Implementation of VaR Estimation Approaches using Python

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In the financial industry, Value-at-risk (VaR) is a new standard measurement of risk. VaR offers asset managers a better understanding of risk comparing to the simple standard deviation measurement. It tackles the weakness of standard deviations by considering only the downside risk rather than both tails of the distribution. Actual VaR calculation is straight-forward, it is the minimum loss that could occur if an unlikely event happens. However, to effectively manage risk, one should estimate the future VaR rather than using the historical VaR. The three common methods to estimate VaR are Non-Parametric, Parametric (Historical Simulation) and the Monte Carlo Simulation. This research paper introduces a new approach to measure VaR using the replicated portfolio. The replicated portfolio is built using parameters from the regression of the portfolio returns and the performance of 11 equity sectors. The estimated VaRs are then compared to the actual VaR. All four approaches usually yield smaller values than the actual VaR. The parametric and the Monte Carlo Simulation methods yield close estimations. The decomposition method fails to capture all the risks of the actual portfolio; hence, it also fails to deliver a fair estimation. While other methods are only sensitive to the parameters of the distribution (mean and standard deviations), the decomposition method can detect both changes in the shape of the distribution and the parameters.

Keyword: Value-at-risk, Sector Investing, Linear Regression.

Introduction

In the financial industry, there are many types of risks, this paper focuses only on the market risk. Since the market risk relates to the risk of the overall performance of the financial market, one can only manage rather than control it. Value-at-risk (VaR) was introduced in 1996 by J.P Morgan and Reuters to develop a standard measurement of risk. Since then, VaR has become an industry standard for risk measurement; it is widely used by regulatory authorities, businesses and academic institutions. For example: the SEC requires banks to report VaR daily to monitor the minimum capital requirement (Basel II), Campbell et.al used VaR constraints to maximize the portfolio selections (Rachel Campbell, 2000). The direct measure of VaR is straight-forward, where it requires the distribution of the asset returns. However, the two main challenges are: actual distribution of the asset returns often have fatter tails than the normal distribution (a usual assumption when modelling VaR), and the actual distribution changes over-time. In this paper, we will test 4 different approaches to calculate value at risk: Non-parametric, parametric, simulations and decomposition (a new approach). While the first three approaches are widely used, the last method has not been addressed in any article.

Literature Review

There are 3 popular methods to estimate VaR: parametric (Historical Simulation), non-parametric, and Monte-Carlo Simulation. This paper also introduces a new approach to measure VaR – decomposition method.

1. Parametric VaR

The earliest version is parametric VaR. One of the famous parametric VaR method is the RiskMetrics which was introduced by JP.Morgan Chase in 1994 (Longerstaey, 1996). Since then, RiskMetrics became the benchmark for measuring risk and providing clients with a way to effectively manage risks. RiskMetrics calculation is based on the Variance-Covariance method under the normality assumption (Investopedia).

Let P_i be the initial value of the portfolio, μ be the mean of the series, σ be the standard deviation of the return series, and $\mathbb{Z}$ be the Z-score related to the confident level under the normality assumption. The Value at Risk of the portfolio's value under the RiskMetrics framework is:

$$VaR = P_i * \alpha$$

Where $\alpha = normal.ppf(1 - confidence)$: the probability density function at (100%-confidence level) of standard normal distribution.

This VaR method was applied widely in both academic fields and businesses. Alexander used Mean-VaR to replace the traditional Mean-Variance of Markowitz to construct the optimal portfolio (Alexander, 2002). The VaR can implement the weakness of variance (similar to standard deviation) as variance considers both sides of the stock movement, while VaR only considers the down-side risks. The variance does not consider the direction of the movement, hence, the optimized portfolio might prevent investors from both gains and losses. On other hand, optimize the portfolio using mean-VaR will only protect investors against the down-side risk (Alexander, 2002).

2. Non- parametric.

Non-parametric VaR is the actual value at risk without the normality assumption. We can calculate VaR directly using left-tail measures (Guojunn Wu, 2002). Given a time-series of assets return: $\{r_t\}_{t=1}^{n}$, VaR is the value such that:

$$Pr(r_t \leq |VAR_t||I_{t-1}) = 1 - c$$

Where I_{t-1} : information available at time t - 1, c: confident level

3. Monte-Carlo Simulation

Monte-Carlo simulation can be applied differently. Monte Carlo simulation can overcome the drawback of parametric approach since it is able to generate different types of distributions without limited to only normal distribution. Using this approach, one will generate a set of future stock prices, then use it to calculate the set of stock return. The VaR is the α quantile from this distribution. In this paper, we will implement Monte-Carlo Simulation using Python and log-normal distribution.

Let S_0 be the initial price of the stocks, n be the sample size, c be the confidence level,$\{S_i\}_{i=1}^{n}$ be the set of future stock paths, $\{R_i\}_{i=1}^{n} = (\{S_i\}_{i=1}^{n} - S_0)/S_0)$ be the set of stock's return

$$\mathbb{P}(X \leq |VaR|) = \alpha = 1 - c$$

$$VAR_\alpha(X) = F_y^{-1}(1 - \alpha)$$

Given the above framework, there are many models can be used to generate the future stock price such as: the Black-Scholes model, the Herton Model. In this paper, we use the NumPy package in Python to generate the log-normal distribution for the future stocks price directly.

4. Decomposition Approach

An alternative approach to analyze VaR using regression analysis. We decompose the portfolio returns to the performance of 11 equity sectors. The replicated portfolio is formed from the corresponding parameters. The VaR of the actual portfolio can be estimated through the replicated portfolio.

Let y is a time-series of returns of a portfolio $y = \{r_t\}_{t=1}^{n}$:

Let $\vec{X} = \{X_i\}_{i=1}^{11}$ are the time-series returns of 11 sectors.

We can decompose the portfolio performances using the multiple regression:

$$y = \sum_{i=1}^{11} \alpha_i X_i + \epsilon$$

The coefficients of the regression are: $\vec{\alpha} = \{\alpha_{i=1}^{11}\}$

The time-series returns of the replicate portfolio $\hat{y} = \vec{X} \bullet \vec{\alpha}$

The VaR for the replicate portfolio can be estimated using parameters: $\mu_{\hat{y}}$ and $\sigma_{\hat{y}}$ given timeframe and level of confidence: c. We can generate the random numbers that follow log-normal distribution using Monte Carlo Simulation. The VaR is estimated such that:

$$\mathbb{P}(X \leq |VaR|) = \alpha = 1 - c$$

$$VAR_\alpha(X) = F_y^{-1}(1 - \alpha)$$

Data

A. Sector

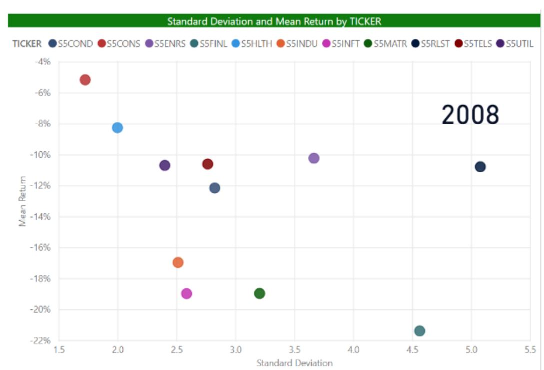
We use the returns of 11 equity sectors from 2002 to 2018. Data is provided by the Bloomberg Terminal in the Lab at The University of Tulsa. Below are the descriptive statistics of 11 equity sectors.

