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Grid search based multi-population particle swarm optimization algorithm for multimodal multi-objective optimization

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ABSTRACT

In the multimodal multi-objective optimization problems (MMOPs), there may exist two or multiple equivalent Pareto optimal sets (PS) with the same Pareto Front (PF). The difficulty of solving MMOPs lies in how to locate more equivalent PS in decision space and maintain a promising balance between the diversity of Pareto optimal solutions in decision space and the convergence of Pareto optimal solutions in objective space at the same time. To address these issues, a grid search based multi-population particle swarm optimization algorithm (GSMPSO-MM) is proposed in this paper to handle MMOPs. Multi-populations based on the k-means clustering method is adopted to locate more equivalent PS in decision space, and a grid is applied to explore high-quality solutions in decision space in GSMPSO-MM. The environmental selection operator, including the removing inefficient solutions operator and the updating non-dominated solutions archive, aims to approach the true non-dominated solutions, where the updating non-dominated solution archive is responsible for developing the diverse solutions in both the decision and objective space, simultaneously. Besides, the purpose of removing inefficient solutions with inferior convergence in objective space is to maintain promising convergence solutions in objective space. GSMPSO-MM is compared with seven state-of-the-art algorithms on a well-known MMOPs benchmark function. Experimental results demonstrate the superior performance of our proposed algorithm in solving MMOPs.

1. Introduction

Multi-objective optimization problems (MOPs) have two or multiple conflicting objectives to be optimized, simultaneously. There is a set of trade-off solutions in MOPs instead of a single optimal solution. Multi-objective optimization problems based on minimization can be formulated as follow:

$$\begin{aligned} &\min f(x) = \left\{ f_1(x), f_2(x), \ldots ., f_n(x) \right\} \\ &\textit{subject to } x \in \Omega \subseteq R^m \end{aligned} \tag{1}$$

where$x = [x_1, x_2, \ldots x_m]$is a solution vector with m dimension decision variables, and $f : \Omega \rightarrow R^n$denotes an objective vector in objective space R^n, including n conflicting objectives.

Due to the conflicting objectives in MOPs, the Pareto dominance relationship is used to estimate the quality of one solution by comparing other solutions [1]. There is a set of trade-off solutions for MOPs, named as *non-dominated solutions*. The set, including all non-dominated solutions in decision space, is called *Pareto optimal solution Set* (PS), while the image mapping of PS is termed as *Pareto optimal Front* (PF) [2,3]. Besides, these solutions which are dominated by non-dominated solutions according to the Pareto dominance relationship are termed as *dominated solutions*.

For MOPs, it probably has multiple Pareto optimal solutions in decision space with the same objective values in objective space. In other words, there can be multiple PS in decision space corresponding to the same PF in objective space for MOPs. This problem is defined as multimodal multi-objective optimization problems (MMOPs) [4,5]. A simple example of MMOPs is presented in Fig. 1. There are PS_1 and PS_2 with the same PF in Fig. 1.

During the past two decades, multi-objective evolutionary algorithms (MOEAs) have demonstrated significantly promising performance in addressing real-world MOPs [6–8]. For instance, the multi-objective genetic algorithm along with elitist (NSGA-II) [9], the multi-objective evolutionary algorithm based on decomposition (MOEA/D) [10], and indicator-based multi-objective evolutionary algorithm (IBEA) [11]. The achievement of such MOEAs owes to emphasis on the distribution of the non-dominated solutions in objective space, resulting in that the performance of the non-dominated solutions in decision space is critical.

Since existing MOEAs cannot find out more equivalent PS in decision space, they are not suitable to handle MMOPs. Multimodal evolution-

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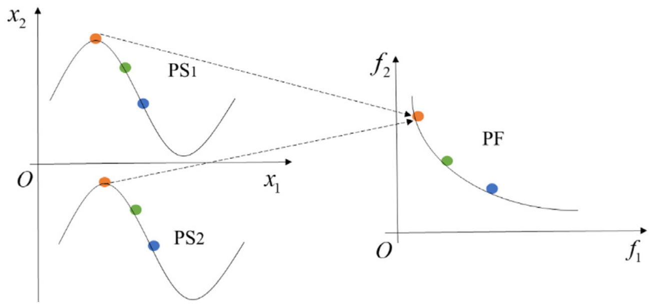


Fig. 1. An example of MMOPs.

ary algorithms (MMEAs) are efficient to exploit manifold solutions in decision space. Fitness sharing [12,13], crowding [14], clearing [15], clustering [16], and other niching mechanisms [17,18] are applied to MOEAs. To supply the shortcomings of MOEAs in solving the MMOPs, some MOEAs based on niching techniques are applied to search for various solutions in decision space, such as Niching-CMA [19], SPEA2+ [20], Omni-optimizer [21]. However, previous work with niching in MOEAs cannot discover some high-quality solutions in both decision and objective space. They are impossible to perform well for MMOPs.

Multimodal multi-objective evolutionary algorithms (MMMEAs) aim to develop more equivalent PS in decision space. Some recent studies are introduced to handle the MMOPs. Decision space based on niching NSGA-II (DN-NSGAII) [5], index-based ring topology MOPSO with special crowding distance (MO_Ring_PSO_SCD) [22], and MOEA/D-AD [23] have obtained outstanding performance in decision space, which are superior to some MOEAs based on niching for MMOPs. Besides, a comprehensive review for the MMMEAs which described the previous development of MMMEAs and some remaining critical issues has been undertaken [24].

Previous studies have demonstrated that most MMMEAs have achieved a great advance in locating diverse PS in decision space. However, the key to handle MMOPs is how to obtain a promising convergence in the premises of locating deviser PS in decision space. It is significant that MMMEAs not only get more equivalent PS in decision space but also maintain the diversity and convergence of PF in objective space. To the best of our knowledge, existing MMMEAs only focus on search more PS in decision space and don't have studies on balancing the diversity of solutions in decision space and objective space and the convergence of solutions in objective space, simultaneously. To tackle these issues, this paper proposes a grid search based multi-populations particle swarm optimization algorithm (GSMPSO-MM) to handle MMOPs. The main contributions of this paper can be summarized as follows. i). In contrast to conventional MOEAs to find the complete PF in objective space, the desirable MMMEAs need to be able to locate diverse PS in decision space. In GSMPSO-MM, the k-means clustering method is used to divide the population into multiple subpopulations in decision space. Each subpopulation searches for more equivalent PS in the different region via the global version PSO. Furthermore, to avoid repeatedly searching for these areas where PS does not exist and to assist in exploiting some new PS for subpopulations, a unique grid is established. The grid search based on the local version PSO in GSMPSO-MM is adopted to find out unexploited PS in other grid areas in decision space, when a grid implied by the global version PSO does not have PS according to the Pareto dominance relation. ii). To balance the diversity of the non-dominated solutions in decision space and the convergence of the non-dominated solutions in objective space, respectively, an appropriate environment selection operator, including removing redundant solutions and updating the non-dominated solutions archive operator, is employed to GSMPSO-MM. In removing redundant solutions operators, an indicator-based hypervolume is applied to estimate the convergence quality of boundary solutions in objective space. These non-dominated solutions in objective space are evaluated by the hypervolume indicator and the solutions which influence the hypervolume indicator are removed from the non-dominated solutions archives to ensure the convergence of solutions in objective space. To survive the diverse non-dominated solutions in both decision and objective space in the archive, a diversity metric is utilized to calculate the diversity of each non-dominated solution, and the highlighting quality of non-dominated solutions is maintained during the environment selection operator. Moreover, some solutions with poor diversity metric in the archive are not really abandoned. On the contrary, these solutions are maintained in an additional archive for the next environment selection.

The remainder of this paper is organized as follows. Section 2 introduces some related work and motivation. The detailed framework of GSMPSO-MM is presented in Section 3. Experiments and results are demonstrated in Section 4. Section 5 concludes this paper and looks ahead to future work.

2. Related works and motivation

2.1. Particle swarm optimization

Particle Swarm Optimization (PSO) was proposed by Kennedy and Eberhart [25], which is originated from research on the foraging behavior of birds. The velocity and position of particles are updated based on the global best particle and historical best particle in the feasible region, and the updated formulas of the velocity and position are shown in Eqs. (2) and (3).

$$v_{i,j}(t+1) = wv_{i,j}(t) + \varphi_1\gamma_1(pbest_{i,j} - x_{i,j}) + \varphi_2\gamma_2(gbest_j - x_{i,j}) \tag{2}$$

$$x_{i,j}(t+1) = x_{i,j}(t) + v_{i,j}(t+1) \tag{3}$$

where $v_{i,j}(t)$ denotes the velocity of jth dimension of ith particle at the time t, $x_{i,j}(t)$ indicates the position of jth dimension of ith particle at the time t, $pbest_{i,j}$ represents jth dimension of historical best position of ith particle, and $gbest_j$ is jth dimension of the global best particle. The$v_{i,j}(t+1)$ and $x_{i,j}(t+1)$ are the new velocity and position of ith particle at the time $t+1$, respectively. φ_1, φ_2 are two acceleration constants, and γ_1, γ_2 are two random numbers within zero and one. ω is the inertia weight factor. This pattern which updates the velocity and position of particles by Eqs. (2) and (3) is called global version PSO (global-best PSO).

Besides, the updating velocity of particles can also be formulated by their neighbors in Eq. (4).

$$v_{i,j}(t+1) = \omega v_{i,j}(t) + \varphi_1\gamma_1(pbest_{i,j} - x_{i,j}) + \varphi_2\gamma_2(nbest_{i,j} - x_{i,j}) \tag{4}$$

where $nbest_{i,j}$ denotes jth dimension of the historical best particle of the neighborhood of ith particle. The position update formula remains unchanged. The updating model of the velocity of particles is called as local version PSO (local-best PSO).

