.

Currently, River offers 6 clustering algorithms, including incremental K-Means, CluStream, DenStream, DBSTREAM, StreamKMeans (O'Callaghan et al., 2002) and evoStream with a clear plan of further implementations.

Includes the most number of clustering algorithms apart from MOA.

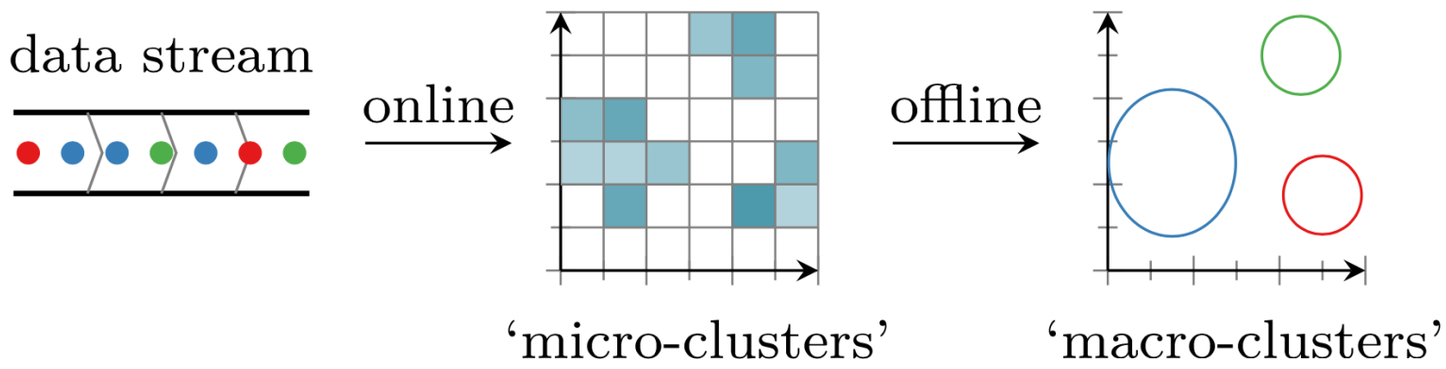
STRATEGY

Basically, finding clustering solutions is an optimization task, with the following principle strategies:

- Minimizing intra-cluster distances or radii of clusters (ensuring that objects within the same cluster are similar);
- Maximizing inter-cluster distances or heterogeneity (ensuring that objects within different clusters are well-separated);
- Maximizing likelihood estimates

ARISING PROBLEMS

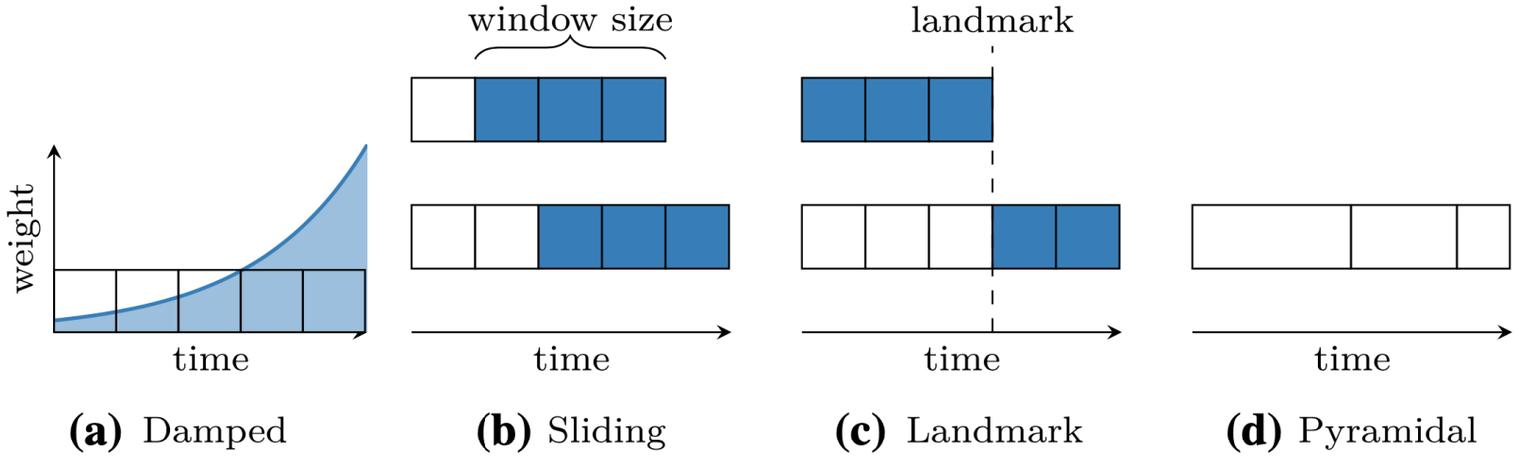
- In online clustering, historical data will be discarded and only information of formed clusters (cluster centers, number of points, linear sum, sum of squares, etc.) will be saved → Clustering algorithms are divided into **two** phases: ONLINE phase and OFFLINE phase.



Two-phase stream clustering with grid-based approach
(Source: Matthias Carnein et al. 2017. An empirical comparison of stream clustering algorithms.)

ARISING PROBLEMS

- Through time, the distribution of the stream will change. (also known as drift or concept drift) → Models can employ time-window models, which only keeps the most few recent data points to avoid bias. This approach can include damped, sliding, landmark or pyramidal models.



