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# Linear Regression

## A Simple Approach

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PhD in Computer Science

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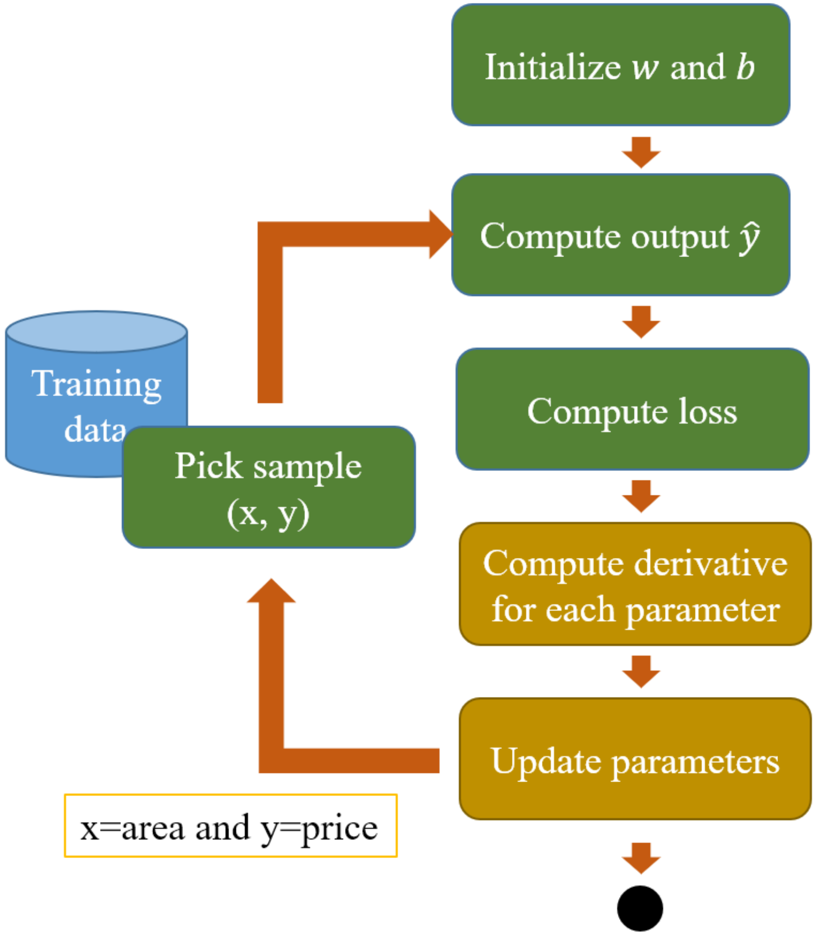
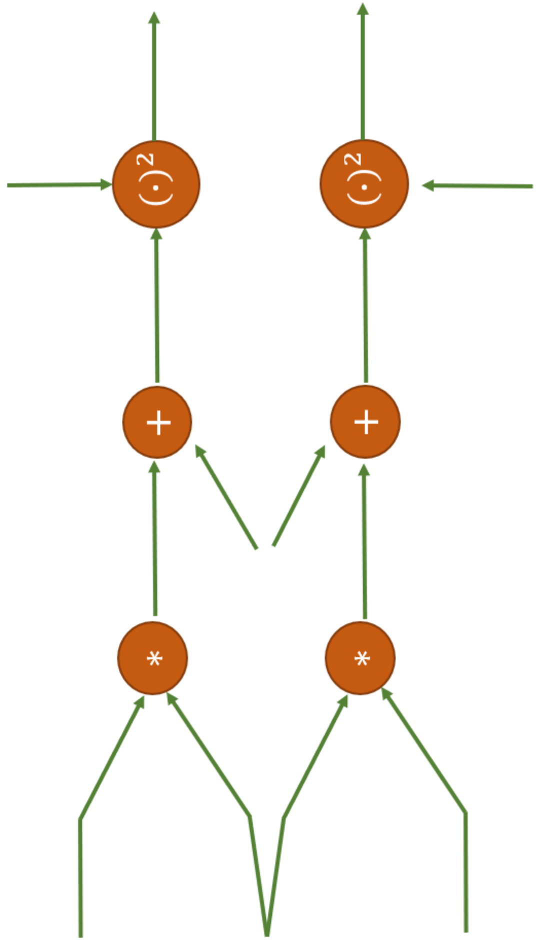
# Objectives

## Linear Regression




## Computational Graph



## Batch Training

1) Pick all the N samples $(x^{(i)}, y^{(i)})$ from training data

2) Compute output $\hat{y}^{(i)}$

$$\hat{y}^{(i)} = wx^{(i)} + b \qquad \text{for } 0 \leq i < N$$

3) Compute loss

$$L^{(i)} = (\hat{y}^{(i)} - y^{(i)})^2 \qquad \text{for } 0 \leq i < N$$

4) Compute derivatives

$$\frac{\partial L^{(i)}}{\partial w} = 2x^{(i)}(\hat{y}^{(i)} - y^{(i)})$$

$$\frac{\partial L^{(i)}}{\partial b} = 2(\hat{y}^{(i)} - y^{(i)}) \qquad \text{for } 0 \leq i < N$$

5) Update

$$w = w - \eta \frac{\sum_i \frac{\partial L^{(i)}}{\partial w}}{N} \qquad b = b - \eta \frac{\sum_i \frac{\partial L^{(i)}}{\partial b}}{N}$$

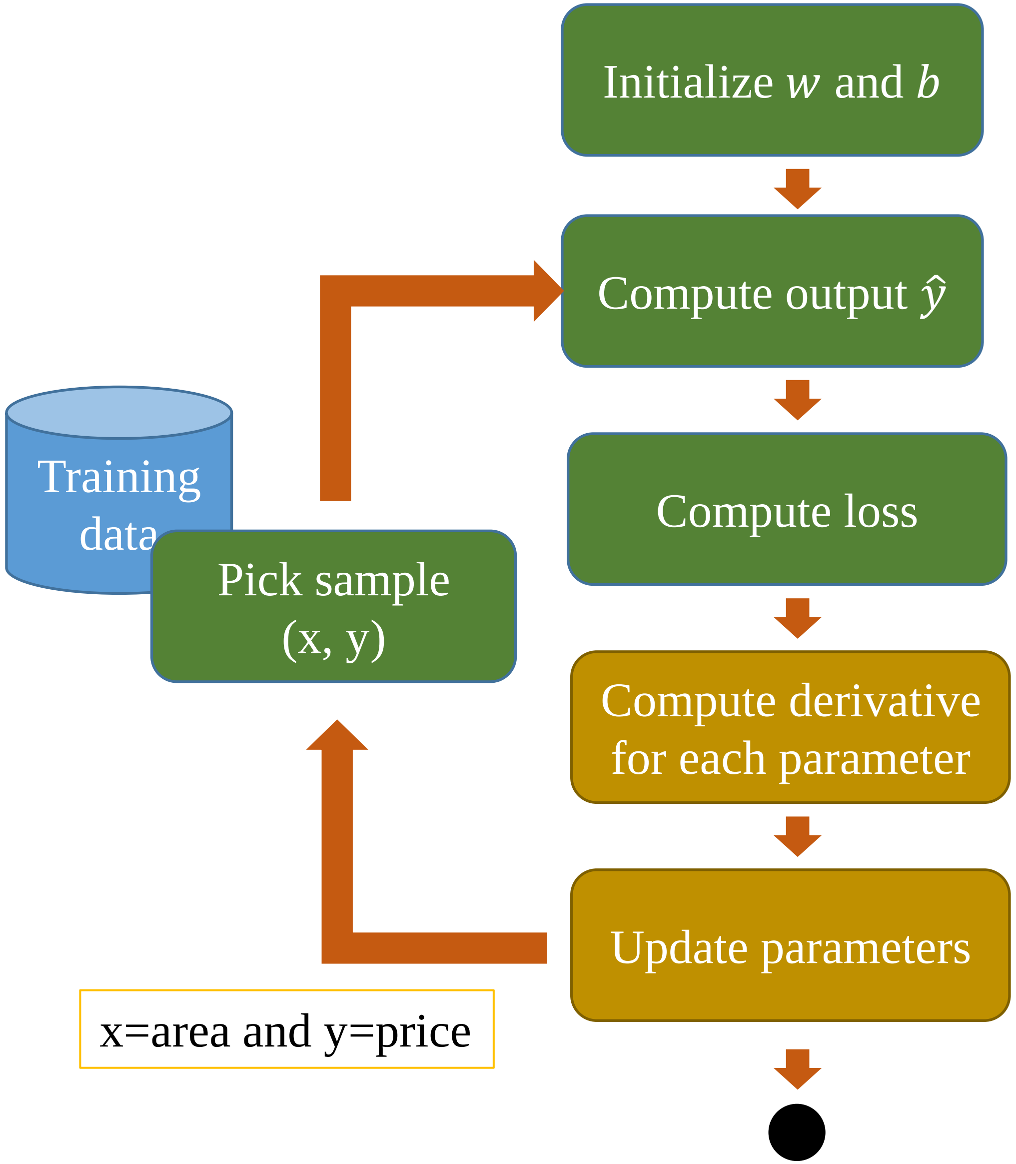
# Outline

SECTION 1
**Linear Regression**

SECTION 2
**Mini-batch Training**

SECTION 3
**Batch Training**

SECTION 4
**Loss Functions**



# Linear Regression

## Introduction

| | Feature | Label |
|---|---|---|
| | area | price |
| | 6.7 | 9.1 |
| | 4.6 | 5.9 |
| | 3.5 | 4.6 |
| | 5.5 | 6.7 |

House price data

| Features | | | Label |
|---|---|---|---|
| TV | Radio | Newspaper | Sales |
| 230.1 | 37.8 | 69.2 | 22.1 |
| 44.5 | 39.3 | 45.1 | 10.4 |
| 17.2 | 45.9 | 69.3 | 12 |
| 151.5 | 41.3 | 58.5 | 16.5 |
| 180.8 | 10.8 | 58.4 | 17.9 |

Advertising data

if area=6.0, price=?

if TV=55.0, Radio=34.0, and Newspaper=62.0, price=?

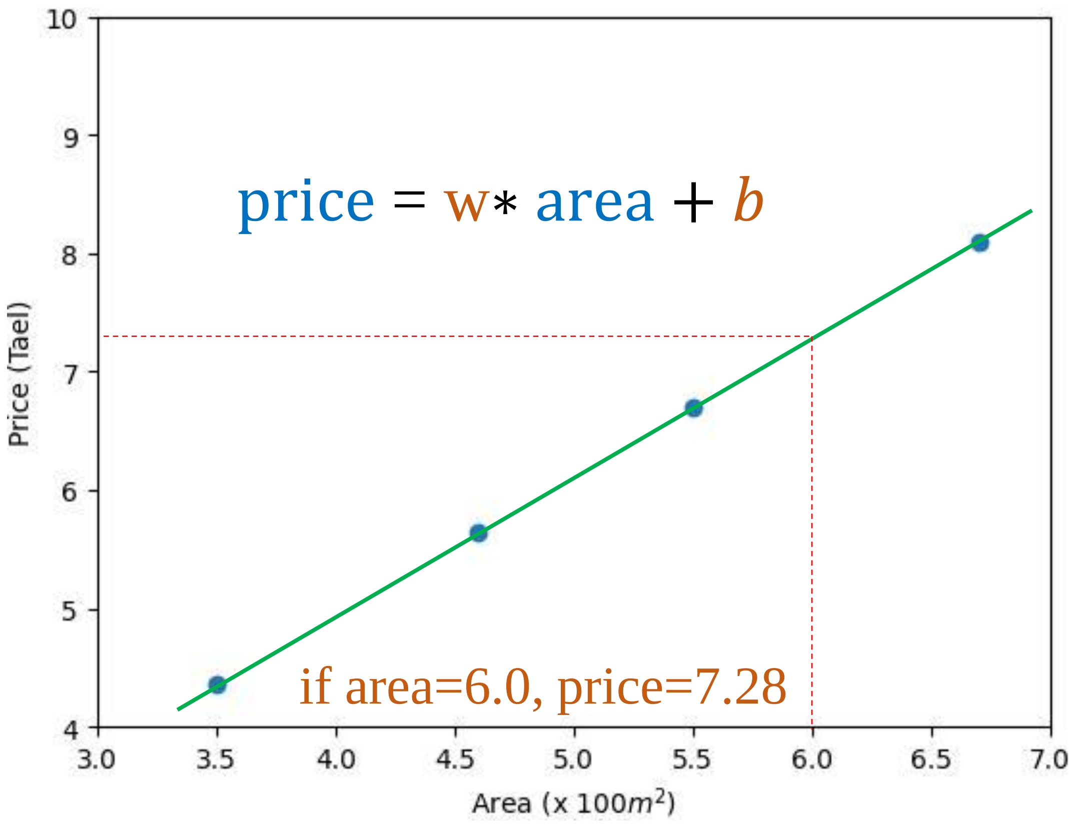
| Features | | | | | | | | | | | | | Label |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| crim | zn | indus | chas | nox | rm | age | dis | rad | tax | ptratio | black | lstat | medv |
| 0.00632 | 18 | 2.31 | 0 | 0.538 | 6.575 | 65.2 | 4.09 | 1 | 296 | 15.3 | 396.9 | 4.98 | 24 |
| 0.02731 | 0 | 7.07 | 0 | 0.469 | 6.421 | 78.9 | 4.9671 | 2 | 242 | 17.8 | 396.9 | 9.14 | 21.6 |
| 0.03237 | 0 | 2.18 | 0 | 0.458 | 6.998 | 45.8 | 6.0622 | 3 | 222 | 18.7 | 394.63 | 2.94 | 33.4 |
| 0.06905 | 0 | 2.18 | 0 | 0.458 | 7.147 | 54.2 | 6.0622 | 3 | 222 | 18.7 | 396.9 | 5.33 | 36.2 |
| 0.08829 | 12.5 | 7.87 | 0 | 0.524 | 6.012 | 66.6 | 5.5605 | 5 | 311 | 15.2 | 395.6 | 12.43 | 22.9 |

Boston House Price Data

# House Price Prediction

| Feature | Label |
|---|---|
| area | price |
| 6.7 | 8.1 |
| 4.6 | 5.6 |
| 3.5 | 4.3 |
| 5.5 | 6.7 |

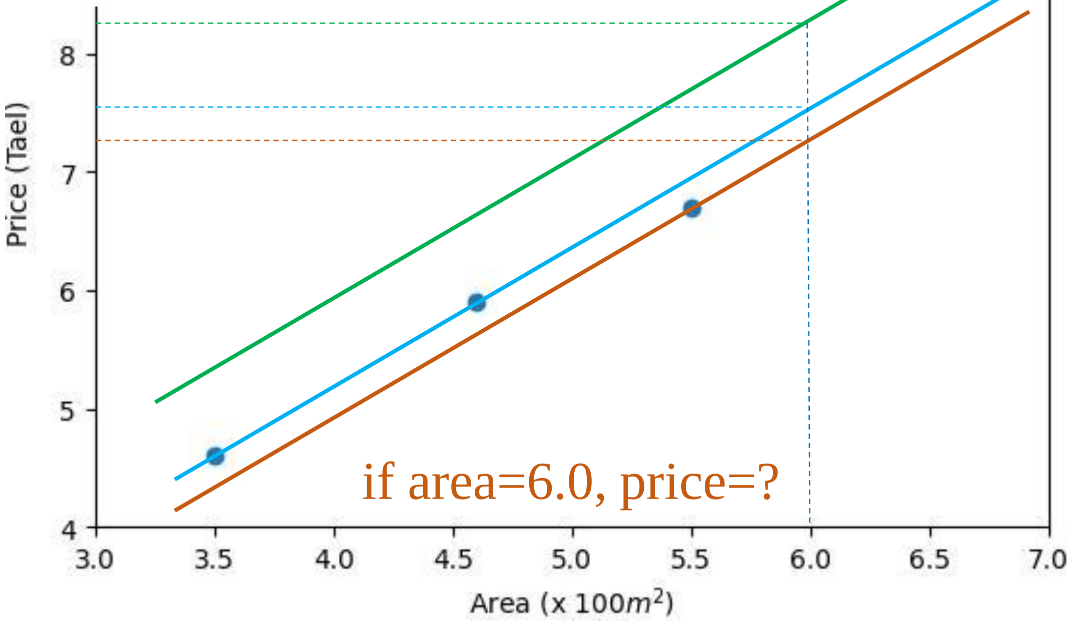
House price data




| Feature | Label |
|---|---|
| area | price |
| 6.7 | 9.1 |
| 4.6 | 5.9 |
| 3.5 | 4.6 |
| 5.5 | 6.7 |
| | |

House price data

$$price = w_1 * area + b_1$$
$$price = w_2 * area + b_2$$
$$price = w_3 * area + b_3$$



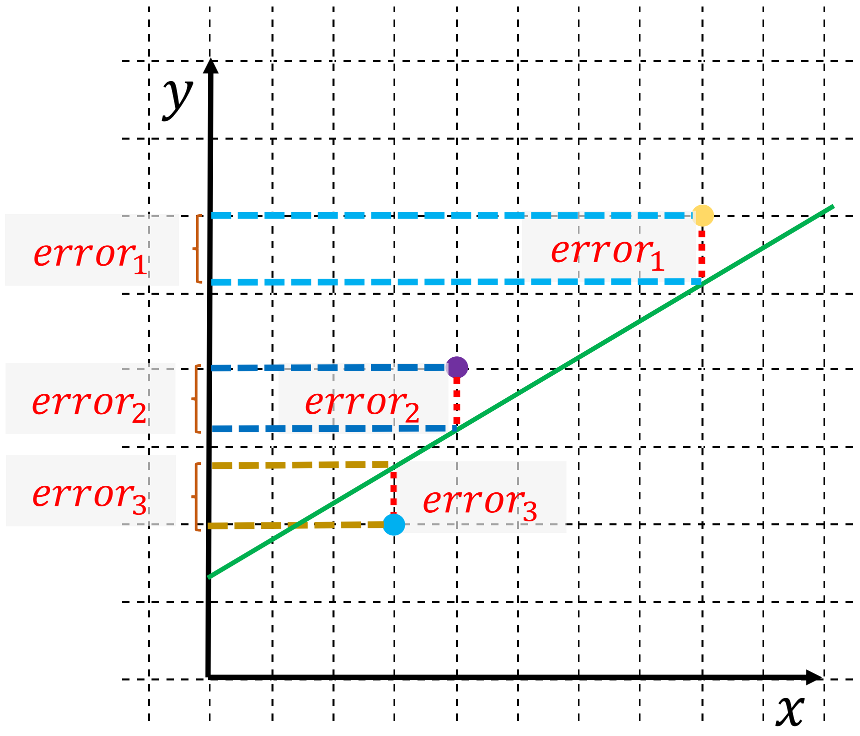
# Linear Regression



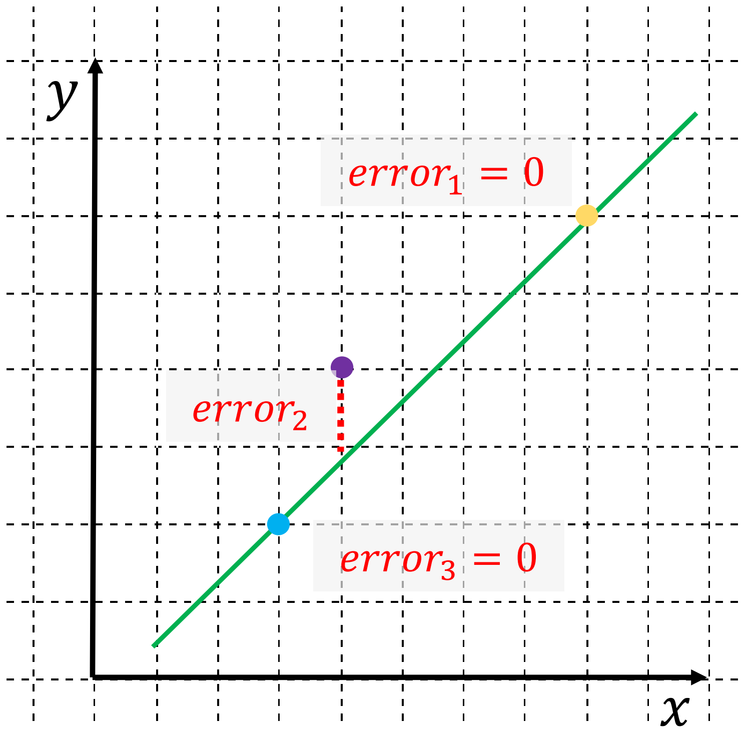
## ❖ Area-based house price prediction

Training data

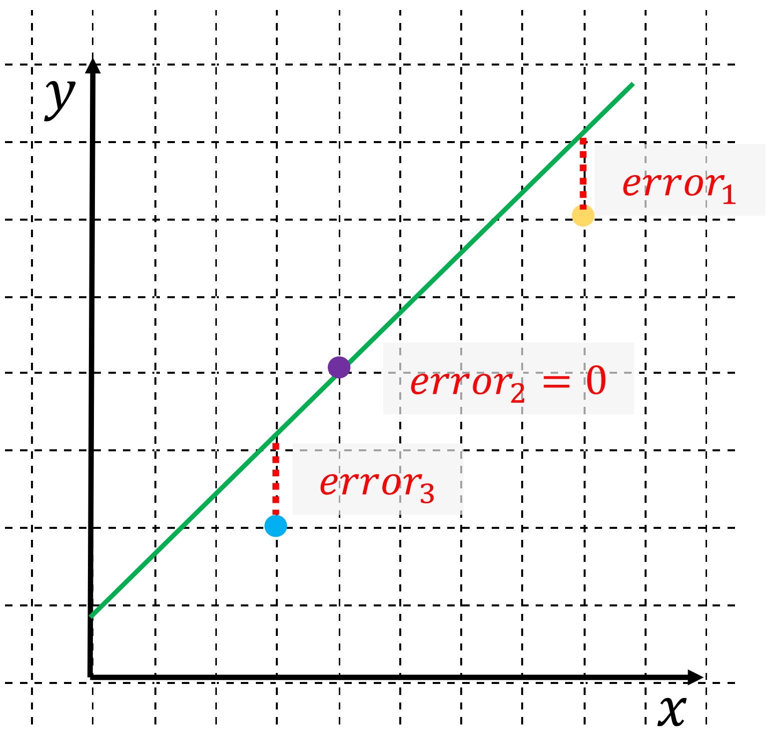
$$error_i = distance(\hat{y}_i, y_i)$$




$\hat{y} = w_1 x + b_1$




$\hat{y} = w_2 x + b_2$




$\hat{y} = w_3 x + b_3$

Find w and b whose model has the smallest error, where $error = \frac{1}{N}\sum_i error_i$ How?

