

Sensors and Sensing

Introduction

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Outline

- 1 Recap on Odometry
- 2 Inertial Measurements
- 3 Absolute Position Measurement

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1 Recap on Odometry

2 Inertial Measurements

3 Absolute Position Measurement

How do we implement an odometry controller?

- Two tasks:
 - Convert from encoder ticks to position and orientation of the vehicle.
 - Convert a velocity command (linear and angular) into velocity command on the wheels.
- Let's try to work out the math and pseudocode of how we do this.

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From ticks to position and orientation

Given:

- t_s : sampling time at which we are computing odometry
- $p_{l,t}, p_{r,t}$: positions of the left and right encoders at time t .
- $v_{l,t}, v_{r,t}$: velocity of the left and right encoders at time t .
- $u_{r,t}, u_{l,t}$: commanded velocity to the encoders at time t .

Note that u should be sampled at a substantially higher frequency.

We also have the equations from the lecture. Note that I have substituted u with v here.

$$\dot{x} = \frac{r}{2}(v_{l,t} + v_{r,t}) \cos \theta \quad (1)$$

$$\dot{y} = \frac{r}{2}(v_{l,t} + v_{r,t}) \sin \theta \quad (2)$$

$$\dot{\theta} = \frac{r}{L}(v_{r,t} - v_{l,t}) \quad (3)$$

From ticks to position and orientation

- Here the coordinate frame is fixed to the robot frame at time $t = 0$.
- In the figure, the x axis points right, the y axis points to the front of the robot, and the current $\theta = 90^\circ$.
- These equations work for computing instantaneous velocities, if we assume the robot always starts at $(0, 0, 0)$.

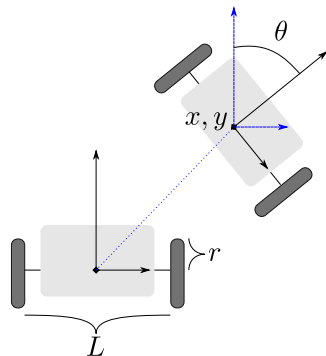


Figure: Differential Drive Kinematic Model

From ticks to position and orientation

Let's calculate the robot motion at time $t + t_s$, relative to its position and orientation at time t . We integrate:

$$dx = t_s \frac{r}{2} (v_{l,t} + v_{r,t}) \cos d\theta \quad (4)$$

$$dy = t_s \frac{r}{2} (v_{l,t} + v_{r,t}) \sin d\theta \quad (5)$$

$$d\theta = t_s \frac{r}{L} (v_{r,t} - v_{l,t}) \quad (6)$$

As we are integrating over t we can re-write as:

$$dx = \frac{r}{2} ((p_{l,t+t_s} - p_{l,t}) + (p_{r,t+t_s} - p_{r,t})) \cos d\theta \quad (7)$$

$$dy = \frac{r}{2} ((p_{l,t+t_s} - p_{l,t}) + (p_{r,t+t_s} - p_{r,t})) \sin d\theta \quad (8)$$

$$d\theta = \frac{r}{L} ((p_{r,t+t_s} - p_{r,t}) - (p_{l,t+t_s} - p_{l,t})) \quad (9)$$

From ticks to position and orientation

We can now write the formula for the relative transformation ${}_nT^{n+1}$ which can be applied to the previous robot pose to obtain the next pose:

$${}_nT^{n+1} = \begin{pmatrix} \cos d\theta & -\sin d\theta & dx \\ \sin d\theta & \cos d\theta & dy \\ 0 & 0 & 1 \end{pmatrix} \quad (10)$$

We obtain the new pose of the robot as ${}_0T^{n+1} = {}_0T^n {}_nT^{n+1}$. How do we calculate that?

Pseudocode

At time t_s :

- Read $p_{r,t+t_s}, p_{l,t+t_s}$
- Compute $d\theta, dx, dy$ according to forward kinematics.
- Rotate dx, dy using θ and add them to x and y .
- Add $d\theta$ to θ .

What about the inverse problem?

Given $\dot{x}, \dot{y}, \dot{\theta}$, find u_l, u_r .

Formulate the linear velocity as the norm of the vector

$$\mathbf{v} = [\dot{x}, \dot{y}]^T.$$

$$v = \sqrt{\left(\frac{r}{2}(v_{l,t} + v_{r,t}) \cos \theta\right)^2 + \left(\frac{r}{2}(v_{l,t} + v_{r,t}) \sin \theta\right)^2} \quad (11)$$

Which simplifies to:

$$v = \frac{r}{2}(v_{l,t} + v_{r,t}) \quad (12)$$

Combining with $\dot{\theta}$ we now have two linear equations with two unknowns, which we can easily solve.