Two-phase stream clustering with grid-based approach
(Source: Zhu Y. and Shasha D. 2002. Statstream: statistical monitoring of thousands of data streams in real life and Silva J. A. et al. 2013. Data stream clustering: a survey.)

APPROACHES

Matthias Carnein and Heike Trautmann. 2019. **Optimizing Data Stream Representation: An Extensive Survey on Stream Clustering Algorithms**. *Business and Information Systems Engineering* 61, 3 (2019), 277-297. https://doi.org/10.1007/s12599-019-00576-5

- **Distance-based approach:** threshold the distance of the new observation to existing clusters, either to insert or initialize new clusters, including:
  - *Clustering Features (CFs), Extended CFs, Time-Fading CFs:* BIRCH, CluStream, SDStream, ClusTree;
  - *Centroids, Medoids:* StreamKM++, STREAM;
  - *Competitive Learning:* DBSTREAM.
- **Density-based (Grid-based) approach:** capture the density of observation in a grid, by separating the data space among all dimension, including:
  - *One-time or recursive partitioning:* DUCStream, D-Stream, Stats-Grid;
  - *Hybrid Grid-Approach:* HDCStream, Mudi-Stream;

APPROACHES

Matthias Carnein and Heike Trautmann. 2019. **Optimizing Data Stream Representation: An Extensive Survey on Stream Clustering Algorithms**. *Business and Information Systems Engineering* 61, 3 (2019), 277-297. https://doi.org/10.1007/s12599-019-00576-5

- **Model-based approach:** Summarize the data stream as a ***statistical model***, with a common area of research based on the Expectation Maximization (EM) algorithm. Including CluDisStream, SWEM, COBWEB, Wstream, etc.

- **Projected approach:** This special approach deals with ***high dimensional data stream***, addressing the curse of dimensionality. Including HPStream, HDDStream, and PreDeConStream along with their extensions.

K-MEANS

K-MEANS(P, k)

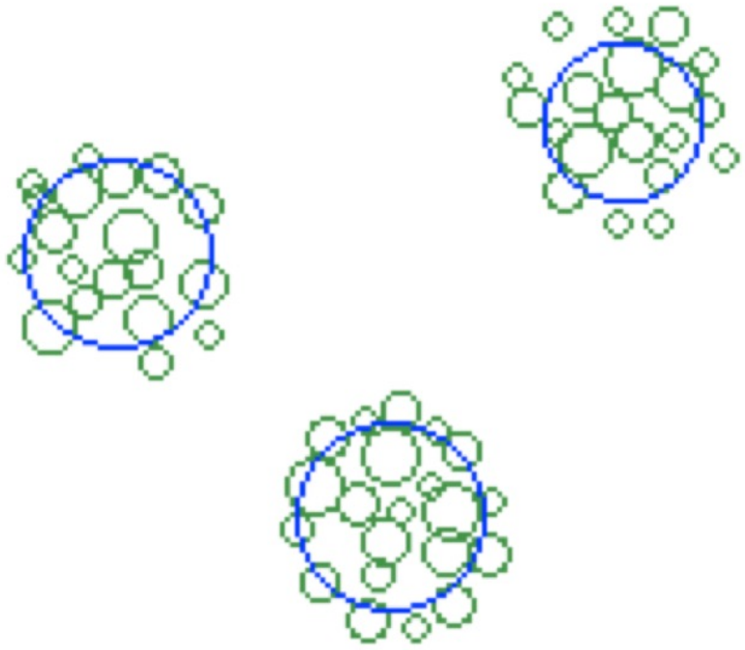
Input: a dataset of points $P = \{p_1, \ldots, p_n\}$, a number of clusters k
Output: centers $\{c_1, \ldots, c_k\}$ implicitly dividing P into k clusters

```
1  choose k initial centers C = {c_1, ..., c_k}
2  while stopping criterion has not been met
3      do ▷ assignment step:
4          for i = 1, ..., N
5              do find closest center c_k ∈ C to instance p_i
6                  assign instance p_i to set C_k
7          ▷ update step:
8          for i = 1, ..., k
9              do set c_i to be the center of mass of all points in C_i
```

MICRO-CLUSTERS

Tian Zhang, Raghu Ramakrishnan, and Miron Livny. 1996. **BIRCH: An Efficient Data Clustering Method for Very Large Databases**. In *SIGMOD'96: Proceedings of the 1996 ACM SIGMOD International Conference on Management of Data*. Association for Computing Machinery, New York, NY, USA. https://doi.org/10.1145/233269.233324

- Cluster Features $\overrightarrow{\mathrm{CF}}$ (statistical summary structure)
- Maintained in online phase, input for offline phase
- Data stream $\langle\overrightarrow{\mathrm{x_i}}\rangle$, d dimensions
- Cluster Features vector includes
  - N: number of points
  - $\mathrm{LS_i}$: sum of values (for dimension j)
  - $\mathrm{SS_i}$: sum of squared values (for dimension j)
- Easy to update, easy to merge
- Constant space irrespective to the number of examples!