# Linear Regression



## ❖ Area-based house price prediction

weight bias

$$\text{predicted\_price} = w * \text{area} + b$$

$$\text{error} = (\text{predicted\_price} - \text{real\_price})^2$$

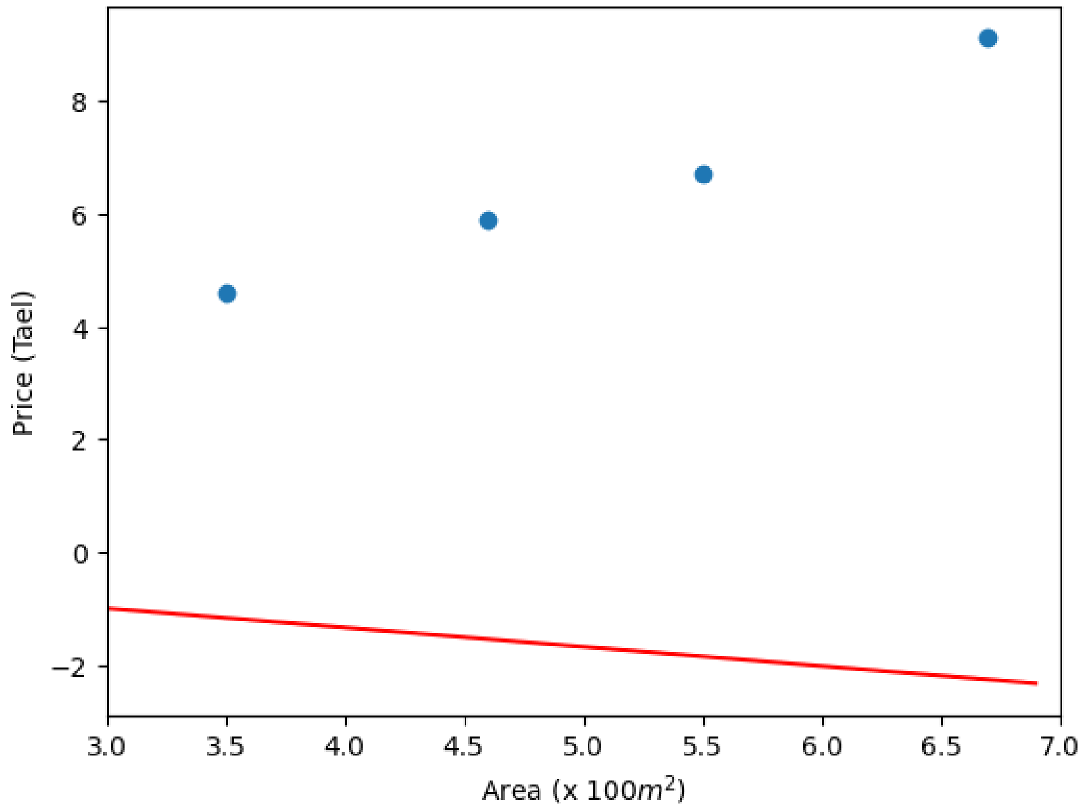
$$\hat{y}_i = wx_i + b$$

$$L(\hat{y}_i, y_i) = (\hat{y}_i - y_i)^2$$

w = -0.34

b = 0.04

| area | price | predicted | error |
|---|---|---|---|
| 6.7 | 9.1 | -2.238 | 128.55 |
| 4.6 | 5.9 | -1.524 | 55.11 |
| 3.5 | 4.6 | -1.15 | 33.06 |
| 5.5 | 6.7 | -1.83 | 72.76 |



# Linear Regression



## ❖ Area-based house price prediction

$$\text{predicted\_price} = w * \text{area} + b$$

$$\text{error} = (\text{predicted\_price} - \text{real\_price})^2$$

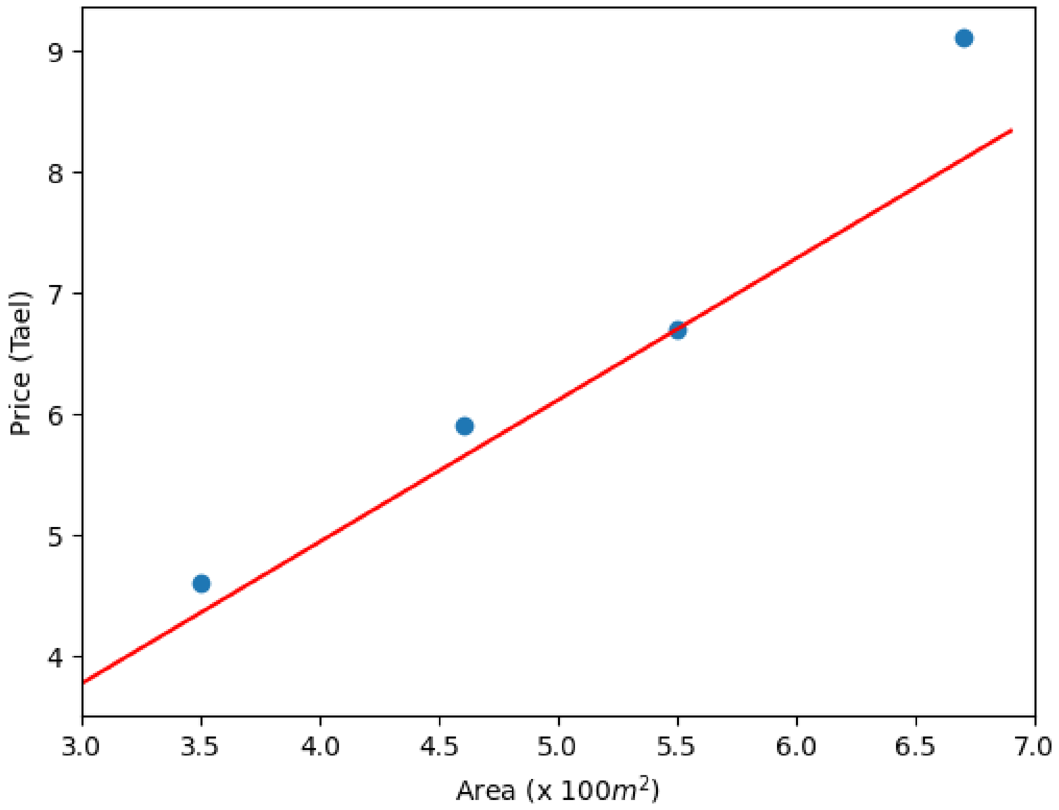
$$\hat{y}_i = wx_i + b$$

$$L(\hat{y}_i, y_i) = (\hat{y}_i - y_i)^2$$

$w = 1.17$

$b = 0.26$

| area | price | predicted | error |
|---|---|---|---|
| 6.7 | 9.1 | 8.099 | 1.002 |
| 4.6 | 5.9 | 5.642 | 0.066 |
| 3.5 | 4.6 | 4.355 | 0.06 |
| 5.5 | 6.7 | 6.695 | 0.00002 |



# Linear Regression

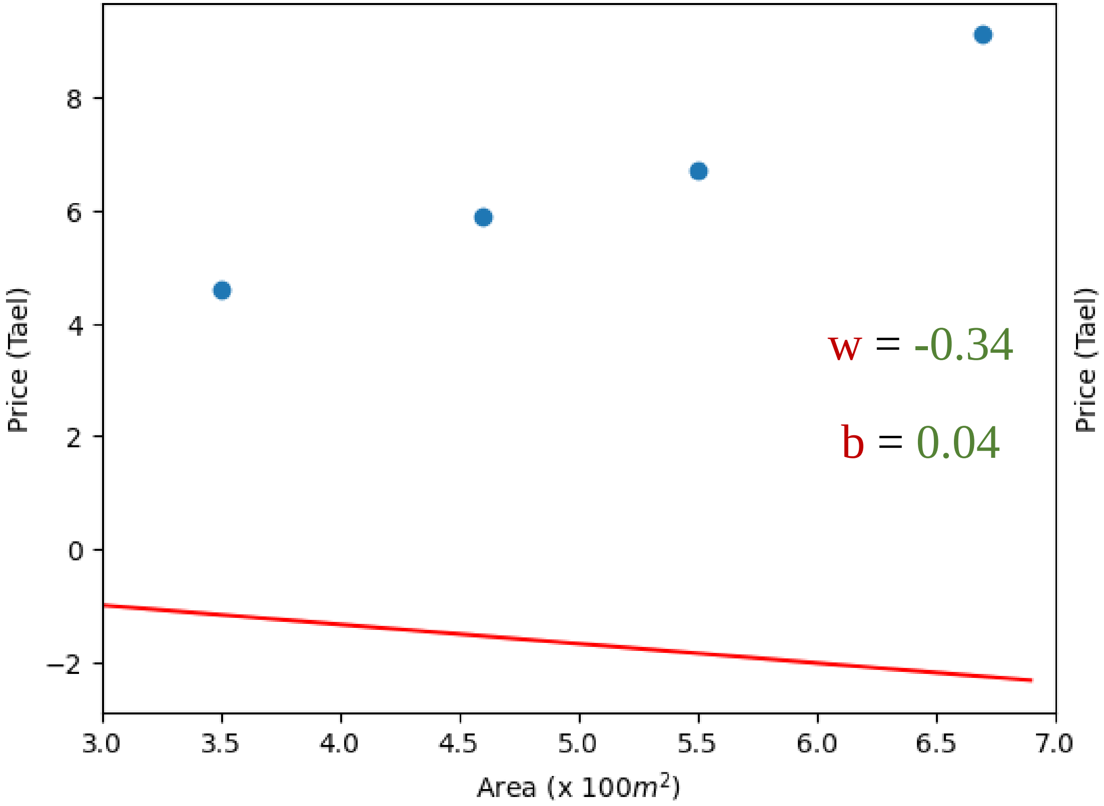


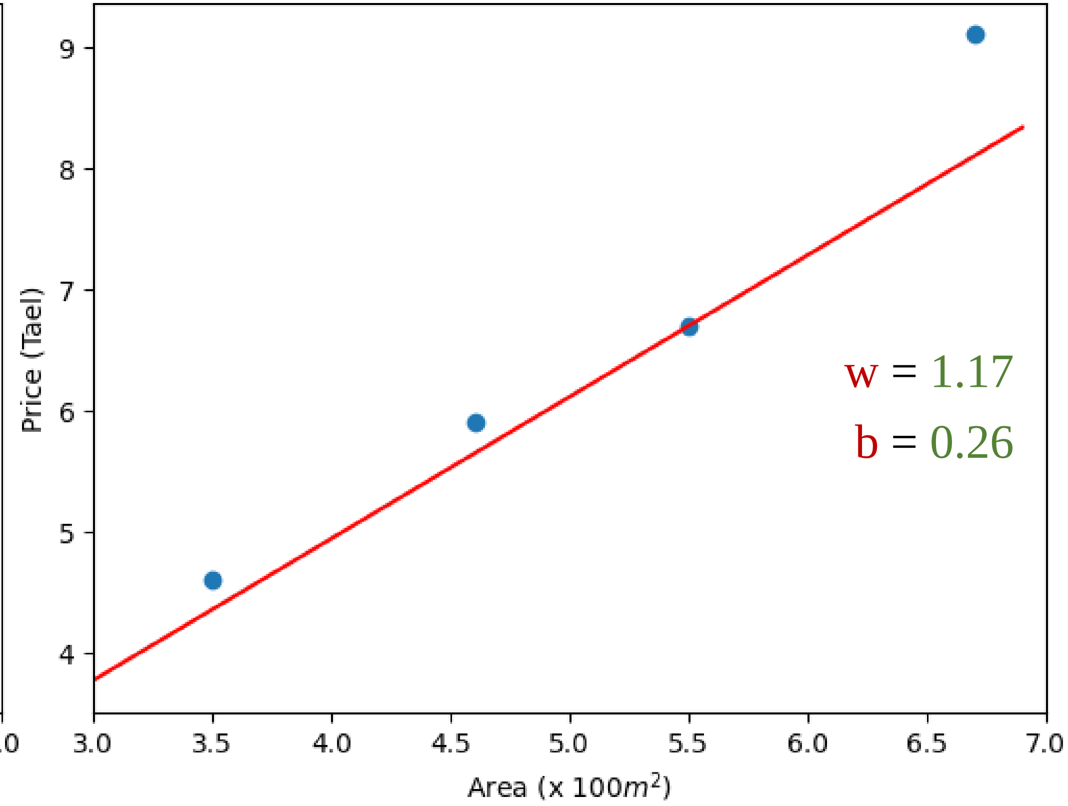
## ❖ Area-based house price prediction

$$\hat{y}_i = wx_i + b$$

$$L(\hat{y}_i, y_i) = (\hat{y}_i - y_i)^2$$

How to change w and b so that $L(\hat{y}_i, y_i)$ reduces



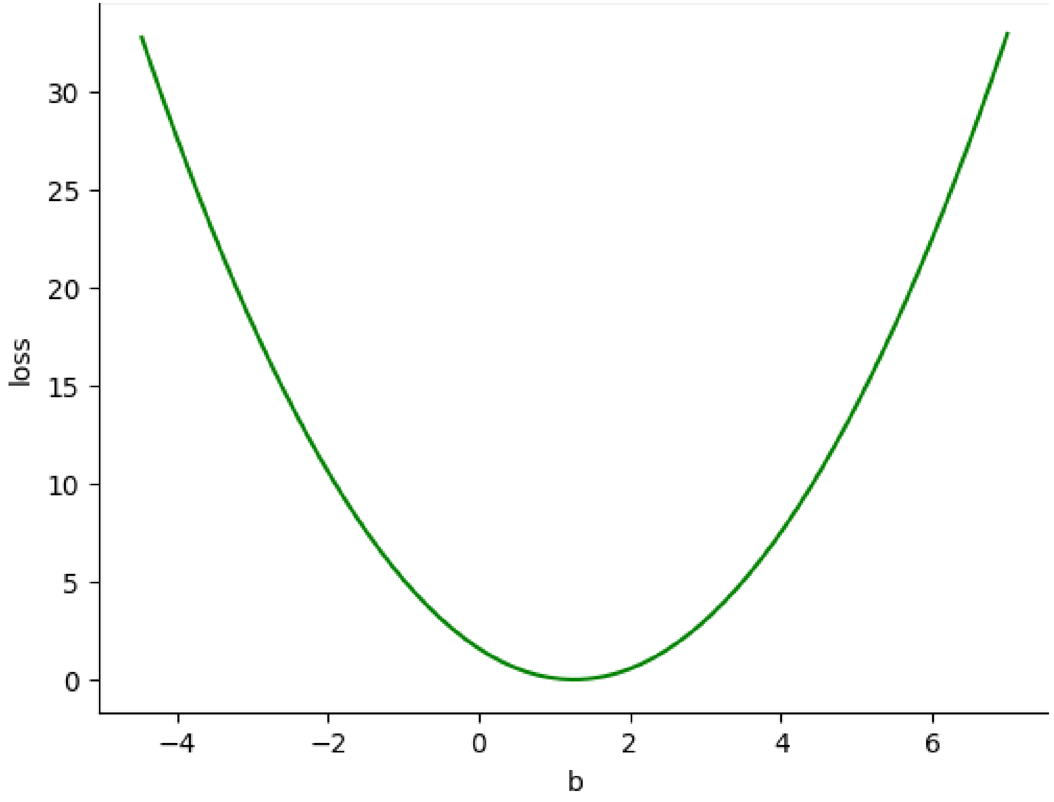
# Linear Regression

$$\hat{y}_i = wx_i + b$$

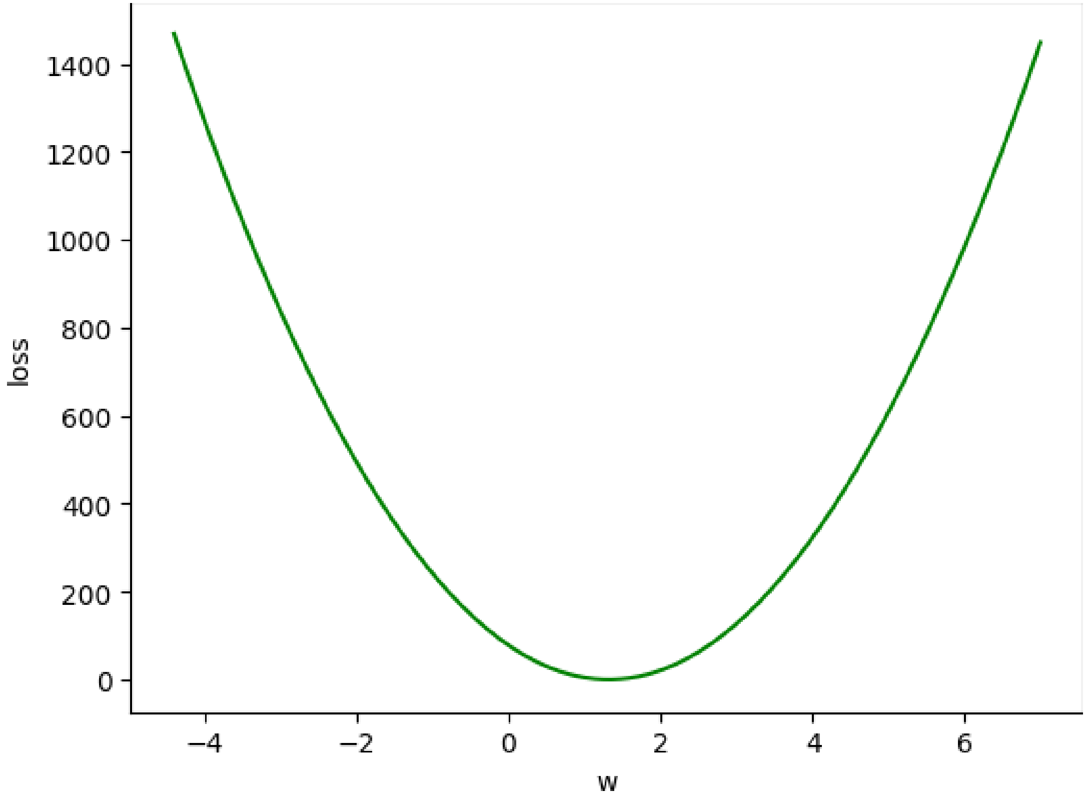
$$L(\hat{y}_i, y_i) = (\hat{y}_i - y_i)^2$$

## ❖ Understanding the loss function

How to change w and b so that $L(\hat{y}_i, y_i)$ reduces




Different b values with a fixed w value




Different w values with a fixed b value

# Linear Regression

## Linear equation

$$\hat{y} = wx + b$$

where $\hat{y}$ is a predicted value,

$w$ and $b$ are parameters

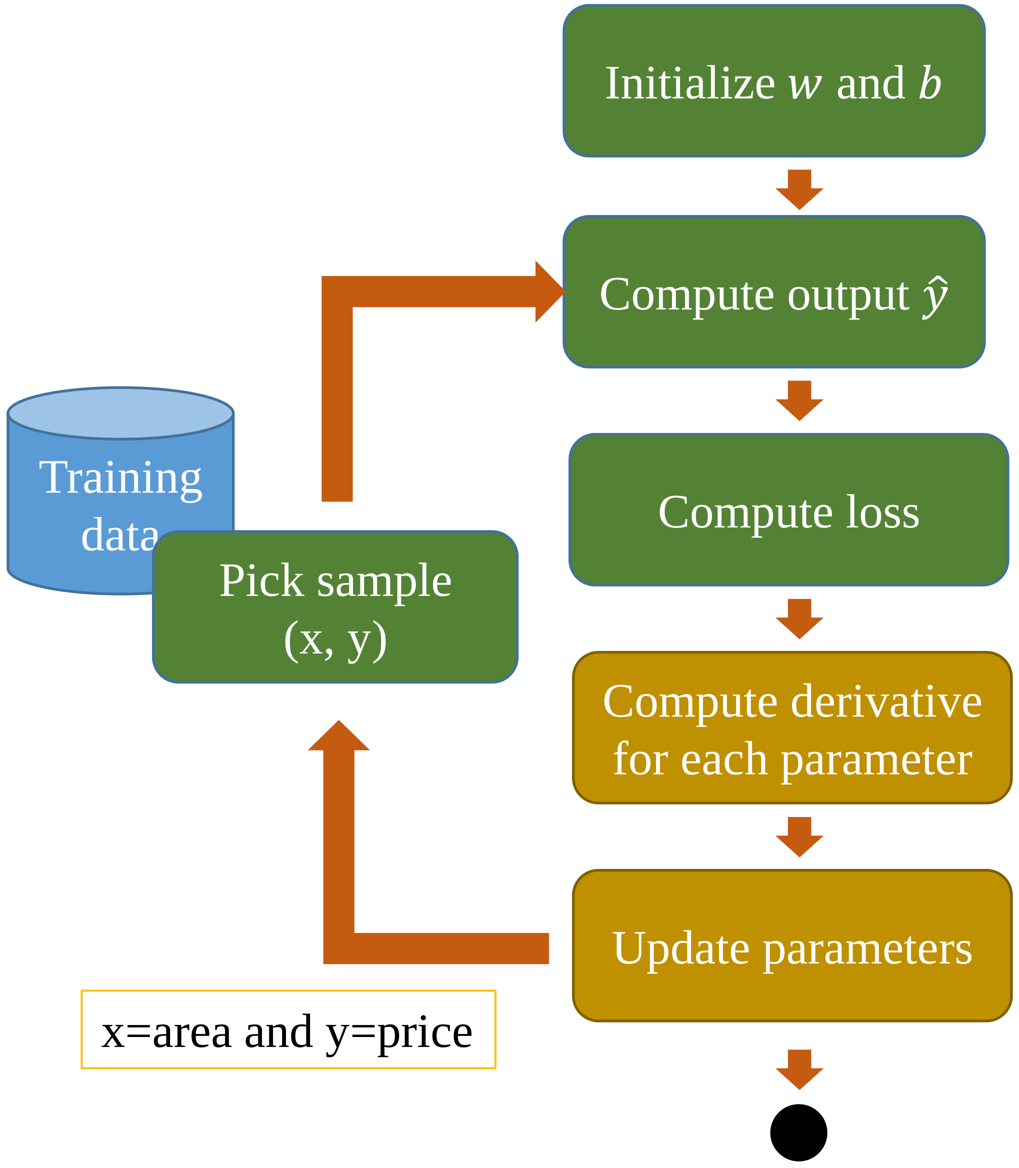
and $x$ is an input feature

## Error (loss) computation

**Idea:** compare predicted values $\hat{y}$ and label values y

Squared loss

$$L(\hat{y}, y) = (\hat{y} - y)^2$$



# Linear Regression



## Linear equation

$$\hat{y} = wx + b$$

where $\hat{y}$ is a predicted value,

$w$ and $b$ are parameters

and $x$ is an input feature

## Error (loss) computation

**Idea:** compare predicted values $\hat{y}$ and label values y

Squared loss

$$\mathrm{L}(\hat{y}, y) = (\hat{y} - y)^2$$

## Find better w and b

Use gradient descent to minimize the loss function

Compute derivate for each parameter

$$\frac{\partial L}{\partial w} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial w} = 2x(\hat{y} - y)$$