YEAR	2002		2003		2004		2005		2006		2007		From 2002-2007 Average	
TICKER	Mean Return	Standard Deviation	Mean Return	Standard Deviation	Mean Return	Standard Deviation	Mean Return	Standard Deviation	Mean Return	Standard Deviation	Mean Return	Standard Deviation	**Mean Return**	**Standard Deviation**
S5COND	-9.20%	1.79	13.44%	1.29	5.26%	0.81	-2.28%	0.81	7.09%	0.74	-5.09%	1.05	**1.54%**	**1.08**
S5CONS	-1.01%	1.16	4.69%	0.83	3.31%	0.63	1.56%	0.57	5.48%	0.52	5.55%	0.72	**3.26%**	**0.74**
S5ENRS	-3.06%	1.81	9.55%	0.99	11.43%	1.04	11.97%	1.51	9.64%	1.41	13.10%	1.44	**8.77%**	**1.37**
S5FINL	-4.43%	1.93	11.47%	1.22	4.40%	0.77	2.75%	0.72	7.25%	0.72	-7.12%	1.48	**2.39%**	**1.14**
S5HLTH	-6.95%	1.62	6.16%	1.09	0.99%	0.81	2.71%	0.67	3.10%	0.64	3.05%	0.77	**1.51%**	**0.93**
S5INDU	-10.48%	1.82	11.70%	1.11	6.89%	0.80	1.17%	0.73	5.23%	0.74	5.03%	1.01	**3.26%**	**1.03**
S5INFT	-14.64%	2.82	16.76%	1.68	1.71%	1.19	0.73%	0.82	3.68%	0.96	6.67%	1.14	**2.48%**	**1.43**
S5MATR	-0.51%	1.86	13.55%	1.19	5.48%	1.06	2.19%	0.99	7.25%	1.08	9.03%	1.40	**6.17%**	**1.26**
S5RLST	-5.84%	1.15	7.92%	0.92	8.50%	1.14	3.40%	1.08	12.97%	0.99	-7.56%	1.77	**3.23%**	**1.17**
S5TELS	-13.56%	2.44	4.05%	1.70	7.59%	0.90	-2.03%	0.74	12.84%	0.83	5.11%	1.11	**2.33%**	**1.28**
S5UTIL	-12.00%	2.07	9.76%	1.02	8.89%	0.72	6.59%	0.91	7.85%	0.71	7.72%	1.15	**4.80%**	**1.10**
Average	**-7.43%**	**1.86**	**9.91%**	**1.18**	**5.86%**	**0.90**	**2.61%**	**0.87**	**7.49%**	**0.85**	**3.23%**	**1.19**	**3.61%**	**1.14**

Figure 1: Return & Standard Deviation (2002 - 2007) for 11 Sectors - Table View

From 2002 to 2007, the average return for 11 sectors is 3.64% and the average standard deviation is 1.14. The return and standard deviation are closed to that of the fund performance during the same period which suggest that our proposal to create a replicate portfolio from sectors is rational.

YEAR	2008	
TICKER	Mean Return	Standard Deviation
S5COND	-12.14%	2.82
S5CONS	-5.15%	1.72
S5ENRS	-10.22%	3.66
S5FINL	-21.38%	4.56
S5HLTH	-8.25%	2.00
S5INDU	-16.95%	2.51
S5INFT	-18.97%	2.58
S5MATR	-18.95%	3.20
S5RLST	-10.77%	5.08
S5TELS	-10.59%	2.76
S5UTIL	-10.68%	2.40
Average	**-13.09%**	**3.03**



In 2008, we observed negative return and high standard deviations.

Figure 2: Return & Standard Deviation 2008 for 11 Sectors - Table View

During financial crisis, all sectors yields negative returns and high standard deviations. Financial sector is the most affected. It is the worse sector to invest during 2008, as it lay in the bottom right corner in the scatter chart above. Interestingly, the real estate sector has highest standard deviations however it not severely damaged as: Information Technology, Material, Industrial, Consumer Discretionary. Since the Real Estate industry are less connected than the Financial Industry, therefore only certain area is affected.

YEAR	2009		2010		2011		2012		2013		2014		2015		2016		2017		2018		From 2009-2018	
TICKER	Mean Return	Standard Deviation	Mean Return	Standard Deviation	Mean Return	Standard Deviation	Mean Return	Standard Deviation	Mean Return	Standard Deviation	Mean Return	Standard Deviation	Mean Return	Standard Deviation	Mean Return	Standard Deviation	Mean Return	Standard Deviation	Mean Return	Standard Deviation	**Mean Return**	**Standard Deviation**
S5COND	15.71%	2.00	10.51%	1.28	3.52%	1.52	8.97%	0.88	14.54%	0.79	4.01%	0.83	4.35%	1.03	2.73%	0.90	8.37%	0.50	1.14%	1.28	**7.38%**	**1.10**
S5CONS	6.06%	1.05	5.52%	0.75	5.62%	0.91	4.26%	0.58	9.47%	0.71	6.07%	0.60	2.91%	0.87	2.35%	0.74	5.16%	0.48	-3.07%	0.91	**4.43%**	**0.76**
S5ENRS	7.23%	2.05	8.38%	1.42	3.67%	1.92	2.40%	1.10	9.24%	0.86	-2.58%	1.13	-8.22%	1.54	10.71%	1.49	-0.06%	0.83	-6.95%	1.41	**2.38%**	**1.37**
S5FINL	15.17%	4.24	5.87%	1.63	-5.10%	2.15	10.75%	1.13	12.50%	0.91	5.94%	0.81	-0.01%	1.11	8.84%	1.18	8.30%	0.81	-4.81%	1.23	**5.74%**	**1.52**
S5HLTH	7.95%	1.27	1.55%	0.92	5.52%	1.24	6.82%	0.69	14.06%	0.76	9.37%	0.90	3.29%	1.14	-0.62%	0.96	8.10%	0.54	3.11%	1.10	**5.91%**	**0.95**
S5INDU	9.71%	2.08	10.35%	1.37	1.19%	1.69	6.16%	0.95	13.86%	0.80	4.07%	0.85	-0.53%	1.00	7.27%	0.91	7.77%	0.57	-4.98%	1.19	**5.49%**	**1.14**
S5INFT	20.63%	1.76	4.62%	1.24	2.09%	1.51	6.05%	1.02	10.24%	0.78	7.64%	0.85	2.95%	1.16	5.69%	1.04	13.33%	0.70	1.01%	1.50	**7.42%**	**1.16**
S5MATR	18.07%	2.16	9.12%	1.52	-2.24%	1.91	6.20%	1.11	9.45%	0.90	3.05%	0.89	-2.79%	1.17	6.76%	1.12	8.74%	0.66	-5.61%	1.20	**5.08%**	**1.27**
S5RLST	17.39%	4.47	11.42%	1.80	4.75%	1.86	6.40%	0.88	-0.11%	1.00	9.44%	0.66	1.07%	1.07	1.00%	1.06	4.28%	0.60	-0.37%	1.02	**5.53%**	**1.44**
S5TELS	4.60%	1.55	7.28%	0.88	3.05%	1.13	7.03%	0.78	4.69%	0.87	1.51%	0.82	1.76%	0.93	8.78%	0.90	-0.06%	0.95	-4.46%	1.32	**3.42%**	**1.01**
S5UTIL	5.36%	1.34	2.56%	0.94	7.76%	1.05	0.71%	0.62	5.23%	0.78	10.47%	0.86	-1.38%	1.08	6.48%	0.99	4.75%	0.62	2.06%	0.95	**4.40%**	**0.92**
Average	**11.62%**	**2.18**	**7.02%**	**1.25**	**2.71%**	**1.53**	**5.98%**	**0.89**	**9.38%**	**0.83**	**5.36%**	**0.84**	**0.31%**	**1.10**	**5.45%**	**1.03**	**6.24%**	**0.66**	**-2.08%**	**1.19**	**5.20%**	**1.15**