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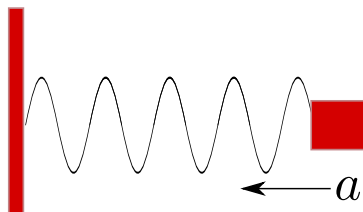
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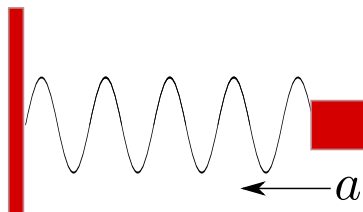
Linear Acceleration: accelerometers

- Accelerometers are sensors that can detect the relative linear acceleration along an axis.
- The basic principle of operation can be thought of as an object suspended on a spring.
- When the system is accelerated along the spring direction, the mass moves, relative to the spring mounting point.



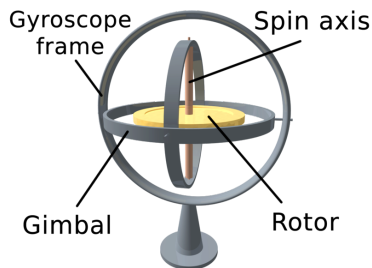
Linear Acceleration: accelerometers

- The displacement of the object is proportional to the spring constant, its mass, and the acceleration.
- Only measure acceleration component along one dimension.
- Often implemented as MEMS.



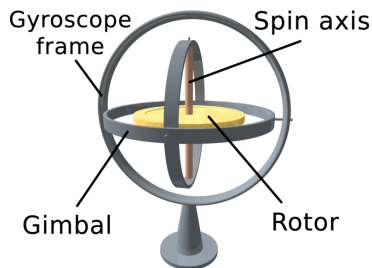
Angular Acceleration: gyros

- Gyroscope sensors rely on the gyroscopic effect to measure angular acceleration.
- The flywheel gyro uses a spinning disc, suspended on a mobile ring.
- When a torque is applied to the input axis, the angular momentum of the wheel transfers the torque to the output axis.
- A sensor on the output axis measures the angular acceleration along the input axis.



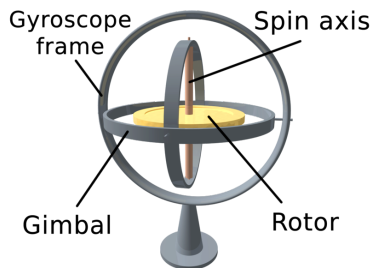
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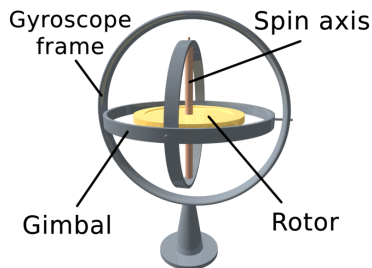
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Physical Implementation

- Both accelerometers and gyroscopes are usually implemented using cheap on-chip systems.
- MEMS implementations, based on vibrations.
- Each sensor only measures along a single axis: linear or rotational.
- If a torque/force is applied to the system, we only see the projection along that axis.

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Integral Measurements and Errors

- Accelerometers and gyros provide instantaneous measurements of linear and angular acceleration.
- We are often interested not in acceleration, but rather speed, or more often position (linear/angular).
- To obtain linear/angular velocity from acceleration, we need to integrate measurements over a time window.
- To obtain position/orientation, we need to integrate linear/angular velocities over time again.
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- Thus, a lot of drift over time. Reliable only over short intervals.

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Magnetic Compass

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- A compass aligns with the magnetic field of earth and measures absolute orientation in the XY-plane (relative to magnetic north).
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Compass error modes

- Compasses are very sensitive to fluctuations in the magnetic field.
- Earth's magnetic field is not perfectly uniform.
- Electronic equipment induces local magnetic fields.
- Metallic and ferro-magnetic objects distort the field.
- Problems with shielding.



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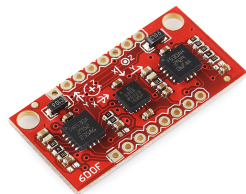
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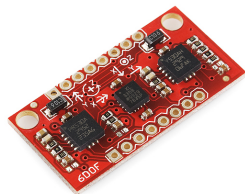
IMUs

- Inertial Measurement Units (IMUs) usually integrate several inertail sensors on a single board.
- Usually 3 accelerometers and 3 gyros for 6DoF pose, may include compass.
- High-end IMUs use redundant additional sensors and perform additional filtering operations to increase reliability.
- Calibration procedures to reduce sensor drift. E.g. common to measure for a time window without moving to remove systematic background noise.