2.2. Multimodal multi-objective optimization problems

Some recent studies have developed an interest in MMOPs. A reasonable definition of MMOPs is introduced in [26], and the definition is described as follows: If there are at least two diverse Pareto optimal solutions or at least a local Pareto optimal solution corresponding to any point in Pareto optimal Front for MOPs, the MOPs is named as MMOPs.

Several typical MMOPs test suites are developed. Liang et al. [5] introduced the concepts in detail and designed some novel MMOPs. Yue et al. [22] proposed 11 significant MMOPs, including MMF1–8, SYM-PART simple, SYM-PART rotated, and Omni-test, where SYM-PART simple, SYM-PART rotated and Omni-test originated in [27] and [21]. Meanwhile, it designed a reasonable indicator to estimate the performance of MMMEAs. The distribution of PS for MMF1 and MMF4–8 is symmetrical. Notice that the equivalent PS of MMF3 and MMF6 is overlapped in each dimension. In addition, Yue et al. [28] redesigned and added some novel test problems with local Pareto optimal solutions to constitute an integrated benchmark function, which includes 22 MMOPs.

A series of novel multimodal multi-objective optimization problems based on the classical multimodal and multi-objective benchmark is developed [29], which contains MMMOP1–6. Different from MMF1–8, MMMOP1–6 provided scalable decision variables, objective numbers, and equivalent Pareto solutions set. To handle the imbalance between the convergence and diversity for MMOPs, the new imbalance minimization distance multimodal multi-objective problems (MMOP-ICDs) are proposed [26], which includes four types of imbalanced distance minimization problem.

2.3. Existing multimodal multi-objective evolutionary algorithms

To search for more Pareto optimal solutions in objective space, it is clear that most MOEAs in previous work pay few attention to the distribution of Pareto optimal solutions in decision space. Fortunately, it is relieved that some MOEAs have been gradually focused on the performance of Pareto optimal solutions in decision space in recent studies. In [21], Omni-optimizer developed the crowding distance operator, which considers both objective space and decision space in the non-dominated ranking technique. DIVA [30] integrated the diversity in decision space into the hypervolume indicator for the purpose of optimizing two sets simultaneously. Interestingly, CMA-MO [19] promoted the diversity in decision space for MOPs by extending an existing CMS-EA niching framework.

Multimodal Multi-Objective Optimization Problems (MMOPs) are defined by Liang [5]. After that, some novel MMMEAs have developed in recent years. DN-NSGAII [5] introduced niching based on NS-GAII to locate different solutions with the same PF in decision space. MO_Ring_PSO_SCD [22] employed a ring topology based on local version PSO and modified the crowding distance to explore and exploit more non-dominant solutions, and put forward MMOPs benchmark functions and performance indicators. It is a great advance for MMOPs. Along with MO_Ring_PSO_SCD, a zoning search method in decision space (ZS-MO_Ring_PSO_SCD) is used to work out MMOPs [31]. The whole decision space is divided into Nr equivalent zoning, then the MO_Ring_PSO_SCD is adopted to find more Pareto optimal solutions in each zoning, and these Pareto optimal solutions in each zoning are merged and ranked to gain the final solutions. The Self-Organizing Map (SOM) is introduced into particle swarm optimization algorithm and pigeon-inspired optimization algorithm in SMPSO-MM [32] and MMO-PIO [33] to address MMOPs, respectively. In [34], a new mutation operator is developed in the multimodal multi-objective Differential Evolution optimization algorithm (MMODE) for locating more PS in the boundary areas in decision space.

The improving performance indicators and benchmark functions are presented in [28]. Several MMMEAs use these indicators and problems to assess the performance of obtained Pareto optimal solutions. Zhang et al. [35] proposed a novel multi-population particle swarm optimization algorithm with leader particle and ring topology (MMO-CLRPSO) to deal with MMOPs. In MMO-CLRPSO, the global version PSO with leader particle in each subpopulation is used to accelerate the convergence rate of solutions, and the local version PSO based on ring topology populations is used to maintain the diverse solutions. A differential evolution based on reinforcement learning with fitness ranking for MMOPs (DE-RFLR) is presented in [36]. In DE-RFLR, each individual is regarded as an agent. Depending on the fitness ranking and Table Q which derive from the Q-learning framework, DE-RFLR selected a mutation operator from three typical operators and generated offspring individuals. Meanwhile, the state of new individuals and Table Q table is updated to guide the movement of individuals based on the reinforcement learning experience.

Liu et al. [29] developed a multimodal multi-objective evolutionary algorithm using double-archives and recombination strategies (TriMOEA-TA&R). TriMOEA-TA&R analyzed the properties and relationships of decision variables at first. Then, a double-archives, including convergence archive and diversity archive, is adopted to guide searching direction cooperatively. Finally, recombine the convergence and diverse archive on behalf of acquiring appropriate Pareto optimal solutions. In addition, Liu et al. [26] introduced a multimodal multi-objective evolutionary algorithm (CPDEA), using a convergence-penalized density method, to handle the imbalance MMOPs with the feature that the complexity of searching for one equivalent Pareto optimal solution in objective space is lower than that of another equivalent Pareto optimal solution for a point in true PF.

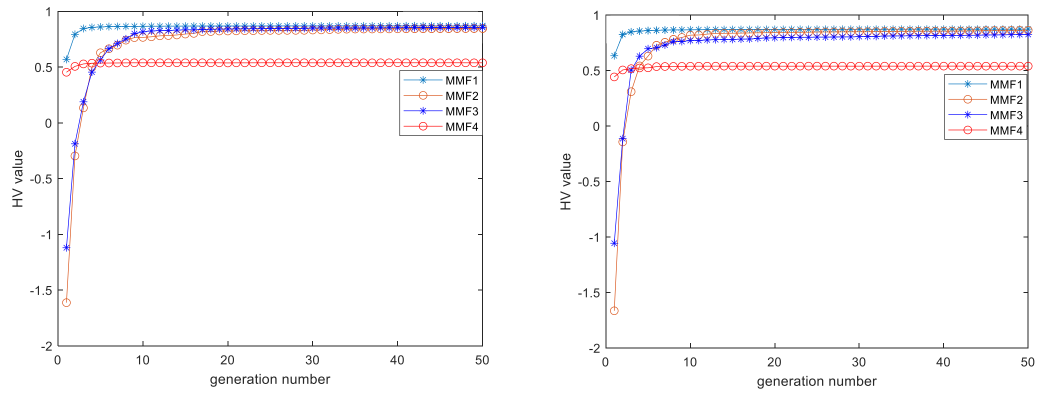
2.4. Motivation

In contrast to some MOEAs, there are the following three challenges in MMMEAs: (i) how to seek out multiple PS which are close to each other in objective space but far from each other in decision space, (ii) how to make solutions survive in the environmental selection and improve the diversity of solutions in decision space and objective space, and (iii) how to enhance the convergence of solutions in objective space under the premise of locating deviser PS in decision space. Among these existing MMMEAs, most efforts were paid to the decision space for the purpose of locating more equivalent PS and overlooking the diversity and convergence performance of solutions in objective space, while the performance of the MMMEAs is decided by the distribution of solutions in both decision and objective space. Moreover, little research has been conducted in applying survival selection and archive maintenance in decision space and objective space, simultaneously.

From the above analysis, it is clear that there is enormous room for enhancing the overall MMMEAs performance. Developing a significant evolutionary model to maintain a promising balance between the diversity of solutions in decision space and the convergence of solutions in objective space at the same time is imperative. However, previous studies fail to provide much attention to decision space. To tackle these issues, an appropriate scheme should be applied. In this paper, multi-population based clustering is adopted to find more equivalent PS in decision space and a grid in decision space is applied to approximate high-quality solutions. Meanwhile, a novel environmental selection is applied to balance the diversity and convergence in decision space and objective space, simultaneously. By integrating the mentioned schemes, GSMPSO-MM is implemented. The proposed GSMPSO-MM can balance the diversity of solutions in decision space and the convergence of solutions in objective space. Therefore, the outstanding performance of GSMPSO-MM is achieved. We introduce the proposed GSMPSO-MM in detail in the next section.

3. Proposed algorithm: GSMPSO-MM

In the proposed GSMPSO-MM algorithm, the four key components are initializing grid G and Tables Q, grid search strategy based on the global-best PSO and local-best PSO, removing unreliable solutions, and surviving diverse solutions in the non-dominated solutions archive, respectively. First, the grid G and Tables Q are initialized. Then, two evolutionary approaches of the basic PSO are adapted to the grid search operator. Next, the poor solutions in objective space are removed by removing unreliable solutions operators. Finally, plenty of diverse solutions in both decision and objective space are survived. The four strategies work together to improve the performance of the proposed GSMPSO-MM. In the following sections, we exhibit the main framework of the proposed GSMPSO-MM algorithm and its four strategies.



(a) The performance of DN-NSGAII on MMMOPs (b) The performance of Omni-optimizer on MMMOPs

Fig. 2. The performance of DN-NSGAII and Omni-optimizer on HV indicator.

3.1. The framework of GSMPSO-MM

Algorithm 1. The main framework of GSMPSO-MM
Input: pop (population), N (the size of the population), Ns (the number of subpopulation)
Output: S
1. Initialize population pop;
2. Generate grid G and initialize Table Q according to the range of decision variables;
3. Divide the population pop into Ns subpopulations $\{subpop_1, subpop_2, ..., subpop_\tau, ...subpop_{Ns}\}$ by k-means clustering method;
4. **for** each subpopulation $subpop_\tau$;
5. Obtain the non-dominated solution set NDS_τ according to the non-dominated sorting with special crowding distance;
6. the global best particle $gbest_\tau$ in $subpop_\tau$←the first particle in NDS_τ;
7. **end**
8. S ←non-dominated solutions inpop;
9. Update the Table Q based on the dominance relation in the population pop;
10. **while** $gen < Maxgen$
11. **for** each subpopulation $subpop_\tau$
12. Grid search based on PSO for $subpop_\tau$;
13. Obtain the non-dominated solution set NDS_τ;
14. **end**
15. S ← Environmental Selection($NDS_1, NDS_2, ..., NDS_\tau, ..., NDS_{Ns}$);
16. Update the Table Q according to S;
17. **end while**
18. **return**S.