Figure 3: Return & Standard Deviation (2009-2018) for 11 Sectors - Table View

From 2009 – 2018, sectors yield higher Sharpe Ratio. The average return is 5.20% while the average standard deviations is 1.15.

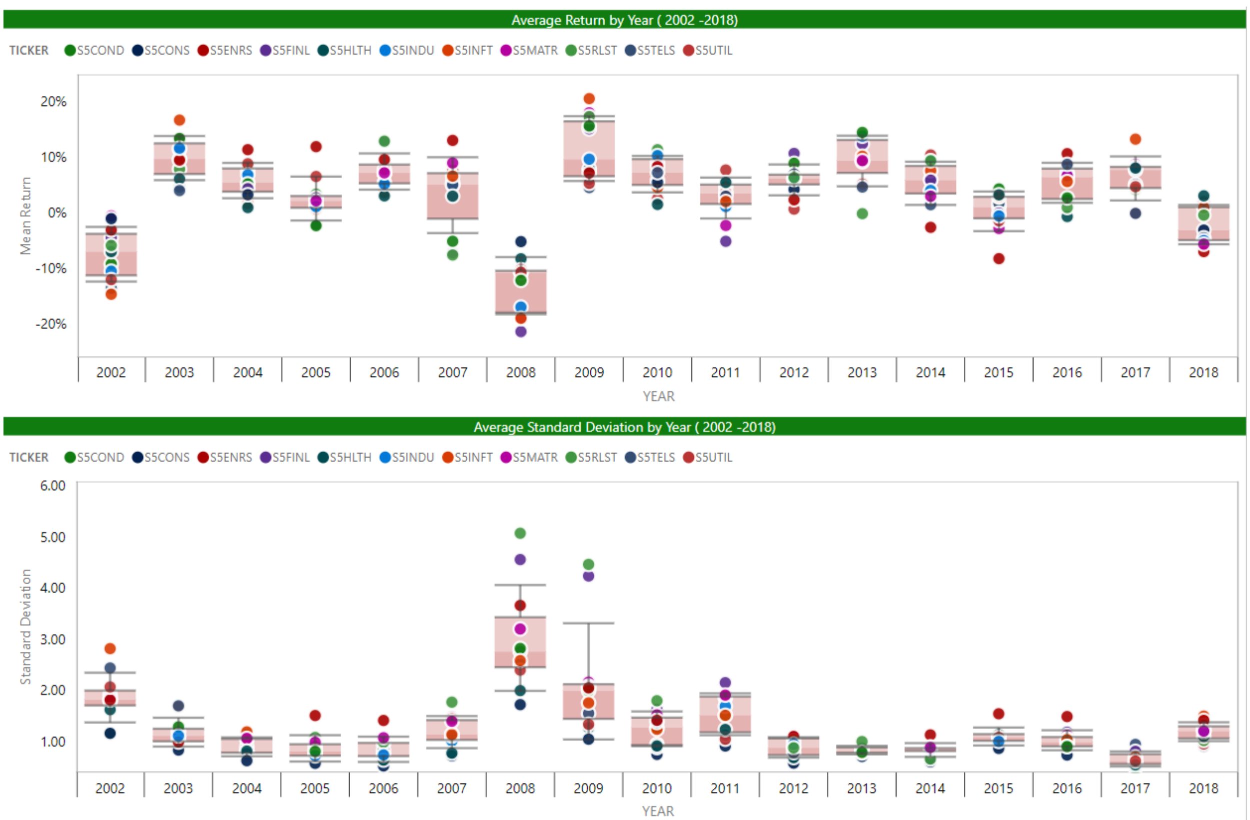


Figure 4: Return & Standard Deviation by Years (2002 - 2018) - Box Plot Type I

During 2002 – 2007: Energy Sector usually outperformed the market, where it yields higher than average (higher than 1 standard deviation from the mean of the sectors return.)

During financial crisis (2008 -2009): Financial and Information Sectors yielded worst return. Financial and Real Estate were the most volatile sectors. Both sectors recovered quickly in 2009.

During 2009 -2018: Information Technology and Real Estates often outperformed, while energy sector usually was the worst sector.