Properties:

- Centroid $= LS/N$
- Radius $= \sqrt{SS/N - (LS/N)^2}$
- Diameter $= \sqrt{\frac{2 \times N * SS - 2 \times LS^2}{N \times (N-1)}}$

CLUSTREAM

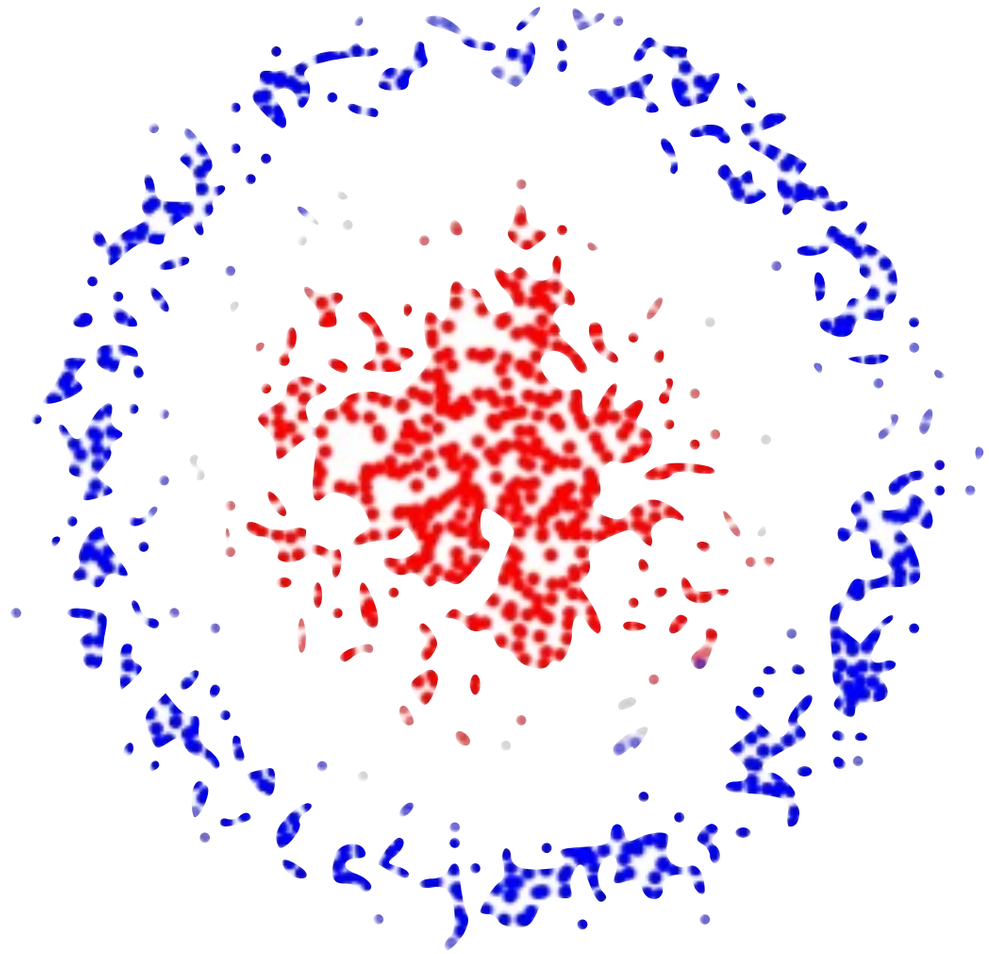
Charu C. Aggarwal, Jiawei Han, Jianyong Wang, and Phillip S. Yu. 2003. A Framework for Clustering Evolving Data Streams. In: *Proceedings of the 29th International Conference on Very Large Data Bases - Volume 29* (Berlin, Germany) (VLDB '03). VLDB Endowment, Berlin, Germany, 81–92.

- Time - stamped data stream $\langle \mathbf{t_i}, \overrightarrow{\mathbf{x_i}} \rangle$, represented in $\mathbf{d} + 1$ dimensions
- Seed algorithm with $\mathbf{q}$ micro-clusters (K-Means on initial data)
- ONLINE phase. For each new point, either:
  - Update one micro-cluster (point within maximum boundary)
  - Create a new micro-cluster (delete/merge other micro-clusters)
- OFFLINE phase. Determine $\mathbf{k}$ macro-clusters on demand:
  - K-Means on micro-clusters (weighted pseudo-points)
  - Time-horizon queries via pyramidal snapshot mechanism

DBSCAN

Martin Ester and Hans-Peter Kriegel and Jörg Sander and Xiaowei Xu. 1996. **A Density-Based Algorithm for Discovering Clusters in Large Spatial Databases with Noise**. In *KDD'96: Proceedings of the Second International Conference on Knowledge Discovery and Data Mining*. AAAI Press, 226-231. https://doi.org/10.5555/3001460.3001507