Algorithm 1 shows the main framework of the proposed GSMPSO-MM algorithm. In Algorithm1, the population pop with N particles is initialized randomly in decision space (line 1). Then, divide the decision space into numerous grids on basis of the range of m decision variables, and then generate a grid G and initialize Table Q (line 2). Next, the population pop in decision space is divided into Ns subpopulations $\{subpop_1, subpop_2, ..., subpop_{Ns}\}$ by the k-means method (line 3). Sort each subpopulation $subpop_\tau$ and obtain the non-dominated solutions set NDS_τ. The first particle in NDS_τ is denoted as the global best particle $gbest_\tau$ of the subpopulation $subpop_\tau$ (line 4-line 7). Meanwhile, sort the population pop according to the non-dominated sorting method and acquire the non-dominated solutions set S (line 8). Update the Table Q based on the dominance relation of the population pop (line 9). A grid search method based on PSO is adopted to seek for more non-dominated solutions in each subpopulation$subpop_\tau$ (line 11-line 14). Next, an environmental selection operator, including removing redundant solutions and double archives strategy, is adopted to maintain a fine balance between the diversity of solutions in decision space and the convergence of solutions in objective space (line 15). Finally, the Table Q is updated based on environmental selection (line 16). Repeat the above procedures until the termination condition is met. Output the final non-dominated solutions S.

3.2. Establish grid G and initialize table Q

The grid is used to search for more non-dominated solutions in objective space in previous MOPs [37,38,39]. As opposed to MOEAs, MMMEAs can quickly find the complete PF in objective space. Here, a simple experiment of DN-NSGAII [5] and Omni-optimizer [21] is conducted on MMF1–4 [22], and the experimental result is presented in Fig. 2. The horizontal axis indicates generation number and the axis of ordinates implies HV value [40]. In Fig. 2, the HV values of true PF for four problems are 0.8741, 0.8741, 0.8741, and 0.5378, respectively. It can be found that HV values emerge a smooth trend after few generation numbers, resulting in that MMMEAs approximate promptly true PF. Distinct from previous studies on multi-objective problems to establish grids in objective space, multimodal multi-objective problems don't need to build a grid in objective space because the entire PF can be found quickly for MMOPs. In contrast, establishing a grid in decision space is more possible to locate more high-quality PS for MMOPs.

In view of there may exist PS in each zone in decision space, the maximum fitness evaluation is restricted. In GSMPSO-MM, a grid is employed to locate the particle position in decision space. The method of dividing the decision space into multiple grids is similar to Vaccine-AIS [41] and the detailed process is described as follows.

The m-dimensional decision space R is divided into $V_1 \times V_2 \times .. \times V_m$ grids, where $V_1 = V_2 = ... = V_m = \lceil maxfit^{\frac{1}{m}} \rceil$, where $maxfit$ is the maximum fitness evaluation. The whole decision space R is divided into $V_1 \times V_2 \times .. \times V_m$ grids, and the grid is named as G. The grid width w_j in the jth dimensional is calculated by Eq. (5).

$$\mathrm{w_j} = \frac{\mathrm{U_j - L_j}}{\mathrm{V_j}}, j = 1, 2, ..., m \tag{5}$$

xwhere U_j and L_j are the upper and lower boundary of the jth dimension in decision space, respectively. For the particle $x = [x_1, x_2, ..., x_j, ..., x_m]$ in the population, the position of the particle x in the grid G is determined by Eq. (6).

$$C(x_j) = \left\lfloor \frac{x_j - L_j}{w_j} \right\rfloor + 1, \mathrm{j} = 1, 2, ...m \tag{6}$$

where $C(x_j)$ denotes the coordinate of jth dimension of the particle x in the grid G. We determine the status of the particles in the Table Q on basis of their position and the dominance relation. Fig. 3 explains the process of constructing the grid G and initializing the Table Q in detail on basis of the decision space.

In Fig. 3, suppose that the maximum fitness evaluation $maxfit$ is set to 100, the decision space R is divided into $V_1 \times V_2$ grids, where $V_1 =$

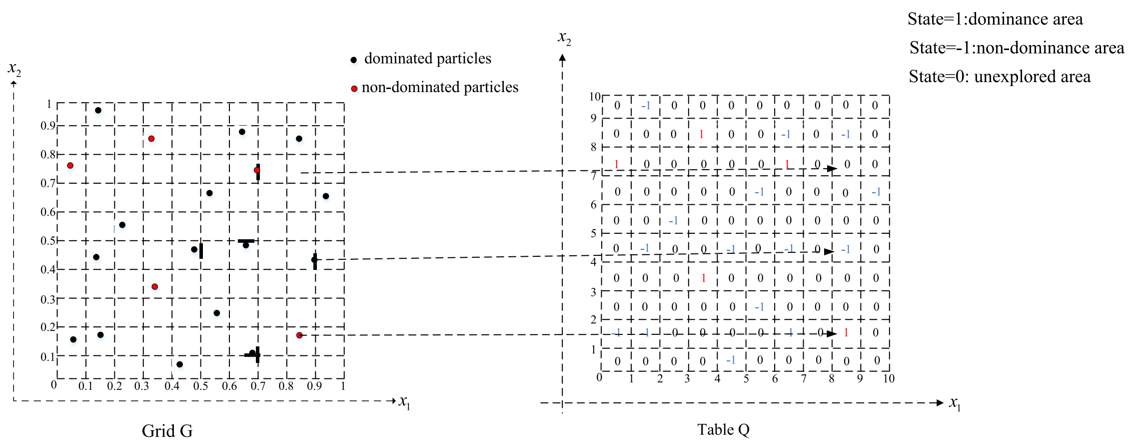


Fig. 3. Structure Grid G and initialize Table Q.

$V_2 = 10$, and each segment of each decision variable is $w_1 = w_2 = 0.1$ according to Eq. (5). The population pop with 20 particles is initialized randomly in two-dimensional space (x_1 and x_2) in the grid G. Then the position of each particle in the grid G can be determined by Eq. (6). Next, initialize the Table Q as zero-matrix. In Fig. 3, the red particles in the grid G indicate the non-dominated particles and the black particles in the grid G denote the dominated particles according to the dominance relationship. If a particle in the subgrid area is a non-dominated particle, the status of the particle in the Table Q is defined as $status = 1$. Otherwise, the status in the Table Q is set to $status = -1$, and it indicates that a newly generated particle in the grid area is of serious possibility to be a domination solution, and prohibits the particle to move into the grid area. Additionally, there are no particles in the subgrid region, and it indicates that the area is undeveloped and the status is denoted as $status = 0$.

3.3. Grid search based on PSO

Algorithm 2. Grid search based on PSO
Input: $subpop_\tau$, S, np(the size of the subpopulation$subpop_\tau$)
Output: NDS_τ
1. **for** each particle p_i in $subpop_\tau$
2. $pbestset_i$ ← non-dominated sorting$pbestset_i$;
3. $pbest_i$ ← the first particle in $pbestset_i$;
4. Update the velocity and position of the particle p_i by Eqs. (2) and (3) respectively;
5. Estimate the Table $Q(p_i{}')$ according to the new position of the particle $p_i{}'$ in the gird G;
6. **if** $Q(p_i{}') = 0$ or $Q(p_i{}') = 1$
7. Calculate the objective values of the particle $p_i{}'$;
8. **else if** $Q(p_i{}') == -1$
9. dis ← calculate the Euclidean distance between p_i and S;
10. $nbest_i$ ←the particle with the minimum Euclidean distance in S;
11. Update again the velocity and position of the particle p_i by Eqs. (4) and (3);
12. Calculate the objective values of the particle $p_i{}'$;
13. **end**
14. $pbestset_i$=[$pbestset_i \cup p_i{}'$];
15. **end**
16. NDS_τ ← non-dominated sorting($pbestset_1 \cup pbestset_2 \cup, ..., pbestset_i \cup, ..., \cup pbestset_{np}$);
17. $gbest_\tau$ ←the first particle in NDS_τ;
18. Update the Table Q according to non-dominated sorting;
19. **return** the non-dominated solution set NDS_τ

To develop more equivalent PS in decision space, a grid search based on PSO is applied to guide the direction in which particles are moving. The global version PSO with $gbest$and $pbest$ is used to update the velocity and position of these particles in the first step. For each particle p_i in $subpop_\tau$, the velocity and position of the particle p_i are updated by Eqs. (2) and (3), respectively. Then, according to the new position of the particle $p_i{}'$ in the grid G, estimate the value $Q(p_i{}')$ in Table Q. If the value $Q(p_i{}')$ is 0 or 1, it implies that the position of the particle $p_i{}'$ in the grid G is unexploited or there exists at least a non-dominated solution. The objective values of $p_i{}'$ being evaluated. If the value $Q(p_i{}')$ is -1, it means that the particle $p_i{}'$ is approximately a dominated solution in the specific area of the grid G. Therefore, the local version PSO with $nbest$ and $pbest$ is adopted to update again the velocity and position of p_i and generate a new $p_i{}'$ by Eqs. (4) and (3). To find the $nbest_i$, the Euclidean distance between p_i and the non-dominated solutions set S is calculated, and the non-dominated solution with the minimum Euclidean distance is located. The non-dominated solution with the minimum Euclidean distance is regarded as $nbest_i$ to update the velocity and position of the particle p_i again. Then, to make the particle p_i move to the direction of the non-dominated solution with the minimum Euclidean distance, the accelerate factors φ_1 and φ_2 are updated by Eq. (7) [42].

$$\varphi_1 = \varphi_1 * \left(1 - \frac{Maxgen - gen}{Maxgen}\right), \quad \varphi_2 = \varphi_2 * \left(1 + \frac{Maxgen - gen}{Maxgen}\right) \tag{7}$$

where gen and $Maxgen$ are the current iteration number and the maximum iteration number, respectively.