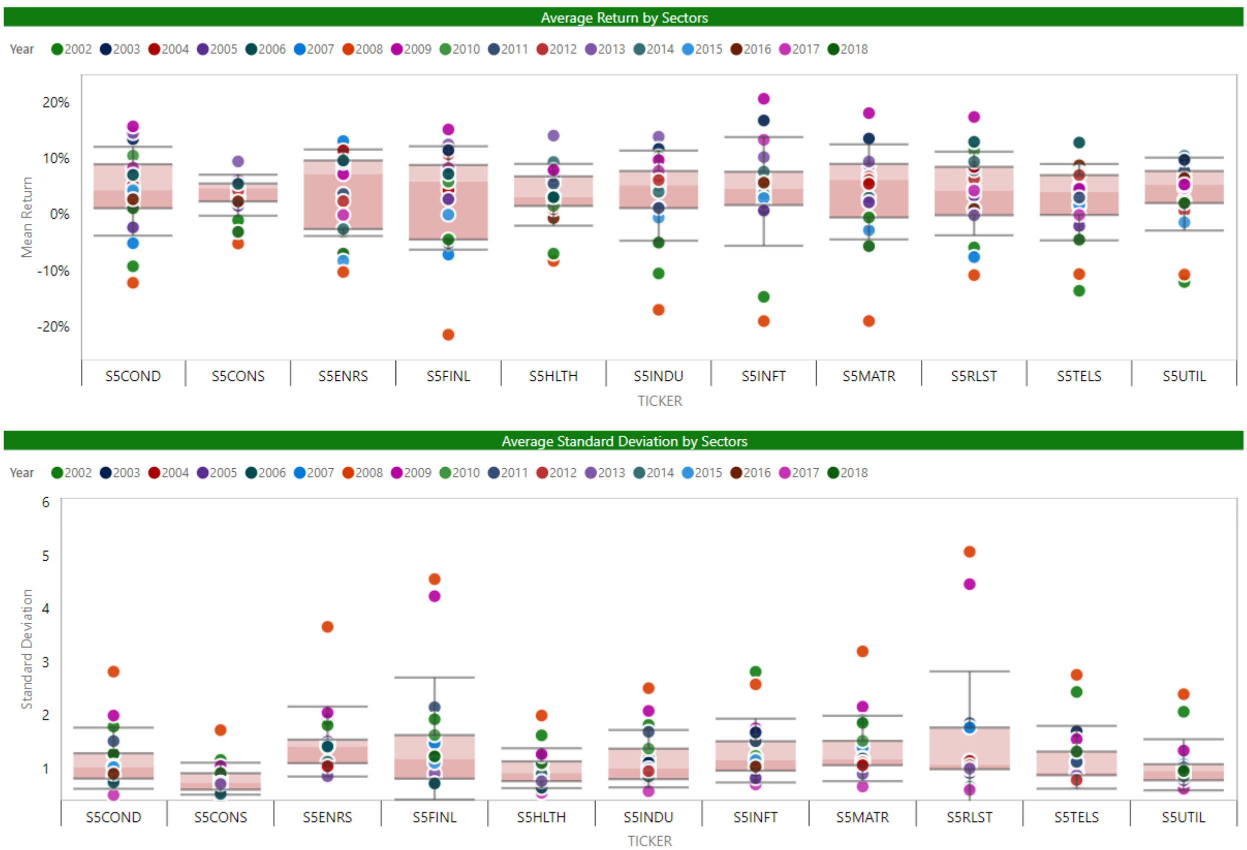


Figure 5: Return & Standard Deviation by Sector (2002 - 2018) Chart view

Some sectors yielded (Consumer Discretionary, Industrial, Information Technology, Telecommunications, Utilities) more symmetric return than other sectors (Energy, Financial, Health Care, Material, Consumer Services). During Financial Crisis, sector returns are more correlated.

B. Funds

We use the return of 09 equity funds from 2002 to 2018. Data is provided by the Bloomberg Terminal in the Lab at The University of Tulsa. Below is the descriptive statistics of 09 mutual funds.

TICKER	NAME	Portfolio Net Assets (M)	Turnover Rate	Exp Ratio	Exp Ratio (Net)	Load
ALSAX	Alger Small Cap Growth Fund Class A	$123	28.68%	1.38%	1.38%	5.25%
BMGAX	BlackRock Mid-Cap Growth Equity Portfolio Investor A Shares	$4,326	43.00%	1.26%	1.05%	5.25%
IALAX	Transamerica Capital Growth Fund Class A	$2,036	40.00%	1.19%	1.19%	5.50%
MSEGX	Morgan Stanley Institutional Fund, Inc. Growth Portfolio Class A	$7,525	41.00%	0.85%	0.85%	5.25%
OLGAX	JPMorgan Large Cap Growth Fund Class A	$15,920	24.00%	1.11%	0.94%	5.23%
RSEGX	Victory RS Small Cap Growth Fund Class A	$693	86.00%	1.45%	1.40%	5.75%
VSCOX	JPMorgan Small Cap Blend Fund Class A	$117	89.00%	1.41%	1.28%	5.25%
WAMCX	Wasatch Ultra Growth Fund	$623	44.00%	1.25%	1.25%	0.00%
WMICX	Wasatch Micro Cap Fund	$472	54.00%	1.65%	1.65%	0.00%

Figure 6: Fund Basic Information

YEAR	2002		2003		2004		2005		2006		2007		From 2002-2007 Average	
TICKER	Mean Return	Standard Deviation	Mean Return	Standard Deviation	Mean Return	Standard Deviation	Mean Return	Standard Deviation	Mean Return	Standard Deviation	Mean Return	Standard Deviation	**Mean Return**	**Standard Deviation**
ALSAX	-11.30%	1.70	14.86%	1.19	6.39%	1.09	6.40%	0.96	7.53%	1.09	6.60%	1.18	**5.08%**	**1.20**
BMGAX	-12.13%	1.68	12.07%	1.10	5.96%	0.87	4.17%	0.79	2.72%	0.90	7.59%	1.06	**3.40%**	**1.06**
IALAX	-10.97%	1.70	13.34%	1.17	3.34%	0.79	2.18%	0.63	6.65%	0.68	0.54%	1.01	**2.51%**	**1.00**
MSEGX	-11.57%	1.63	9.84%	1.12	3.16%	0.79	5.99%	0.77	1.92%	0.91	8.67%	1.24	**3.00%**	**1.08**
OLGAX	-11.91%	1.70	10.06%	1.06	2.83%	0.73	2.07%	0.69	2.64%	0.84	9.08%	1.18	**2.46%**	**1.03**
RSEGX	-18.52%	1.92	16.28%	1.45	6.52%	1.35	0.76%	0.99	4.28%	1.17	5.91%	1.19	**2.54%**	**1.35**
VSCOX	-9.05%	1.55	13.55%	1.12	4.28%	1.06	2.54%	1.00	6.37%	1.22	5.67%	1.20	**3.89%**	**1.19**
WAMCX	-6.51%	1.84	15.53%	1.34	0.31%	1.13	1.78%	0.88	3.29%	0.98	6.26%	1.14	**3.44%**	**1.22**
WMICX	-4.84%	1.57	16.66%	1.09	5.20%	0.96	4.46%	0.79	6.62%	0.83	1.42%	1.04	**4.92%**	**1.05**
Average	**-10.75%**	**1.70**	**13.58%**	**1.18**	**4.22%**	**0.97**	**3.37%**	**0.83**	**4.67%**	**0.96**	**5.75%**	**1.14**	**3.47%**	**1.13**

Figure 7: Return & Standard Deviation (2002 - 2007) for 9 Funds - Table View

From 2002 to 2007, the average return for 9 funds is 3.47% and the average standard deviation is 1.13. In 2008 (Global Financial Crisis), we observed (-21.25%) return and 2.56 standard deviation, a 126% increase from last period. Therefore, risk management can be challenging as thing could change rapidly in the financial market. Given the historical data from first period (2002 -2007), we could not capture enough potential risks in 2008.