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial b} = 2(\hat{y} - y)$$

Update parameters

$$w = w - \eta \frac{\partial L}{\partial w} \qquad b = b - \eta \frac{\partial L}{\partial b}$$

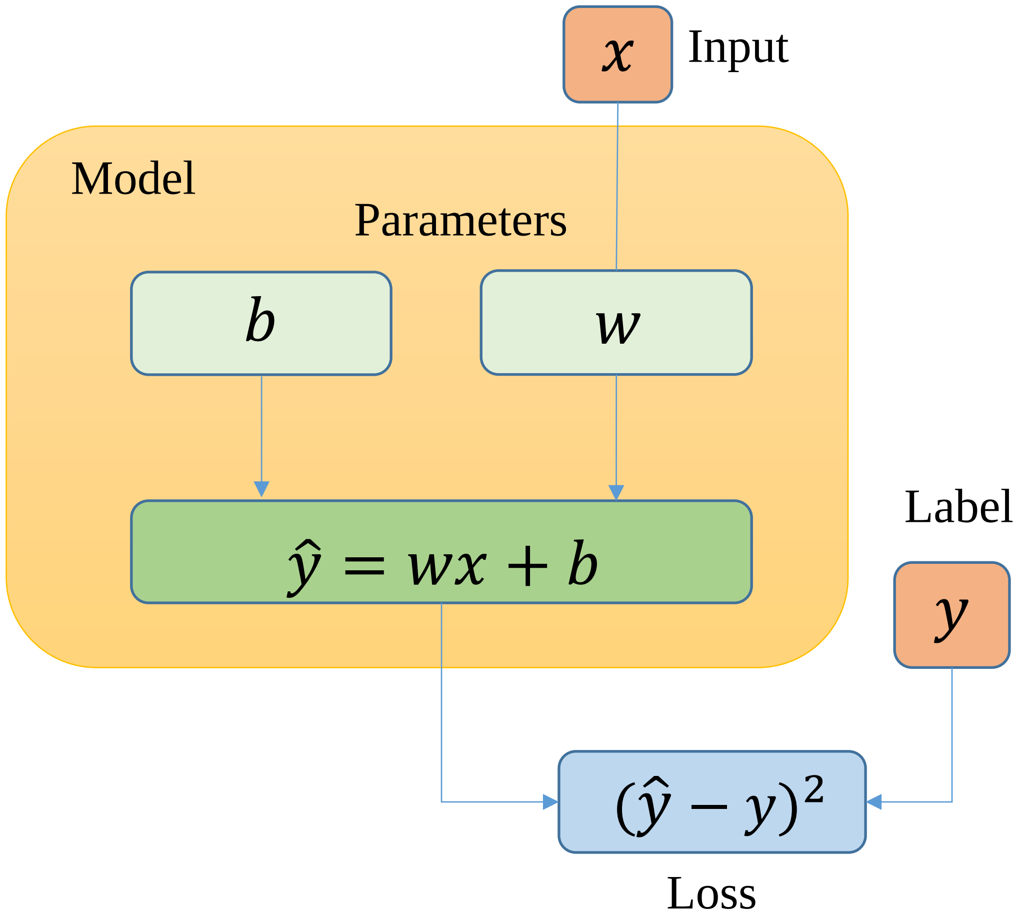
$\eta$ is learning rate

# Linear Regression



## ❖ Simple example

### Diagram




### Cheat sheet

Compute the output $\hat{y}$

$$\hat{y} = wx + b$$

Compute the loss

$$L = (\hat{y} - y)^2$$

Compute derivative

$$\frac{\partial L}{\partial w} = 2x(\hat{y} - y)$$

$$\frac{\partial L}{\partial b} = 2(\hat{y} - y)$$

Update parameters

$$w = w - \eta \frac{\partial L}{\partial w}$$

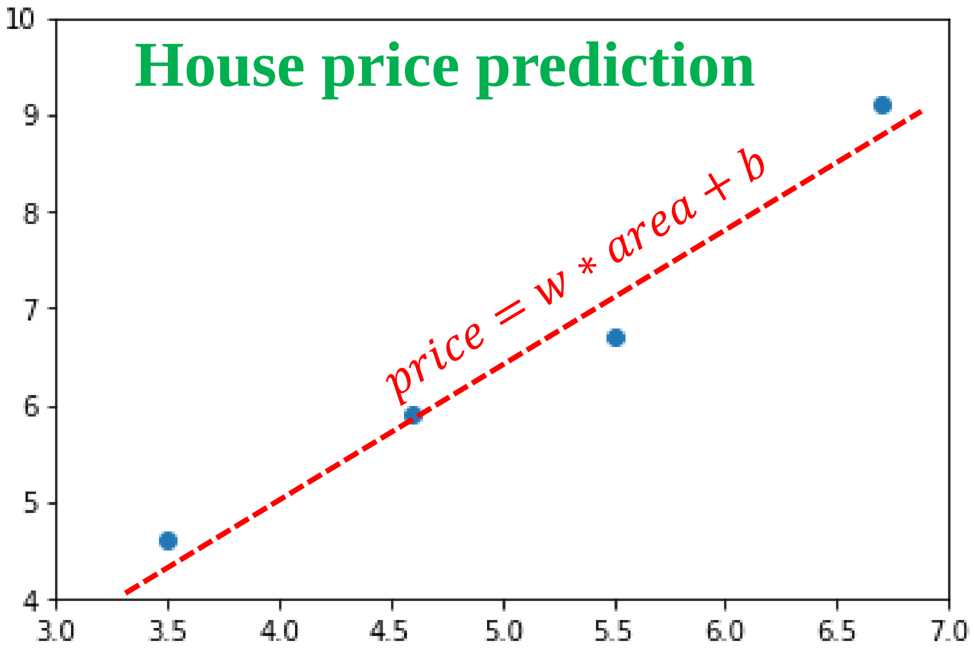
$$b = b - \eta \frac{\partial L}{\partial b}$$

# Linear Regression

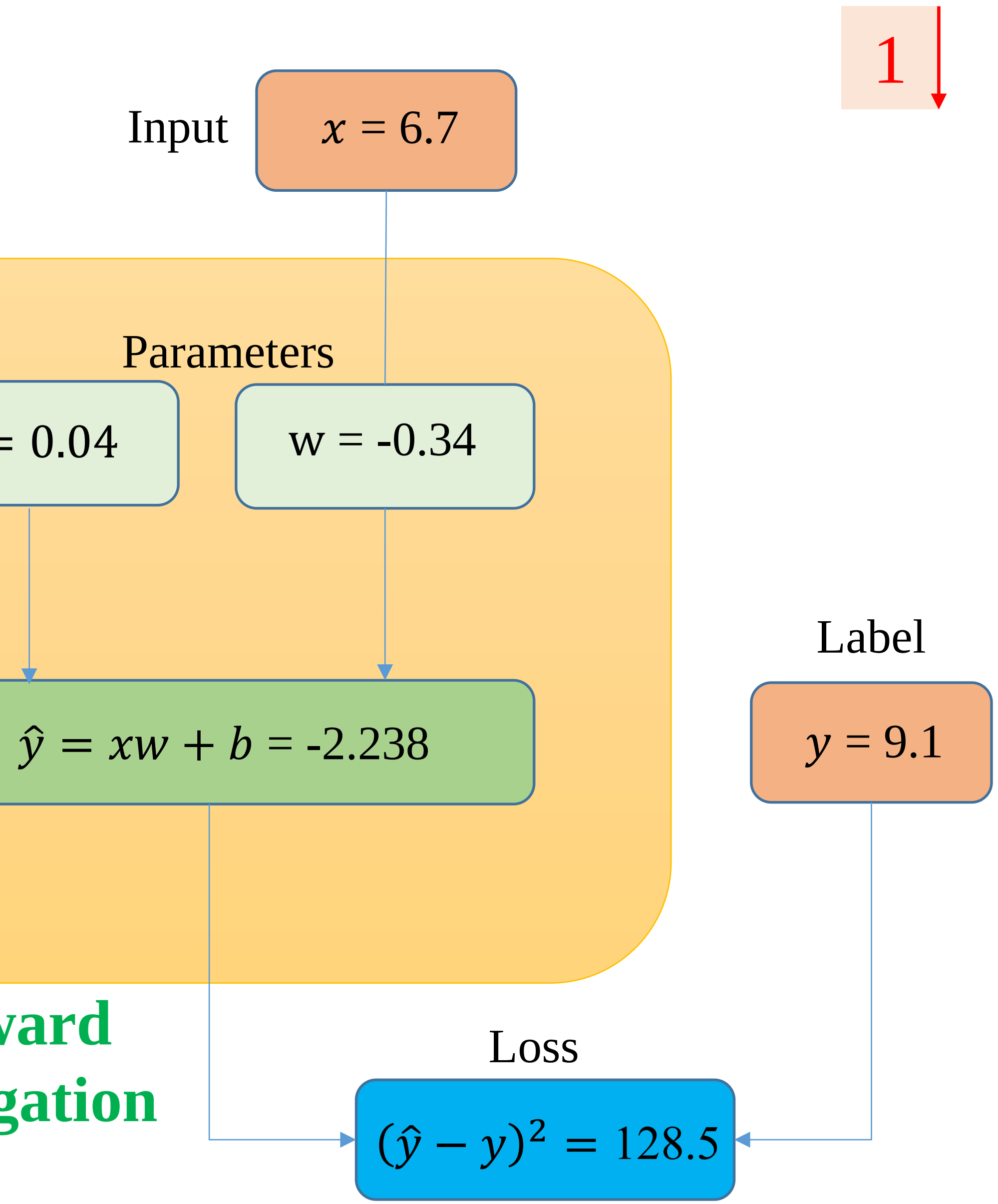


**Given sample data**

| | Feature | Label |
|---|---|---|
| | area | price |
| | 6.7 | 9.1 |
| | 4.6 | 5.9 |
| | 3.5 | 4.6 |
| | 5.5 | 6.7 |




Initialize
b=0.04 and
w=-0.34




**Forward propagation**

# Linear Regression



2

Input $x = 6.7$

**Backpropagation**

Model

Parameters

$b = 0.26676$ $w = 1.17929$

$b = b - \eta \frac{\partial L}{\partial b}$ $\quad$ $w = w - \eta \frac{\partial L}{\partial w}$

$\hat{y} = xw + b = -2.238$

$\eta = 0.01$

Label $y = 9.1$

$$\frac{\partial L}{\partial w} = 2x(\hat{y} - y) = -151.9292$$

$$\frac{\partial L}{\partial b} = 2(\hat{y} - y) = -22.676$$

Loss $(\hat{y} - y)^2 = 128.5$

3

Input $x = 6.7$

**Forward propagation**

Model

Parameters

$b = 0.26676$ $w = 1.17929$

$b = b - \eta \frac{\partial L}{\partial b}$ $\quad$ $w = w - \eta \frac{\partial L}{\partial w}$

$\hat{y} = xw + b = 8.168$

Label $y = 9.1$

New w and b help the loss reduce

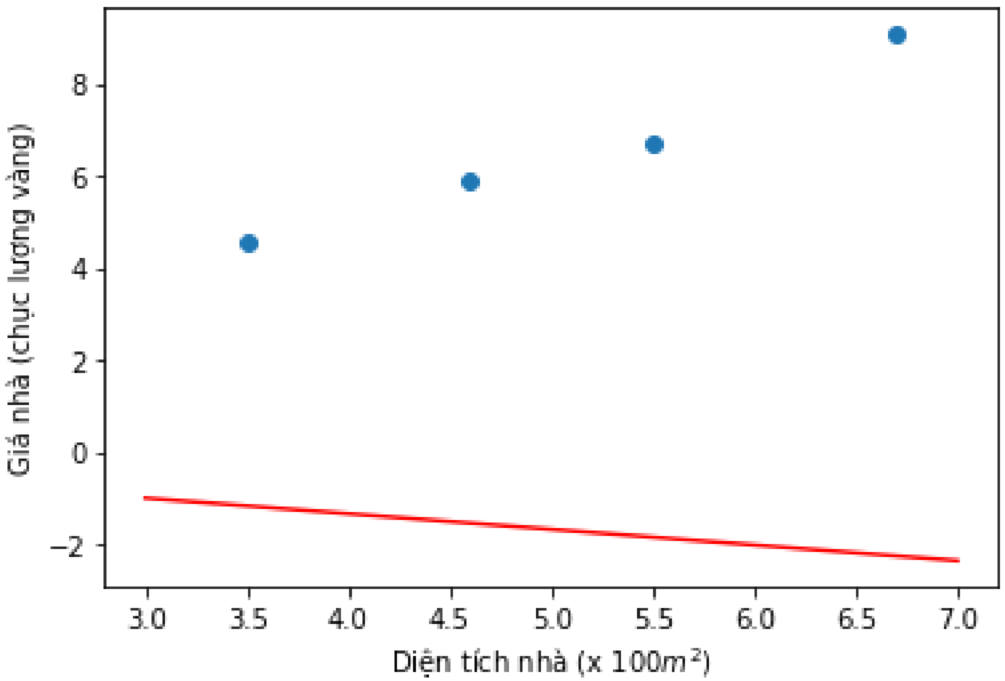
Loss $(\hat{y} - y)^2 = 0.868$

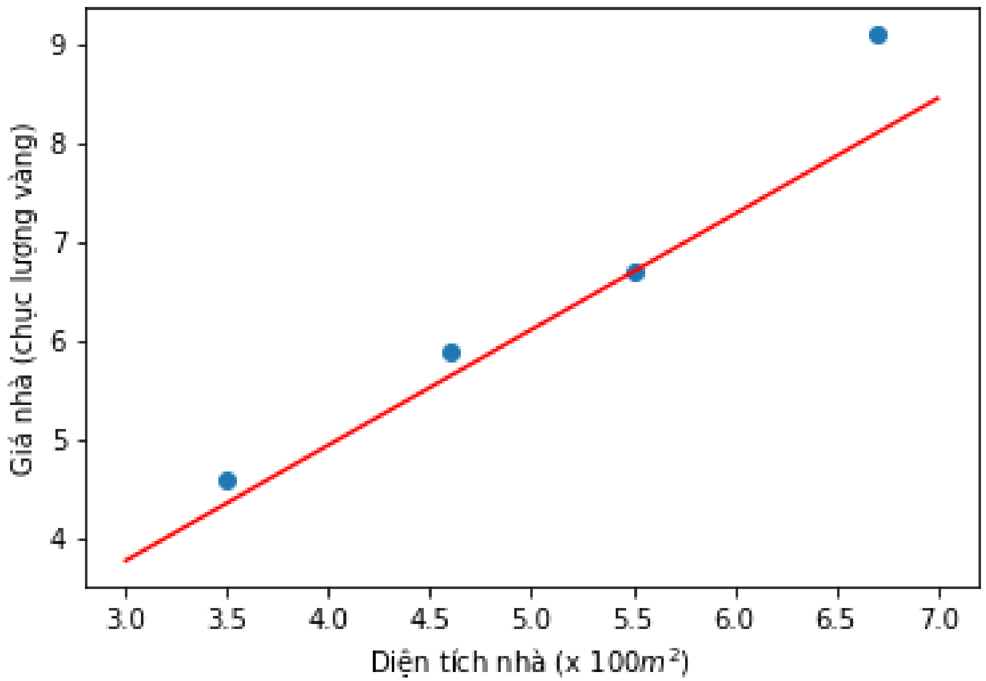
# Linear Regression



## ❖ Simple example

Model prediction before and after the first update




w = -0.34 b = 0.04 L = 128.55

Before updating

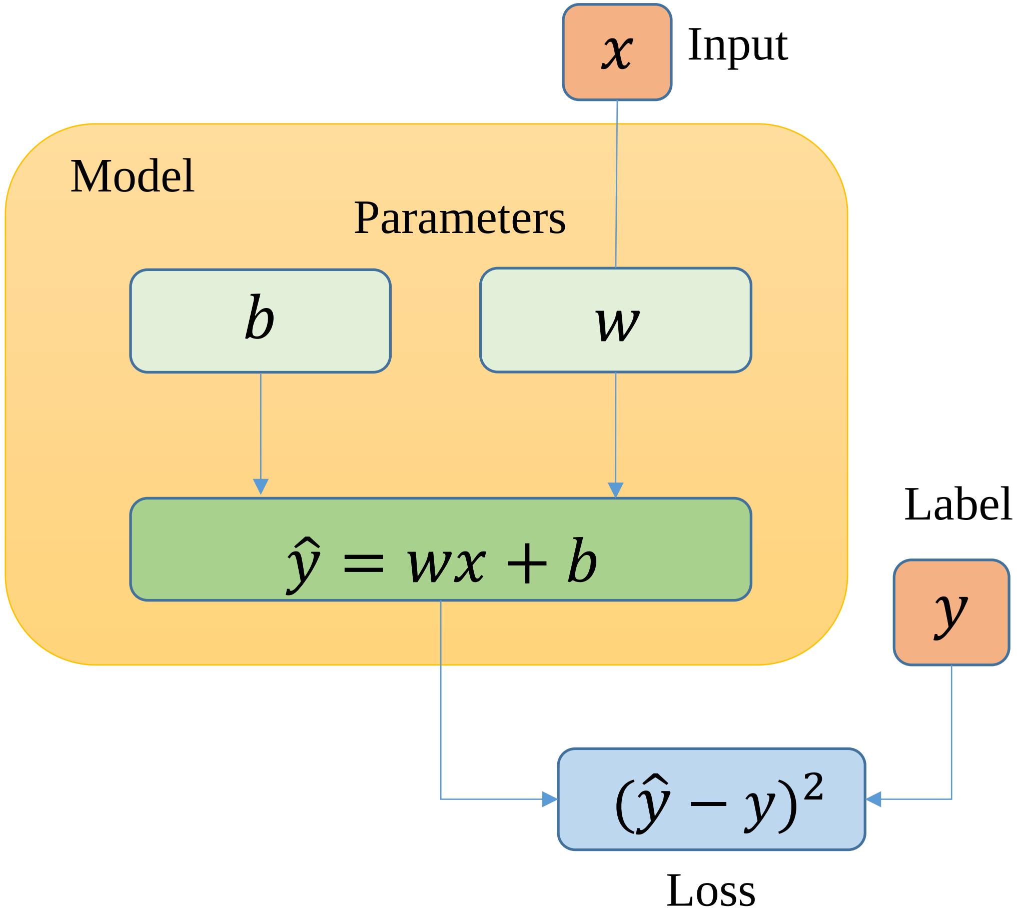
w = 1.179292 b = 0.26676 L = 0.868

After updating

# Linear Regression



## ❖ Summary (simple version)




1) Pick a sample $(x, y)$ from training data

2) Compute the output $\hat{y}$

$$\hat{y} = wx + b$$

3) Compute loss

$$L = (\hat{y} - y)^2$$

4) Compute derivative

$$\frac{\partial L}{\partial w} = 2x(\hat{y} - y) \qquad \frac{\partial L}{\partial b} = 2(\hat{y} - y)$$

5) Update parameters

$$w = w - \eta \frac{\partial L}{\partial w} \qquad b = b - \eta \frac{\partial L}{\partial b}$$

$\eta$ is learning rate

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# Linear Regression



## ❖ Implementation

### Cheat sheet

Compute the output $\hat{y}$

$$\hat{y} = wx + b$$

Compute the loss

$$L = (\hat{y} - y)^2$$

Compute derivative

$$\frac{\partial L}{\partial w} = 2x(\hat{y} - y)$$

$$\frac{\partial L}{\partial b} = 2(\hat{y} - y)$$

Update parameters

$$w = w - \eta \frac{\partial L}{\partial w}$$

$$b = b - \eta \frac{\partial L}{\partial b}$$

```python
# forward
def predict(x, w, b):
    return x*w + b

# compute gradient
def gradient(y_hat, y, x):
    dw = 2*x*(y_hat-y)
    db = 2*(y_hat-y)

    return (dw, db)

# update weights
def update_weight(w, b, lr, dw, db):
    w_new = w - lr*dw
    b_new = b - lr*db

    return (w_new, b_new)
```