The velocity and position of each particle in $subpop_\tau$ are updated. To preserve more potential non-dominated solutions, all personal best position of the particle p_i is preserved in the set $pbestset_i$. Then, the non-dominated solution set NDS_τ is obtained by merging all $pbestset_i$ and the non-dominated ranking. The first non-dominated solution in NDS_τ is regarded as the best particle $gbest_\tau$ in $subpop_\tau$. Meanwhile, the Table Q is updated based on the dominant relationship. The pseudo-code of grid search based on PSO is shown in Algorithm 2.

3.4. Environmental selection in archive

In MMMEAs, a specific archive is used to maintain the diversity and convergence of solutions in decision space and the convergence of solutions in objective space. However, the critical issue is how to choose high-performance solutions in the non-dominated solutions archive. We introduce a novel environmental selection scheme in GSMPSO-MM, including removing redundant solutions based on objective space and updating the non-dominated solutions archive.

3.4.1. Remove redundant solutions based on objective space

For MMMEAs, the convergence of the non-dominated solutions in objective space is ignored to pay attention to search more equivalent PS in decision space. For example, in Fig. 4, the red line denotes the true PF in objective space, and the dots and stars are obtained PF by MMMEAs. Although the stars are non-dominated solutions, they are not unreliable solutions and prejudice the whole convergence of solutions. A similar solution like these is defined as an inefficient (redundant) solution. To address these issues, we propose a novel method to deal with

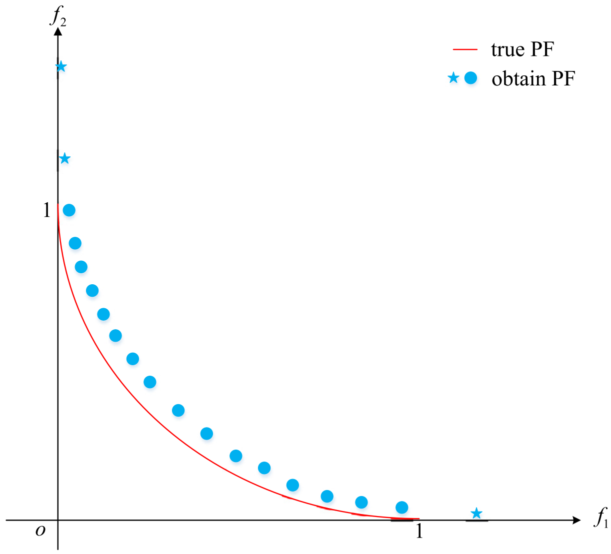


Fig. 4. The distribution of true PF and obtain PF in objective space.

the unreasonable boundary non-dominated solutions in objective space. In Algorithm 3, δth objective value of the non-dominated solutions S is sorted by descending, and the difference vector $vd_{\delta,q}$ of δth objective value of qth non-dominated solution in S is calculated by the following equation:

$$vd_{\delta,\,q} = f_\delta(S_q) - f_\delta(S_{q+1}) \tag{8}$$

where $f_\delta(S_q)$ is the δth objective value of qth non-dominated solution in S. Then, the maximum difference vector $Maxvd$ is obtained by Eq. (9).

$$Maxvd = \max_{\delta\in[1,n]} \max_{q\in[1,|S|]} (vd_{\delta,q}) \tag{9}$$

The non-dominated solution u with the maximum difference vector $Maxvd$ is located in S, and it is more probable to be a reliable non-dominant solution in S. Inspired by the HV indicator [40], the $HV_{previous}$ with the non-dominated solution u and the HV_{delete} without the non-dominated solution u in S are calculated by Eq. (10), respectively.

$$HV(\mathrm{S},\mathrm{rf}) = hypervolum\left(\bigcup_{\mathrm{u}\in\mathrm{S}} v(\mathrm{S},\mathrm{rf})\right) \tag{10}$$

where rf is a reference point. If the HV_{delete} value is greater than the $HV_{previous}$value, it indicates that the special non-dominated solution u is infeasible and delete it from S. Thus, the operator is equipped to eliminate those unreliable solutions (obtain PF with stars) in Fig. 4, and conducive to the convergence of solutions in objective space.

Furthermore, to determine the reference point rf in Eq. (10), the maximum value of each objective of the non-dominated solutions set S at the gen-st iteration called $Maxobj_{\delta,gen}$ is recorded in Mr, then the reference point rf is confirmed by Eq. (11) according to the average value of each objective in Mr.

$$\mathrm{rf}_\delta = \overline{\mathrm{Mr}_\delta}\delta \in [1, n] \tag{11}$$

It is worth noting that the maximum value which is equal to one objective of the non-dominated solution u is removed from Mr when the non-dominated solution u is removed from S.

Algorithm 3. Remove redundant solutions based on objective space
Input: S, rf, n
Output: S
1. **for** each objective value δ in S
2. the difference vector $vd_{\delta,q}$ is calculated by Eq. (8);
3. **end**
4. the maximum difference vector $Maxvd$ is obtained by Eq. (9);
5. Locate the non-dominated solution u with the maximum difference vector $Maxvd$ in S;
6. $S' = S\backslash u$;
7. $HV_{previous} = hypervolume(\mathrm{S}, \mathrm{rf})$;
8. $HV_{delete} = hypervolume(\mathrm{S}', \mathrm{rf})$;
9. **if** $HV_{delete} > HV_{previous}$
10. $S = S\backslash u$;
11. **end**
12. **return** S.

3.4.2. Update non-dominated solutions archive

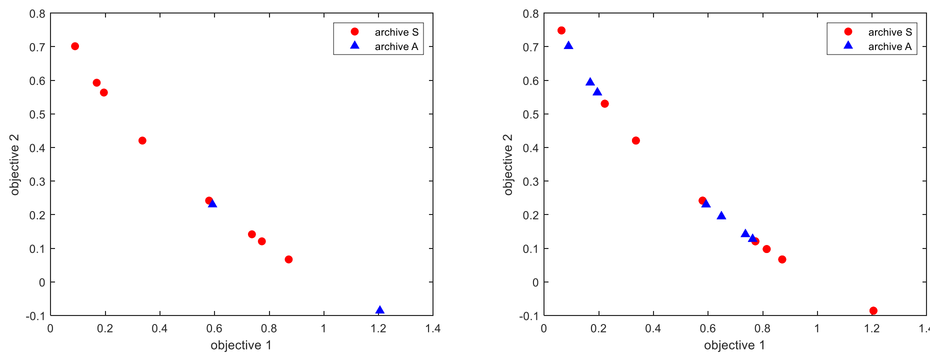
An archive is adopted to most MMMEAs for the purpose of maintaining the diversity of solutions, and the archive size is fixed. It is supposed that the number of solutions in MMMEAs is greater than the number of the population N. And it is difficult to decide which supererogatory solutions should be deleted from the archive. In [26], it presented a double $k-$nearest neighbor mode to estimate the diversity of solutions based on both decision and objective space, and proposed a fitness calculation method to assess the diversity quality of solutions. The fitness method based $k-$nearest neighbor for qth non-dominated solution S_q in S is shown by the following formula:

$$f_{diversity}(\mathrm{S}_q) = \frac{1}{1 + \frac{\sum_{a=1}^{k} d_{q,a}^{obj}}{d_{mean}^{obj}} + \frac{\sum_{a=1}^{k} d_{q,a}^{dec}}{d_{mean}^{dec}}} \tag{12}$$

where $d_{q,a}^{dec}(d_{q,a}^{obj})$ is the Euclidean distance in decision (objective) space between S_q and $k-$nearest neighbors, $d_{mean}^{dec}(d_{mean}^{obj})$ is the mean Euclidean distance for all $d_{q,a}^{dec}(d_{q,a}^{obj})$ in decision (objective) space. The larger $f_{diversity}(\mathrm{S}_q)$ indicates the inferiorer diversity for solutions.

According to the diversity ranking of the non-dominated solutions, some non-dominated solutions with inferior diversity solutions are removed from the non-dominated solutions archive. However, it doesn't mean that the diversity of those non-dominated solutions in the next or later non-dominated solutions archive is poor. For example, Fig. 5 shows the process of updating the non-dominated solutions archive in the current environment and the next environment. The solutions in the archive S are denoted as red points, and other solutions within the archive S (blue triangles) are reserved for the extra archive A. Suppose that there are 10 non-dominated solutions in the archive S and the archive size is 8 in the current environment in Fig. 5(a). It is observed that a special solution (1.20, −0.08) in the extra archive A in Fig. 5(a) is turned into the archive S in the later environment in Fig. 5(b) owing to the extra archive A is added to S in next archive S. Thus, we develop some modifications based on the double $k-$nearest neighbor mode to maintain the diversity of non-dominated solutions in the archive S. The procedure of updating the non-dominated solutions archive S is presented in Algorithm 4.

Algorithm 4. Update non-dominated solutions archive S
Input: S, A, N(the size of the archive), m(the number of decision variable), n (the number of the objectives), k (the closest number of the non-dominated solutions)
Output: S
1. $S \leftarrow$non-dominated sorting($S \cup A$)
2. **if** $|\mathrm{S}| > N$
3. **for** each non-dominated solution S_q in S
4. $f_{diversity} \leftarrow$ Calculate $f_{diversity}(\mathrm{S}_q)$ by Eq. (12) and sort;
5. **end**
6. $D \leftarrow |\mathrm{S}| - N$ non-dominated solutions with inferior diversity $f_{diversity}$ in S;
7. $S = S\backslash D$;
8. $A = A \cup D$;
9. end
10. return S



(a) The archive S and extra archive in the current environment

(b) The archive S and extra archive in the next environment

Fig. 5. The process of updating the archive S.