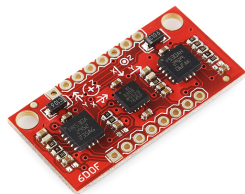
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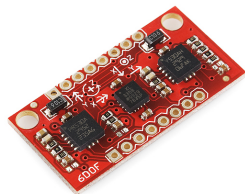
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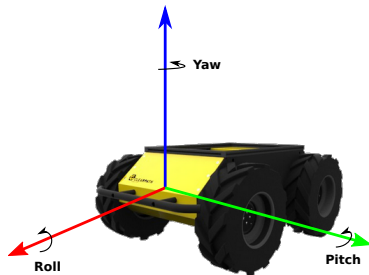
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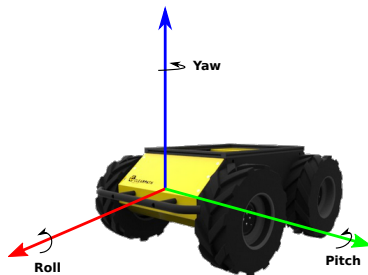
Representing orientations

- There are several alternative ways to represent the relative orientation between two coordinate frames.
- From the sensor point of view, a Gyro / IMU sensor usually measures rotations relative to a fixed sensor-centric frame.
- The Roll, Pitch and Yaw (RPY) of a vehicle are common concepts.



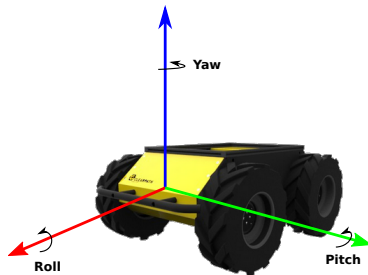
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Representing orientations

- RPY is an Euler angles parameterization of orientation.
- Transferring from Euler angles to rotation matrices can be done by stacking the rotation matrices along each axis.
- Euler angles however suffer from singularities.
- In mechanical gyros this is known also as gimbal lock : two of the gimbals are aligned in the same plane, losing a degree of freedom.
- Depending on Euler angles chosen (XYZ, YXZ, etc.) singularities are at different orientations
- An alternative is to use unit quaternions.

$$q = ix + jy + kz + w \quad (13)$$

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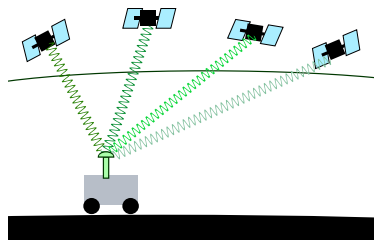
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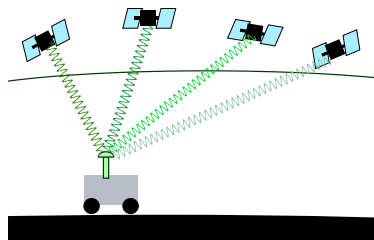
Global Positioning System (GPS)

- Satellite-based localization is often used for outdoor robotic platforms, as well as on ships/airplanes/etc.
- Systems like GPS/ GLONASS / GALILEO operate a fleet of satellites in lower earth orbit.
- GPS receivers measure the signal from visible satellites and use it to deduce an absolute 3D position in geo-reference frame.



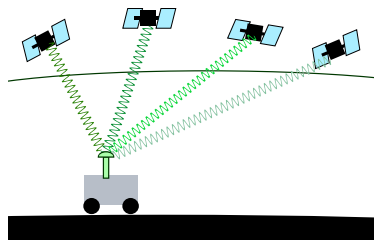
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GPS operation principle

- How does it work exactly?
- Each satellite sends a pseudo-random sequence of bits, encoded on top of a carrier signal.
- The code transmitted is related to the internal clock of the satellite.
- Receivers generate the same sequence based on their local clocks. By aligning the two codes, the receiver can measure the time of travel of the signal.
- The time of travel of the signal t_t is:

$$t_t = t_r - t_s + t_{\text{off}}$$

where t_r is the measured time of receiving the signal, t_s is the measured time of sending and t_{off} is an unknown offset between the receiver and sender clocks.