The period after crisis (2009 -2018) yield higher Sharpe Ratio as it has average return of 6.71% and average standard deviation of 1.23.

YEAR	2008	
TICKER	Mean Return	Standard Deviation
ALSAX	-20.87%	2.61
BMGAX	-20.90%	2.71
IALAX	-15.59%	2.53
MSEGX	-23.50%	2.94
OLGAX	-17.31%	2.31
RSEGX	-21.03%	2.45
VSCOX	-19.44%	2.54
WAMCX	-28.67%	2.52
WMICX	-23.91%	2.40
Average	**-21.25%**	**2.56**

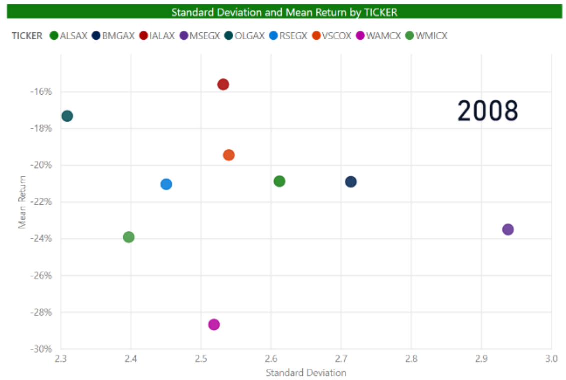


Figure 8: Return & Standard Deviation (2008) for 9 Funds - Table View

YEAR	2009		2010		2011		2012		2013	
TICKER	Mean Return	Standard Deviation	Mean Return	Standard Deviation	Mean Return	Standard Deviation	Mean Return	Standard Deviation	Mean Return	Standard Deviation
ALSAX	16.25%	1.83	9.96%	1.42	0.95%	2.02	5.16%	1.12	11.99%	0.95
BMGAX	17.63%	1.92	6.50%	1.37	0.40%	1.87	4.59%	1.03	15.67%	0.94
IALAX	12.97%	1.95	10.53%	1.31	-1.09%	1.57	5.92%	0.94	15.98%	0.84
MSEGX	21.21%	1.94	9.00%	1.31	-0.09%	1.56	6.16%	0.94	15.98%	0.84
OLGAX	12.93%	1.53	8.71%	1.20	2.18%	1.49	4.95%	0.96	11.53%	0.83
RSEGX	17.06%	1.78	10.75%	1.44	1.04%	1.98	6.12%	1.10	16.33%	0.94
VSCOX	11.25%	1.82	12.52%	1.47	0.55%	2.05	5.13%	1.14	15.83%	0.96
WAMCX	18.79%	1.66	12.87%	1.19	-0.90%	1.60	5.17%	0.97	12.47%	0.85
WMICX	16.34%	1.88	11.13%	1.26	-1.01%	1.79	4.94%	0.96	15.05%	0.87
Average	**16.05%**	**1.81**	**10.22%**	**1.33**	**0.23%**	**1.77**	**5.35%**	**1.02**	**14.54%**	**0.89**

YEAR	2014		2015		2016		2017		2018		From 2009-2018	
TICKER	Mean Return	Standard Deviation	Mean Return	Standard Deviation	Mean Return	Standard Deviation	Mean Return	Standard Deviation	Mean Return	Standard Deviation	**Mean Return**	**Standard Deviation**
ALSAX	0.61%	1.17	-0.79%	1.12	2.89%	1.21	10.25%	0.80	1.83%	1.50	**5.91%**	**1.31**
BMGAX	2.94%	1.11	3.00%	0.99	1.64%	1.05	11.90%	0.59	1.76%	1.23	**6.60%**	**1.21**
IALAX	2.80%	1.09	4.95%	1.11	-0.59%	1.14	14.76%	0.71	3.77%	1.54	**7.00%**	**1.22**
MSEGX	3.01%	1.09	4.93%	1.09	-0.28%	1.09	14.64%	0.71	4.08%	1.57	**7.86%**	**1.22**
OLGAX	4.51%	0.99	3.46%	1.10	-0.41%	0.95	12.96%	0.64	1.12%	1.48	**6.19%**	**1.12**
RSEGX	4.26%	1.18	0.75%	1.19	1.13%	1.25	12.82%	0.79	-2.74%	1.41	**6.75%**	**1.31**
VSCOX	0.60%	1.25	-0.18%	1.20	3.99%	1.40	14.17%	0.83	-0.72%	1.24	**6.32%**	**1.34**
WAMCX	1.97%	1.12	1.73%	1.11	3.86%	1.25	11.27%	0.82	4.85%	1.30	**7.21%**	**1.19**
WMICX	0.62%	1.05	0.11%	1.05	3.60%	1.11	12.58%	0.78	1.72%	1.19	**6.51%**	**1.19**
Average	**2.37%**	**1.12**	**2.00%**	**1.11**	**1.76%**	**1.16**	**12.82%**	**0.74**	**1.74%**	**1.38**	**6.71%**	**1.23**

Figure 9: Return & Standard Deviation (2009 - 2018) for 9 Funds - Table View

The chart in Figure 9 shows average return distribution by years. There are two years (2002, 2008) we observed negative returns, out of 17 years. We also observed an interesting trend, as average return decreased slowly from 2009 to 2011, the standard deviation decreased (from 2009 – 2010), then increased substantially (2010 - 2011). During this period, the changes in return is less correlated with the changes in standard deviations. During other periods, changes in returns has negative correlation with changes in standard deviations. This effect might be explained by the investor's confidence level. When the market performs better than the previous years, investors are more confident. Therefore, investors are willing to buy when stock price decreases to a level, they could pay for it previously. However, when the stock price increases, investor will also buy as they do not want to miss the chance. Vice versa, when the market performs worse than the previous years, investors less likely to buy in either cases. If the stock price falls, investors will wait as it may keep falling because of the downtrend. Even if the stock price goes up in a big downtrend, it does not look appealing since everyone is doubtful about the market. Obviously, the financial market performance should be closely attached to the fundamental reasons: Earning, Revenue, Profit, Tax...etc. However, the confidence level does affect the market, as we observed non-symmetric distribution of stock returns.