When the amount of the non-dominated solutions S is larger than N, the diversity value $f_{diversity}(\mathrm{S}_q)$ of each non-dominated solution S_q in the archive S is estimated by Eq. (12), and the diversity value $f_{diversity}(\mathrm{S}_q)$ is sorted based on descending. Then, the $(|\mathrm{S}| - N)$ non-dominated solutions with max $f_{diversity}(\mathrm{S}_q)$ value are deleted from the archive S and survived in D. It's worth noting that we do not abandon those non-dominated solutions. On the contrary, those solutions are reserved in the special archive D for the purpose of the diversity of non-dominated solutions in the next updating S.

3.5. Computational complexity

The complexity of the proposed GSMPSO-MM algorithm is influenced by the number of decision variables, the objective variables, and the population size. Taking a multimodal multi-objective problem with m decision variables, n objective variables, and N particles as an example, the complexity of GSMPSO-MM is composed of clustering, grid search based on PSO, environmental selection. The complexity of k-means clustering based on decision space is $O(mN)$ and the complexity of the grid search operator is $Ns(m+n)N^2$, where Ns is the number of subpopulations. The environmental selection involves removing redundant solution and updating archive operators, and the complexity is $O(N^2)$ and $O(N^2)$, respectively. And the complexity of the environmental selection is $O(N^2)$. Combining the complexity of clustering, grid search based on PSO, and environmental selection, and the number of population evolution g, the total complexity of GSMPSO-MM is $O(Ns(m+n)gN^2)$. Since Ns, m, and n are far lower than N, the final complexity of the proposed GSMPSO-MM is $O(gN^2)$.

4. Experimental results and analysis

In this section, the experiment is conducted to demonstrate the performance of the proposed GSMPSO-MM. At first, the benchmark functions and performance metrics of MMMEAs are introduced. Then, the proposed GSMPSO-MM is compared with other state-of-the-art algorithms. Finally, the influence of different parameters and strategies for the performance of GSMPSO-MM is analyzed.

4.1. Benchmark function and compared algorithms

To test the performance of the proposed GSMPSO-MM, the benchmark function is presented at the IEEE Congress on Evolutionary Computation 2019 (CEC2019) competition on MMOPs, including MMF1–15, MMF1_e, MMF1_z, MMF14_a, MMF15_a, SYM-PART simple, SYM-PART rotated, and Omni-optimizer. The characteristics and parameters of those problems are presented in Table 1, where m and n indicates the number of decision variables and objective variables, respectively. N and $maxfit$ denotes severally the population size and maximum fitness evaluations, which are influenced by the number of decision variables.

Table 1
The characteristics and parameters of benchmark functions.

Problems	m	n	PS shape	PF shape	N	$maxfit$
MMF1	2	2	Nonlinear	Convex	200	10,000
MMF2	2	2	Nonlinear	Convex	200	10,000
MMF3	2	2	Nonlinear	Convex	200	10,000
MMF4	2	2	Nonlinear	Concave	200	10,000
MMF5	2	2	Nonlinear	Convex	200	10,000
MMF6	2	2	Nonlinear	Convex	200	10,000
MMF7	2	2	Nonlinear	Convex	200	10,000
MMF8	2	2	Nonlinear	Concave	200	10,000
MMF9	2	2	Linear	Convex	200	10,000
MMF10	2	2	Linear	Convex	200	10,000
MMF11	2	2	Linear	Convex	200	10,000
MMF12	2	2	Linear	Convex	200	10,000
MMF13	3	2	Nonlinear	Convex	300	15,000
MMF14	3	3	Linear	Concave	300	15,000
MMF15	3	3	Linear	Concave	300	15,000
MMF1_z	2	2	Nonlinear	Convex	200	10,000
MMF1_e	2	2	Nonlinear	Convex	200	10,000
MMF14_a	3	3	Nonlinear	Concave	300	15,000
MMF15_a	3	3	Nonlinear	Concave	300	15,000
SYM-PART simple	2	2	Linear	Convex	200	10,000
SYM-PART rotated	2	2	Linear	Convex	200	10,000
Omni-test	3	2	Linear	Convex	300	15,000

Beyond that, there are convex and non-convex PS, and nonlinear and linear PF for 22 problems. Detailed information about 22 test problems are shown in [28].

In this experiment, seven state-of-the-art algorithms are adopted to competing algorithms, including Omni-optimizer [21], DN-NSGAII [5], MO_Ring_PSO_SCD [22], TriMOEA-TA&R [29], MMO-CLRPSO [35], MMO-ClusteringPSO [43] and DE-RLFR [36]. It is worth noting that the MMO-ClusteringPSO won the championship in the CEC 2019 MMOP competition. The parameter k in the environmental selection operator is set to $k = 2^m$. Besides, the parameters of competing algorithms are set in terms of their original literature.

4.2. Performance indicators

Four indicators, PSP [22], HV [40], IGDX [22], and IGDF [28], are adopted to estimate the performance of the proposed GSMPSO-MM and its competing algorithms. PSP indicates the similarity between obtained PS and true PS; HV reflects the convergence of obtained PF; IGDX and IGDF measure the distance between obtained PS and PF and true PS and PF, respectively. PSP and IGDX demonstrate the diversity and convergence performance of the obtained solutions in decision space, while HV and IGDF reflect the convergence and diversity of the obtained solutions in objective space. For PSP and HV, the higher value is better, while the smaller value is better for IGDX and IGDF.

Table 2
The mean value and standard deviation of PSP among different algorithms.

Problems	A1mean(std)	A2mean(std)	A3mean(std)	A4mean(std)	A5mean(std)	A6mean(std)	A7mean(std)	A8mean(std)
MMF1	10.25081	10.17090	20.15354	13.31079	24.10391	23.66735	19.45402	**27.59851**
	(1.80750)+	(1.48455)+	(0.99374)+	(1.82779)+	(1.06776)+	(0.93907)+	(1.36191)+	**(0.80342)**
MMF2	10.09215	10.92424	26.70976	11.75658	28.38704	28.92565	14.38965	**38.97152**
	(3.18733)+	(6.36226)+	(5.35658)+	(6.52870)+	(6.38801)+	(8.07264)+	(8.92593)+	**(6.18228)**
MMF3	10.62583	12.71149	29.78587	11.77395	34.20761	37.31895	18.27939	**48.15204**
	(4.65452)+	(5.34395)+	(8.40280)+	(5.13748)+	(5.99253)+	(8.66066)+	(5.37283)+	**(5.78472)**
MMF4	14.29727	11.94874	36.26345	25.93145	39.81991	38.27729	33.36986	**53.28601**
	(3.42360)+	(2.60976)+	(1.64077)+	(9.13600)+	(2.71338)+	(3.46166)+	(3.83564)+	**(1.40279)**
MMF5	5.90596	5.89413	11.89052	8.28469	13.40065	13.299	11.66757	**15.83213**
	(0.78488)+	(0.63446)+	(0.54497)+	(1.49876)+	(0.66418)+	(0.69186)+	(0.78053)+	**(0.56415)**
MMF6	7.15123	7.03776	13.91657	9.91210	15.23746	15.10769	13.09328	**18.29409**
	(0.58414)+	(0.72298)+	(0.63575)+	(1.90294)+	(0.66702)+	(0.64561)+	(1.01833)+	**(0.42064)**
MMF7	21.25522	21.44364	37.29341	23.87857	41.42121	41.10418	27.35538	**51.75962**
	(4.03386)+	(4.67168)+	(1.84198)+	(7.97444)+	(3.68780)+	(2.36432)+	(6.29156)+	**(2.59655)**
MMF8	3.78964	3.72264	14.89878	3.38975	17.15407	16.86992	13.03892	**18.99134**
	(0.69727)+	(1.38520)+	(0.93856)+	(1.38333)+	(1.31930)+	(1.61995)+	(2.62693)+	**(1.82251)**
MMF9	53.49230	50.59137	124.36800	0.00000	145.25900	141.16694	152.82252	**154.45799**
	(19.6144)+	(14.6143)+	(6.47734)+	(0.00000)+	(10.59712)+	(13.81868)+	(23.19621)~	**(4.91884)**
MMF10	3.27144	5.41801	5.57553	0.00000	5.52164	**6.20382**	2.47770	5.93279
	(3.14847)+	(3.58699)~	(1.39792)~	(0.00000)+	(0.50177)~	**(0.97202)-**	(2.69752)+	(0.20961)
MMF11	0.57082	0.57610	**4.02168**	0.00000	3.16103	3.37544	0.28427	2.34249
	(0.04700)+	(0.05300)+	**(1.84835)-**	(0.00000)+	(2.01525)-	(1.93632)-	(0.06273)+	(2.00064)
MMF12	1.40896	0.99193	4.26536	0.04264	4.27300	3.66187	0.27624	**4.62351**
	(2.03448)+	(1.70879)+	(2.76922)+	(0.11073)+	(2.77329)+	(2.58462)+	(0.15217)+	**(2.76205)**
MMF13	1.64275	1.64448	2.88699	0.91825	**3.43380**	3.14421	1.78882	3.10228
	(0.06664)+	(0.05576)+	(0.71077)+	(0.59168)+	**(0.57300)-**	(0.7012)~	(0.03953)+	(0.80546)
MMF14	10.90673	10.88364	18.79428	0.00000	19.13385	18.57547	–	**22.67918**
	(0.79467)+	(0.78063)+	(0.46135)+	(0.00000)+	(0.37747)+	(0.67431)+		**(0.35784)**
MMF15	3.50166	3.99658	6.62347	0.00000	6.68595	6.42895	–	**6.82091**
	(1.12745)+	(1.09334)+	(0.70339)~	(0.00000)+	(0.40432)~	(0.73288)+		**(0.88647)**
MMF1_z	13.52376	13.35200	27.38083	13.14600	32.49753	31.88127	24.34419	**37.05000**
	(2.93724)+	(3.07130)+	(1.60735)+	(2.87591)+	(2.44762)+	(2.35137)+	(3.04675)+	**(1.31470)**
MMF1_e	0.85634	0.72839	1.78920	0.50578	1.63485	1.72262	0.40900	**2.50939**
	(0.47998)+	(0.35452)+	(0.62150)+	(0.25216)+	(0.55547)+	(0.66812)+	(0.30270)+	**(0.62804)**
MMF14_a	9.05450	8.30738	16.50176	13.94988	**16.72610**	16.68737	–	14.93069
	(0.50022)+	(0.57834)+	(0.41658)-	(0.55913)+	**(0.44800)-**	(0.43646)-		(0.24273)
MMF15_a	4.06924	4.86311	6.01720	3.64500	6.06934	5.92257	–	**12.93301**
	(0.69367)+	(0.69477)+	(0.49600)+	(0.06130)+	(0.50868)+	(0.59819)+		**(0.35168)**
SYM-PART	0.21385	0.25964	6.02447	0.00000	6.45665	8.18865	**12.60591**	4.78579
simple	(0.07288)+	(0.09446)+	(1.08423)-	(0.00000)+	(1.19804)-	(1.8215)-	**(4.30700)-**	(1.07199)
SYM-PART	0.19915	0.20567	5.06973	1.34400	5.44395	6.25578	**6.88766**	4.32778
rotated	(0.09724)+	(0.13149)+	(0.87101)-	(1.71860)+	(1.60293)-	(2.0712)-	**(2.18828)-**	(0.55189)
Omni-test	0.58773	0.66642	2.40478	2.04484	2.62640	2.86511	**6.41531**	2.29549
	(0.12598)+	(0.09289)+	(0.49194)~	(0.81713)~	(0.54050)-	(0.65155)-	**(1.30779)-**	(0.55332)
+/-/~	22/0/0	21/0/1	15/4/3	21/0/1	14/6/2	15/6/1	18/3/1	