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GPS operation principle

- The clocks of all satellites are synchronized by the central operations point, correcting for relativistic effects.
- Given two satellite signal travel times t_{t1} and t_{t2} , we can compute

$$t_{t1} - t_{t2} = t_{r1} - t_{s1} + t_{r2} - t_{s2}$$

- this eliminates the clock offset t_{off} .
- With at least 4 satellites, we can compute 3 time differentials and triangulate the position of the receiver.
- Using more than 4 satellites allows for a least-squares solution to minimize the error in position.

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- The clocks of all satellites are synchronized by the central operations point, correcting for relativistic effects.
- Given two satellite signal travel times t_{t1} and t_{t2} , we can compute

$$t_{t1} - t_{t2} = t_{r1} - t_{s1} + t_{r2} - t_{s2}$$

- this eliminates the clock offset t_{off} .
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GPS Errors

GPS systems have several common issues:

- Clouds and stratospheric effects can alter significantly the measurements as they cause refractions.
- Multi-path reflections can cause sporradic jumps in the position estimate.
- The height estimate is usually substaantially more unreliable than the xy-position.
- Low visibility of sattelites causes issues in proximity to tall buildings and of course indoors.
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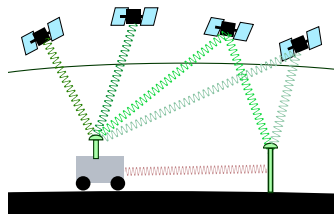
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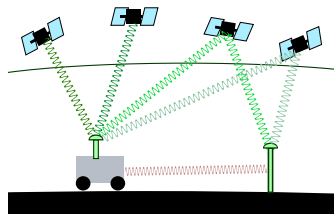
Differential GPS

- Differential GPS relies on additional ground-based stations.
- Each ground station observes the same satellites as the receivers.
- Positions of ground stations are precisely known.
- Ground stations compute a differential between measured and known position and transmit corrections.
- Different ways to correct signals. Usually D-GPS refers to code-space correction.



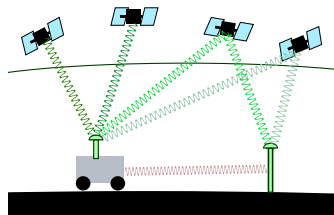
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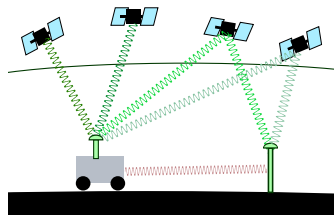
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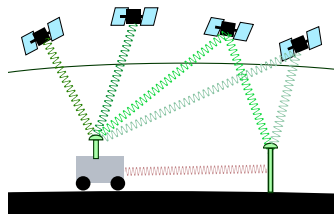
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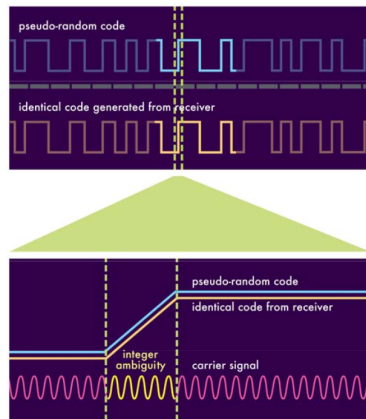
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Real-time Kinematic (RTK) GPS

- RTK-GPS is a form of differential GPS.
- RTK receivers and base stations use the carrier signal, instead of the code.
- Carrier signals are modulated at $\sim 15\text{MHz}$, code signal is modulated at $\sim 1\text{MHz}$.
- Precise alignment of carrier signals gives a more accurate estimate of the travel time t_t .
- Accuracy in centimeter range.



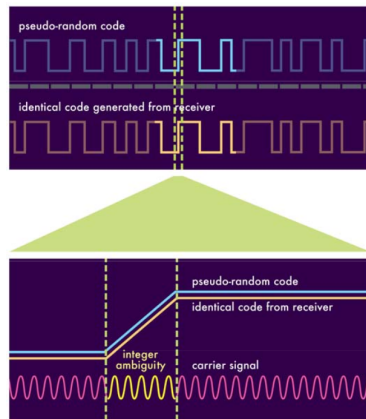
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¹https://extension.usu.edu/nasa/files/uploads/GTK-tuts/RTK_DGPS.pdf

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Real-time Kinematic (RTK) GPS