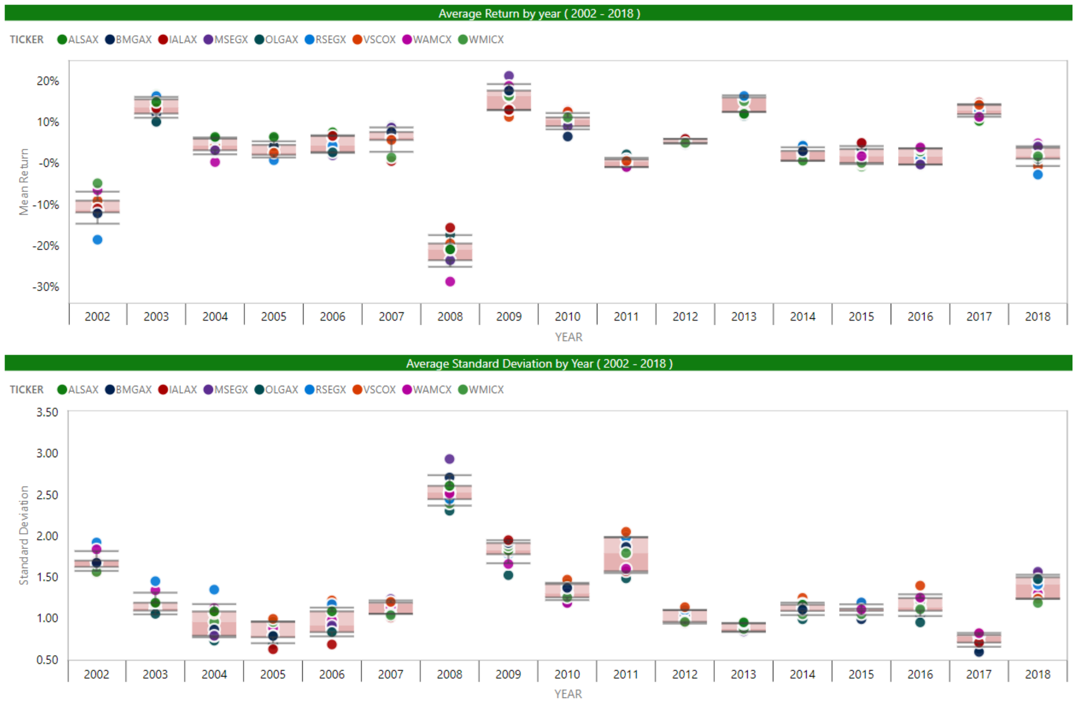


Figure 10: Return & Standard Deviation for 9 Funds by Years (2002 - 2018) - Box Plot Type I

Empirical Results

Actual VaR

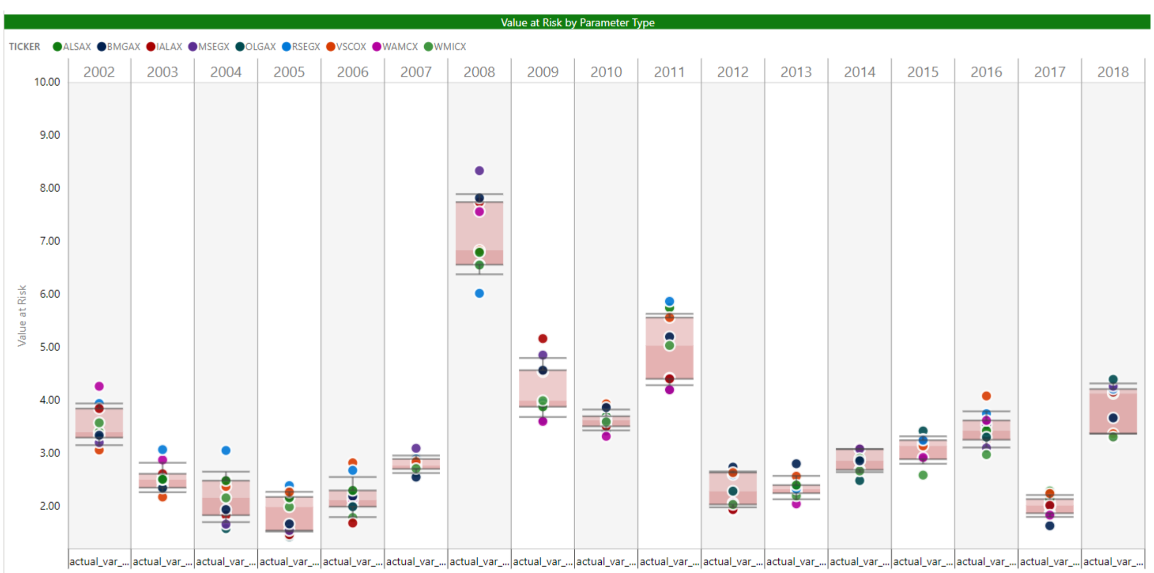


Figure 11: Actual VaR from 2002 – 2018 - Box Plot Type I

The actual VaR in figure 11 shows similar characteristics with the standard deviations in Figure 10. Both VaR and Standard Deviation are standard measure of risk. However, while standard deviations consider both tails of the distribution while VaR only consider the left-tail. Moreover, the potential loss of portfolio can be interpreted directly from VaR, while we need to perform further calculation on standard deviations. We also noted that future VaR does not correlated to the historical VaR although some trend can be detected. However, it is very dangerous to solely use the historical value to simulate the VaR of the current portfolio. Rather, for adequate risk management solution, one should also perform stress test.

Different Methods to Estimate VaR

We implement different methods to estimate VaR using Python. In this paper, we used 4 methods: Covariance-Variance (Risk Metrics) VaR, Parameter VaR, Monte Carlo Simulation and Decomposition VaR. The results then are compared to the actual VaR. We perform the calculation for 09 funds, and 17 years period (from 2002 -2018). Due to the size of the data, we only show the VaR estimation for two periods: 2008 (Financial Crisis period) and 2018 (Last Data Points).

From the figure 11, we can see that in general, all methods yield small VaR than the actual VaR. Note that the actual VaR has wider range than other simulation values. The parameters, Cov-variance and Monte Carlo methods estimate VaR based on the historical value of the mean and standard deviations which consider both tails of the distribution, therefore the simulated distribution will be adjusted to the right compare to the actual distribution. As the result, the simulated value often smaller than the actual value. The decomposition method does capture the VaR with acceptable range. However, it also failed to capture the tail risk. Besides, the fact that decomposition method always yields the smallest estimation of VaR compare to other methods suggest that the replicate portfolio fail to capture all the risk of the actual portfolio. One could improve the decomposition method using the factors return (Value, Quality, Volatility) rather than the sectors return.