The result is formatted as the mean value (mean) and standard deviation (std) of PSP on each problem. A1, A2, A3, A4, A5, A6, A7 and A8 indicate Omni-optimizer [21], DN-NSGAII [5], MO_Ring_PSO_SCD [22], TriMOEA-TA&R [29], MMO-CLRPSO [35], MMO-ClusteringPSO [43], DE-RLFR [36] and GSMPSO-MM respectively.

4.3. General comparisons on benchmark function

For a fair comparison, the population number, the maximum fitness evaluations, and the size of the archive of all the peer algorithms are set to 100 $*m$, 5000 $*$ m and 100 $*m$, respectively, where m is the number of decision variables. All compared algorithms are independently conducted 21 times on 22 test problems. Furthermore, the Wilcoxon rank-sum test [44] at the significance level $p = 0.05$ is applied to compare the results gained by the proposed algorithms. Symbols "+", "-", and "$\sim$" denote that the proposed GSMPSO-MM algorithm is significantly better than, worse than, and similar to its competing algorithms. The performance metrics of the proposed GSMPSO-MM and its competing algorithms is shown in Tables 2–5. The best performance of compared algorithms on the same problems is shown in bold.

4.3.1. Analysis of the performance of solutions on decision space

In Table 2, it reveals the mean value and standard deviation of the PSP metric on 22 test problems. It can be observed that GSMPSO-MM achieves remarkable performance in decision space and obtains the best values on 15 out of 22 problems among seven competitive algorithms. Compared with Omni-optimizer, DN-NSGAII, and TriMOEA-TA&R, the proposed GSMPSO-MM outperforms the three algorithms on almost all of these problems. MO_Ring_PSO_SCD makes the highest value on MMF11, followed by MMO-CLRPSO. The difference between MMO-CLRPSO and the top-performing algorithms (MMO-ClusteringPSO) in the CEC 2019 MMOP competition is that a ring topology is established in subpopulations in MMO-CLRPSO, while their performance is similar for the PSP metric. Compared with MO_Ring_PSO_SCD, MMO-CLRPSO, and MMO-ClusteringPSO, GSMPSO-MM gets the best of them on almost all test functions except MMF10, MMF11, MMF13, MMF14_a, SYM-PART simple, SYM-PART rotated, and Omni-test. MMO-CLRPSO performs the best on MMF13 and MMF14_a, and DE-RLFR achieves the higher values on SYM-PART simple, SYM-PART rotated, and Omni-test. Compared with TriMOEA-TA&R and DE-RLFR, they are surpassed by the proposed GSMPSO-MM, MO_Ring_PSO_SCD, MMO-CLRPSO, and MMO-ClusteringPSO in most cases.

Table 3
The mean value and standard deviation of IGDX among different algorithms.

Problems	A1mean(std)	A2mean(std)	A3mean(std)	A4mean(std)	A5mean(std)	A6mean(std)	A7mean(std)	A8mean(std)
MMF1	0.09889 (0.01960)+	0.09899 (0.01502)+	0.04941 (0.00252)+	0.07505 (0.01005)+	0.04127 (0.00194)+	0.04201 (0.00167)+	0.05115 (0.00362)+	**0.03612 (0.00118)**
MMF2	0.09769 (0.03100)+	0.10628 (0.05261)+	0.03855 (0.01249)+	0.11727 (0.09588)+	0.03529 (0.0085)+	0.0357 (0.01064)+	0.08216 (0.04177)+	**0.02525 (0.00457)**
MMF3	0.10474 (0.06042)+	0.08322 (0.033)+	0.03418 (0.00987)+	0.11494 (0.11326)+	0.02907 (0.0063)~	0.02684 (0.00679)~	0.05324 (0.01364)+	**0.02046 (0.0028)**
MMF4	0.07479 (0.02058)+	0.08737 (0.02057)+	0.02737 (0.00132)+	0.07932 (0.13484)+	0.02514 (0.0018)+	0.02621 (0.00249)+	0.02986 (0.00349)+	**0.01870 (0.00054)**
MMF5	0.17013 (0.02343)+	0.16924 (0.01795)+	0.08375 (0.00396)+	0.12291 (0.02707)+	0.07464 (0.00422)+	0.07499 (0.00393)+	0.08527 (0.00573)+	**0.06308 (0.00238)**
MMF6	0.13790 (0.01040)+	0.14245 (0.01779)+	0.07151 (0.00337)+	0.10369 (0.02394)+	0.06534 (0.00313)+	0.06595 (0.00284)+	0.07618 (0.00582)+	**0.05449 (0.00125)**
MMF7	0.04718 (0.00865)+	0.04768 (0.00961)+	0.02663 (0.00144)+	0.04360 (0.01837)+	0.02437 (0.00264)~	0.0243 (0.00144)~	0.03614 (0.00724)+	**0.01941 (0.0012)**
MMF8	0.26067 (0.0502)+	0.31658 (0.15895)+	0.06706 (0.00454)+	0.30617 (0.09716)+	0.05848 (0.00500)~	0.0594 (0.00606)~	0.07866 (0.01707)+	**0.05297 (0.0059)**
MMF9	0.02278 (0.01152)+	0.02153 (0.0065)+	0.00805 (0.00046)+	0.25157 (0.00006)+	0.00693 (0.00055)~	0.00715 (0.00075)~	0.00660 (0.00109)~	**0.00647 (0.00023)**
MMF10	0.18361 (0.04333)+	**0.14698 (0.04437)-**	0.16801 (0.02179)~	0.20137 (0.00005)+	0.17304 (0.01217)+	0.15726 (0.01812)-	0.18698 (0.01814)+	0.16599 (0.00637)
MMF11	0.25024 (0.00041)+	0.25045 (0.00045)+	**0.20837 (0.02431)-**	0.25240 (0.00009)+	0.21926 (0.02536)-	0.2152 (0.02548)-	0.25152 (0.00042)+	0.22717 (0.02662)
MMF12	0.23360 (0.03010)+	0.23572 (0.02992)+	0.18718 (0.04454)~	0.24798 (0.00033)+	0.18572 (0.04416)~	0.19546 (0.04257)+	0.24659 (0.00072)+	**0.18134 (0.04394)**
MMF13	0.28814 (0.0121)+	0.28703 (0.00967)+	0.23951 (0.01291)~	0.32447 (0.04306)+	0.23159 (0.01692)~	**0.22788 (0.0128)-**	0.25969 (0.00314)+	0.23809 (0.01571)
MMF14	0.09213 (0.00689)+	0.09255 (0.00719)+	0.05320 (0.00134)+	0.26915 (0.00027)+	0.05226 (0.0012)+	0.0539 (0.002)+	–	**0.04409 (0.00073)**
MMF15	0.24569 (0.01918)+	0.23255 (0.02891)+	0.15294 (0.01785)~	0.27093 (0.00028)+	0.14986 (0.0094)~	0.15759 (0.01742)~	–	**0.14974 (0.02074)**
MMF1_z	0.07544 (0.01526)+	0.07772 (0.01849)+	0.03646 (0.00237)+	0.07843 (0.01707)+	0.03078 (0.00245)~	0.03132 (0.00236)+	0.04132 (0.0054)+	**0.02689 (0.00099)**
MMF1_e	1.08485 (0.53223)+	1.20098 (0.59331)+	0.54047 (0.17247)+	1.53562 (0.53703)+	0.58410 (0.20177)+	0.56231 (0.18361)+	1.83817 (0.80786)+	**0.39523 (0.09998)**
MMF14_a	0.11069 (0.00614)+	0.12102 (0.00889)+	0.06064 (0.00173)~	0.07180 (0.00282)~	**0.05975 (0.00174)~**	0.05988 (0.00158)~	–	0.06700 (0.00121)
MMF15_a	0.21788 (0.01937)+	0.20232 (0.02225)+	0.16493 (0.01401)+	0.21999 (0.00243)+	0.16298 (0.01293)+	0.16883 (0.01665)+	–	**0.07733 (0.00222)**
SYM-PART simple	4.96812 (1.53356)+	4.05105 (1.0955)+	0.17263 (0.0356)-	9.13719 (1.44346)+	0.16024 (0.03163)-	**0.12943 (0.03366)-**	0.17297 (0.30946)-	0.28326 (0.26195)
SYM-PART rotated	4.55974 (1.76618)+	4.15980 (1.32257)+	**0.20092 (0.03574)-**	1.73268 (1.08119)+	0.26451 (0.27238)+	0.23819 (0.27401)~	0.21976 (0.26882)~	0.23383 (0.03794)
Omni-test	1.60070 (0.24579)+	1.50251 (0.22156)+	0.42619 (0.08603)~	0.56413 (0.26214)+	0.39399 (0.08379)-	0.36398 (0.07554)-	**0.16095 (0.03135)-**	0.45020 (0.10494)
+/-/~	22/0/0	21/1/0	13/3/6	21/0/1	10/3/9	10/5/7	18/2/2	

