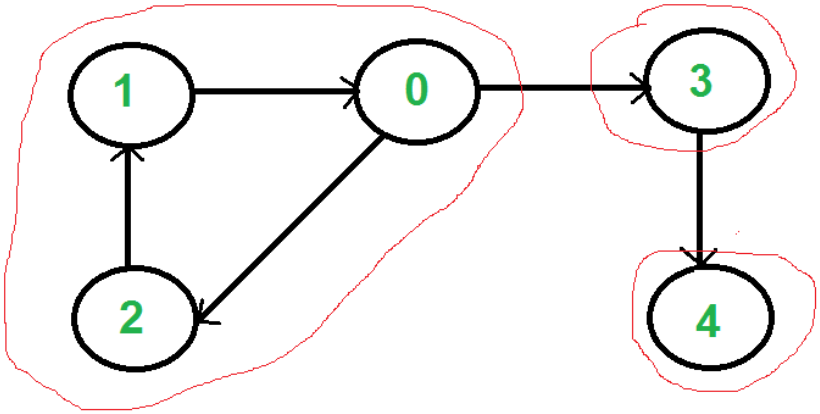
Strongly Connected Components

A directed graph is strongly connected if there is a path between all pairs of vertices. A strongly connected component (**SCC**) of a directed graph is a maximal strongly connected subgraph. For example, there are 3 SCCs in the following graph.



We can find all strongly connected components in O(V+E) time using Kosaraju's algorithm. Following is detailed Kosaraju's algorithm.

1) Create an empty stack 'S' and do DFS traversal of a graph. In DFS traversal, after calling recursive DFS for adjacent vertices of a vertex, push the vertex to stack. In the above graph, if we start DFS from vertex 0, we get vertices in stack as 1, 2, 4, 3, 0.

2) Reverse directions of all arcs to obtain the transpose graph.

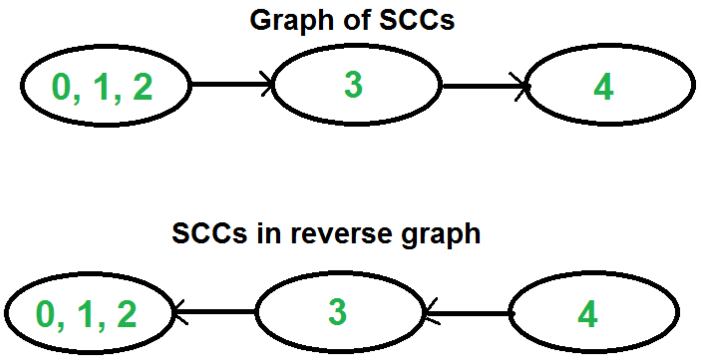
3) One by one pop a vertex from S while S is not empty. Let the popped vertex be 'v'. Take v as source and do DFS (call DFSUtil(v)). The DFS starting from v prints strongly connected component of v. In the above example, we process vertices in order 0, 3, 4, 2, 1 (One by one popped from stack).

How does this work?

The above algorithm is DFS based. It does DFS two times. DFS of a graph produces a single tree if all

vertices are reachable from the DFS starting point. Otherwise DFS produces a forest. So DFS of a graph with only one SCC always produces a tree. The important point to note is DFS may produce a tree or a forest when there are more than one SCCs depending upon the chosen starting point. For example, in the above diagram, if we start DFS from vertices 0 or 1 or 2, we get a tree as output. And if we start from 3 or 4, we get a forest. To find and print all SCCs, we would want to start DFS from vertex 4 (which is a sink vertex), then move to 3 which is sink in the remaining set (set excluding 4) and finally any of the remaining vertices (0, 1, 2). So how do we find this sequence of picking vertices as starting points of DFS? Unfortunately, there is no direct way for getting this sequence. However, if we do a DFS of graph and store vertices according to their finish times, we make sure that the finish time of a vertex that connects to other SCCs (other that its own SCC), will always be greater than finish time of vertices in the other SCC (See this for proof). For example, in DFS of above example graph, finish time of 0 is always greater than 3 and 4 (irrespective of the sequence of vertices considered for DFS). And finish time of 3 is always greater than 4. DFS doesn't guarantee about other vertices, for example finish times of 1 and 2 may be smaller or greater than 3 and 4 depending upon the sequence of vertices considered for DFS. So to use this property, we do DFS traversal of complete graph and push every finished vertex to a stack. In stack, 3 always appears after 4, and 0 appear after both 3 and 4.