# Computational Graph
# (A different viewpoint)

# Computational graph

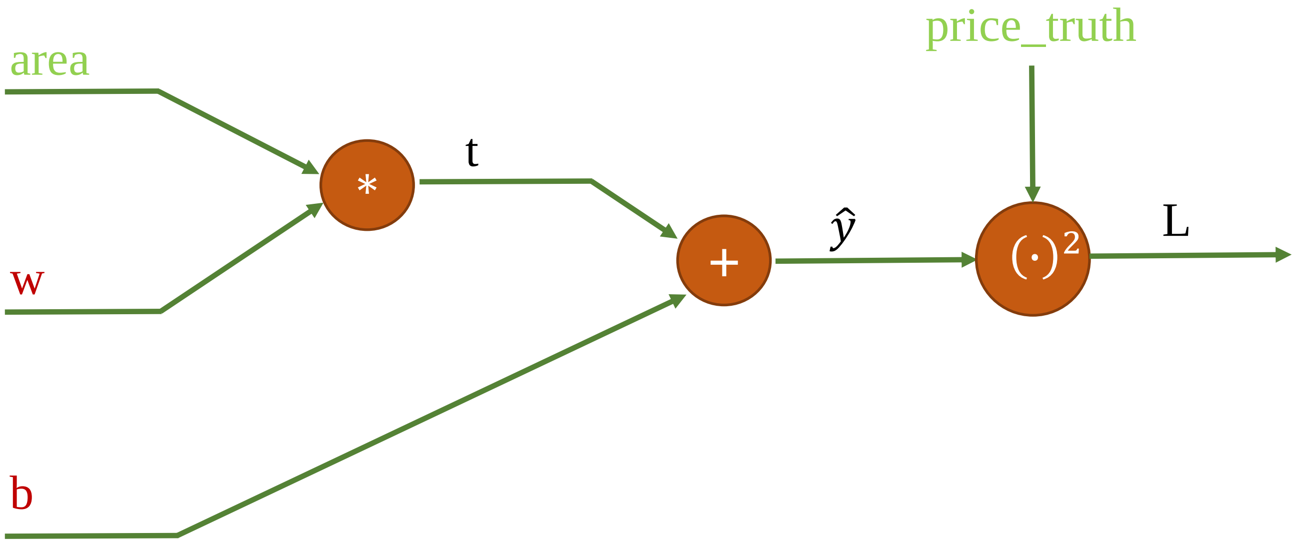


## ❖ House price prediction

### ❖ One-sample training

$$price = w * area + b$$

$$t = w * area$$



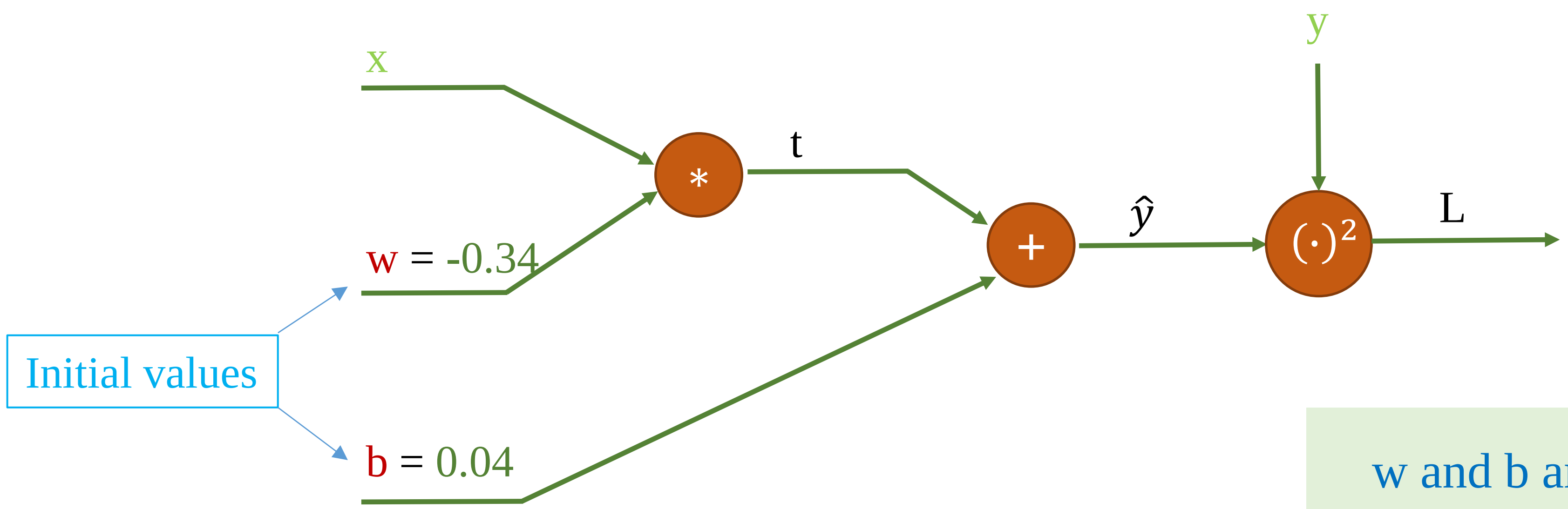
# Computational graph



## ❖ House price prediction

### ❖ One-sample training

$t = wx$ $\qquad \hat{y} = t + b$ $\qquad L = (\hat{y} - y)^2$




w and b are usually set by random values

For example, $N(0, \sigma)$ where $\sigma$ is small

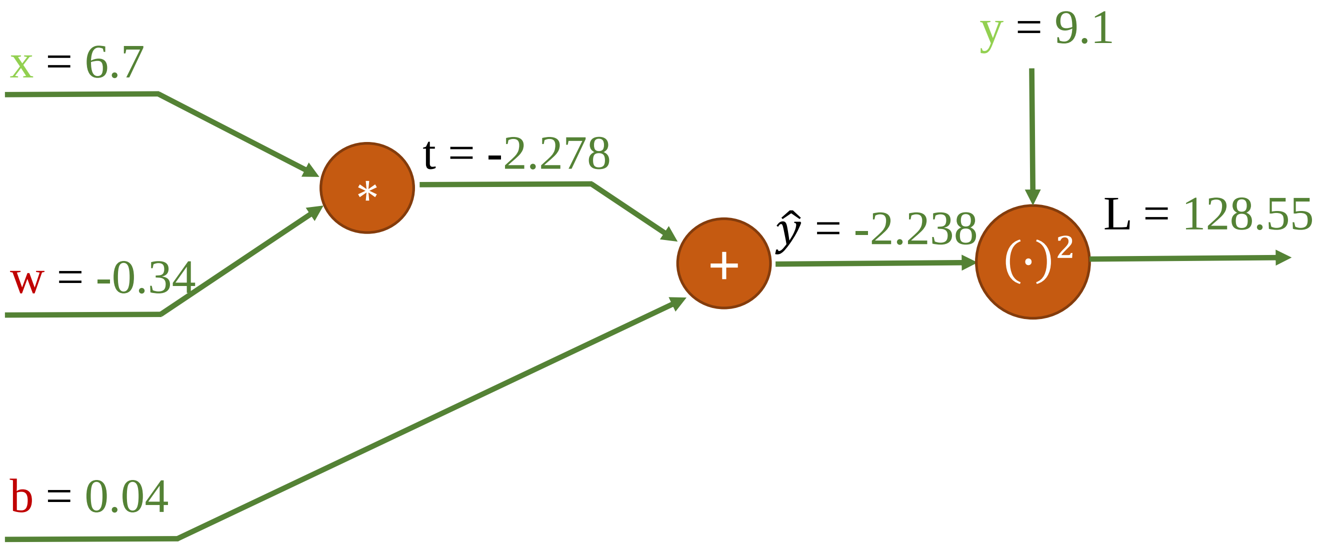
# Computational graph



## ❖ House price prediction

### ❖ One-sample training

$t = wx$ $\hat{y} = t + b$ $L = (\hat{y} - y)^2$




| Feature | Label |
|---|---|
| area | price |
| 6.7 | 9.1 |
| 4.6 | 5.9 |
| 3.5 | 4.6 |
| 5.5 | 6.7 |

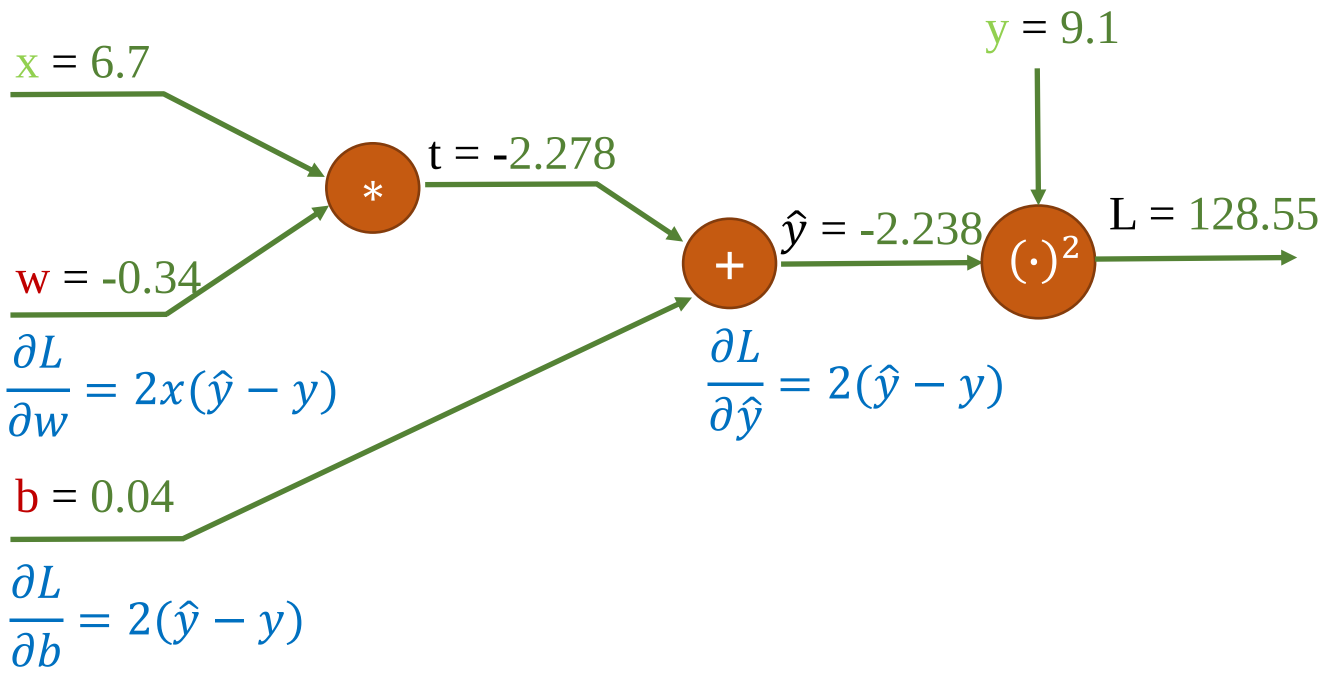
# Computational graph



## ❖ House price prediction

### ❖ One-sample training

$t = wx$ $\quad\quad$ $\hat{y} = t + b$ $\quad\quad$ $L = (\hat{y} - y)^2$

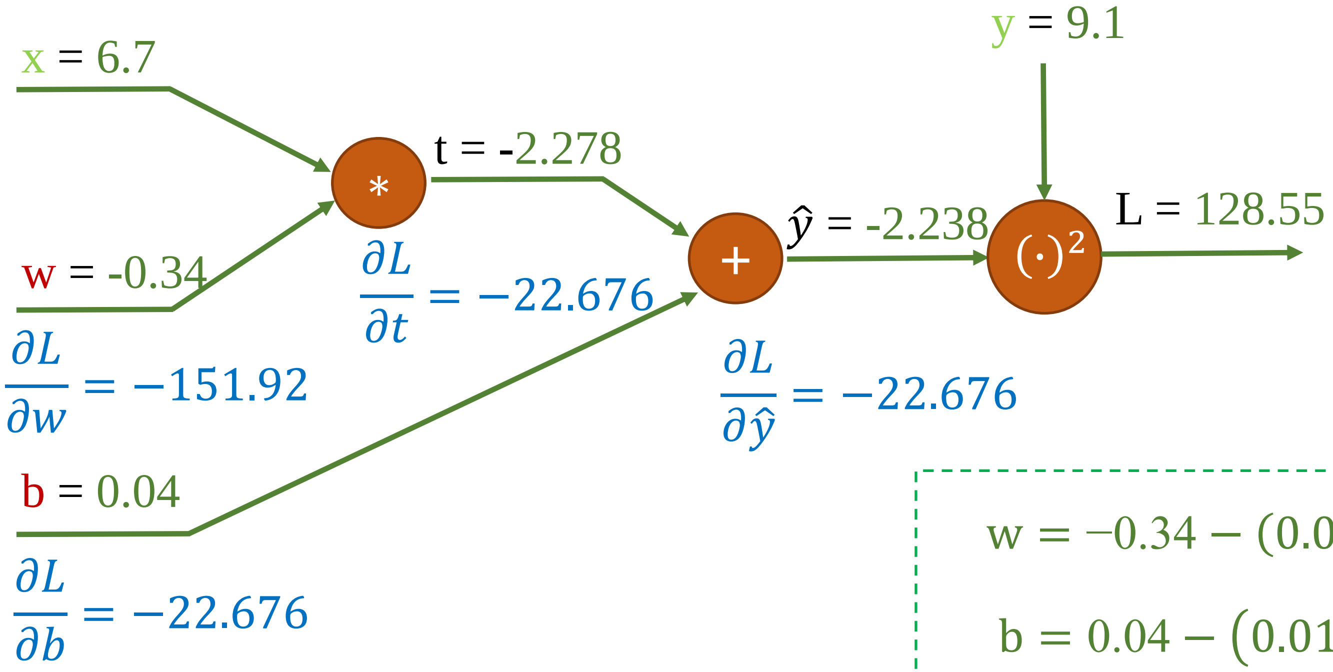


# Computational graph

## ❖ House price prediction

### ❖ One-sample training

$t = wx$ $\quad\quad$ $\hat{y} = t + b$ $\quad\quad$ $L = (\hat{y} - y)^2$




**Update w and b**

$$w = w - \eta * \frac{\partial L}{\partial w}$$

$$b = b - \eta * \frac{\partial L}{\partial b}$$

Learning rate $\eta = 0.01$

$$w = -0.34 - (0.01 * (-151.9)) = 1.179$$

$$b = 0.04 - \left(0.01 * (-22.67)\right) = 0.266$$

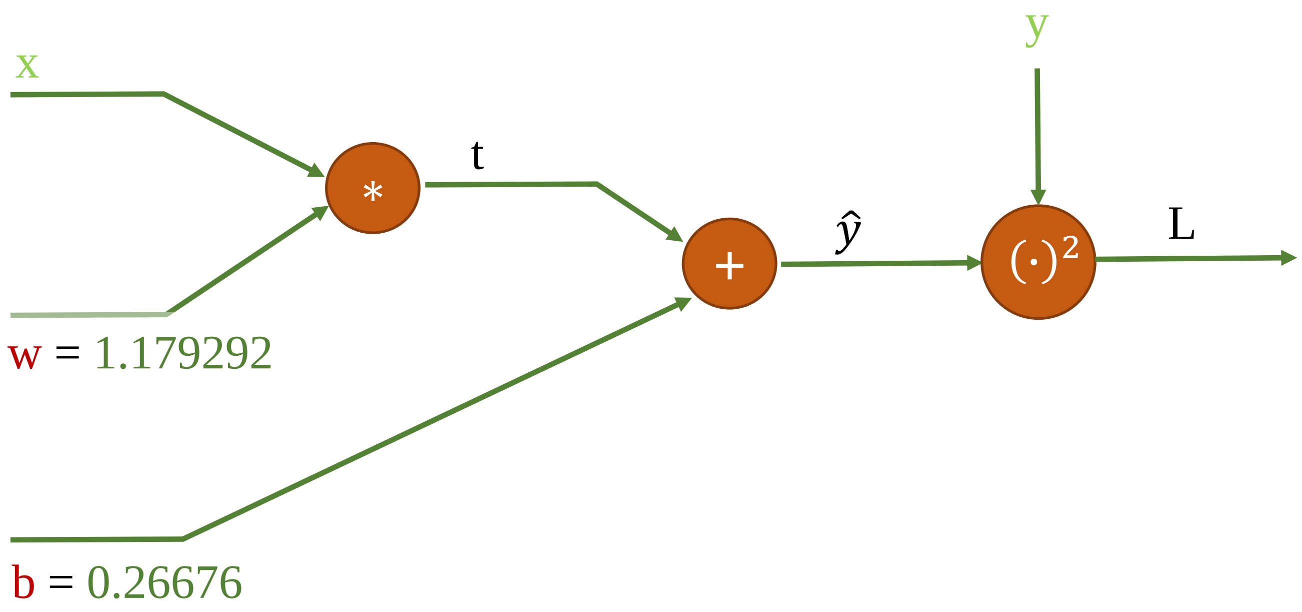
# Computational graph



## ❖ House price prediction

### ❖ One-sample training

$t = wx$ $\hat{y} = t + b$ $L = (\hat{y} - y)^2$




| Feature | Label |
|---|---|
| area | price |
| 6.7 | 9.1 |
| 4.6 | 5.9 |
| 3.5 | 4.6 |
| 5.5 | 6.7 |

# Computational graph

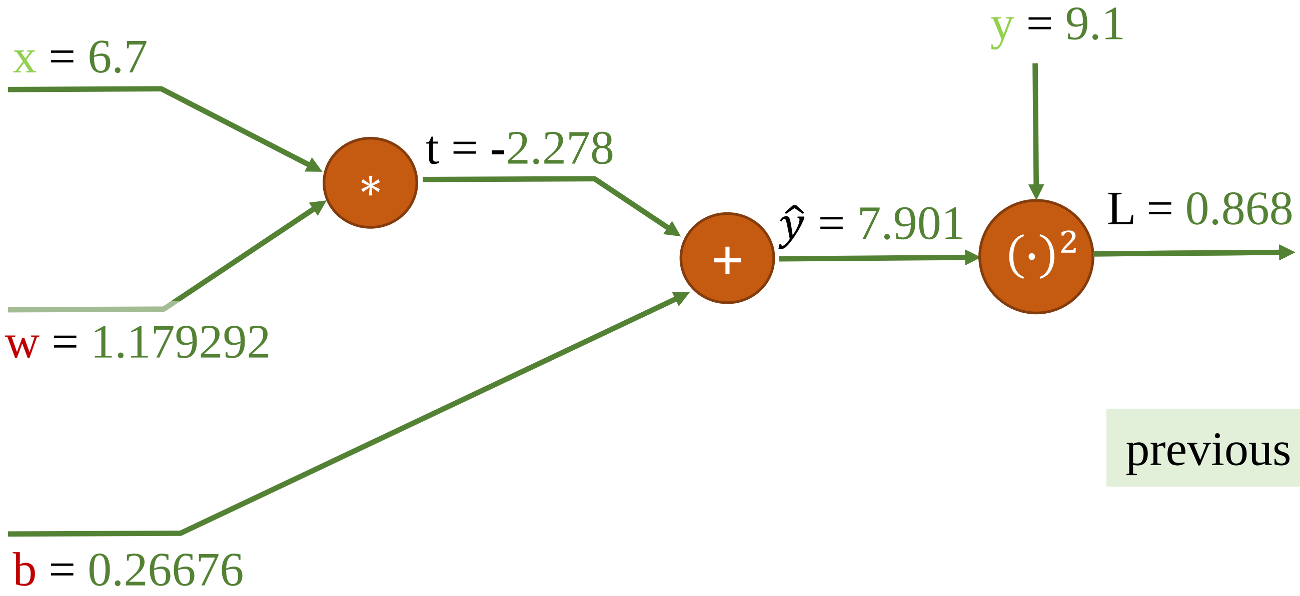


## ❖ House price prediction

### ❖ One-sample training

| Feature | Label |
|---|---|
| area | price |
| 6.7 | 9.1 |
| 4.6 | 5.9 |
| 3.5 | 4.6 |
| 5.5 | 6.7 |

$t = wx$ $\qquad \hat{y} = t + b$ $\qquad L = (\hat{y} - y)^2$




previous L = 128.55

Updated a and b values help to reduce the L value

# Implementation

❖ **One-sample training**

| Feature | Label |
|---|---|
| area | price |
| 6.7 | 9.1 |
| 4.6 | 5.9 |
| 3.5 | 4.6 |
| 5.5 | 6.7 |

column index=0 column index=1

```python
# data preparation
import numpy as np
import matplotlib.pyplot as plt

def get_column(data, index):
    result = [row[index] for row in data]
    return result

data = np.genfromtxt('data.csv',
                     delimiter=',').tolist()

x_data = get_column(data, 0)
y_data = get_column(data, 1)
N = len(x_data)

print(f'areas: {x_data}')
print(f'prices: {y_data}')
print(f'data_size: {N}')
```

```
areas: [6.7, 4.6, 3.5, 5.5]
prices: [9.1, 5.9, 4.6, 6.7]
data_size: 4
```

# Implementation



## ❖ House price prediction

### ❖ One-sample training

1) Pick a sample $(x, y)$ from training data

2) Compute the output $\hat{y}$

$$\hat{y} = wx + b$$

3) Compute loss

$$L = (\hat{y} - y)^2$$

4) Compute derivative

$$\frac{\partial L}{\partial w} = 2x(\hat{y} - y) \qquad \frac{\partial L}{\partial b} = 2(\hat{y} - y)$$

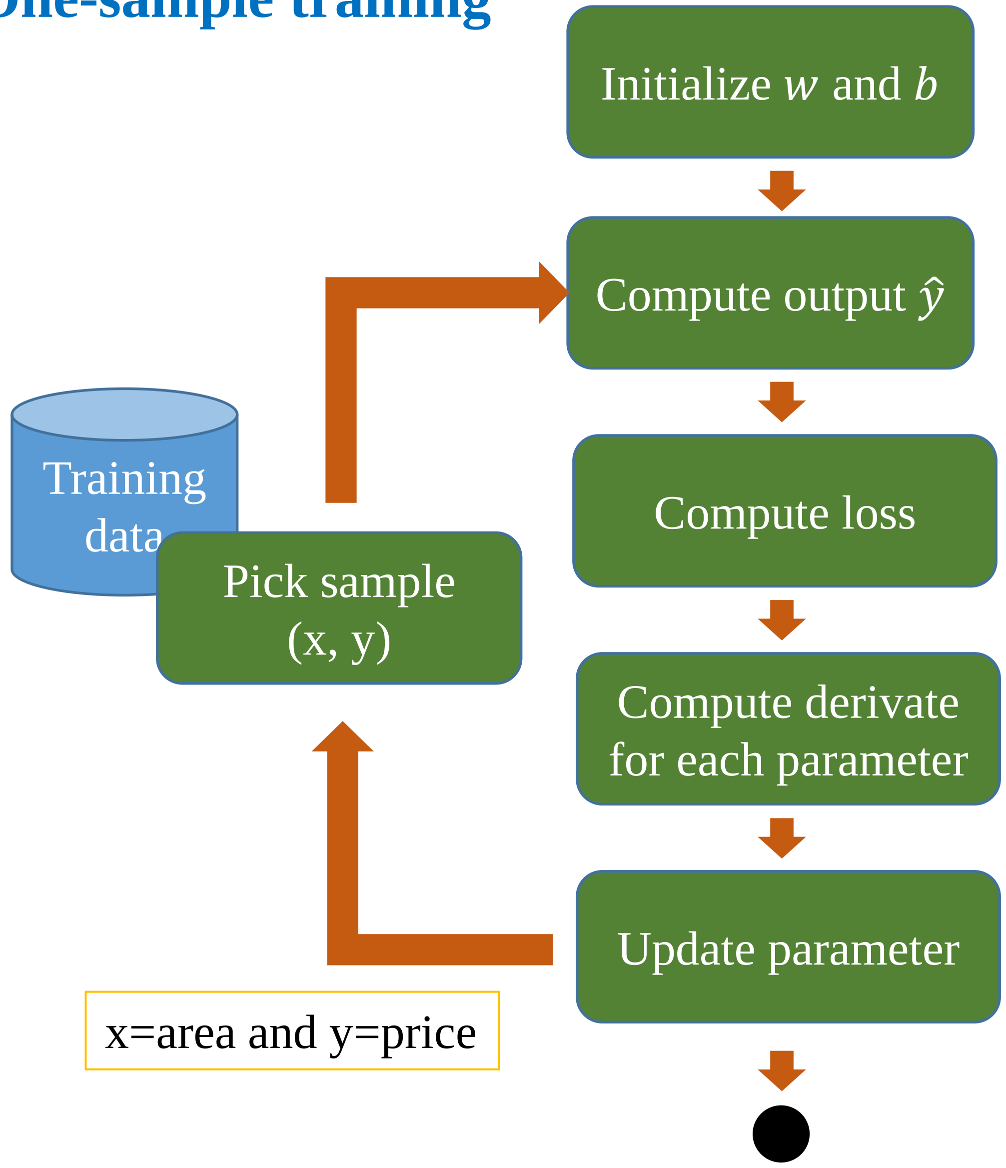
5) Update parameters

$$w = w - \eta \frac{\partial L}{\partial w} \qquad b = b - \eta \frac{\partial L}{\partial b}$$

```python
# forward
def predict(x, w, b):
    return x*w + b

# compute gradient
def gradient(y_hat, y, x):
    dw = 2*x*(y_hat-y)
    db = 2*(y_hat-y)

    return (dw, db)

# update weights
def update_weight(w, b, lr, dw, db):
    w_new = w - lr*dw
    b_new = b - lr*db

    return (w_new, b_new)
```