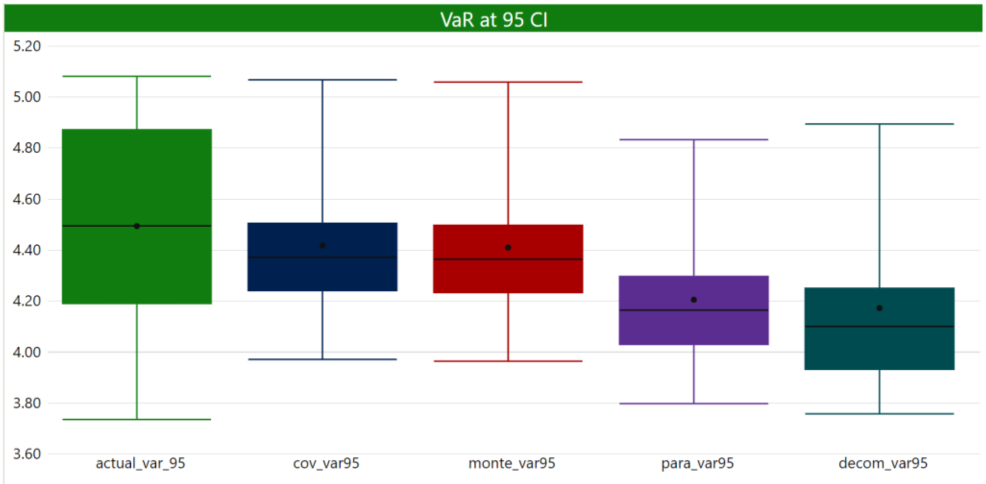


Figure 12: VaR by Type from 2008 – - Box Plot Type II

Figure 12 shows different estimations VaR for FY 2018. We see the same patterns with the Figure 11, except for the range of the decomposition VaR which is closed to that of the actual VaR in this figure and wider than the others. It might suggest that the decomposition method is more sensitive to the characteristic of the return since it's regression-based but fail to capture all the risk. For example, given two portfolios have different empirical distributions but same mean and standard deviations: the actual and decomposition VaR will yield different values, but other estimation methods will deliver the same VaR.

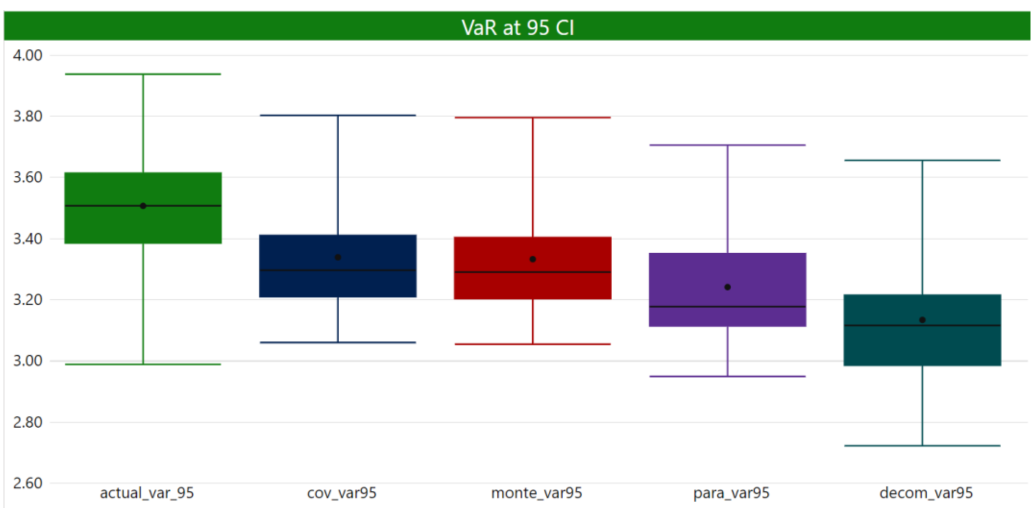


Figure 13: VaR by Type from 2008 - Box Plot Type II

VaR predictability

To address the change of VaR (both actual and simulated) over time, we analyze two periods: from 2007 – 2009 and 2016 -2019.

From 2007 -2009, the 95CI VaR estimation are close to the actual VaR. During the crisis, the accuracy of the estimations for 95CI VaR is not affected. However, from figure 15, we can see that the accuracy of the estimates for 99CI VaR is heavy affected in 2008. Both figure 14 and 15 shows no predictability for the future VaR given the previous year data.

Figure 14: VaR95 estimation from 2007 -2009 - Box Plot Type I

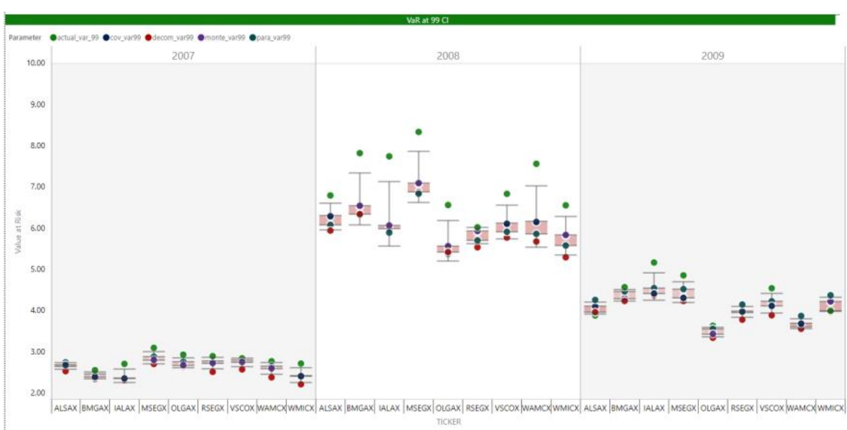


Figure 15: VaR99 estimation from 2007 -2009 - Box Plot Type I

From 2016 -2018, the 95CI VaR estimation are close to the actual VaR except in 2018, the actual VaR is significantly higher than the estimated value. Estimation for 99CI Var performed poorly during the whole period. Decomposition method was the worst estimator, while other methods yields similar results.