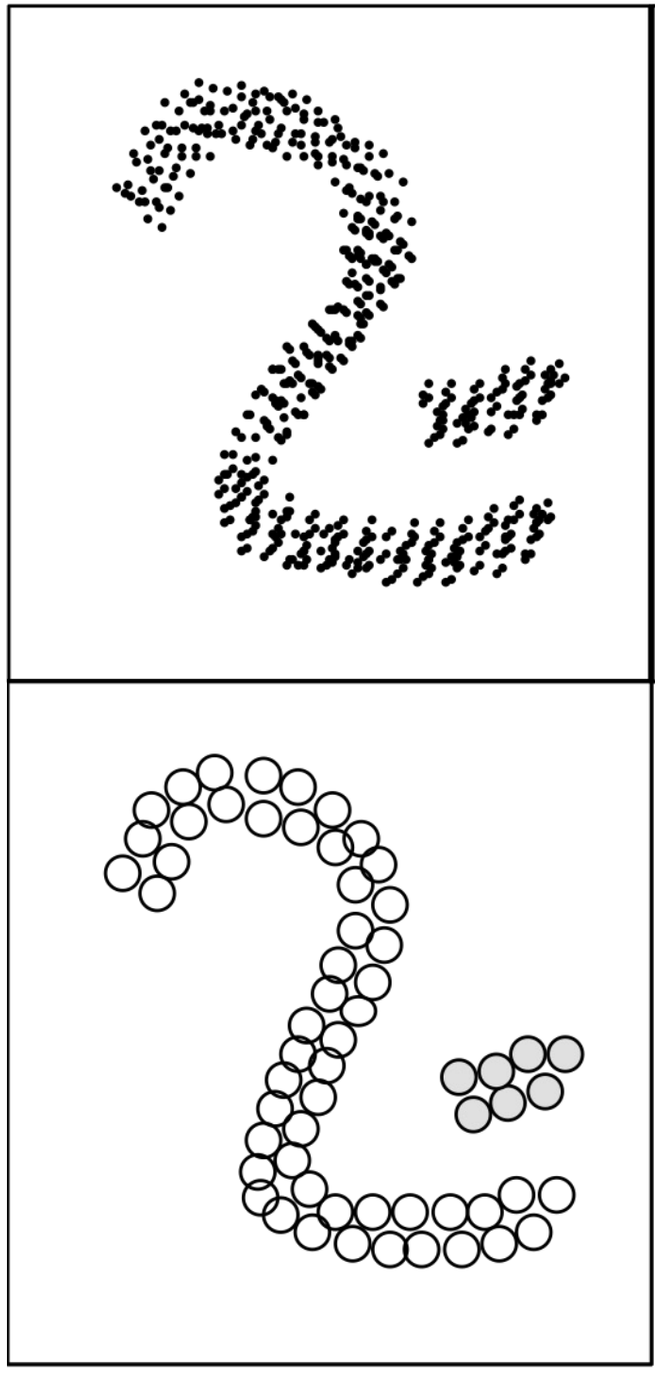
- $\mathbf{N}_{\epsilon}(\mathbf{p})$: set of points at distance $\leq \epsilon$ from $\mathbf{p}$
- Core object $\mathbf{p}$ **iff** $\mathbf{N}_{\epsilon}(\mathbf{p})$ has weight $\geq \mu$
- $\mathbf{p}$ is directly density-reachable from $\mathbf{q}$ **iff** $\mathbf{p} \in \mathbf{N}_{\epsilon}(\mathbf{q})$ **and** $\mathbf{q}$ is a core object
- $\mathbf{p_n}$ is density-reachable from $\mathbf{p_1}$ **iff** there exists chain of points $\mathbf{p_1}, \ldots, \mathbf{p_n}$ such that $\mathbf{p_{i+1}}$ is directly $\mathbf{d} - \mathbf{r}$ from $\mathbf{p_i}$
- **Cluster**: set of points that are mutually density-connected



DENSTREAM

Feng Cao and Martin Ester and Weining Qian and Aoying Zhou. **Density-Based Clustering over an Evolving Data Stream with Noise**. In: *Proceedings of the 2006 SIAM International Conference on Data Mining (SDM)*. Society for Industrial and Applied Mathematics (SIAM), 328-339. https://doi.org/10.1137/1.9781611972764.29

- Based on DBSCAN
- Core-micro-cluster: $\mathrm{CMC}(\omega, \mathrm{c}, \mathrm{r})$ with weight $\omega > \mu$, center c, radius $\mathrm{r} < \epsilon$
- Potential/Outlier micro-clusters
- **Online phase:** merge point into p (or o) micro-cluster if new radius $\mathrm{r}' < \epsilon$
  - Promote outlier to potential if $\omega > \beta\mu$; else, create a new o-micro-cluster
- **Offline phase:** DBSCAN



DENSTREAM

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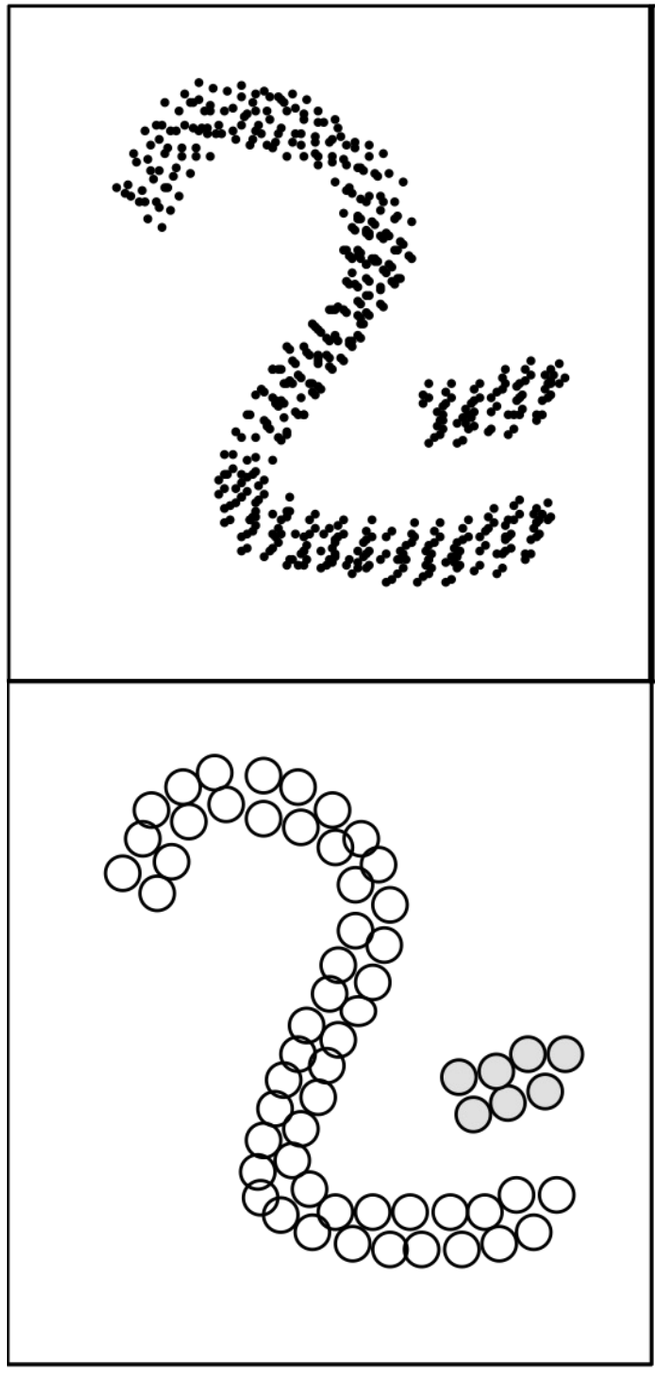
DEN-STREAM(*Stream*, λ, μ, β)