The result is formatted as the mean value (mean) and standard deviation (std) of IGDX on each problem. A1, A2, A3, A4, A5, A6, A7 and A8 indicate Omni-optimizer [21], DN-NSGAII [5], MO_Ring_PSO_SCD [22], TriMOEA-TA&R [29], MMO-CLRPSO [35], MMO-ClusteringPSO [43], DE-RLFR [36] and GSMPSO-MM respectively.

The performance of the IGDX metric for eight algorithms in Table 3 is similar to the PSP indicator. GSMPSO-MM performs the best on 15 out of 22 problems. Compared with Omni-optimizer, DN-NSGAII, and TriMOEA-TA&R, GSMPSO-MM outperforms the three competing algorithms on almost all of the problems apart from MMF10. MO_Ring_PSO_SCD performs the best on MMF11 and SYM-PART rotated. It is obvious that GSMPSO-MM is superior to MO_Ring_PSO_SCD in almost all cases. DE-RLFR is unavailable to perform on MMF14, MMF14_a, MMF15, and MMF15_a. Multi-populations are also adopted to MMO-CLRPSO and MMO-ClusteringPSO to develop more equivalent PS in decision space, and MMO-CLRPSO and MMO-ClusteringPSO perform the best on MMF13, MMF14_a, and SYM-PART simple. However, it is observed that the proposed GSMPSO-MM performs better than MMO-CLRPSO and MMO-ClusteringPSO in most cases, owing to the grid search and environmental selection.

Based on the above analysis on the performance of the PSP and IGDX metrics, it is confirmed that GSMPSO-MM performs better than other algorithms in most cases, and GSMPSO-MM can find more equivalent PS in decision space.

4.3.2. Analysis of the performance of solutions on objective space

It is observed in Table 4 that the HV values of different algorithms among the same problems are approximately equal and only have slight differences. GSMPSO-MM obtains a promising performance, which achieves the best performance on 9 out of 22. Omni-optimizer performs the best results on MMF8, MMF11, MMF13, SYM-PART simple, SYM-PART rotated, and Omni-test; DN-NSGAII obtains the highest values on MMF14 and MMF14_a; TriMOEA-TA&R achieves the optimal value on MMF10, MMF12 MMF15, and MMF15_a. DE-RLFR gets only one best value on MMF1_e, while MO_Ring_PSO_SCD, MMO-CLRPSO, and GSMPSO-MM fail to obtain the highest values in any problems. Compared with all competing algorithms, GSMPSO-MM performs the best results on MMF1–7, MMF9, and MMF1_z, while GSMPSO-MM has a poor performance on SYM-PART simple, SYM-PART rotated, and Omni-test. Overall speaking, GSMPSO-MM gets better performance than its competing algorithms in terms of the HV metric.

As shown in Table 5, GSMPSO-MM achieves significant performance and performs the best results on 15 out of 22. Omni-optimizer obtains the best performance on MMF8, SYM-PART simple, SYM-PART rotated,

Table 4
The mean value and standard deviation of HV among different algorithms.

Problems	A1mean(std)	A2 mean(std)	A3mean(std)	A4mean(std)	A5mean(std)	A6mean(std)	A7mean(std)	A8mean(std)
MMF1	0.87069	0.86981	0.87065	0.86928	0.87152	0.87109	0.87081	**0.87234**
	(0.00084)+	(0.00093)+	(0.00024)+	(0.00117)+	(0.00049)~	(0.00097)+	(0.00124)+	**(0.00022)**
MMF2	0.84843	0.84868	0.84376	0.82135	0.84641	0.84719	0.82070	**0.85210**
	(0.01850)+	(0.01613)+	(0.00633)+	(0.08853)+	(0.00631)+	(0.00548)+	(0.02346)+	**(0.00358)**
MMF3	0.84887	0.85079	0.84797	0.81891	0.85301	0.85322	0.83688	**0.85687**
	(0.02289)+	(0.01482)~	(0.00641)+	(0.11451)+	(0.00309)~	(0.00323)~	(0.01185)+	**(0.00241)**
MMF4	0.53902	0.53838	0.53722	0.52851	0.53747	0.53668	0.53663	**0.53911**
	(0.00018)~	(0.00023)~	(0.0006)~	(0.0474)+	(0.00061)~	(0.00106)~	(0.001)~	**(0.00021)**
MMF5	0.87182	0.87060	0.87091	0.86914	0.87159	0.87137	0.87116	**0.87241**
	(0.00079)~	(0.00155)+	(0.00033)~	(0.00333)+	(0.00028)~	(0.00046)~	(0.00065)~	**(0.00019)**
MMF6	0.87150	0.87019	0.87077	0.86964	0.87123	0.87113	0.87107	**0.87259**
	(0.00097)~	(0.00115)+	(0.00054)~	(0.00282)+	(0.00077)~	(0.00083)~	(0.00054)~	**(0.00017)**
MMF7	0.87173	0.86967	0.87080	0.87097	0.87039	0.87057	0.87082	**0.87254**
	(0.00038)~	(0.00153)+	(0.00044)~	(0.00145)~	(0.0008)~	(0.00077)~	(0.00064)~	**(0.0002)**
MMF8	**0.42119**	0.41969	0.41544	0.41991	0.41841	0.41732	0.40536	0.41949
	(0.00015)~	(0.00054)~	(0.00287)~	(0.00054)~	(0.00164)~	(0.00286)~	(0.04189)+	(0.0006)
MMF9	9.68171	9.67625	9.67101	9.55664	9.66609	9.65961	9.67006	**9.68261**
	(0.00191)~	(0.00215)+	(0.00292)+	(0.01752)+	(0.00685)+	(0.01261)+	(0.01434)+	**(0.00126)**
MMF10	12.26546	12.16388	12.56292	**12.81071**	12.57993	12.52605	12.63349	12.62395
	(0.47281)+	(0.40451)+	(0.09147)+	**(0.00461)-**	(0.06707)~	(0.10886)+	(0.4663)-	(0.06426)
MMF11	**14.51374**	14.50570	14.49326	14.37933	14.50364	14.49785	14.49255	14.50782
	(0.00165)~	(0.00284)~	(0.00434)+	(0.01548)+	(0.00567)~	(0.01177)+	(0.01373)+	(0.00415)
MMF12	1.49646	1.55215	1.56543	**1.57223**	1.56434	1.56418	1.56114	1.56560
	(0.15304)+	(0.0755)+	(0.00272)~	**(0.00017)-**	(0.00466)~	(0.00784)~	(0.01703)~	(0.00314)
MMF13	**18.44279**	18.43388	18.38900	18.20586	18.40148	18.41036	18.4265	18.40505
	(0.00116)-	(0.00406)-	(0.00704)+	(0.02013)+	(0.01032)~	(0.00735)~	3(0.0078)-	(0.01144)
MMF14	2.92616	**3.06486**	2.88662	3.05515	2.88662	2.87846	–	2.93621
	(0.06741)~	**(0.12933)-**	(0.22835)+	(0.12138)-	(0.20526)+	(0.18071)+		(0.17541)
MMF15	4.27143	4.31231	4.22107	**4.49067**	4.16889	4.19633	–	4.19160
	(0.14952)-	(0.16077)-	(0.15103)-	**(0.11746)-**	(0.1904)~	(0.17619)~		(0.11689)
MMF1_z	0.87188	0.87072	0.87063	0.86881	0.87165	0.87156	0.87098	**0.87243**
	(0.00067)~	(0.00102)~	(0.00041)~	(0.00227)+	(0.00026)~	(0.00039)~	(0.00074)~	**(0.00019)**
MMF1_e	0.84696	0.83240	0.84390	0.86404	0.85082	0.85354	**0.86733**	0.83808
	(0.03862)~	(0.01927)~	(0.012)~	(0.00366)-	(0.00935)-	(0.00949)-	**(0.0038)-**	(0.01339)
MMF14_a	3.03341	**3.14569**	3.01469	2.95480	2.94232	3.0453	–	3.01428
	(0.0958)-	**(0.12182)-**	(0.24489)~	(0.10638)+	(0.26668)+	(0.2615)-		(0.19113)
MMF15_a	4.29475	4.37175	4.17212	**4.40684**	4.21657	4.25535	–	4.23418
	(0.17366)~	(0.17617)-	(0.19746)~	**(0.11388)-**	(0.16806)~	(0.18956)~		(0.18813)
SYM-PART	**16.64552**	16.64458	16.53231	16.63476	16.56724	16.59732	16.63838	16.47387
simple	**(0.00201)-**	(0.00204)-	(0.02532)-	(0.00366)-	(0.01058)~	(0.01441)-	(0.00481)-	(0.02625)
SYM-PART	**16.64238**	16.63545	16.49907	16.6235	16.55448	16.57902	16.61245	16.46381
rotated	**(0.00198)-**	(0.0035)-	(0.01979)~	(0.00415)-	(0.01752)-	(0.01092)-	(0.00828)-	(0.01914)
Omni-test	**52.80232**	52.79692	52.57667	52.70333	52.60834	52.66345	52.67265	52.56290
	(0.00142)-	(0.00209)-	(0.05304)~	(0.03687)-	(0.0404)~	(0.02722)~	(0.04894)-	(0.04236)
+/-/~	5/6/11	8/8/6	8/2/12	11/9/2	4/2/16	6/4/12	10/6/6	