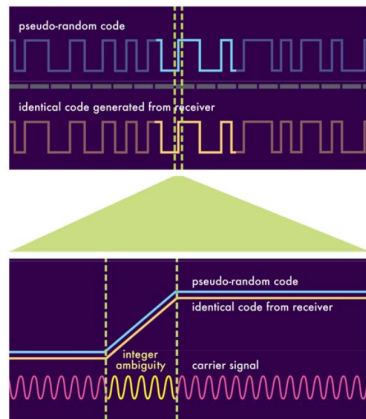
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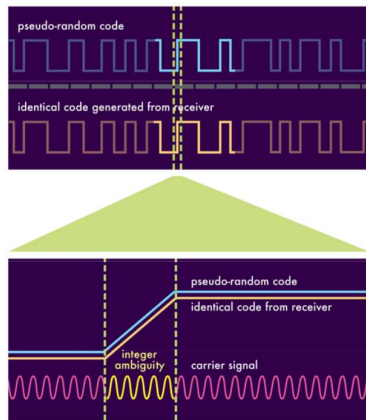
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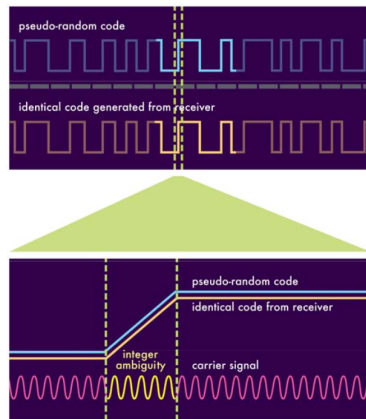
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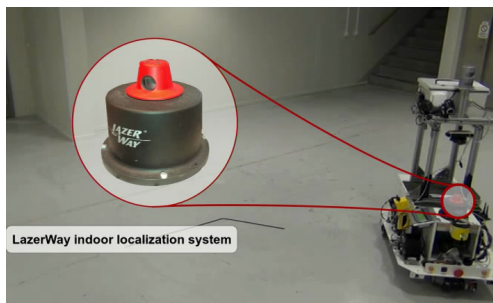
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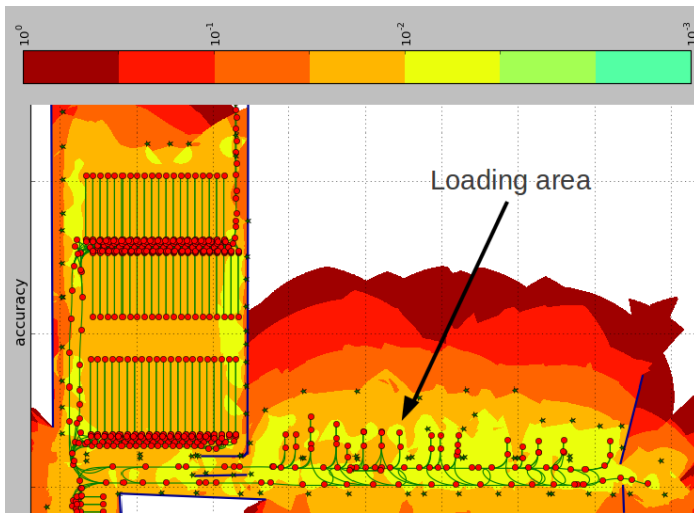
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Indoor Positioning Systems

- Indoor global positioning systems use a set of landmarks, distributed in the environment.
- Landmark observations are fused in a filter.

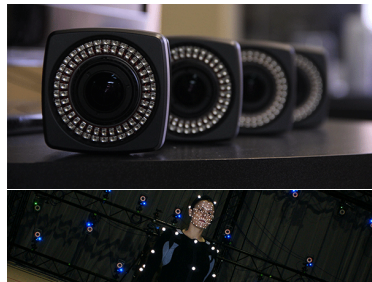


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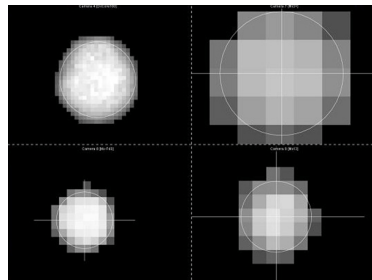
Motion capture systems

- Motion capture systems are sometimes used for tracking in robotics as well.
- Targeted application is in filming: capture motion for a digital actor.
- A number of active infrared cameras and markers.



Motion capture systems

- Markers are triangulated to obtain 3D position.
- Distribution of markers on the model is important to guarantee correct data association
- So is camera resolution and good calibration.
- In robotics, often used for ground truth motion measurement.



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Sensors and Sensing

Introduction

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