Input: a stream of points, decaying factor λ,
core weight threshold μ, tolerance factor β

▷ Online phase
$T_p \leftarrow \lceil \frac{1}{\lambda} \log(\frac{\beta\mu}{\beta\mu-1}) \rceil$
for each new point that arrives
    do try to merge to a p-microcluster; if not possible,
        merge to nearest o-microcluster
            if o-microcluster weight $> \beta\mu$
                then convert the o-microcluster to p-microcluster
                else create a new o-microcluster

▷ Offline phase
if ($t \bmod T_p = 0$)
    then for each p-microcluster c_p
            do if $w_p < \beta\mu$
                then remove c_p
        for each o-microcluster c_o
            do if $w_o < (2^{-\lambda(t-t_o+T_p)} - 1)/(2^{-\lambda T_p} - 1)$
                then remove c_o
        apply DBSCAN using microclusters as points



CLUSTREE

Philipp Kranen, Ira Assent, Corinna Baldauf, and Thomas Seidl. 2010. **The ClusTree: indexing micro-clusters for anytime stream mining**. *Knowledge and Information Systems* 29, 2 (2011). Springer, 249-272. https://doi.org/10.1007/s10115-010-0342-8

- **ClusTree:** anytime clustering
- Hierarchical data structure: logarithmic insertion complexity
- Buffer and hitchhiker concept: enable anytime clustering
- Exponential decay
- Aggregation: for very fast streams

STREAMKM++

Marcel R. Ackermann, Marcus Märtens, Christoph Raupach, Kamil Swierkot, Christiane Lammersen, and Christian Sohler. 2012. StreamKM++: A clustering algorithm for data streams. *ACM J. Exp. Algorithmics* 17, 1 (2012) 2.1-2.30. https://doi.org/10.1145/2133803.2184450

Coreset of a set P with respect to some problem

Small subset that approximates the original set P.

- Solving the problem for the coreset provides an approximate solution for the problem on P.

(k, ϵ)-coreset

A (k, ϵ)-coreset S of P is a subset of P that for each C of size k

$$(1 - \epsilon)cost(P, C) \leq cost_w(S, C) \leq (1 + \epsilon)cost(P, C)$$

STREAMKM++

Marcel R. Ackermann, Marcus Märtens, Christoph Raupach, Kamil Swierkot, Christiane Lammersen, and Christian Sohler. 2012. StreamKM++: A clustering algorithm for data streams. *ACM J. Exp. Algorithmics* 17, 1 (2012) 2.1-2.30. https://doi.org/10.1145/2133803.2184450

Coreset Tree

- Choose a leaf l node at random
- Choose a new sample point denoted by q_{t+1} from P_l according to d^2
- Based on q_l and q_{t+1}, split P_l into two subclusters and create two child nodes

StreamKM++

- Maintain $L = \lceil \log_2(\frac{n}{m}) + 2 \rceil$ buckets $B_0, B_1, \ldots, B_{L-1}$

DBSTREAM

Michael Hashler and Matthew Bolanos. 2016. **Clustering Data Streams Based on Shared Density between Micro-Clusters**. *IEEE Transactions on Knowledge and Data Engineering* 28, 6 (2016), 1449-1461. https://doi.org/10.1109/TKDE.2016.2522412

- DBSTREAM is the first micro-cluster-based algorithm that explicitly captures the density between micro clusters via a shared density graph.
- Online phase:
  - A new micro cluster is generated from the newly coming point. If one or more micro clusters are found within a fixed radius from it, these micro clusters will be updated. Else, the newly generated one will be added into the list of existing micro clusters.
  - The density graph is then updated. Also, to prevent micro-clusters from collapsing, movement will be restricted.
  - Cleanup process is then initiated after each $\mathbf{t_{gap}}$ interval.