# Implementation

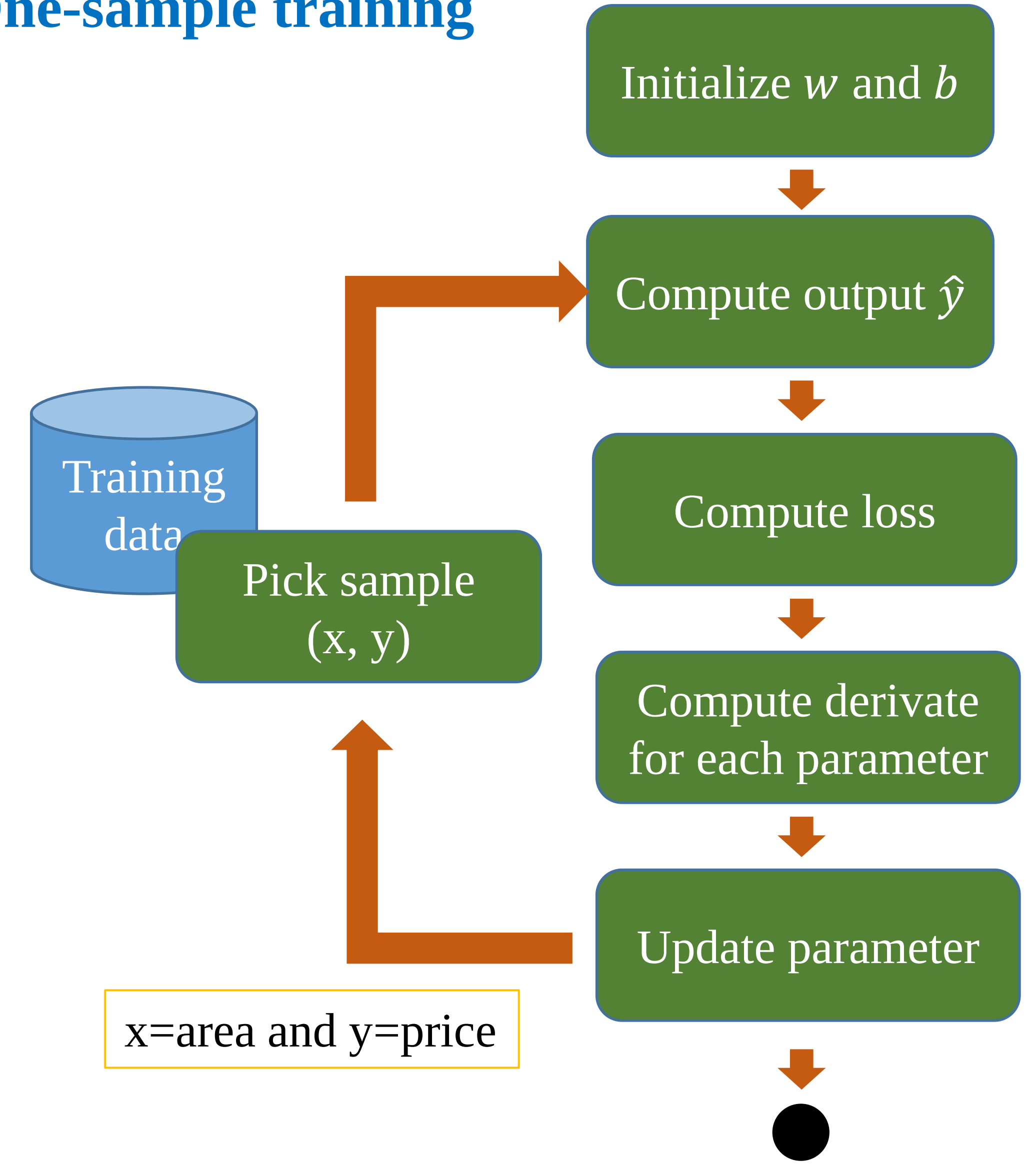
❖ **One-sample training**




```python
# init weights
b = 0.04
w = -0.34
lr = 0.01

# how long
epoch_max = 10

for epoch in range(epoch_max):
    for i in range(data_size):
        # get a sample
        # ...

        # predict y_hat
        # ...

        # compute loss
        # ...

        # compute gradient
        # ...

        # update weights
        # ...

```

# Implementation

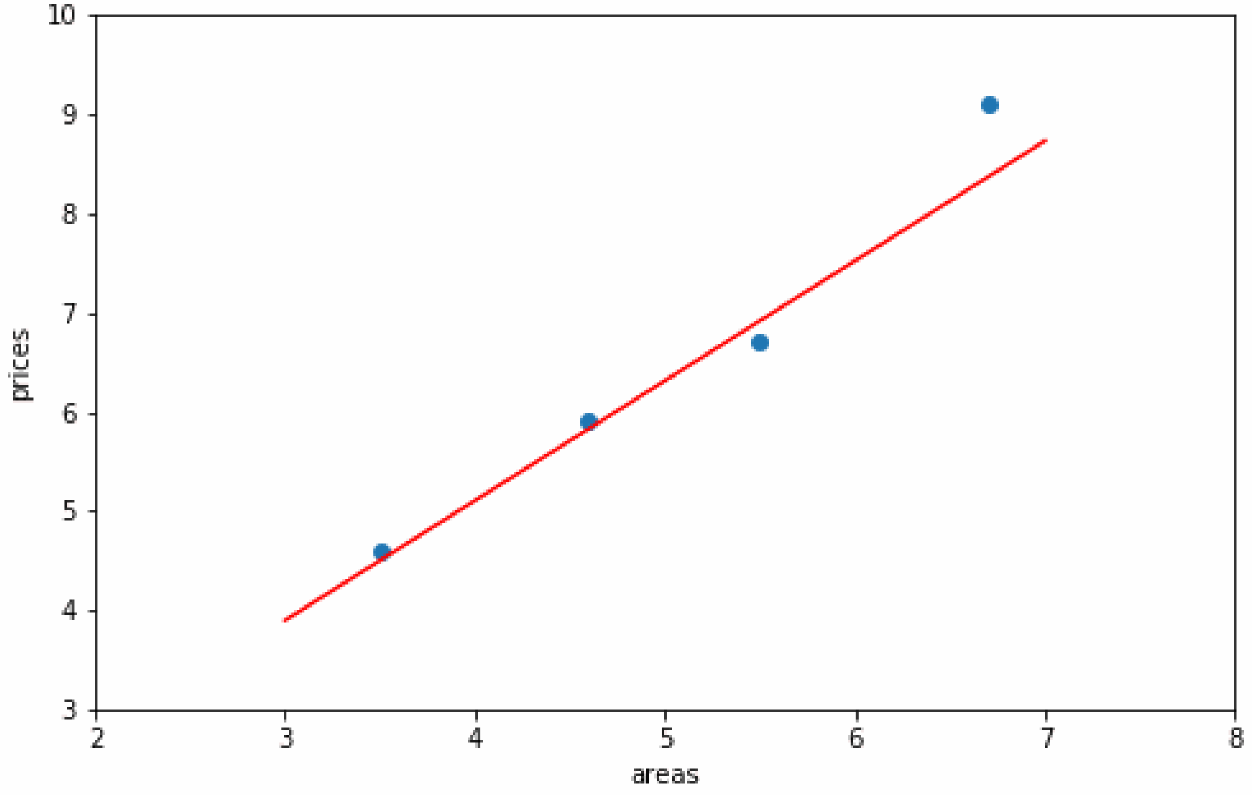
## ❖ One-sample training



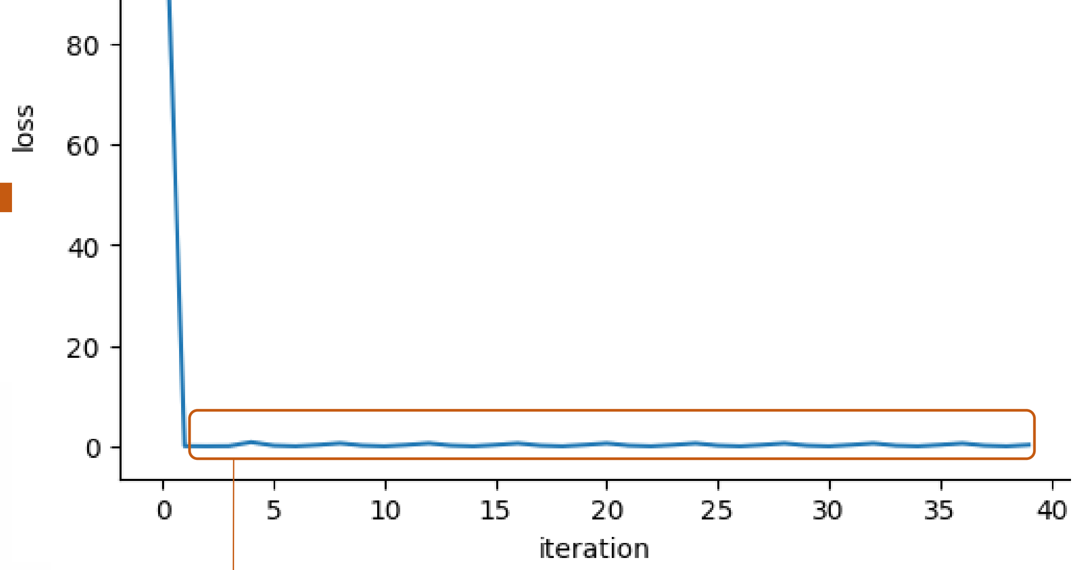

```python
# init weights
b = 0.04
w = -0.34
lr = 0.01

# how long
epoch_max = 10
data_size = 4

for epoch in range(epoch_max):
    for i in range(data_size):
        # get a sample
        x = areas[i]
        y = prices[i]

        # predict y_hat
        y_hat = predict(x, w, b)

        # compute loss
        loss = (y_hat-y)*(y_hat-y)

        # compute gradient
        (dw, db) = gradient(y_hat, y, x)

        # update weights
        (w, b) = update_weight(w, b, lr, dw, db)
```

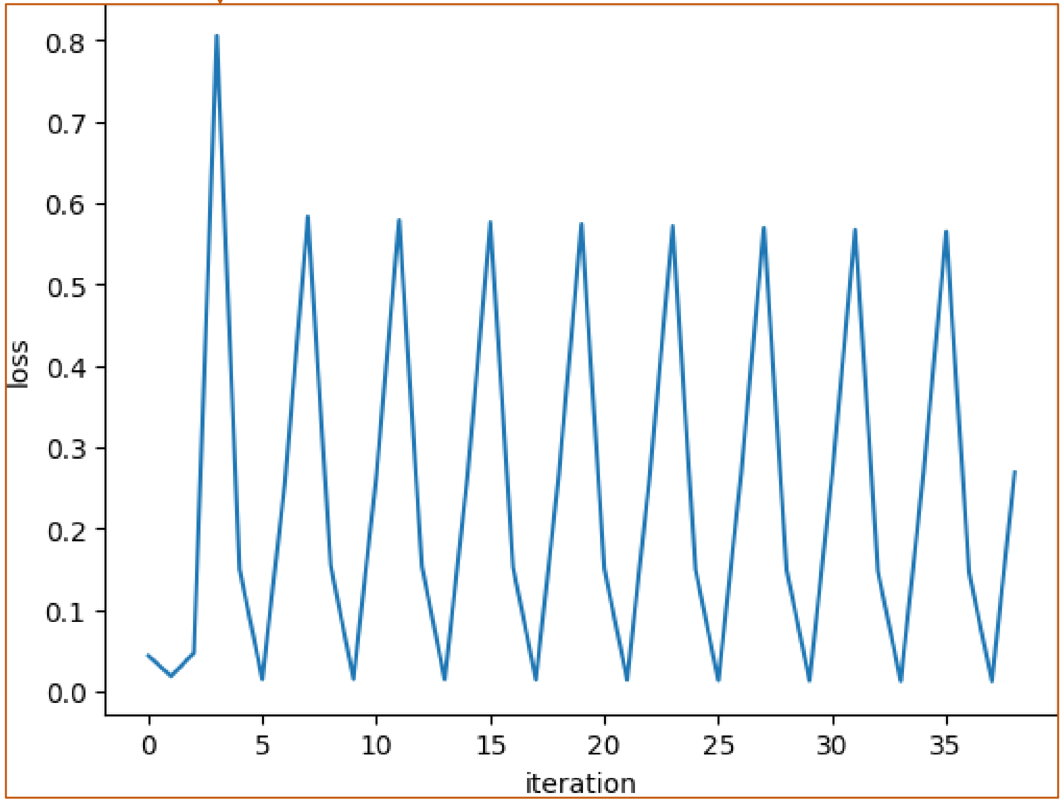
# Implementation

- **House price prediction**
  - **One-sample training**




Model training



## Quiz 1: Construct for the following data

| Features | | Label |
|---|---|---|
| Radio | Newspaper | Sales |
| 37.8 | 69.2 | 22.1 |
| 39.3 | 45.1 | 10.4 |
| 45.9 | 69.3 | 12 |
| 41.3 | 58.5 | 16.5 |
| 10.8 | 58.4 | 17.9 |

# Outline

SECTION 1
**Linear Regression**

SECTION 2
**Mini-batch Training**

SECTION 3
**Batch Training**

SECTION 4
**Loss Functions**

1) Pick m samples $(x^{(i)}, y^{(i)})$ from training data

2) Compute output $\hat{y}^{(i)}$

$$\hat{y}^{(i)} = wx^{(i)} + b \qquad \text{for } 0 \leq i < m$$

3) Compute loss

$$L^{(i)} = (\hat{y}^{(i)} - y^{(i)})^2 \quad \text{for } 0 \leq i < m$$

4) Compute derivatives

$$\frac{\partial L^{(i)}}{\partial w} = 2x^{(i)}(\hat{y}^{(i)} - y^{(i)})$$
$$\frac{\partial L^{(i)}}{\partial b} = 2(\hat{y}^{(i)} - y^{(i)}) \qquad \text{for } 0 \leq i < m$$

5) Update

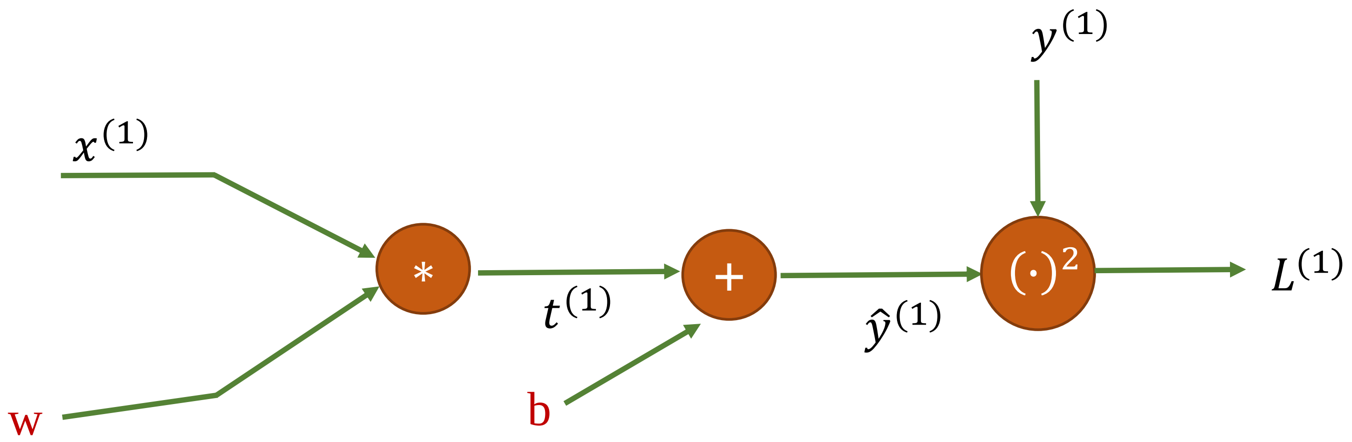
$$w = w - \eta \frac{\sum_i \frac{\partial L^{(i)}}{\partial w}}{m} \qquad b = b - \eta \frac{\sum_i \frac{\partial L^{(i)}}{\partial b}}{m}$$

