

Dynamic Programming | Set 10 (0-1 Knapsack Problem)

3.3

Given weights and values of n items, put these items in a knapsack of capacity W to get the maximum total value in the knapsack. In other words, given two integer arrays $val[0..n-1]$ and $wt[0..n-1]$ which represent values and weights associated with n items respectively. Also given an integer W which represents knapsack capacity, find out the maximum value subset of $val[]$ such that sum of the weights of this subset is smaller than or equal to W . You cannot break an item, either pick the complete item, or don't pick it (0-1 property).

0-1 Knapsack Problem

```
value[] = {60, 100, 120};  
weight[] = {10, 20, 30};  
W = 50;
```

Solution: 220

```
Weight = 10; Value = 60;  
Weight = 20; Value = 100;  
Weight = 30; Value = 120;  
Weight = (20+10); Value = (100+60);  
Weight = (30+10); Value = (120+60);  
Weight = (30+20); Value = (120+100);  
Weight = (30+20+10) > 50
```

Recommended: Please solve it on “PRACTICE” first, before moving on to the solution.

A simple solution is to consider all subsets of items and calculate the total weight and value of all subsets. Consider the only subsets whose total weight is smaller than W . From all such subsets, pick the maximum value subset.

1) Optimal Substructure:

To consider all subsets of items, there can be two cases for every item: (1) the item is included in the

optimal subset, (2) not included in the optimal set.

Therefore, the maximum value that can be obtained from n items is max of following two values.

- 1) Maximum value obtained by n-1 items and W weight (excluding nth item).
- 2) Value of nth item plus maximum value obtained by n-1 items and W minus weight of the nth item (including nth item).

If weight of nth item is greater than W, then the nth item cannot be included and case 1 is the only possibility.

2) Overlapping Subproblems

Following is recursive implementation that simply follows the recursive structure mentioned above.

C/C++

```
/* A Naive recursive implementation of 0-1 Knapsack problem */
#include<stdio.h>

// A utility function that returns maximum of two integers
int max(int a, int b) { return (a > b)? a : b; }

// Returns the maximum value that can be put in a knapsack of capacity W
int knapSack(int W, int wt[], int val[], int n)
{
    // Base Case
    if (n == 0 || W == 0)
        return 0;

    // If weight of the nth item is more than Knapsack capacity W, then
    // this item cannot be included in the optimal solution
    if (wt[n-1] > W)
        return knapSack(W, wt, val, n-1);

    // Return the maximum of two cases:
    // (1) nth item included
    // (2) not included
    else return max( val[n-1] + knapSack(W-wt[n-1], wt, val, n-1),
                    knapSack(W, wt, val, n-1)
                );
}

// Driver program to test above function
int main()
{
    int val[] = {60, 100, 120};
    int wt[] = {10, 20, 30};
    int W = 50;
    int n = sizeof(val)/sizeof(val[0]);
    printf("%d", knapSack(W, wt, val, n));
    return 0;
}
```

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Java

```
/* A Naive recursive implementation of 0-1 Knapsack problem */
class Knapsack
{

```

```
// A utility function that returns maximum of two integers
static int max(int a, int b) { return (a > b)? a : b; }

// Returns the maximum value that can be put in a knapsack of capacity W
static int knapSack(int W, int wt[], int val[], int n)
{
    // Base Case
    if (n == 0 || W == 0)
        return 0;

    // If weight of the nth item is more than Knapsack capacity W, then
    // this item cannot be included in the optimal solution
    if (wt[n-1] > W)
        return knapSack(W, wt, val, n-1);

    // Return the maximum of two cases:
    // (1) nth item included
    // (2) not included
    else return max( val[n-1] + knapSack(W-wt[n-1], wt, val, n-1),
                    knapSack(W, wt, val, n-1)
                );
}

// Driver program to test above function
public static void main(String args[])
{
    int val[] = new int[]{60, 100, 120};
    int wt[] = new int[]{10, 20, 30};
    int W = 50;
    int n = val.length;
    System.out.println(knapSack(W, wt, val, n));
}
/*This code is contributed by Rajat Mishra */
```