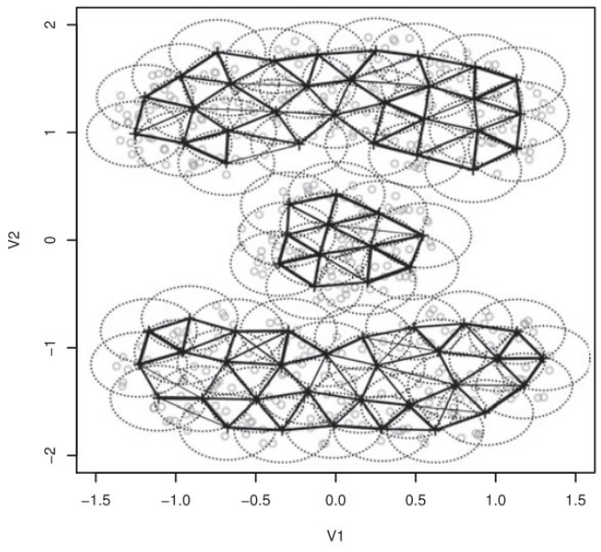
DBSTREAM

Michael Hashler and Matthew Bolanos. 2016. **Clustering Data Streams Based on Shared Density between Micro-Clusters**. *IEEE Transactions on Knowledge and Data Engineering* 28, 6 (2016), 1449-1461. https://doi.org/10.1109/TKDE.2016.2522412

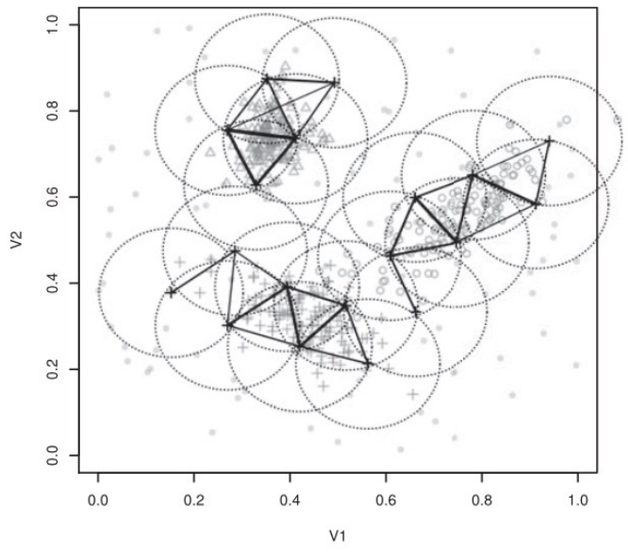
- DBSTREAM is the first micro-cluster-based algorithm that explicitly captures the density between micro clusters via a shared density graph.
- Offline phase:
  - The connectivity graph is constructed using shared density entries between strong micro clusters. The edges in this connectivity graph with a connectivity value greater than the intersection threshold (α) are used to find connected components representing the final cluster.
  - After the connectivity graph is generated, a version of DBSCAN by Ester et al. is applied to form all macro-clusters from α-connected micro clusters.

DBSTREAM

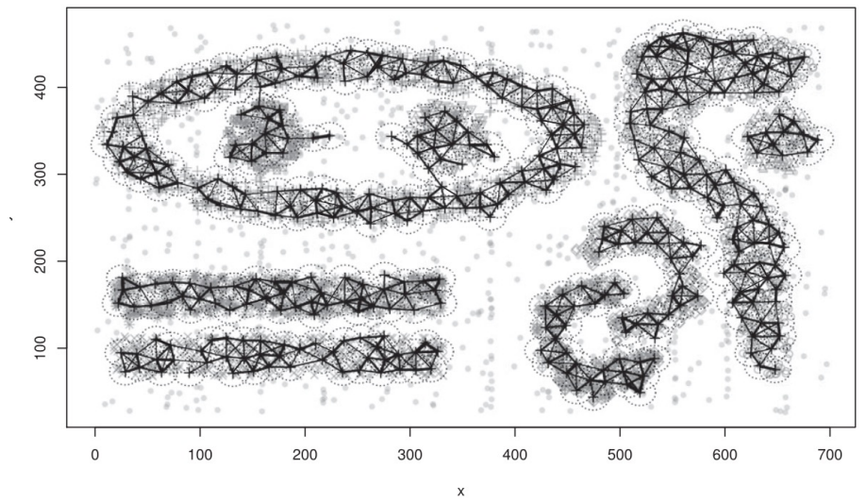
Michael Hashler and Matthew Bolanos. 2016. **Clustering Data Streams Based on Shared Density between Micro-Clusters**. *IEEE Transactions on Knowledge and Data Engineering* 28, 6 (2016), 1449-1461. https://doi.org/10.1109/TKDE.2016.2522412



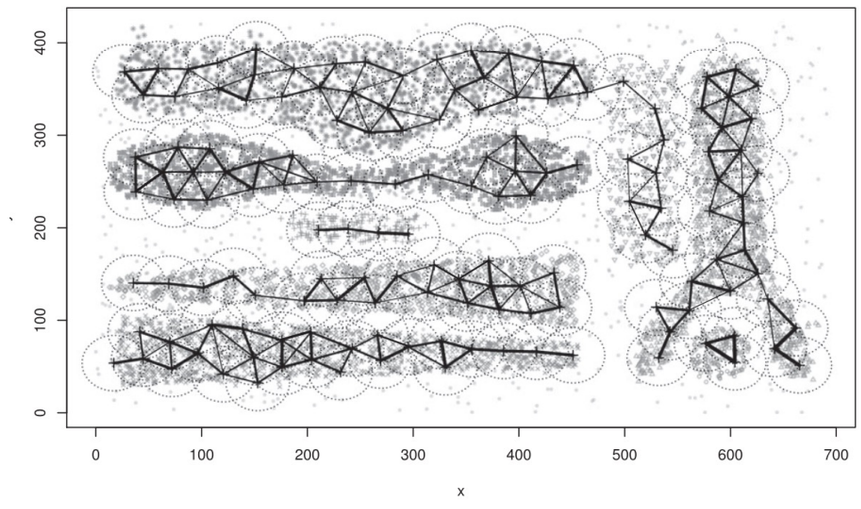
(a) Cassini



(b) Noisy mixture of Gaussians



(c) Chameleon dataset DS3



(d) Chameleon dataset DS4

EVOSTREAM

Matthias Carnein and Heike Trautmann. 2018. **evoStream – Evolutionary Stream Clustering Utilizing Idle Time**. *Big Data Research* 14 (2018), 101-111. https://doi.org/10.1016/j.bdr.2018.05.005