# Computational graph



## ❖ Compute derivate for w and b

mini-batch m = 2

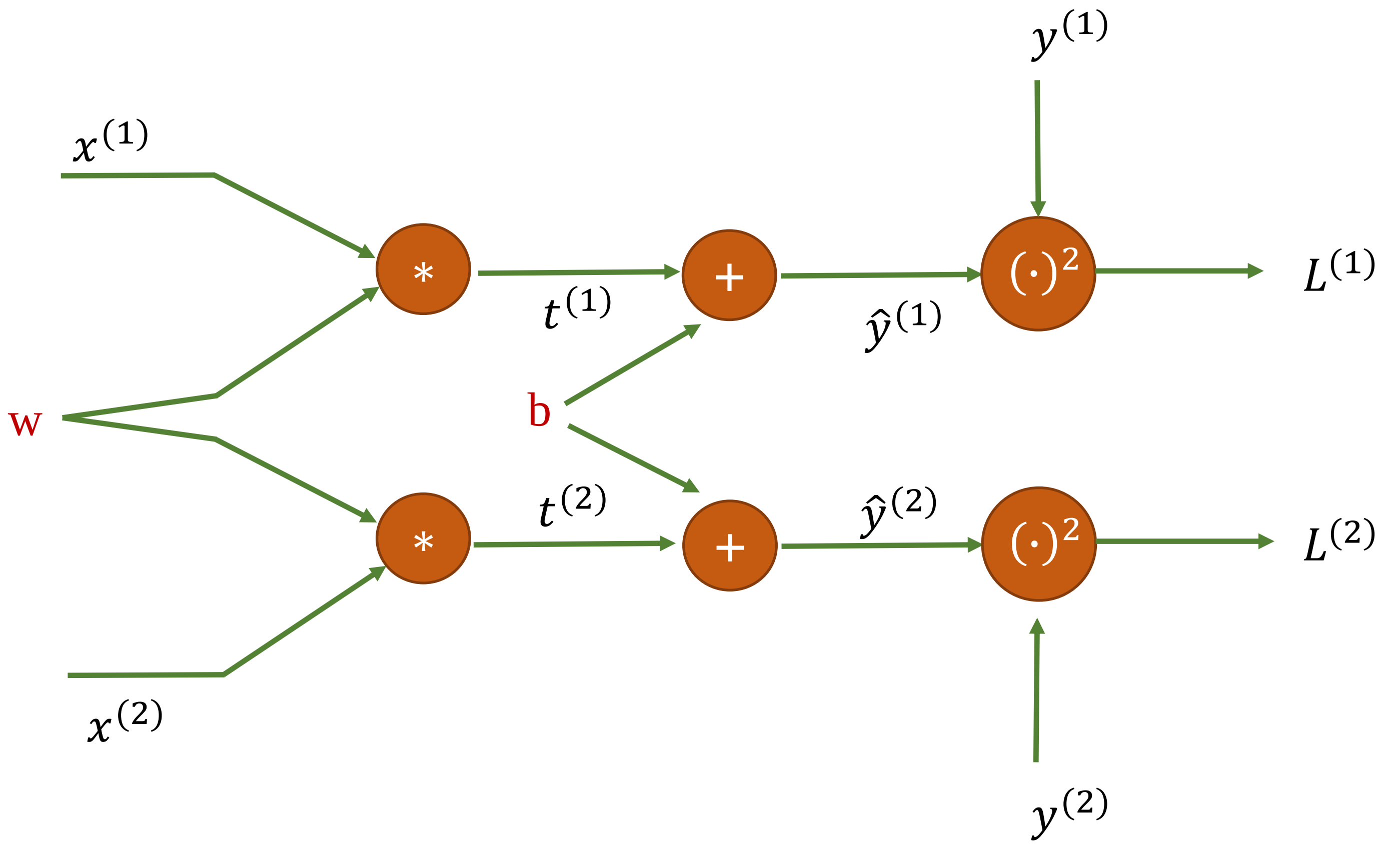


# Computational graph



## ❖ Compute derivate for w and b

mini-batch m = 2

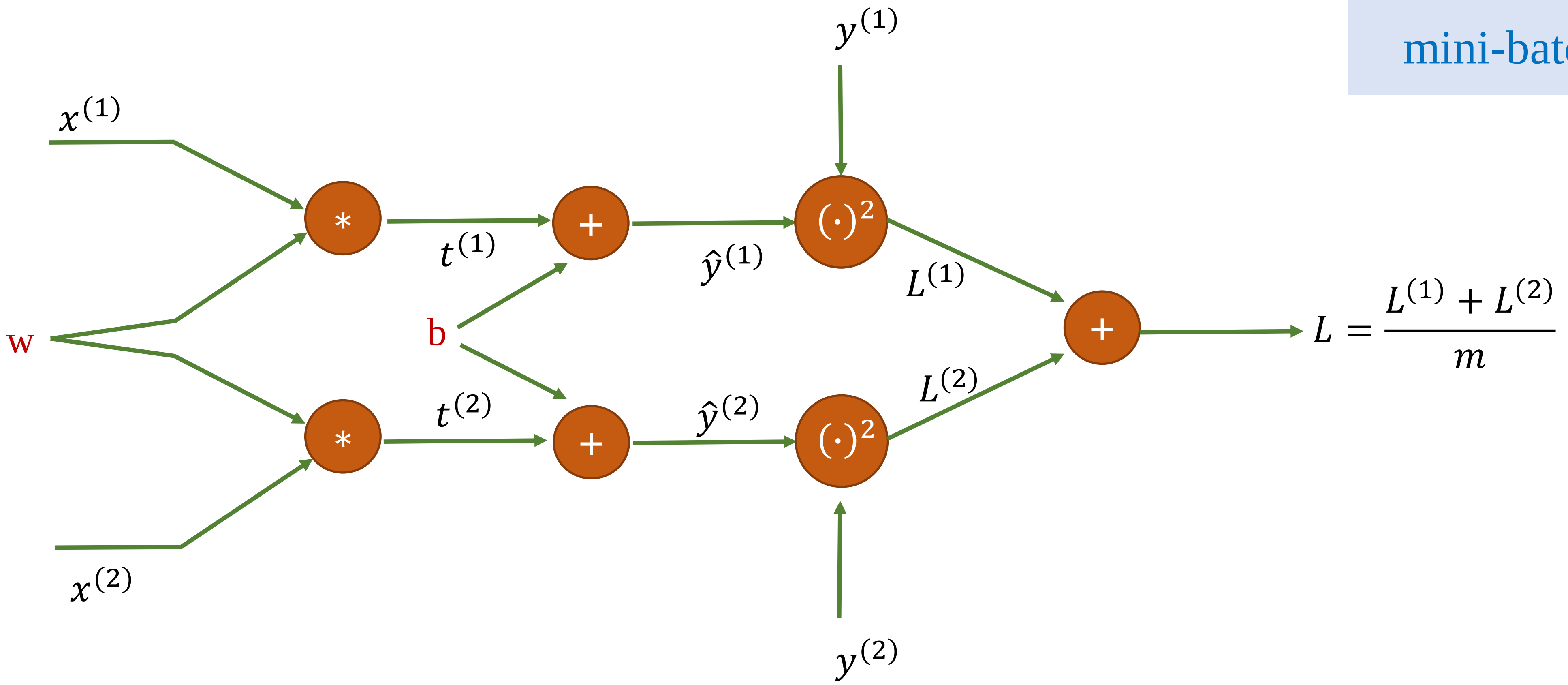


# Computational graph



## ❖ Compute derivate for w and b

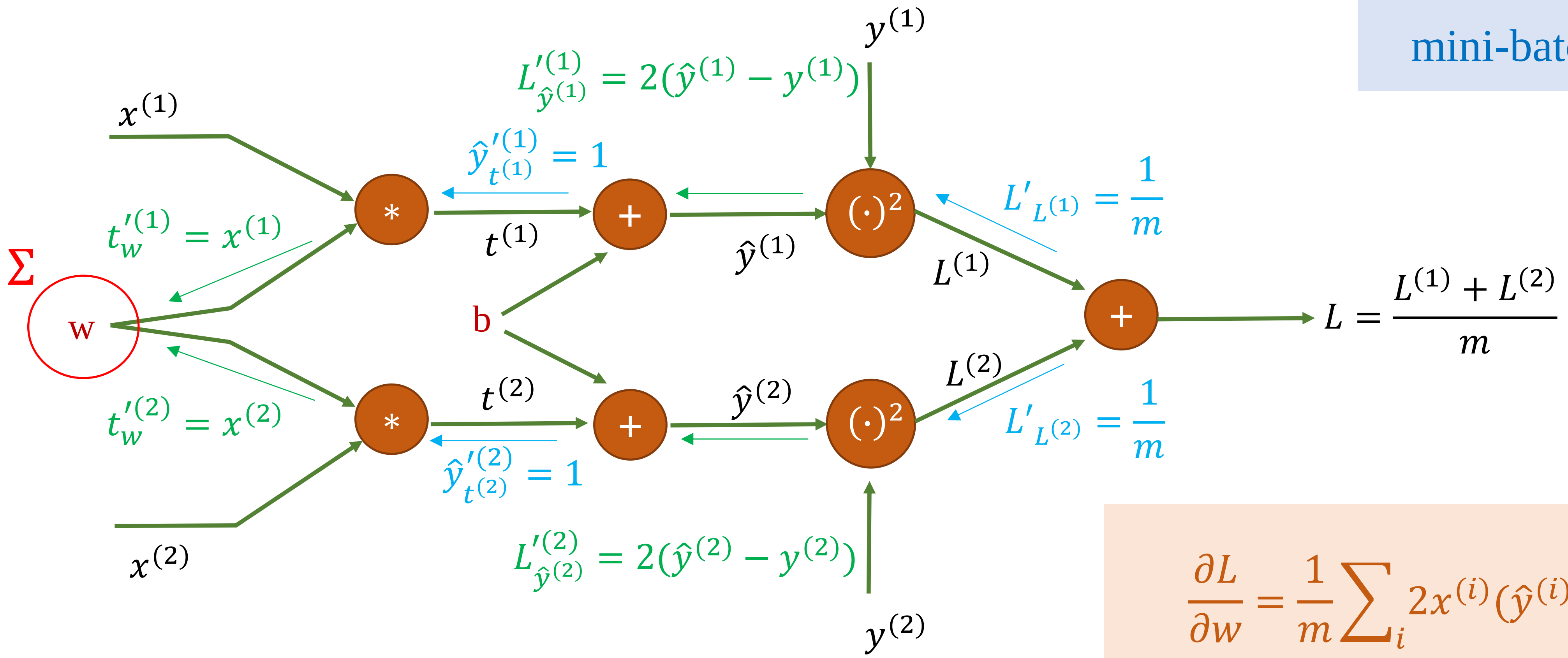
mini-batch m = 2



# Computational graph



## ❖ Compute derivate for w and b




mini-batch m = 2

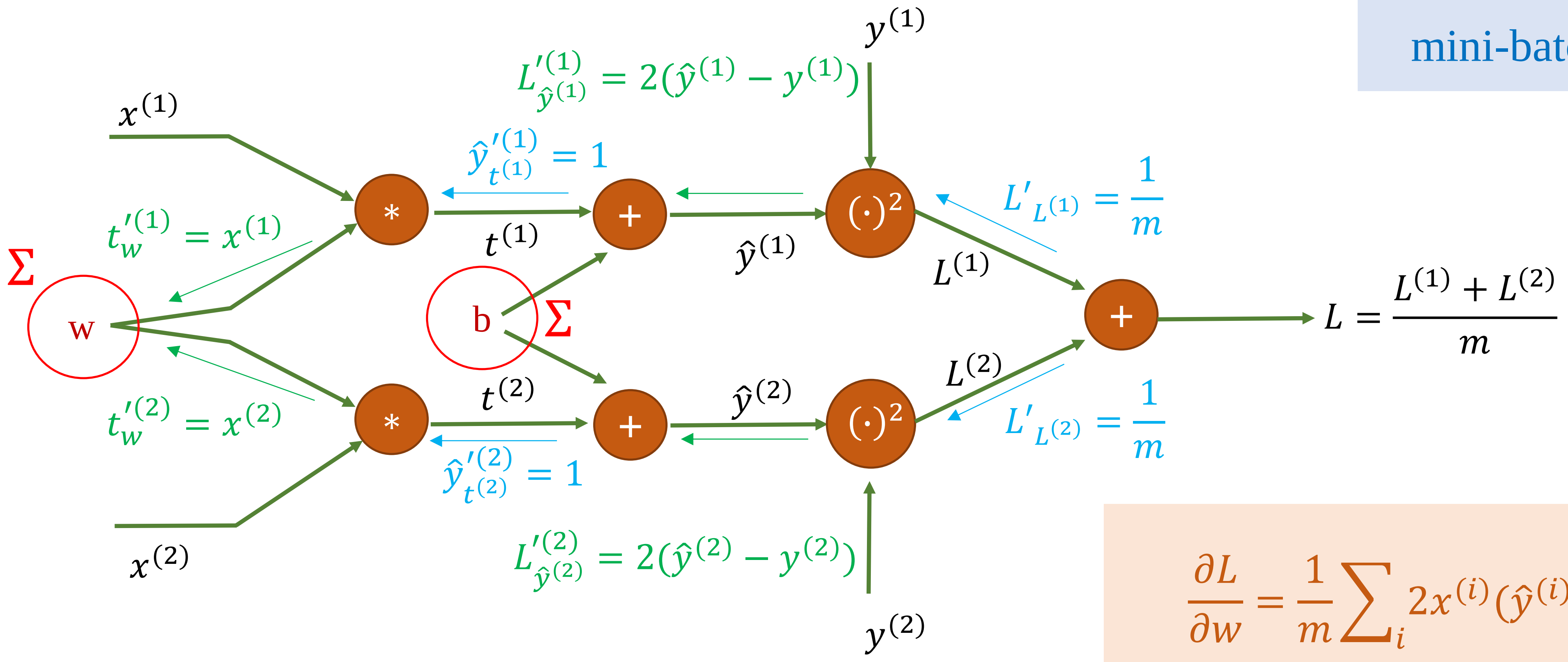
$$\frac{\partial L}{\partial w} = \frac{1}{m}\sum_i 2x^{(i)}(\hat{y}^{(i)} - y^{(i)})$$

# Computational graph



## ❖ Compute derivate for w and b

mini-batch m = 2




$$\frac{\partial L}{\partial w} = \frac{1}{m}\sum_i 2x^{(i)}(\hat{y}^{(i)} - y^{(i)})$$

$$\frac{\partial L}{\partial b} = \frac{1}{m}\sum_i 2(\hat{y}^{(i)} - y^{(i)})$$

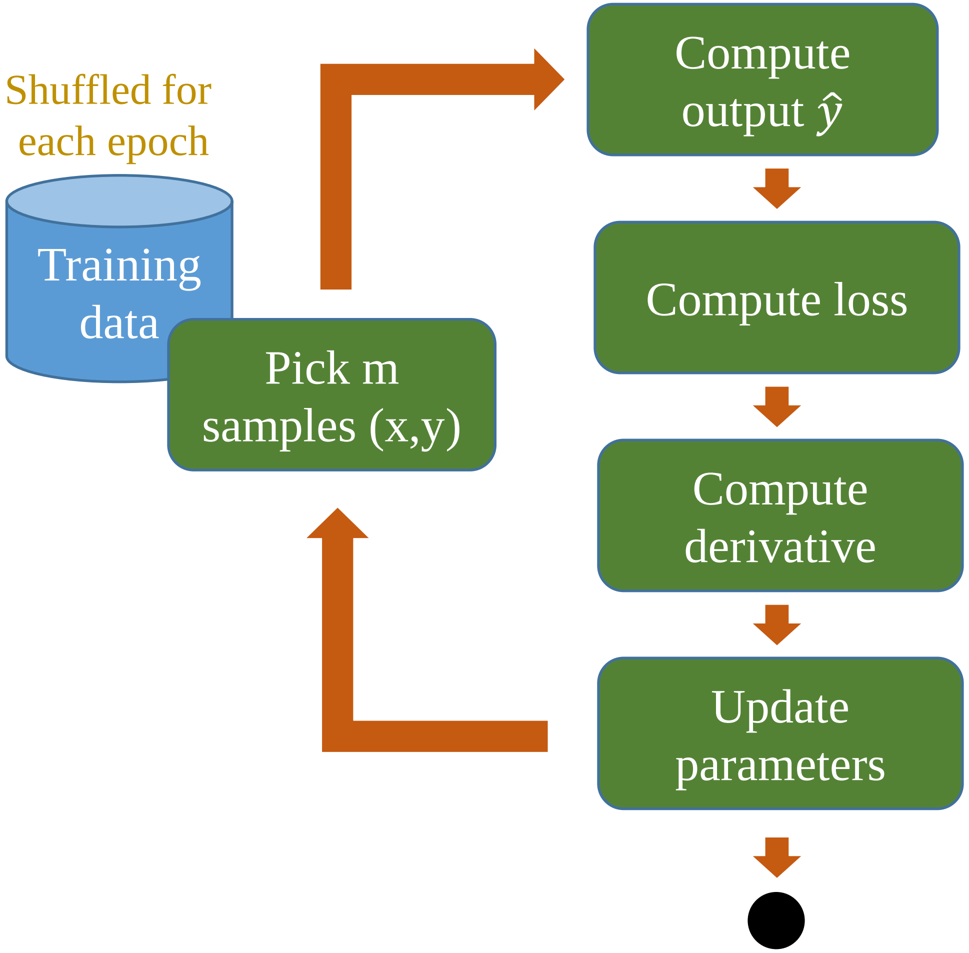
# Computational graph

simplified



## ❖ House price prediction

### ❖ m-sample training (1<m<N)




1) Pick m samples $(x^{(i)}, y^{(i)})$ from training data

2) Compute output $\hat{y}^{(i)}$

$$\hat{y}^{(i)} = wx^{(i)} + b \qquad \text{for } 0 \le i < m$$

3) Compute loss

$$L = \frac{1}{m}\sum_i (\hat{y}^{(i)} - y^{(i)})^2$$

4) Compute derivatives

$$\frac{\partial L^{(i)}}{\partial w} = 2x^{(i)}(\hat{y}^{(i)} - y^{(i)})$$

$$\frac{\partial L^{(i)}}{\partial b} = 2(\hat{y}^{(i)} - y^{(i)})$$

for $0 \le i < m$

5) Update

$$w = w - \eta \frac{\sum_i \frac{\partial L^{(i)}}{\partial w}}{m} \qquad b = b - \eta \frac{\sum_i \frac{\partial L^{(i)}}{\partial b}}{m}$$

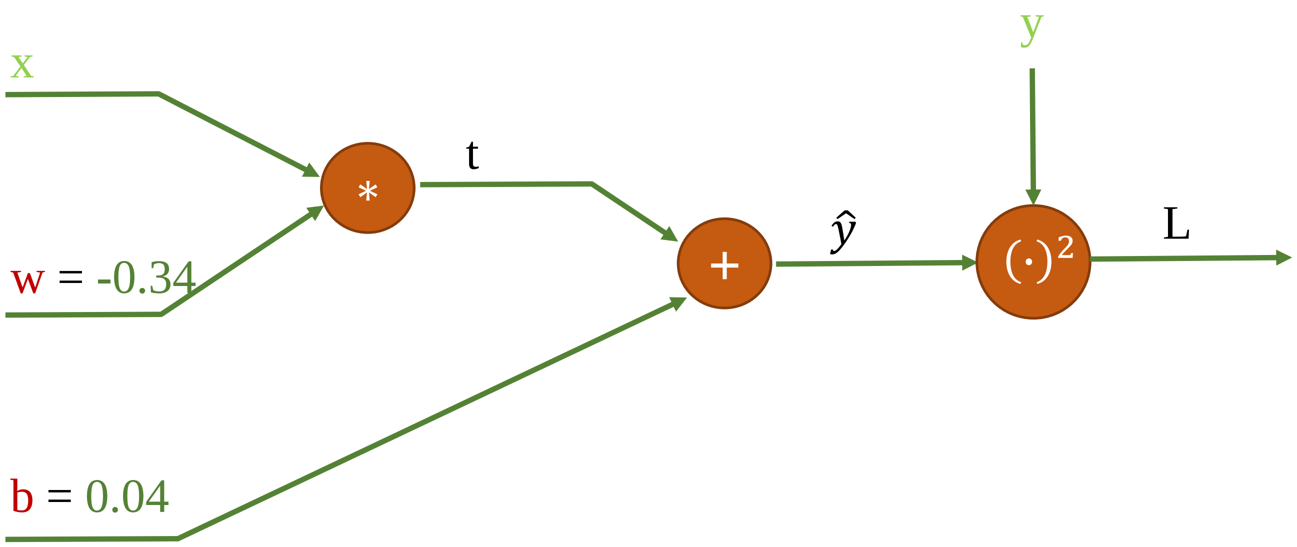
# Computational graph



## ❖ House price prediction

### ❖ m-sample training (1<m<N)

$t = wx$ $\hat{y} = t + b$ $L = (\hat{y} - y)^2$

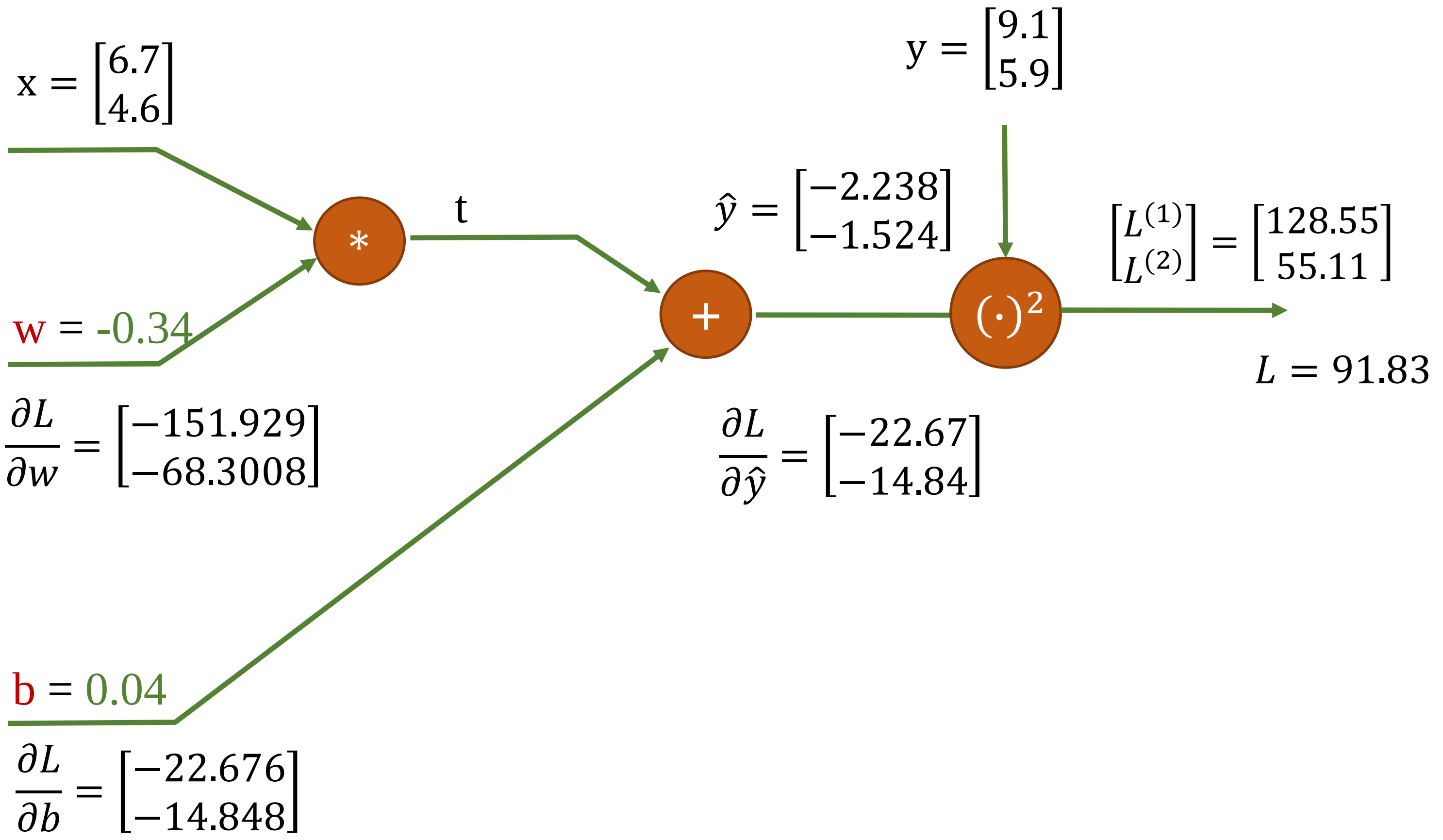


# Computational graph

- **House price prediction**
  - **m-sample training (1<m<N)**

| | Feature | Label |
|---|---|---|
| | area | price |
| | 6.7 | 9.1 |
| | 4.6 | 5.9 |
| | 3.5 | 4.6 |
| | 5.5 | 6.7 |

m = 2




$$\frac{sum(L'_w)}{m} = -110.115$$

$$\frac{sum(L'_b)}{m} = -18.762$$

$$t = wx \qquad \hat{y} = t + b \qquad L = (\hat{y} - y)^2$$

# Computational graph

- **House price prediction**
  - **m-sample training (1<m<N)**

| Feature | Label |
| --- | --- |
| area | price |
| 6.7 | 9.1 |
| 4.6 | 5.9 |
| 3.5 | 4.6 |
| 5.5 | 6.7 |