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Python

```
#A naive recursive implementation of 0-1 Knapsack Problem

# Returns the maximum value that can be put in a knapsack of
# capacity W
def knapSack(W , wt , val , n):

    # Base Case
    if n == 0 or W == 0 :
        return 0

    # If weight of the nth item is more than Knapsack of capacity
    # W, then this item cannot be included in the optimal solution
    if (wt[n-1] > W):
        return knapSack(W , wt , val , n-1)

    # return the maximum of two cases:
    # (1) nth item included
    # (2) not included
    else:
        return max(val[n-1] + knapSack(W-wt[n-1] , wt , val , n-1),
                    knapSack(W , wt , val , n-1))

# end of function knapSack

# To test above function
val = [60, 100, 120]
```

```

wt = [10, 20, 30]
W = 50
n = len(val)
print knapSack(W , wt , val , n)

# This code is contributed by Nikhil Kumar Singh

```

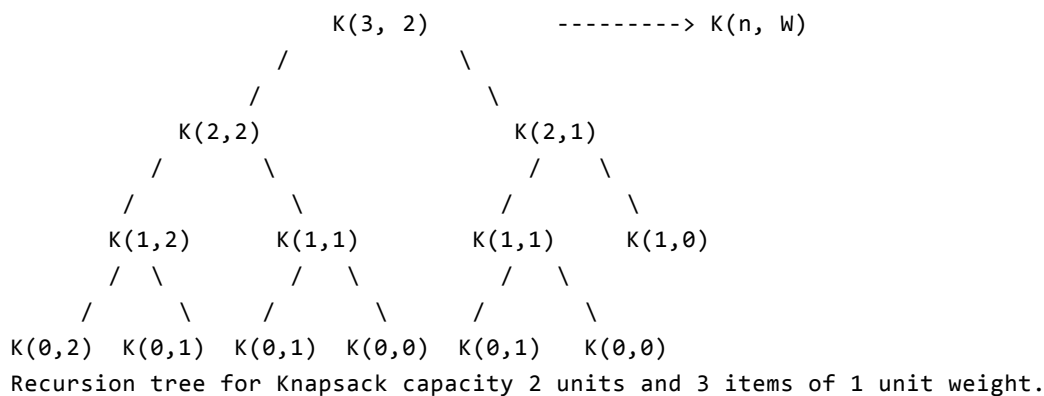
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Output:

220

It should be noted that the above function computes the same subproblems again and again. See the following recursion tree, $K(1, 1)$ is being evaluated twice. Time complexity of this naive recursive solution is exponential (2^n).

In the following recursion tree, $K()$ refers to $\text{knapSack}()$. The two parameters indicated in the following recursion tree are n and W . The recursion tree is for following sample inputs.
 $\text{wt}[] = \{1, 1, 1\}$, $W = 2$, $\text{val}[] = \{10, 20, 30\}$



Since subproblems are evaluated again, this problem has Overlapping Subproblems property. So the 0-1 Knapsack problem has both properties (see [this](#) and [this](#)) of a dynamic programming problem. Like other typical [Dynamic Programming\(DP\) problems](#), recomputations of same subproblems can be avoided by constructing a temporary array $K[][]$ in bottom up manner. Following is Dynamic Programming based implementation.