In the next step, we reverse the graph. Consider the graph of SCCs. In the reversed graph, the edges that connect two components are reversed. So the SCC {0, 1, 2} becomes sink and the SCC {4} becomes source. As discussed above, in stack, we always have 0 before 3 and 4. So if we do a DFS of the reversed graph using sequence of vertices in stack, we process vertices from sink to source (in reversed graph). That is what we wanted to achieve and that is all needed to print SCCs one by one.



Recommended: Please solve it on "*PRACTICE*" first, before moving on to the solution.

Following is C++ implementation of Kosaraju's algorithm.

C++

```cpp
// C++ Implementation of Kosaraju's algorithm to print all SCCs
#include <iostream>
#include <list>
#include <stack>
using namespace std;

class Graph
{
    int V;    // No. of vertices
    list<int> *adj;    // An array of adjacency lists

    // Fills Stack with vertices (in increasing order of finishing
    // times). The top element of stack has the maximum finishing
    // time
    void fillOrder(int v, bool visited[], stack<int> &Stack);

    // A recursive function to print DFS starting from v
    void DFSUtil(int v, bool visited[]);
public:
    Graph(int V);
    void addEdge(int v, int w);

    // The main function that finds and prints strongly connected
    // components
    void printSCCs();

    // Function that returns reverse (or transpose) of this graph
    Graph getTranspose();
};

Graph::Graph(int V)
{
    this->V = V;
    adj = new list<int>[V];
}

// A recursive function to print DFS starting from v
void Graph::DFSUtil(int v, bool visited[])
{
    // Mark the current node as visited and print it
    visited[v] = true;
    cout << v << " ";

    // Recur for all the vertices adjacent to this vertex
    list<int>::iterator i;
    for (i = adj[v].begin(); i != adj[v].end(); ++i)
        if (!visited[*i])
            DFSUtil(*i, visited);
}

Graph Graph::getTranspose()
{
    Graph g(V);
    for (int v = 0; v < V; v++)
    {
        // Recur for all the vertices adjacent to this vertex
        list<int>::iterator i;
        for(i = adj[v].begin(); i != adj[v].end(); ++i)
        {
            g.adj[*i].push_back(v);
        }
    }
    return g;
}

void Graph::addEdge(int v, int w)
{
    adj[v].push_back(w); // Add w to v's list.
}
```

```
void Graph::fillOrder(int v, bool visited[], stack<int> &Stack)
{
    // Mark the current node as visited and print it
    visited[v] = true;

    // Recur for all the vertices adjacent to this vertex
    list<int>::iterator i;
    for(i = adj[v].begin(); i != adj[v].end(); ++i)
        if(!visited[*i])
            fillOrder(*i, visited, Stack);

    // All vertices reachable from v are processed by now, push v
    Stack.push(v);
}

// The main function that finds and prints all strongly connected
// components
void Graph::printSCCs()
{
    stack<int> Stack;

    // Mark all the vertices as not visited (For first DFS)
    bool *visited = new bool[V];
    for(int i = 0; i < V; i++)
        visited[i] = false;

    // Fill vertices in stack according to their finishing times
    for(int i = 0; i < V; i++)
        if(visited[i] == false)
            fillOrder(i, visited, Stack);

    // Create a reversed graph
    Graph gr = getTranspose();

    // Mark all the vertices as not visited (For second DFS)
    for(int i = 0; i < V; i++)
        visited[i] = false;

    // Now process all vertices in order defined by Stack
    while (Stack.empty() == false)
    {
        // Pop a vertex from stack
        int v = Stack.top();
        Stack.pop();

        // Print Strongly connected component of the popped vertex
        if (visited[v] == false)
        {
            gr.DFSUtil(v, visited);
            cout << endl;
        }
    }
}

// Driver program to test above functions
int main()
{
    // Create a graph given in the above diagram
    Graph g(5);
    g.addEdge(1, 0);
    g.addEdge(0, 2);
    g.addEdge(2, 1);
    g.addEdge(0, 3);
    g.addEdge(3, 4);

    cout << "Following are strongly connected components in "
            "given graph \n";
    g.printSCCs();

    return 0;
```

```
}
```