m = 2

**Update w and b**

$$w = w - \eta * \frac{\partial L}{\partial w}$$

$$b = b - \eta * \frac{\partial L}{\partial b}$$

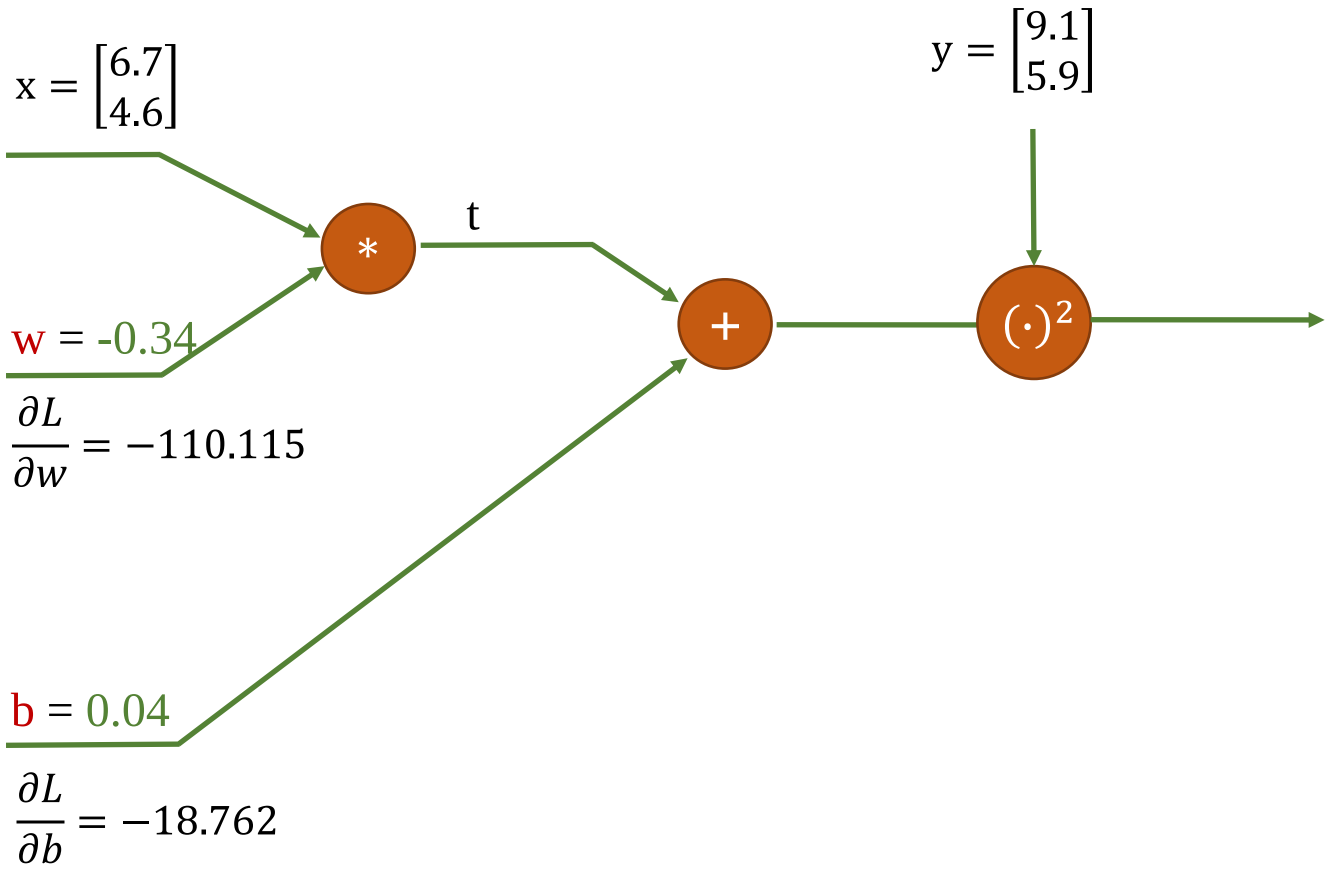
Learning rate $\eta = 0.01$

$$w = -0.34 - (0.01 * (-110.115)) = \boxed{0.761}$$

replace

$$b = 0.04 - \big(0.01 * (-18.762)\big) = \boxed{0.227}$$

replace



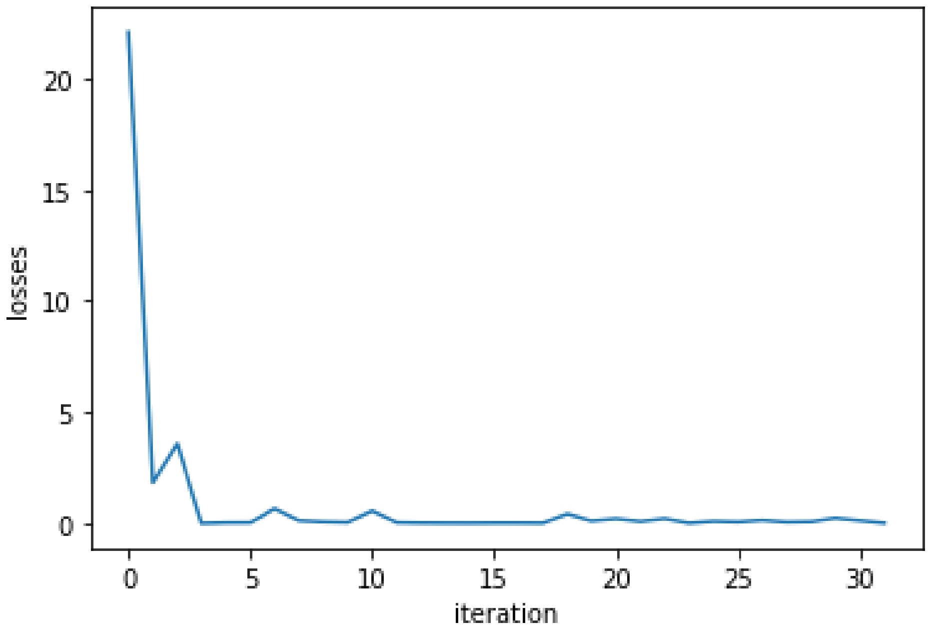

$t = wx$ $\quad \hat{y} = t + b$ $\quad L = (\hat{y} - y)^2$

# Computational graph

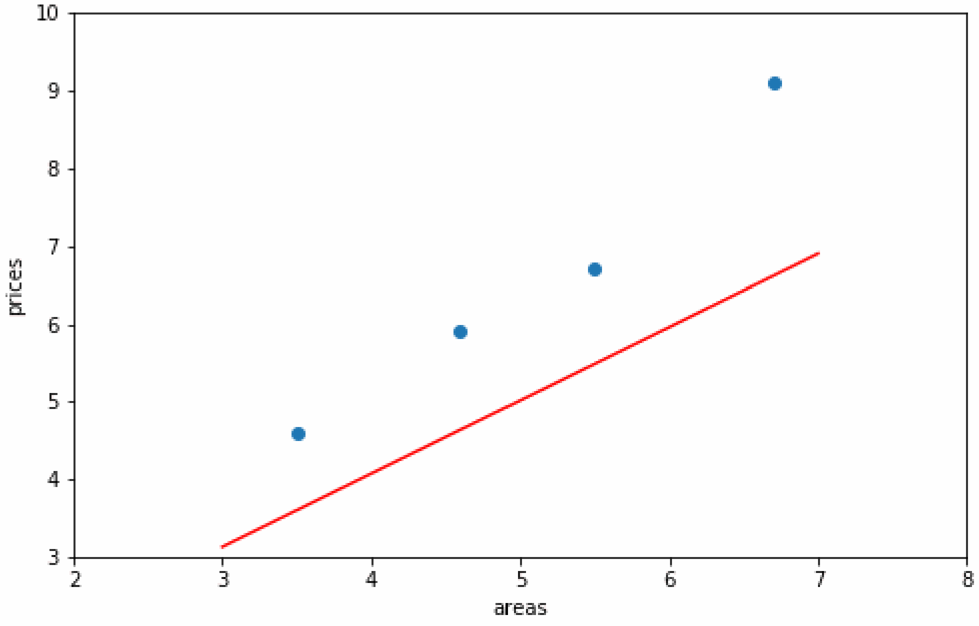


## ❖ House price prediction

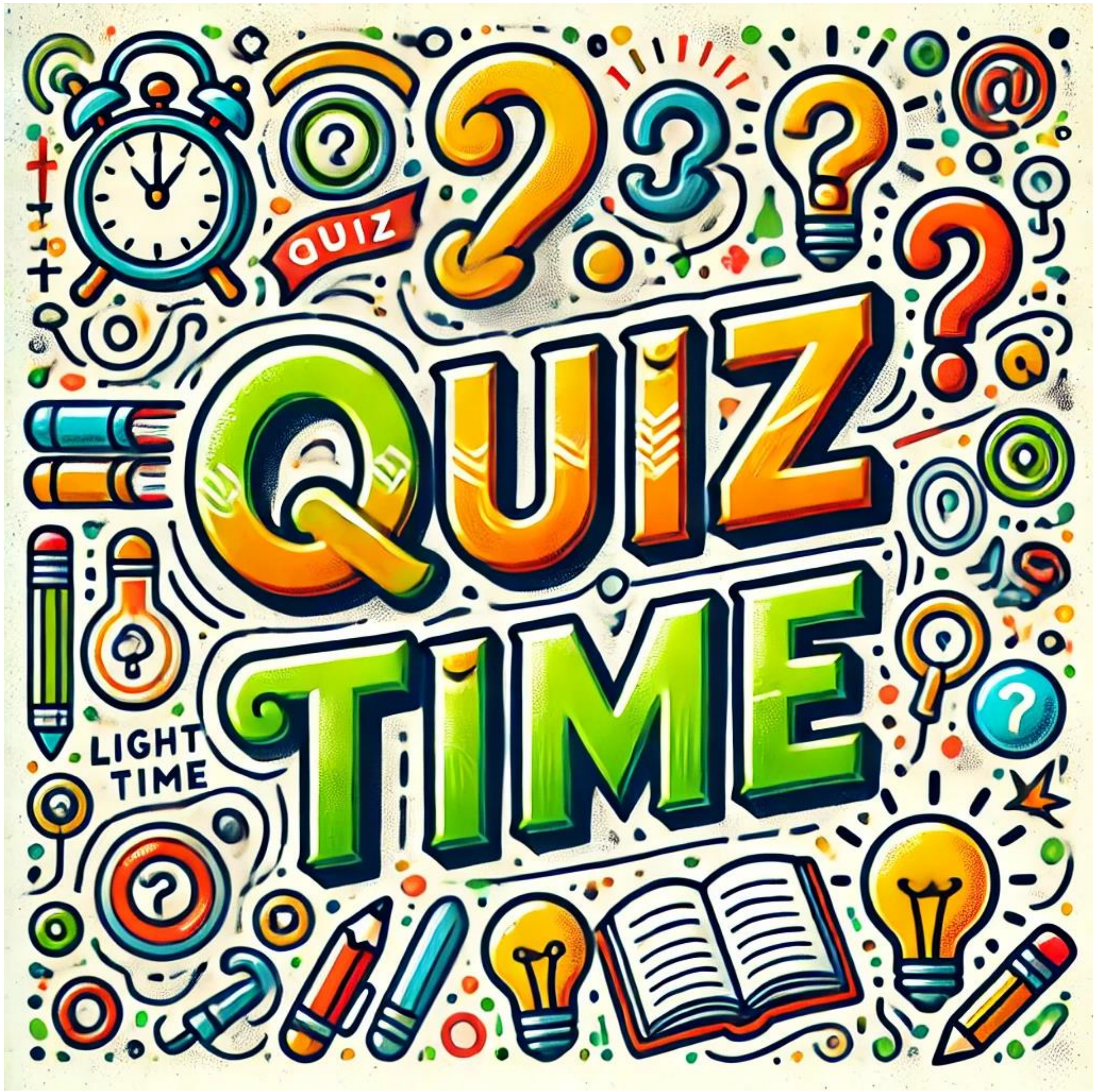
### ❖ m-sample training (1<m<N)




Losses for 30 iterations




Model updating for different iterations


QUIZ
QUIZ
TIME
LIGHT
TIME

# Outline

SECTION 1
## Linear Regression

SECTION 2
## Mini-batch Training

SECTION 3
## Batch Training

SECTION 4
## Loss Functions

1) Pick all the N samples $(x^{(i)}, y^{(i)})$ from training data

2) Compute output $\hat{y}^{(i)}$

$$\hat{y}^{(i)} = wx^{(i)} + b \qquad \text{for } 0 \le i < N$$

3) Compute loss

$$L^{(i)} = (\hat{y}^{(i)} - y^{(i)})^2 \quad \text{for } 0 \le i < N$$

4) Compute derivatives

$$\frac{\partial L^{(i)}}{\partial w} = 2x^{(i)}(\hat{y}^{(i)} - y^{(i)})$$

$$\frac{\partial L^{(i)}}{\partial b} = 2(\hat{y}^{(i)} - y^{(i)})$$

for $0 \le i < N$

5) Update

$$w = w - \eta \frac{\sum_i \frac{\partial L^{(i)}}{\partial w}}{N} \qquad b = b - \eta \frac{\sum_i \frac{\partial L^{(i)}}{\partial b}}{N}$$

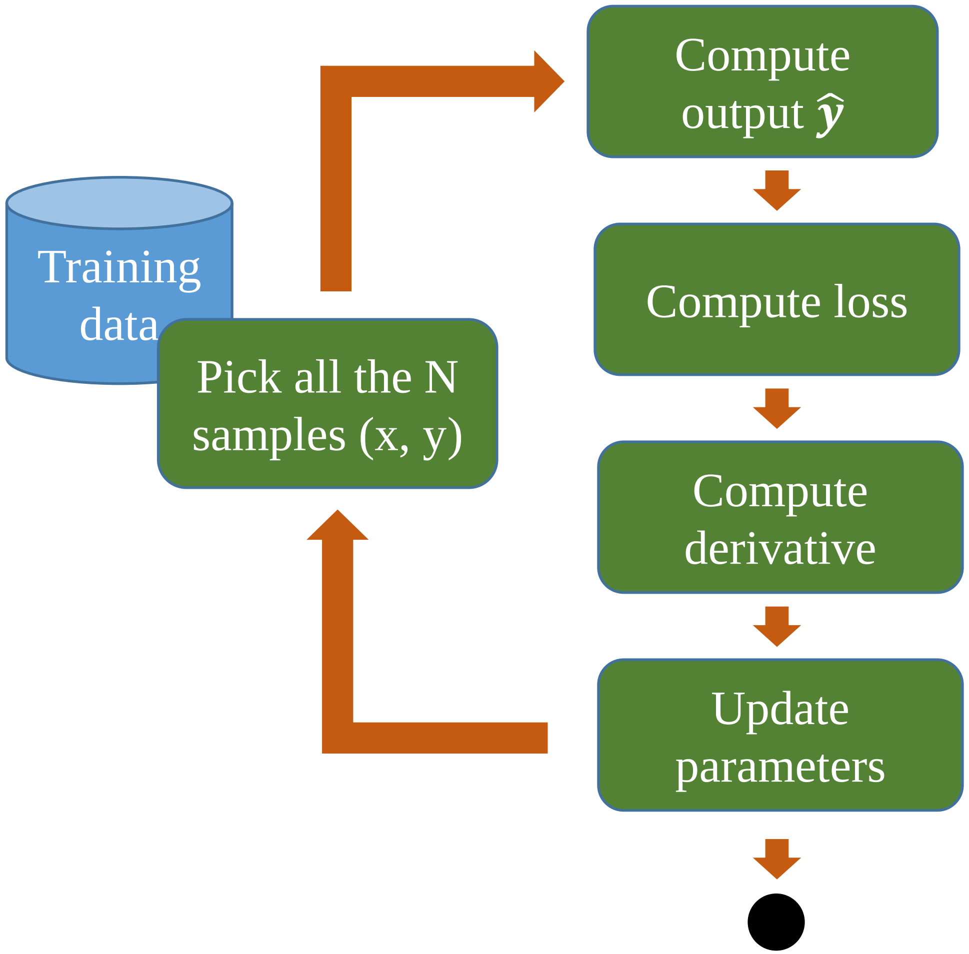
# Computational graph

simplified



## ❖ House price prediction

### ❖ N-sample training




1) Pick all the N samples $(x^{(i)}, y^{(i)})$ from training data

2) Compute output $\hat{y}^{(i)}$

$$\hat{y}^{(i)} = wx^{(i)} + b \qquad \text{for } 0 \leq i < N$$

3) Compute loss

$$L = \frac{1}{N}\sum_i (\hat{y}^{(i)} - y^{(i)})^2$$

4) Compute derivatives

$$\frac{\partial L^{(i)}}{\partial w} = 2x^{(i)}(\hat{y}^{(i)} - y^{(i)})$$

$$\frac{\partial L^{(i)}}{\partial b} = 2(\hat{y}^{(i)} - y^{(i)})$$

for $0 \leq i < N$

5) Update

$$w = w - \eta \frac{\sum_i \frac{\partial L^{(i)}}{\partial w}}{N} \qquad b = b - \eta \frac{\sum_i \frac{\partial L^{(i)}}{\partial b}}{N}$$

