NNDL2024: Exercices & Assignments
Session 0 (14 Mar)

Preliminaries

Exceptionally these exercises are just considered during the Thursday session, no reports are handed in, no points are given.

Mathematical exercices

1. Matrix square root. Consider a symmetric matrix $\mathbf{C}$. Denote the eigen-decomposition of $\mathbf{C}$ as

$$\mathbf{C} = \mathbf{U}\text{diag}(\lambda_1, \dots, \lambda_n)\mathbf{U}^T. \tag{1}$$

   Consider the matrix $\mathbf{M}$ defined using that EVD as:

$$\mathbf{M} = \mathbf{U}\text{diag}(\sqrt{\lambda_1}, \dots, \sqrt{\lambda_n})\mathbf{U}^T. \tag{2}$$

   Show that

$$\mathbf{MM} = \mathbf{C}. \tag{3}$$

2. Consider a bivariate Gaussian distribution where the two (scalar) variables x, y have zero mean, unit variance, and covariance equal to c. Hint1: The pdf uses the inverse covariance, and we have

$$\begin{pmatrix} 1 & c \\ c & 1 \end{pmatrix}^{-1} = \frac{1}{1-c^2}\begin{pmatrix} 1 & -c \\ -c & 1 \end{pmatrix}. \tag{4}$$

   Calculate

   (a) the marginal distribution of x

   (b) the conditional distribution of x given y

   The point here is to do the integral calculations in detail, not use some formulas readily available on the internet. So, please show the detailed derivation. Hint2: in (a), use the "completion of square formula" on y

$$y^2 - 2ay = (y-a)^2 - a^2, \tag{5}$$

   where a is something that contains the other variable (y or x) and the covariance. In (b), you may need a similar formula as well but for x.

3. Consider the Laplace distribution

$$p(z|\alpha) = \frac{\alpha}{2} \exp(-\alpha|z|). \tag{6}$$

The parameter α determines how likely large the values are in absolute value. Given a sample $z_1, \ldots, z_N$ of observations, derive the maximum likelihood estimator for α.

Computer assignments In this session we will go through a tutorial covering PyTorch etc.