Run on IDE

Java

```java
// Java implementation of Kosaraju's algorithm to print all SCCs
import java.io.*;
import java.util.*;
import java.util.LinkedList;

// This class represents a directed graph using adjacency list
// representation
class Graph
{
    private int V;   // No. of vertices
    private LinkedList<Integer> adj[]; //Adjacency List

    //Constructor
    Graph(int v)
    {
        V = v;
        adj = new LinkedList[v];
        for (int i=0; i<v; ++i)
            adj[i] = new LinkedList();
    }

    //Function to add an edge into the graph
    void addEdge(int v, int w)  { adj[v].add(w); }

    // A recursive function to print DFS starting from v
    void DFSUtil(int v,boolean visited[])
    {
        // Mark the current node as visited and print it
        visited[v] = true;
        System.out.print(v + " ");

        int n;

        // Recur for all the vertices adjacent to this vertex
        Iterator<Integer> i =adj[v].iterator();
        while (i.hasNext())
        {
            n = i.next();
            if (!visited[n])
                DFSUtil(n,visited);
        }
    }

    // Function that returns reverse (or transpose) of this graph
    Graph getTranspose()
    {
        Graph g = new Graph(V);
        for (int v = 0; v < V; v++)
        {
            // Recur for all the vertices adjacent to this vertex
            Iterator<Integer> i =adj[v].listIterator();
            while(i.hasNext())
                g.adj[i.next()].add(v);
        }
        return g;
    }

    void fillOrder(int v, boolean visited[], Stack stack)
    {
        // Mark the current node as visited and print it
```

```
        visited[v] = true;

        // Recur for all the vertices adjacent to this vertex
        Iterator<Integer> i = adj[v].iterator();
        while (i.hasNext())
        {
            int n = i.next();
            if(!visited[n])
                fillOrder(n, visited, stack);
        }

        // All vertices reachable from v are processed by now,
        // push v to Stack
        stack.push(new Integer(v));
    }

    // The main function that finds and prints all strongly
    // connected components
    void printSCCs()
    {
        Stack stack = new Stack();

        // Mark all the vertices as not visited (For first DFS)
        boolean visited[] = new boolean[V];
        for(int i = 0; i < V; i++)
            visited[i] = false;

        // Fill vertices in stack according to their finishing
        // times
        for (int i = 0; i < V; i++)
            if (visited[i] == false)
                fillOrder(i, visited, stack);

        // Create a reversed graph
        Graph gr = getTranspose();

        // Mark all the vertices as not visited (For second DFS)
        for (int i = 0; i < V; i++)
            visited[i] = false;

        // Now process all vertices in order defined by Stack
        while (stack.empty() == false)
        {
            // Pop a vertex from stack
            int v = (int)stack.pop();

            // Print Strongly connected component of the popped vertex
            if (visited[v] == false)
            {
                gr.DFSUtil(v, visited);
                System.out.println();
            }
        }
    }

    // Driver method
    public static void main(String args[])
    {
        // Create a graph given in the above diagram
        Graph g = new Graph(5);
        g.addEdge(1, 0);
        g.addEdge(0, 2);
        g.addEdge(2, 1);
        g.addEdge(0, 3);
        g.addEdge(3, 4);

        System.out.println("Following are strongly connected components "+
                           "in given graph ");
        g.printSCCs();
    }
```

```
}
// This code is contributed by Aakash Hasija
```