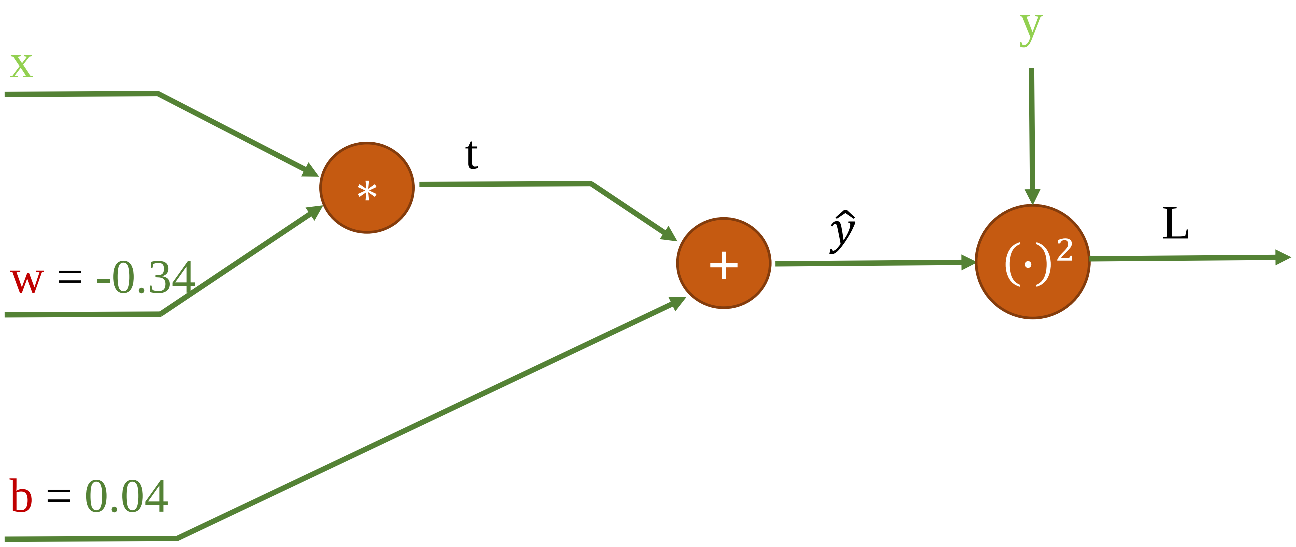
# Computational graph



## ❖ House price prediction

### ❖ N-sample training

$t = wx$ $\hat{y} = t + b$ $L = (\hat{y} - y)^2$

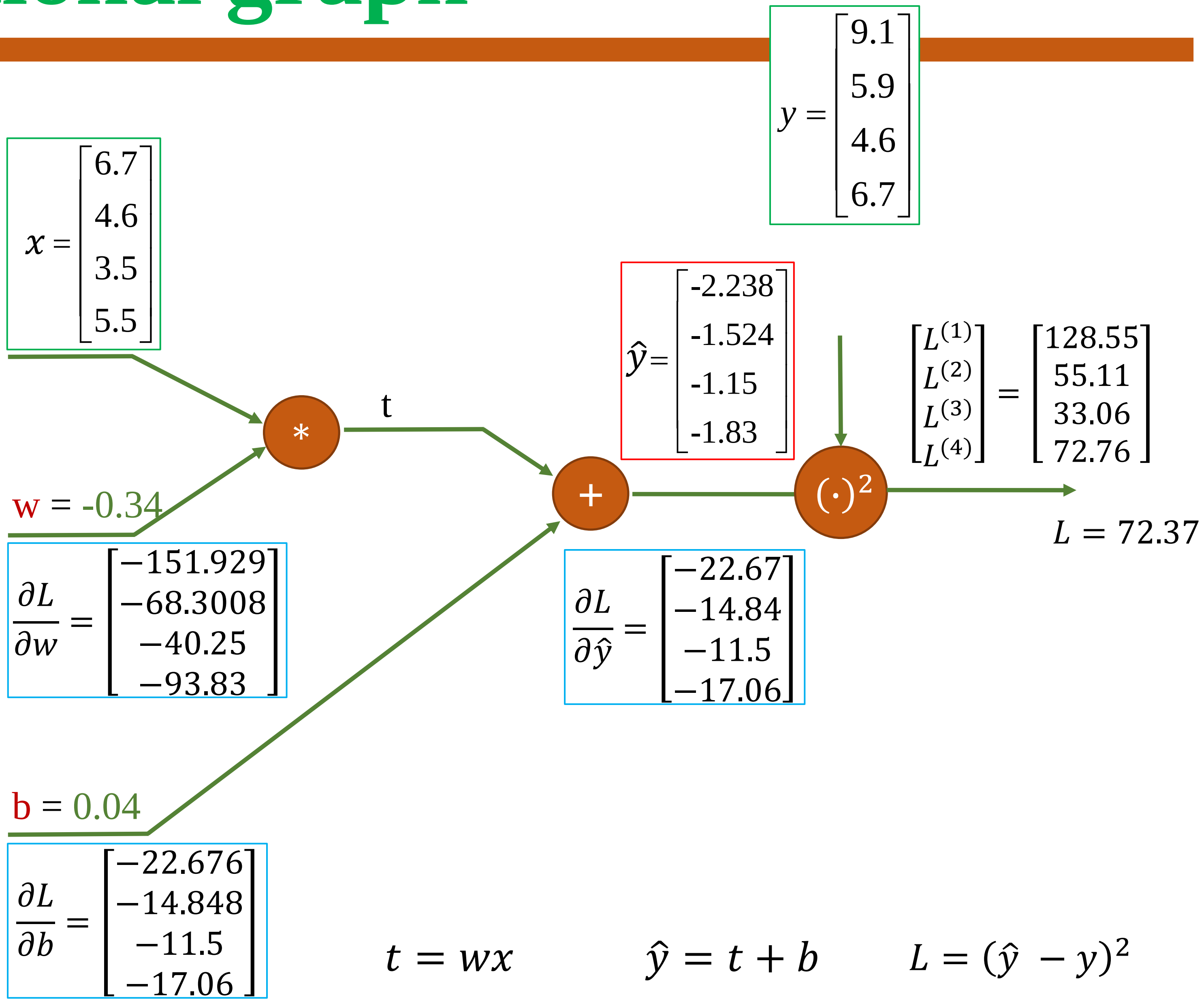


# Computational graph

- **House price prediction**
  - **N-sample training**

$$\frac{sum(\frac{\partial L}{\partial w})}{4} = -88.5775$$

$$\frac{sum(\frac{\partial L}{\partial b})}{4} = -16.521$$



# Computational graph



## ❖ House price prediction

### ❖ N-sample training

**Update w and b**

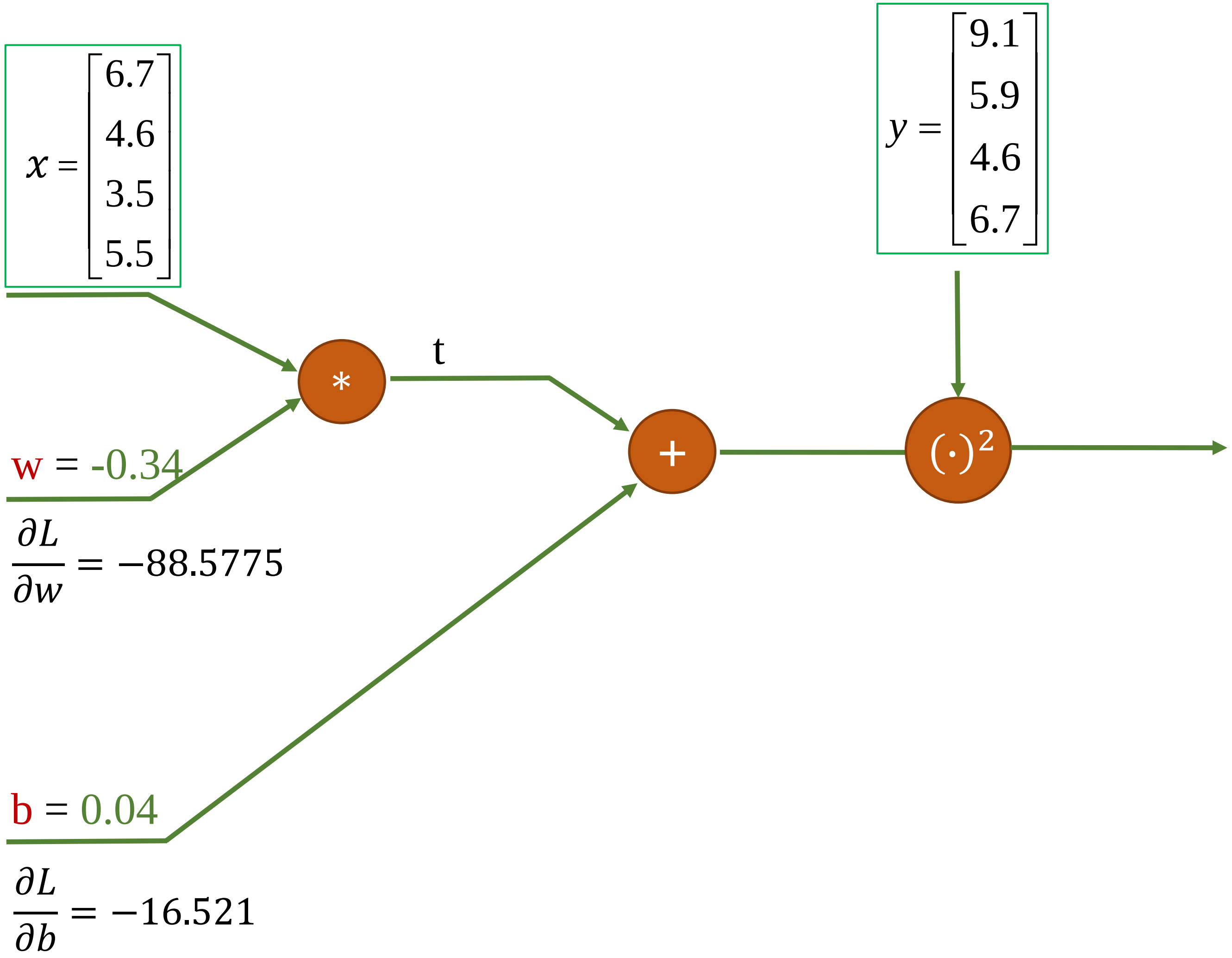
$$w = w - \eta * \frac{\partial L}{\partial w}$$

$$b = b - \eta * \frac{\partial L}{\partial b}$$

Learning rate $\eta = 0.01$

$$w = -0.34 - (0.01 * (-88.5775)) = 0.54$$

$$b = 0.04 - \left(0.01 * (-16.521)\right) = 0.205$$



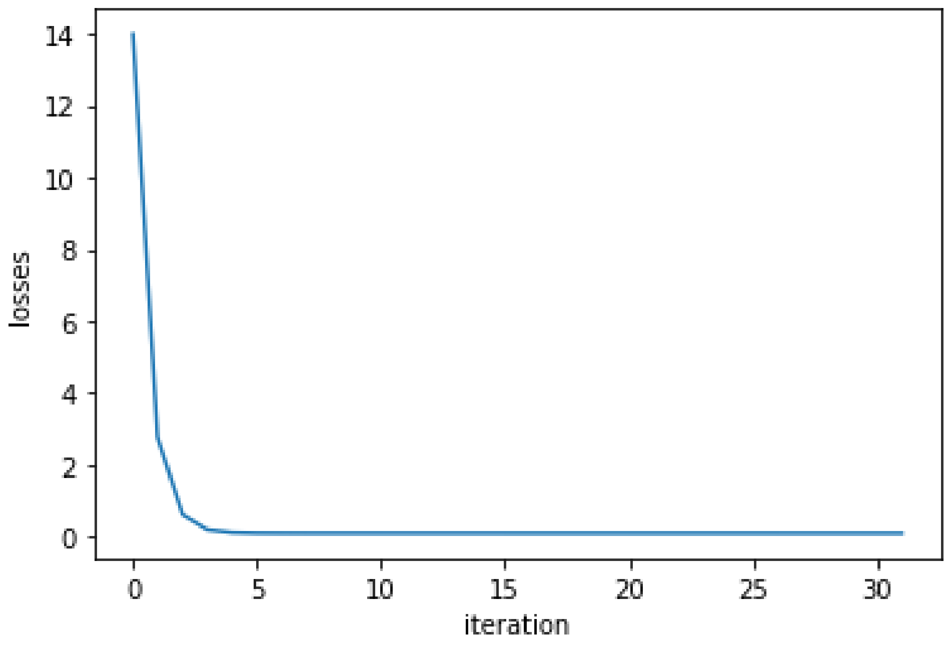

$t = wx$ $\quad \hat{y} = t + b$ $\quad L = (\hat{y} - y)^2$

# Computational graph

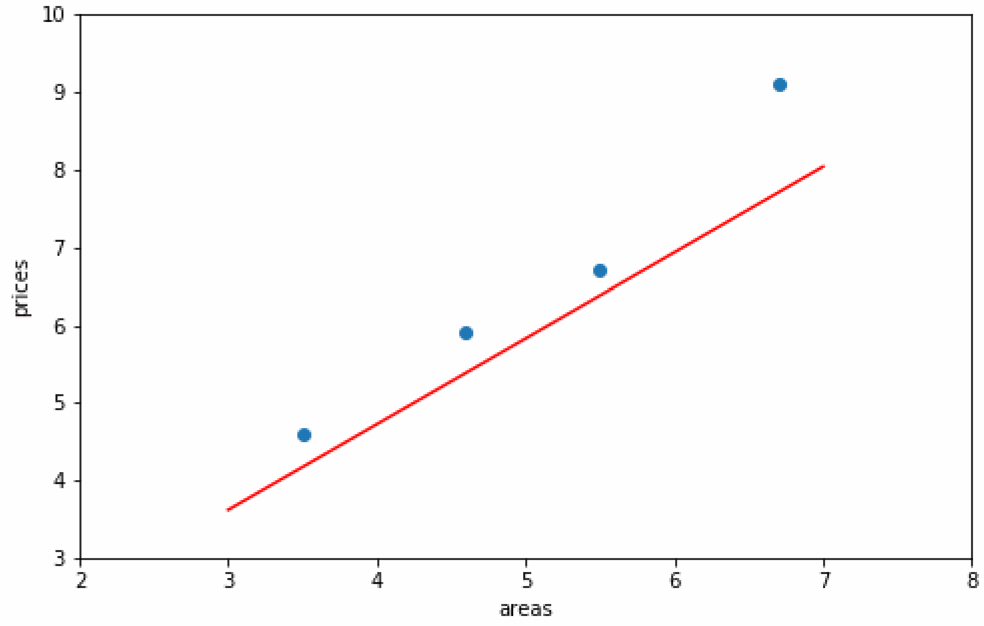


## ❖ House price prediction

### ❖ N-sample training




Losses for 30 iterations




Model updating for different iterations

# Extension

| Features | | | Label |
|---|---|---|---|
| **TV** | **Radio** | **Newspaper** | **Sales** |
| 230.1 | 37.8 | 69.2 | 22.1 |
| 44.5 | 39.3 | 45.1 | 10.4 |
| 17.2 | 45.9 | 69.3 | 12 |
| 151.5 | 41.3 | 58.5 | 16.5 |
| 180.8 | 10.8 | 58.4 | 17.9 |

Advertising data

Model: $\hat{y} = w_1x_1 + w_2x_2 + w_3x_3 + b$

Sale = $w_1 * TV + w_2 * Radio + w_3 * Newspaper + b$

# Linear Regression



## ❖ General formula

| Feature | Label |
|---|---|
| area | price |
| 6.7 | 9.1 |
| 4.6 | 5.9 |
| 3.5 | 4.6 |
| 5.5 | 6.7 |

House price data

Model: $\hat{y} = w_1 x_1 + b$

price = a $* \, area + b$

| Features | | | Label |
|---|---|---|---|
| TV | Radio | Newspaper | Sales |
| 230.1 | 37.8 | 69.2 | 22.1 |
| 44.5 | 39.3 | 45.1 | 10.4 |
| 17.2 | 45.9 | 69.3 | 12 |
| 151.5 | 41.3 | 58.5 | 16.5 |
| 180.8 | 10.8 | 58.4 | 17.9 |

Advertising data

Model: $\hat{y} = w_1 x_1 + w_2 x_2 + w_3 x_3 + b$

Sale = $w_1 * TV + w_2 * Radio + w_3 * Newspaper + b$

# Linear Regression

1) Pick a sample $(x_1, x_2, x_3, y)$ from training data

2) Compute the output $\hat{y}$

$$\hat{y} = w_1 * TV + w_2 * R + w_3 * N + b$$

$$\hat{y} = w_1 * x_1 + w_2 * x_2 + w_3 * x_3 + b$$

3) Compute loss

$$L = (\hat{y} - y)^2$$

4) Compute derivative

$$\frac{\partial L}{\partial w_1} = 2x_1(\hat{y} - y) \qquad \frac{\partial L}{\partial w_3} = 2x_3(\hat{y} - y)$$

$$\frac{\partial L}{\partial w_2} = 2x_2(\hat{y} - y) \qquad \frac{\partial L}{\partial b} = 2(\hat{y} - y)$$

5) Update parameters

$$w_1 = w_1 - \eta \frac{\partial L}{\partial w_1} \qquad w_3 = w_3 - \eta \frac{\partial L}{\partial w_3}$$

$$w_2 = w_2 - \eta \frac{\partial L}{\partial w_2} \qquad b = b - \eta \frac{\partial L}{\partial b}$$

| Features | | | Label |
|---|---|---|---|
| TV | Radio | Newspaper | Sales |
| 230.1 | 37.8 | 69.2 | 22.1 |
| 44.5 | 39.3 | 45.1 | 10.4 |
| 17.2 | 45.9 | 69.3 | 12 |
| 151.5 | 41.3 | 58.5 | 16.5 |
| 180.8 | 10.8 | 58.4 | 17.9 |

Advertising data

Model

$$\text{Sale} = w_1 * TV + w_2 * Radio + w_3 * Newspaper + b$$

$$\hat{y} = w_1x_1 + w_2x_2 + w_3x_3 + b$$

1) Pick a sample $(x_1, x_2, x_3, y)$ from training data

2) Compute the output $\hat{y}$

$$\hat{y} = w_1 * TV + w_2 * R + w_3 * N + b$$

$$\hat{y} = w_1 * x_1 + w_2 * x_2 + w_3 * x_3 + b$$

3) Compute loss

$$L = (\hat{y} - y)^2$$

4) Compute derivative

$$\frac{\partial L}{\partial w_1} = 2x_1(\hat{y} - y) \qquad \frac{\partial L}{\partial w_3} = 2x_3(\hat{y} - y)$$

$$\frac{\partial L}{\partial w_2} = 2x_2(\hat{y} - y) \qquad \frac{\partial L}{\partial b} = 2(\hat{y} - y)$$

5) Update parameters

$$w_1 = w_1 - \eta \frac{\partial L}{\partial w_1} \qquad w_3 = w_3 - \eta \frac{\partial L}{\partial w_3}$$

$$w_2 = w_2 - \eta \frac{\partial L}{\partial w_2} \qquad b = b - \eta \frac{\partial L}{\partial b}$$

```python
# compute output and loss
def predict(x1, x2, x3, w1, w2, w3, b):
    return w1*x1 + w2*x2 + w3*x3 + b
def compute_loss(y_hat, y):
    return (y_hat - y)**2

# compute gradient
def compute_gradient_wi(xi, y, y_hat):
    dl_dwi = 2*xi*(y_hat-y)
    return dl_dwi
def compute_gradient_b(y, y_hat):
    dl_db = 2*(y_hat-y)
    return dl_db

# update weights
def update_weight_wi(wi, dl_dwi, lr):
    wi = wi - lr*dl_dwi
    return wi
def update_weight_b(b, dl_db, lr):
    b  = b - lr*dl_db
    return b
```

| Features | | | Label |
|---|---|---|---|
| TV | Radio | Newspaper | Sales |
| 230.1 | 37.8 | 69.2 | 22.1 |
| 44.5 | 39.3 | 45.1 | 10.4 |
| 17.2 | 45.9 | 69.3 | 12 |
| 151.5 | 41.3 | 58.5 | 16.5 |
| 180.8 | 10.8 | 58.4 | 17.9 |

```python
def initialize_params():
    w1 = random.gauss(mu=0.0, sigma=0.01)
    w2 = random.gauss(mu=0.0, sigma=0.01)
    w3 = random.gauss(mu=0.0, sigma=0.01)
    b  = 0

    return w1, w2, w3, b

# initialize model's parameters
w1, w2, w3, b = initialize_params()
print(w1, w2, w3, b)
```

```
0.01609506469549467 0.00607778501208891 0.0023344573891806507 0
```

```python
import numpy as np
import random

def get_column(data, index):
    result = [row[index] for row in data]
    return result

data = np.genfromtxt('advertising.csv',
                     delimiter=',',
                     skip_header=1).tolist()

# get tv (index=0)
tv_data = get_column(data, 0)

# get radio (index=1)
radio_data = get_column(data, 1)

# get newspaper (index=2)
newspaper_data = get_column(data, 2)

# get sales (index=0)
sales_data = get_column(data, 3)
```

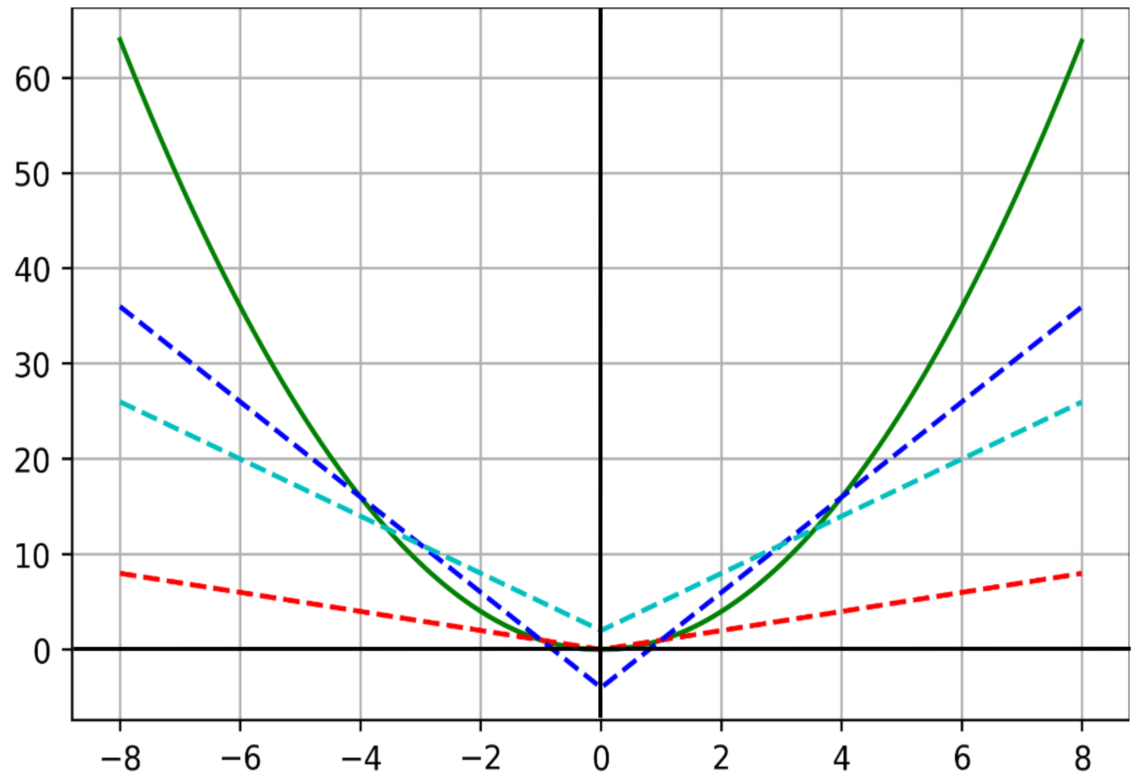
# Outline

SECTION 1
**Linear Regression**

SECTION 2
**Mini-batch Training**

SECTION 3
**Batch Training**

SECTION 4
**Loss Functions (Optional)**



Discussion 1: Is it OK to use the following loss function?

$$L = \frac{1}{2}(\hat{y} - y)^2$$

Discussion 2: if so, construct formulas

Discussion 3: What about the following loss function?

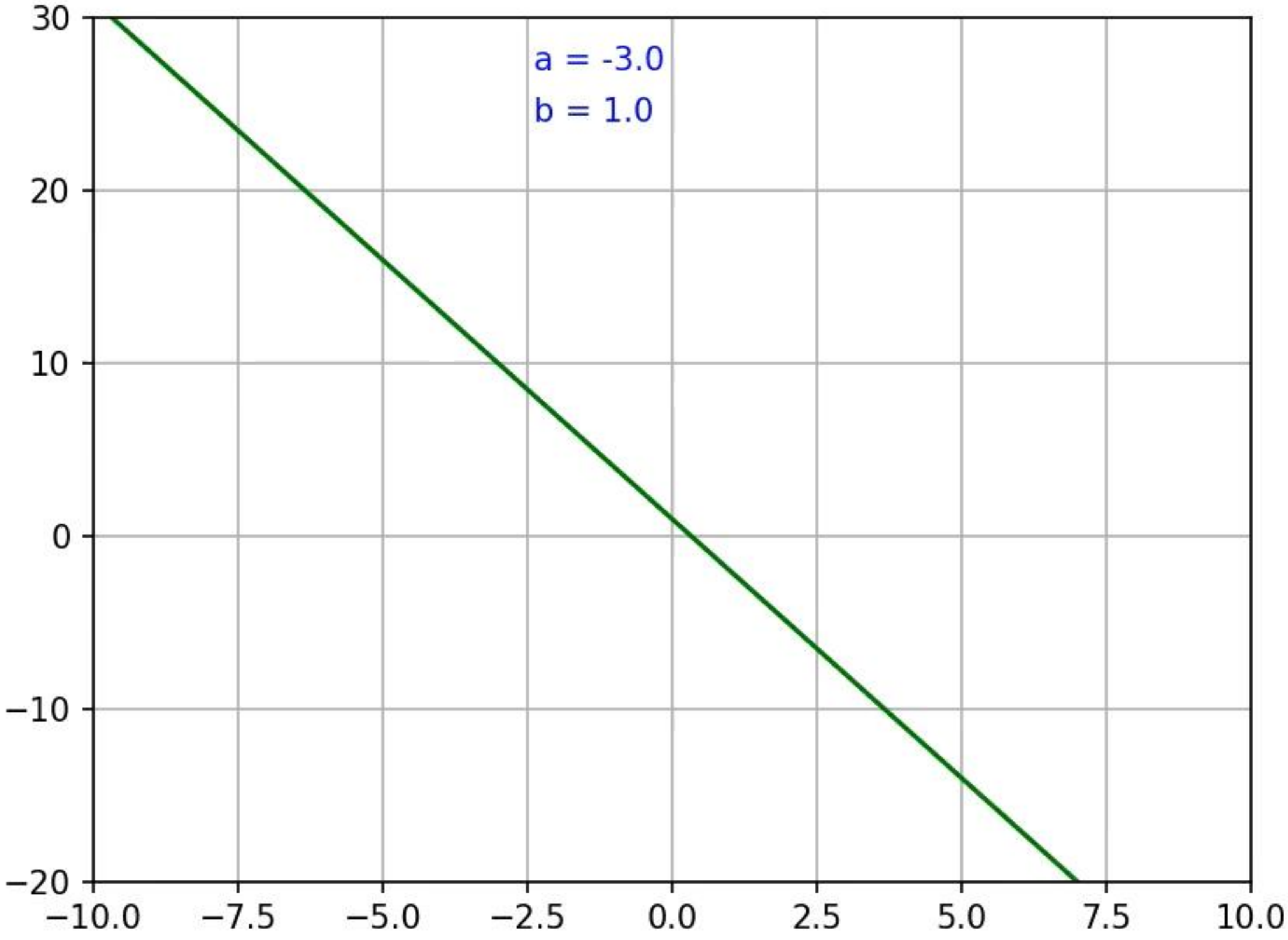
$$L = \frac{1}{2}(y - \hat{y})^2$$

Discussion 4: Construct the connection between the two following losses?

$$\mathrm{L}_1 = \frac{1}{2}(\hat{y} - y)^2 \qquad\qquad \mathrm{L}_2 = (\hat{y} - y)^2$$

$$\hat{y} = wx + b$$

Discussion 5:
Can we remove b?



$$\hat{y} = wx + b$$

Discussion 5:
Can we remove b?