Binary Relations: A binary relation R from a set A to a set B is a subset R ⊆ A x B.

Binary Relation on a Set: A binary relation R on a set A is a subset of A x A or a relation from A to A.

Reflexive Relations: R is reflexive iff(a,a) ∊ R for every element a ∊ A. Written symbolically, R is reflexive if and only if ∀x[x∊U⟶ (x,x) ∊ R].

Symmetric Relations: R is symmetric iff (b,a) ∊ R whenever (a,b) ∊ R for all a,b ∊ A. Written symbolically, R is symmetric if and only if ∀x∀y[(x,y) ∊R⟶ (y,x) ∊ R]

Antisymmetric Relations: A relation R on a set A such that for all a,b ∊ A if (a,b) ∊ R and (b,a) ∊ R, then a = b is called antisymmetric. [ Written symbolically, ∀x∀y[(x,y) ∊R∧ (y,x) ∊ R ⟶ x= y]

Transitive Relations: A relation R on a set A is called transitive if whenever (a,b) ∊ R and (b,c) ∊ R, then (a,c) ∊ R, for all a,b,c ∊ A. Written symbolically, R is transitive if and only if ∀x∀y∀z[(x,y) ∊R∧ (y,z) ∊ R ⟶ (x,z) ∊ R]

Composition: Suppose [ 1) R1 is a relation from a set A to a set B [ 2) R2 is a relation from B to a set C [ Then the composition (or composite) of R2 with R1, is a relation from A to C where if (x,y) is a member of R1 and (y,z) is a member of R2, then (x,z) is a member of R2 ∘ R1.

Powers of a Relation: Let R be a binary relation on A. Then the powers R^n of the relation R can be defined inductively by: [ 1) Basis Step: R^1 = R [ 2) Inductive Step: R^(n+1) = R^n ∘ R

Digraphs: A directed graph, or digraph, consists of a set V of vertices (or nodes) together with a set E of ordered pairs of elements of V called edges (or arcs). The vertex a is called the initial vertex of the edge (a,b), and the vertex b is called the terminal vertex of this edge. An edge of the form (a,a) is called a loop.

Reflexivity: A loop must be present at all vertices in the graph.

Symmetry: If (x,y) is an edge, then so is (y,x).

Antisymmetry: If (x,y) with x ≠ y is an edge, then (y,x) is not an edge.

Transitivity: If (x,y) and (y,z) are edges, then so is (x,z).

Equivalence Relations: A relation on a set A is called an equivalence relation if it is reflexive, symmetric, and transitive. Two elements a, and b that are related by an equivalence relation are called equivalent. The notation a~b is often used to denote that a and b are equivalent elements with respect to a particular equivalence relation.

Equivalence Classes: Let R be an equivalence relation on a set A. The set of all elements that are related to an element a of A is called the equivalence class of a. The equivalence class of a with respect to R is denoted by [a]R.

Partition of a Set: A partition of a set S is a collection of disjoint nonempty subsets of S that have S as their union. In other words, the collection of subsets Ai, where i ∈ I (where I is an index set), forms a partition of S if and only if [ Ai ≠ ∅ for i ∈ I, [ Ai ∩ Aj = ∅ when i ≠ j