C++

```

// A Dynamic Programming based solution for 0-1 Knapsack problem
#include<stdio.h>

// A utility function that returns maximum of two integers
int max(int a, int b) { return (a > b)? a : b; }

// Returns the maximum value that can be put in a knapsack of capacity W
int knapSack(int W, int wt[], int val[], int n)

```

```

{
    int i, w;
    int K[n+1][W+1];

    // Build table K[][] in bottom up manner
    for (i = 0; i <= n; i++)
    {
        for (w = 0; w <= W; w++)
        {
            if (i==0 || w==0)
                K[i][w] = 0;
            else if (wt[i-1] <= w)
                K[i][w] = max(val[i-1] + K[i-1][w-wt[i-1]], K[i-1][w]);
            else
                K[i][w] = K[i-1][w];
        }
    }

    return K[n][W];
}

int main()
{
    int val[] = {60, 100, 120};
    int wt[] = {10, 20, 30};
    int W = 50;
    int n = sizeof(val)/sizeof(val[0]);
    printf("%d", knapSack(W, wt, val, n));
    return 0;
}

```

[Run on IDE](#)

Java

```

// A Dynamic Programming based solution for 0-1 Knapsack problem
class Knapsack
{
    // A utility function that returns maximum of two integers
    static int max(int a, int b) { return (a > b)? a : b; }

    // Returns the maximum value that can be put in a knapsack of capacity W
    static int knapSack(int W, int wt[], int val[], int n)
    {
        int i, w;
        int K[][] = new int[n+1][W+1];

        // Build table K[][] in bottom up manner
        for (i = 0; i <= n; i++)
        {
            for (w = 0; w <= W; w++)
            {
                if (i==0 || w==0)
                    K[i][w] = 0;
                else if (wt[i-1] <= w)
                    K[i][w] = max(val[i-1] + K[i-1][w-wt[i-1]], K[i-1][w]);
                else
                    K[i][w] = K[i-1][w];
            }
        }

        return K[n][W];
    }

    // Driver program to test above function

```

```

public static void main(String args[])
{
    int val[] = new int[]{60, 100, 120};
    int wt[] = new int[]{10, 20, 30};
    int W = 50;
    int n = val.length;
    System.out.println(knapSack(W, wt, val, n));
}
/*This code is contributed by Rajat Mishra */

```

Run on IDE

Python

```

# A Dynamic Programming based Python Program for 0-1 Knapsack problem
# Returns the maximum value that can be put in a knapsack of capacity W
def knapSack(W, wt, val, n):
    K = [[0 for x in range(W+1)] for x in range(n+1)]

    # Build table K[][] in bottom up manner
    for i in range(n+1):
        for w in range(W+1):
            if i==0 or w==0:
                K[i][w] = 0
            elif wt[i-1] <= w:
                K[i][w] = max(val[i-1] + K[i-1][w-wt[i-1]], K[i-1][w])
            else:
                K[i][w] = K[i-1][w]

    return K[n][W]

# Driver program to test above function
val = [60, 100, 120]
wt = [10, 20, 30]
W = 50
n = len(val)
print(knapSack(W, wt, val, n))

# This code is contributed by Bhavya Jain

```

Run on IDE

Output:

220

Time Complexity: $O(nW)$ where n is the number of items and W is the capacity of knapsack.

Asked in: Amazon, Flipkart, GreyOrange , Microsoft, Mobicip, Morgan-Stanley, Oracle, Payu, Snapdeal, Visa

References:

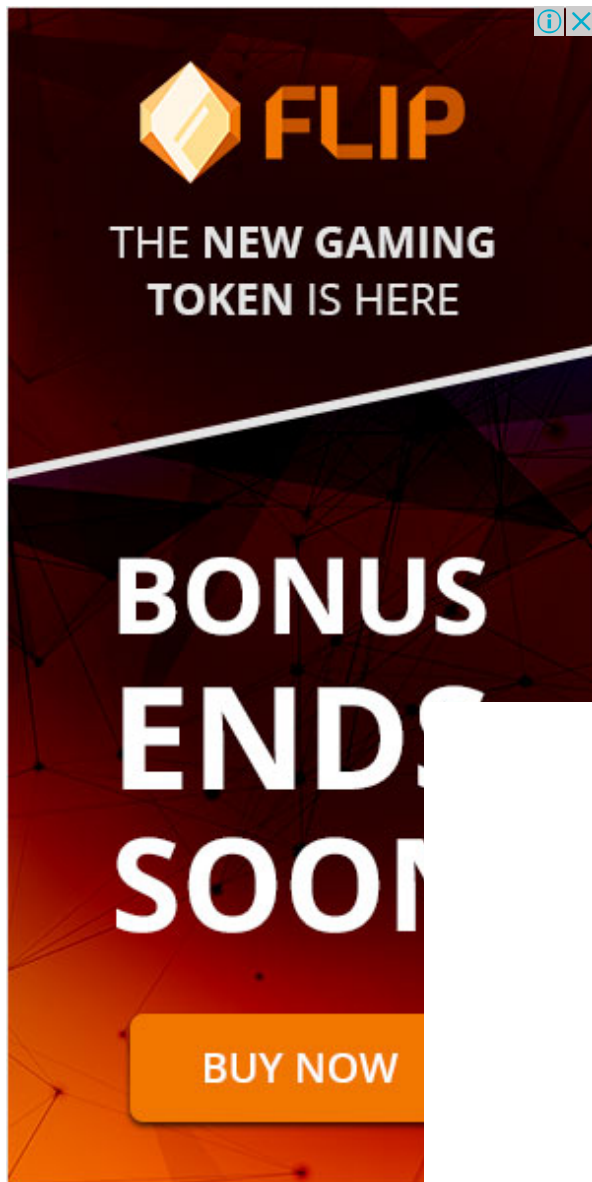
<http://www.es.ele.tue.nl/education/5MC10/Solutions/knapsack.pdf>

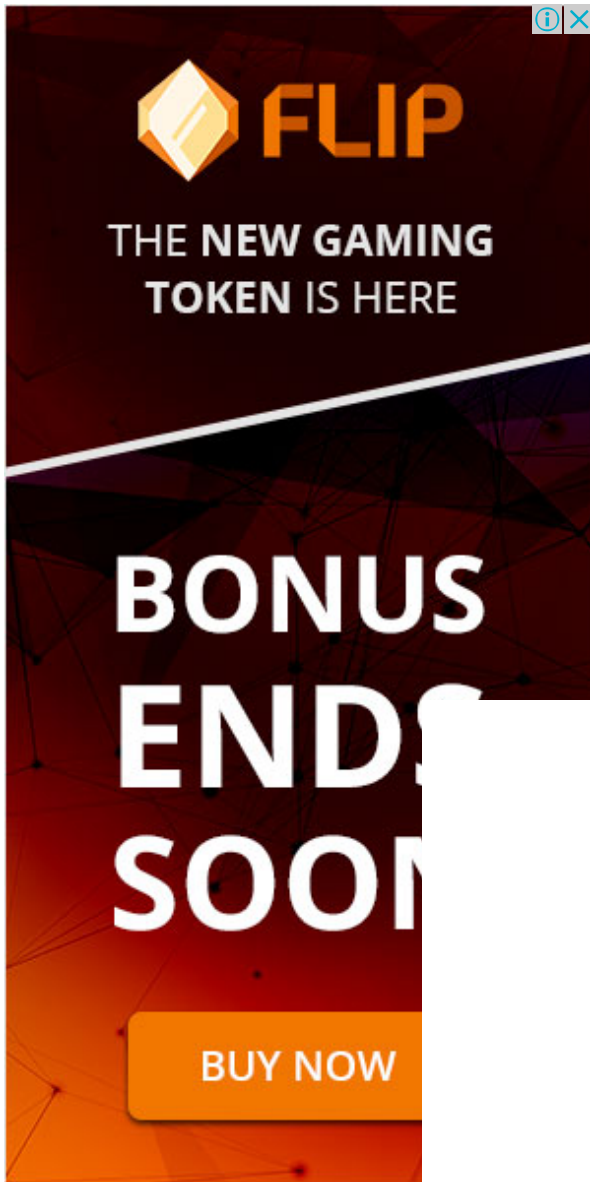
<http://www.cse.unl.edu/~goddard/Courses/CSCE310J/Lectures/Lecture8-DynamicProgramming.pdf>

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