- A fairly new online clustering algorithm.
- evoStream employs an evolutionary algorithm, first introduced by Maulik U. and Bandyopadhyay S. (2000), to utilize “idle” time efficiently to find better macro-cluster solutions.
- In the evolution algorithm, promising solutions are combined to create offsprings which can combine the best attributes of both parents.
- Include two phases: **Micro-cluster maintenance** (online learning phase) and **evolutionary step of micro-cluster generation** (offline phase)

EVOSTREAM

Matthias Carnein and Heike Trautmann. 2018. evoStream – Evolutionary Stream Clustering Utilizing Idle Time. *Big Data Research* 14 (2018), 101-111. https://doi.org/10.1016/j.bdr.2018.05.005

Require: radius r, decay rate λ, cleanup interval t_{gap}, initialization threshold γ, Population size P, number of clusters k
Initialize: $t = 0$, $MC = \emptyset$, $\boldsymbol{C} = \emptyset$

1: **while** stream is active **do**
2: read $\boldsymbol{x}$ from stream
3: $t \leftarrow t + 1$
4: $new \leftarrow (\boldsymbol{x}, t, 1)$ ▷ Temporary micro-cluster
5: **for** $mc \in MC$ **do** ▷ $mc := (\boldsymbol{c}, t, \omega)$
6: **if** DIST$(mc, new) < r$ **then** ▷ Absorb observation
7: $mc[\boldsymbol{c}] \leftarrow mc[\boldsymbol{c}] + h(new[\boldsymbol{c}], mc[\boldsymbol{c}]) \cdot (new[\boldsymbol{c}] - mc[\boldsymbol{c}])$ ▷ h as in Equation (1)
8: $mc[t] \leftarrow t$
9: $mc[\omega] \leftarrow mc[\omega] \cdot 2^{-\lambda(t - mc[t])} + 1$
10: **if** new has not been absorbed by any $mc \in MC$ **then** ▷ Initialize new micro-cluster
11: $MC \leftarrow MC \cup new$
12: **if** $t \mod t_{gap} = 0$ **then** ▷ Periodic adjustments
13: CLEANUP$(\cdot)$ ▷ see Algorithm 2
14: **if** $|MC| = \gamma$ **and not** initialized **then** ▷ Initialize macro-clusters
15: **for** $i \leftarrow 1, \ldots, P$ **do**
16: $C_i \leftarrow k$ randomly chosen micro-cluster
17: **while** idle **do** ▷ Until new example available
18: EVOLUTION$(\cdot)$ ▷ Repeat evolutionary step, see Algorithm 3

---

evoStream algorithm (with both online and offline phase)

EVOSTREAM

Matthias Carnein and Heike Trautmann. 2018. **evoStream – Evolutionary Stream Clustering Utilizing Idle Time**. *Big Data Research* 14 (2018), 101-111. https://doi.org/10.1016/j.bdr.2018.05.005

1: **function** CLEANUP($\cdot$)
2: **for each** $mc \in MC$ **do**
3: $mc[\omega] \leftarrow mc[\omega] \cdot 2^{-\lambda(t-mc[t])}$ ▷ Update weight
4: **if** $mc[\omega] \leq 2^{-\lambda t_{gap}}$ **then**
5: Remove mc from MC ▷ Remove outdated
6: Merge all mc_i, mc_j where DIST$(mc_i, mc_j) \leq r$ ▷ Merge colliding clusters

---

Cleanup phase (after each $\mathbf{t_{gap}}$ time interval)

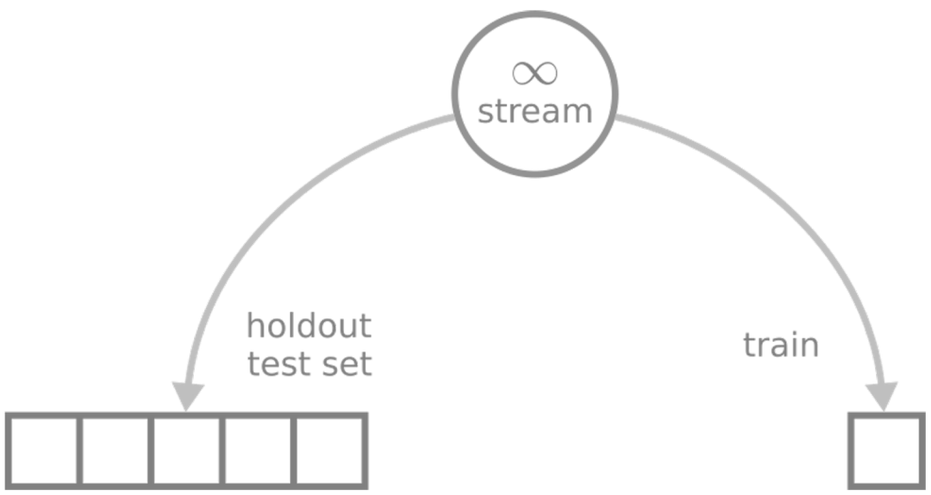
EVOSTREAM

Matthias Carnein and Heike Trautmann. 2018. **evoStream – Evolutionary Stream Clustering Utilizing Idle Time**. *Big Data Research* 14 (2018), 101-111. https://doi.org/10.1016/j.bdr.2018.05.005