Run on IDE

Python

```
# Python implementation of Kosaraju's algorithm to print all SCCs

from collections import defaultdict

#This class represents a directed graph using adjacency list representation
class Graph:

    def __init__(self,vertices):
        self.V= vertices #No. of vertices
        self.graph = defaultdict(list) # default dictionary to store graph

    # function to add an edge to graph
    def addEdge(self,u,v):
        self.graph[u].append(v)

    # A function used by DFS
    def DFSUtil(self,v,visited):
        # Mark the current node as visited and print it
        visited[v]= True
        print v,
        #Recur for all the vertices adjacent to this vertex
        for i in self.graph[v]:
            if visited[i]==False:
                self.DFSUtil(i,visited)


    def fillOrder(self,v,visited, stack):
        # Mark the current node as visited
        visited[v]= True
        #Recur for all the vertices adjacent to this vertex
        for i in self.graph[v]:
            if visited[i]==False:
                self.fillOrder(i, visited, stack)
        stack = stack.append(v)


    # Function that returns reverse (or transpose) of this graph
    def getTranspose(self):
        g = Graph(self.V)

        # Recur for all the vertices adjacent to this vertex
        for i in self.graph:
            for j in self.graph[i]:
                g.addEdge(j,i)
        return g



    # The main function that finds and prints all strongly
    # connected components
    def printSCCs(self):

        stack = []
        # Mark all the vertices as not visited (For first DFS)
        visited =[False]*(self.V)
        # Fill vertices in stack according to their finishing
        # times
        for i in range(self.V):
            if visited[i]==False:
```

```
                self.fillOrder(i, visited, stack)

        # Create a reversed graph
        gr = self.getTranspose()

        # Mark all the vertices as not visited (For second DFS)
        visited =[False]*(self.V)

        # Now process all vertices in order defined by Stack
        while stack:
            i = stack.pop()
            if visited[i]==False:
                gr.DFSUtil(i, visited)
                print""

# Create a graph given in the above diagram
g = Graph(5)
g.addEdge(1, 0)
g.addEdge(0, 2)
g.addEdge(2, 1)
g.addEdge(0, 3)
g.addEdge(3, 4)


print ("Following are strongly connected components " +
                            "in given graph")
g.printSCCs()
#This code is contributed by Neelam Yadav
```

Run on IDE

Output:

```
Following are strongly connected components in given graph
0 1 2
3
4
```

Time Complexity: The above algorithm calls DFS, fins reverse of the graph and again calls DFS. DFS takes O(V+E) for a graph represented using adjacency list. Reversing a graph also takes O(V+E) time. For reversing the graph, we simple traverse all adjacency lists.

The above algorithm is asymptotically best algorithm, but there are other algorithms like Tarjan's algorithm and path-based which have same time complexity but find SCCs using single DFS. The Tarjan's algorithm is discussed in the following post.

Tarjan's Algorithm to find Strongly Connected Components

Applications:

SCC algorithms can be used as a first step in many graph algorithms that work only on strongly connected graph.

In social networks, a group of people are generally strongly connected (For example, students of a class or any other common place). Many people in these groups generally like some common pages or play common games. The SCC algorithms can be used to find such groups and suggest the

commonly liked pages or games to the people in the group who have not yet liked commonly liked a page or played a game.

References:

http://en.wikipedia.org/wiki/Kosaraju%27s_algorithm

https://www.youtube.com/watch?v=PZQ0Pdk15RA

You may also like to see Tarjan's Algorithm to find Strongly Connected Components.

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