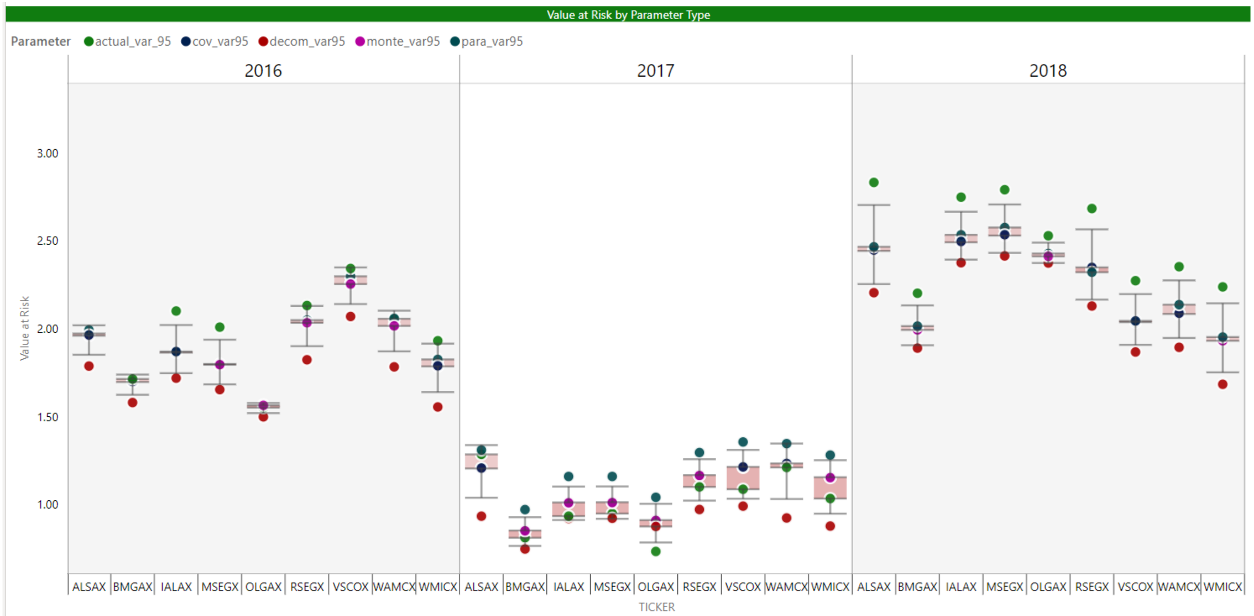


Figure 16: VaR95 estimation from 2016 -2018 - Box Plot Type I

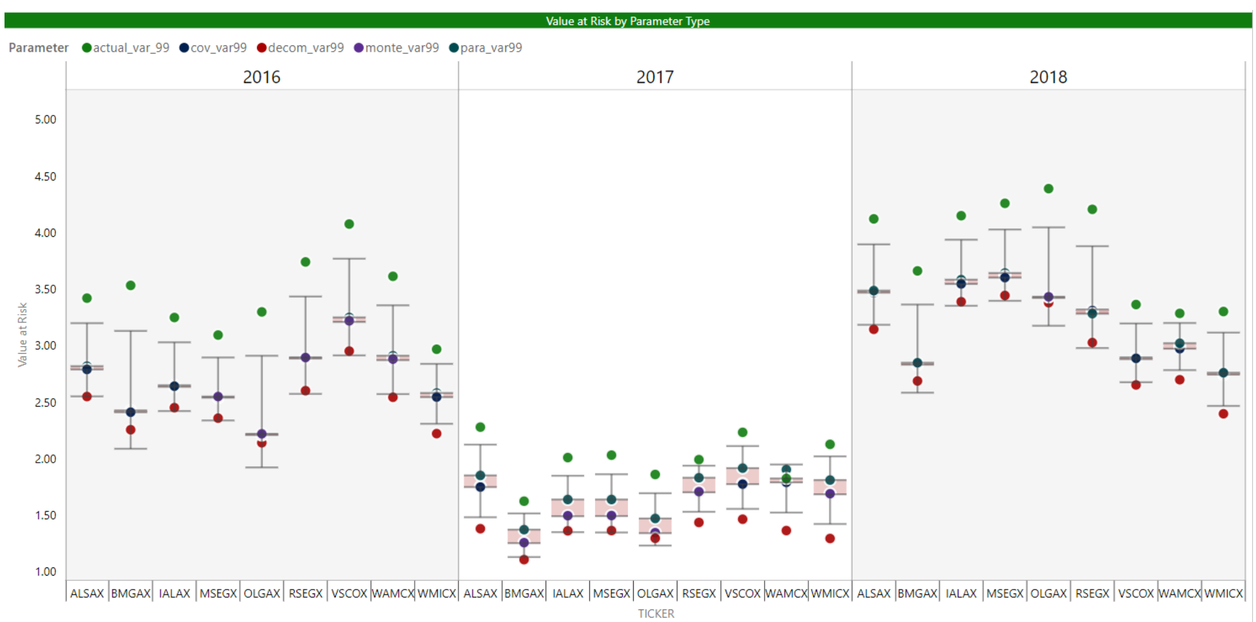


Figure 17: VaR99 estimation from 2016 – 2018 - Box Plot Type I

Conclusion

We successful evaluate 04 different VaR estimation approaches. The parameters VaR and Monte Carlo simulation often yields closed estimation due to similar distributions are generated (Normal vs Log Normal Distribution). The decomposition VaR failed to capture all to risk from actual portfolio, hence always yield smaller VaR than the actual one. However, it offers a new approach to measure risk. This method can implement differently if we decompose the portfolio performance by risk factors rather than equity sectors' return. From the empirical results, we can confirm the danger of these estimations as all methods failed to capture the actual VaR during financial crisis (2008). Moreover, we do not observe any trend that helps forecast future VaR based on historical value. Therefore, due to the nature of the financial market, measure and forecast VaR remains challenging regardless of the measurement approaches.

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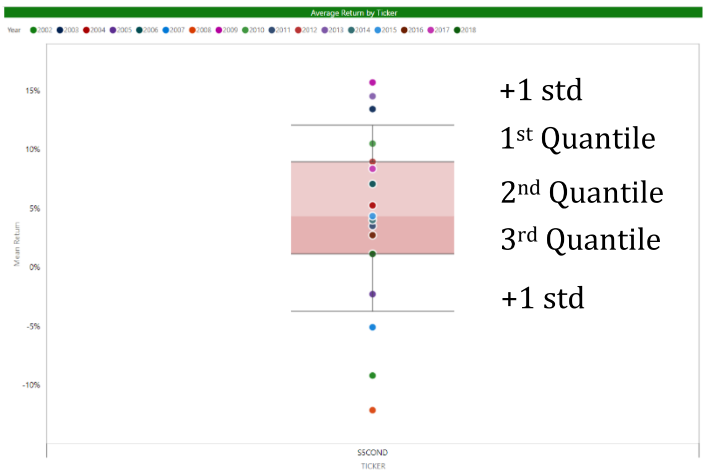
Appendix

Box- Whisker Plot



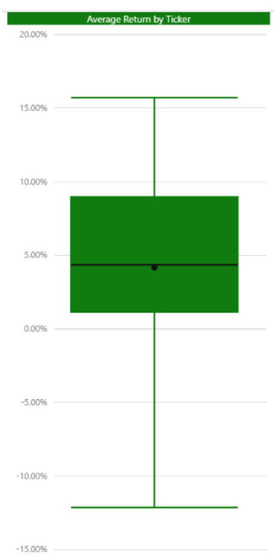
Box part:

- 1st line: 1st Quantile (25%)
- 2nd line: Median (50%)
- 3rd line: 3rd Quantile (75%)
- Mean: Middle point of the whisker boundary (not shown here)

Whisker part:

- Boundary: 1 standard deviations from the mean

Figure 18: Box Plot Type 1



Box part:

- Box Top: 1st Quantile (25%)
- 2nd line: Median (50%)
- Box Bottom: 3rd Quantile (75%)
- Mean: Dot

Whisker part:

- Boundary: Min/ Max value

Figure 19: Box Plot Type 2

Full data:

To view full data please visit:

https://app.powerbi.com/view?r=eyJrIjoiMjU3NjYzY2UtYzM1MC00YjhmLThjNGMtNGJiN2I1MjM1MGVmIiwidCI6ImQ0ZmYwMTNjLTYyYjctNDE2Ny05MjRmLTViZDkzZTgyMDJkMyIsImMiOjN9