1: **function** EVOLUTION($\cdot$)
2: $p_1, p_2 \leftarrow$ Select two solutions proportionally to their fitness from $\boldsymbol{C}$
3: $o_1, o_2 \leftarrow$ Create offsprings of p_1, p_2 using binary crossover
4: **for each** g_i **in** o_1, o_2 **do** ▷ For each child-gene
5: **if** RANDOM(0,1) < P_m **then** ▷ Mutate with probability P_m
6: **if** $g_i = 0$ **then**
7: $g_i \leftarrow 2\delta$
8: **else**
9: $g_i \leftarrow 2\delta \cdot g_i$
10: Add o_1, o_2 to $\boldsymbol{C}$ and discard the two least fittest solutions

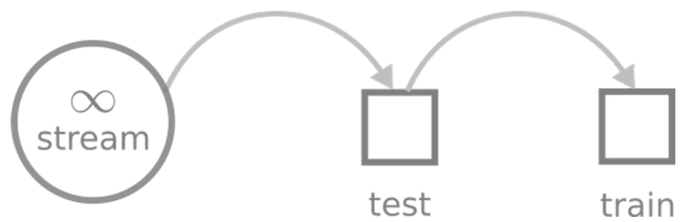
Evolution algorithm (during idle time)

EVALUATION



Holdout an independent test set

- Apply the current model to the test set, at regular time intervals
- *Unbiased* performance estimation
- Popular in *batch* and *stream* learning



Prequential

- Test *then* train each new instance
  - Order matters!
  - All data is used for training
- Performance is estimated on the sequence
- Popular in the *stream* setting

STATIC EVALUATION

4 basic internal (validation) metrics include

- Cohesion: The average distance from a point in the dataset to its assigned cluster centroid. The smaller the better.
- SSQ: The sum of squared distances from data points to their assigned centroids. Closely related to cohesion. The smaller the better.
- Separation: Average distance from a point to the points assigned to other clusters. The larger the better.
- Silhouette coefficient: the ratio between cohesion and the average distances from the points to their second-closest centroid.

STATIC EVALUATION

External validation metrics, requiring ground truth values, is mostly based on the following concepts

- **Accuracy:** Fraction of the points assigned to their "correct" cluster.
- **Recall:** Fraction of the points of a cluster that are in fact assigned to it.
- **Precision:** Fraction of the points assigned to a cluster that truly belong to it.
- **Purity:** In a maximally pure clustering, all points in the cluster belong to the same ground-truth class or cluster. Formally, purity is

$$\frac{1}{N}\sum_{c=1}^{k}(\text{number of points in cluster } c \text{ in the majority class for } c).$$

STATIC EVALUATION

In River, static evaluation metrics can be modified to continuously evaluate data streams as follows:

- External metrics: All external metrics implemented in River share the same confusion matrix, thus reducing the amount of occupied storage and computational time. This confusion matrix can easily be updated once a new predicted label and its ground truth arrive.

- Internal metrics: Since information on previous data points are totally discarded once a new data point arrives, traditional internal metrics cannot be used incrementally. Instead, they are slightly modified by saving the most essential information based on requirement, for example linear sum and sum of squares of data points in the same cluster, distance from data point to assigned and/or second closest cluster centre at the time of update, etc.

STATIC EVALUATION

With **20 internal metrics** and **18 external metrics**, River is currently the package with the highest number of metrics offered for data stream continuous or incremental validation.

- Internal metrics: Cohesion, SSB, SSW, Separation, Silhouette, Ball-Hall, CH, Hartigan, WB, Xie-Beni, Xu, (Root) Mean Squared Standard Deviation, R-Squared, I Index, Davies-Bouldin, Partition Separation, Dunn's indices 43 and 53, SD Validation Index, and Bayesian Information Criterion.

- External metrics: Completeness, Homogeneity, VBeta, (Adjusted, Expected, Normalized) Mutual Information, Q0 and Q2, Fowlkes-Mallows, Markedness, Informedness, Matthews Correlation Coefficient, (Adjusted) Rand Index, Purity, Prevalence Threshold, and Sorensen-Dice index.

STATIC EVALUATION

Every metric (both internal and external) in River contains the following attributes:

- `cm`: Confusion matrix;
- `update` and `revert`: Allow the metric to be updated with a new observation, or reverted to the previous state;
- `get`: Obtain the exact value of the metric;
- `bigger-is-better`: Indicate whether the metric has the property of the bigger, the better the clustering solution is;
- `work_with`: Indicate whether the metrics work with algorithms of which type (clustering, classification, regression, etc.);

STREAMING EVALUATION

Hardy Kremer, Philipp Kranen, Timm Jansen, Thomas Seidl, Albert Bifet, Geoff Holmes, Bernhard Pfahringer. 2011. **An effective evaluation measure for clustering on evolving data streams**. In: *KDD '11 Proceedings of the 17th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*. Association for Computing Machinery, New York, NY, USA, 868-876. https://doi.org/10.1145/2020408.2020555

- Clusters may: appear, fade, move, merge
  - Missed points (unassigned)
  - Misplaced points (assigned to different cluster)
  - Noise
- Cluster Mapping Measure CMM
  - External (ground truth)
  - Normalized sum of penalties of these errors

Thank you for your attention!

ONLINE CLUSTERING: ALGORITHMS, EVALUATION, METRICS, APPLICATIONS AND BENCHMARKING

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