The result is formatted as the mean value (mean) and standard deviation (std) of HV on each problem. A1, A2, A3, A4, A5, A6, A7 and A8 indicate Omni-optimizer [21], DN-NSGAII [5], MO_Ring_PSO_SCD [22], TriMOEA-TA&R [29], MMO-CLRPSO [35], MMO-ClusteringPSO [43], DE-RLFR [36] and GSMPSO-MM respectively.

and Omni-test; MMO-ClusteringPSO gets the best result on MMF12 and MMF13; MMO-CLRPSO and DE-RLRF gain the best performance on MMF11 and MMF1_e, respectively. DN-NSGAII, MO_Ring_PSO_SCD, and TriMOEA-TA&R haven't achieved the best results on any problems, but the whole performance on all problems is accepted. Compared with these competing algorithms, GSMPSO-MM demonstrates a promising performance on almost all problems and outperforms its competing algorithms.

For the performance of the HV and IGDF metrics in Tables 4 and 5, it is reasonable to deduce that GSMPSO-MM performs better than its competing algorithms in most cases, being able to maintain the diversity and convergence of obtained solutions in objective space.

In conclusion, it is confirmed that the proposed GSMPSO-MM algorithm obtains the best performance in both decision space and objective space through the above discussion, and GSMPSO-MM is suitable to solve MMOPs and locate more PS in decision space.

4.4. Effectiveness of clustering and grid in GSMPSO-MM

Clustering is an effective niching method for multimodal optimization. To obtain more equivalent PS in decision space, the k-means clustering method is adopted to divide the population into multiple subpopulations in GSMPSO-MM. Furthermore, the grid search cooperates with clustering in the proposed GSMPSO-MM to find more PS in decision space. Here, an experiment is designed to demonstrate the contribution of the clustering and grid for the proposed GSMPSO-MM. Fig. 6 shows the PSP and IGDX values of GSMPSO-MM with only clustering, GSMPSO-MM with only grid, and GSMPSO-MM with clustering and grid on 11 problems. Compared with the performance of the PSP indicator, it is observed that the proposed GSMPSO-MM with clustering and grid outperforms the GSMPSO-MM with only clustering or grid in most cases and the performance of GSMPSO-MM with only clustering and GSMPSO-MM with the only grid is similar. Compared with the performance of the IGDX metric, it is clear that the IGDX values of GSMPSO-MM are significantly better than the GSMPSO-MM with only clustering or grid, and the performance of the GSMPSO-MM with only clustering and the GSMPSO-MM with the only grid is close on MMF1–8, but the performance of the GSMPSO-MM with only clustering is inferior to the GSMPSO-MM with the only grid on SYM-PART simple, SYM-PART rotated, and Omni-test. Combining the performance of GSMPSO-MM, GSMPSO-MM with only clustering, GSMPSO-MM with only grid on PSP and IGDX metrics, it

Table 5
The mean value and standard deviation of IGDF among different algorithms.

Problems	A1mean(std)	A2 mean(std)	A3 mean(std)	A4 mean(std)	A5 mean(std)	A6 mean(std)	A7 mean(std)	A8 mean(std)
MMF1	0.00392	0.00438	0.00374	0.00480	0.00313	0.00335	0.00359	**0.00276**
	(0.00049)+	(0.0006)+	(0.00015)+	(0.00073)+	(0.00018)+	(0.00049)+	(0.00037)+	**(0.00014)**
MMF2	0.02404	0.02171	0.02106	0.05305	0.01801	0.01842	0.04930	**0.01322**
	(0.01877)+	(0.01661)+	(0.00512)+	(0.08458)+	(0.00402)+	(0.00473)+	(0.02987)+	**(0.00142)**
MMF3	0.02497	0.01898	0.01808	0.05269	0.01428	0.01436	0.03091	**0.01053**
	(0.02494)+	(0.01273)+	(0.00538)+	(0.09786)+	(0.00198)~	(0.00304)~	(0.0125)+	**(0.00085)**
MMF4	0.00283	0.00319	0.0036	0.01974	0.00350	0.00402	0.00368	**0.00255**
	(0.00018)~	(0.00027)+	(0.00035)+	(0.05265)~	(0.00039)+	(0.00075)+	(0.00026)+	**(0.00018)**
MMF5	0.00325	0.00370	0.00361	0.00510	0.00310	0.00323	0.00354	**0.00268**
	(0.00038)+	(0.00032)+	(0.00018)+	(0.00242)+	(0.00016)+	(0.00023)+	(0.00027)+	**(0.00010)**
MMF6	0.00320	0.00378	0.00358	0.00457	0.00311	0.00317	0.00355	**0.00256**
	(0.00038)+	(0.00032)+	(0.00022)+	(0.00185)+	(0.00023)~	(0.00025)~	(0.00021)+	**(0.00008)**
MMF7	0.00314	0.00400	0.00381	0.00418	0.00397	0.00388	0.00410	**0.00252**
	(0.00024)+	(0.00041)+	(0.00032)+	(0.0013)+	(0.00046)+	(0.00045)+	(0.00058)+	**(0.00006)**
MMF8	**0.00319**	0.00419	0.00488	0.00346	0.00384	0.00342	0.00403	0.00375
	(0.00027)~	(0.00054)~	(0.00045)+	(0.00011)~	(0.00027)~	(0.00022)~	(0.00046)~	(0.00040)
MMF9	0.01281	0.01430	0.01604	0.06827	0.01851	0.02123	0.01927	**0.01090**
	(0.00117)~	(0.00135)+	(0.00179)+	(0.00739)+	(0.00307)+	(0.00526)+	(0.00571)+	**(0.00060)**
MMF10	0.19381	0.18257	0.20042	0.22830	0.19965	0.20679	0.18295	**0.18103**
	(0.04313)+	(0.0415)~	(0.01247)+	(0.0026)+	(0.01769)+	(0.01976)+	(0.02575)~	**(0.01450)**
MMF11	0.09595	0.09800	0.08895	0.16332	**0.08518**	0.08814	0.10745	0.08537
	(0.00091)+	(0.00143)+	(0.007)~	(0.00783)+	**(0.00569)~**	(0.00815)~	(0.0059)+	(0.00602)
MMF12	0.09198	0.08654	0.06419	0.08634	0.06498	**0.06405**	0.08877	0.06439
	(0.01544)+	(0.00773)+	(0.01331)~	(0.00159)+	(0.01354)~	**(0.01165)~**	(0.00415)+	(0.01268)
MMF13	0.14687	0.15121	0.09927	0.24403	0.08933	**0.08402**	0.15566	0.09705
	(0.0014)+	(0.00357)+	(0.02721)~	(0.00726)+	(0.01722)-	**(0.01054)-**	(0.00899)+	(0.01975)
MMF14	0.09825	0.11173	0.08060	0.08664	0.07974	0.08111	–	**0.06471**
	(0.0035)+	(0.00947)+	(0.00268)+	(0.00099)+	(0.00256)+	(0.00353)+		**(0.00123)**
MMF15	0.20244	0.21502	0.17442	0.20634	0.17238	0.1738	–	**0.16319**
	(0.0078)+	(0.00834)+	(0.00379)+	(0.00081)+	(0.00284)+	(0.00329)+		**(0.00337)**
MMF1_z	0.00305	0.00368	0.00375	0.00504	0.00301	0.00311	0.00353	**0.00261**
	(0.00028)~	(0.00054)+	(0.00023)+	(0.00145)+	(0.00014)~	(0.00015)~	(0.00027)+	**(0.00009)**
MMF1_e	0.01548	0.03175	0.01210	0.00767	0.01182	0.01052	**0.00580**	0.01181
	(0.00833)+	(0.03016)+	(0.0015)~	(0.00198)-	(0.00203)~	(0.0019)-	**(0.00274)-**	(0.00135)
MMF14_a	0.10463	0.11957	0.07755	0.09201	0.07727	0.07648	–	**0.06218**
	(0.00619)+	(0.00929)+	(0.00168)+	(0.00234)+	(0.00169)+	(0.00145)+		**(0.00104)**
MMF15_a	0.20733	0.22558	0.17379	0.19539	0.17454	0.17323	–	**0.16611**
	(0.00728)+	(0.00913)+	(0.00329)~	(0.00353)+	(0.00308)~	(0.00313)~		**(0.00408)**
SYM-PART	**0.01223**	0.01254	0.04192	0.03312	0.03288	0.02572	0.02448	0.05219
simple	**(0.00125)-**	(0.00158)-	(0.00624)-	(0.00317)-	(0.00412)-	(0.00538)-	(0.00852)-	(0.00718)
SYM-PART	**0.01294**	0.01480	0.04705	0.02767	0.03592	0.02997	0.02683	0.05725
rotated	**(0.00155)-**	(0.00203)-	(0.00802)-	(0.00400)-	(0.00535)-	(0.00495)-	(0.00587)-	(0.00764)
Omni-test	**0.00671**	0.00797	0.04225	0.02085	0.03389	0.0292	0.02274	0.04362
	(0.00029)-	(0.00054)-	(0.00568)~	(0.00374)-	(0.00417)-	(0.00285)-	(0.00422)-	(0.00360)
+/-/~	15/3/4	17/3/2	14/2/6	16/4/2	10/4/8	10/5/7	16/4/2	

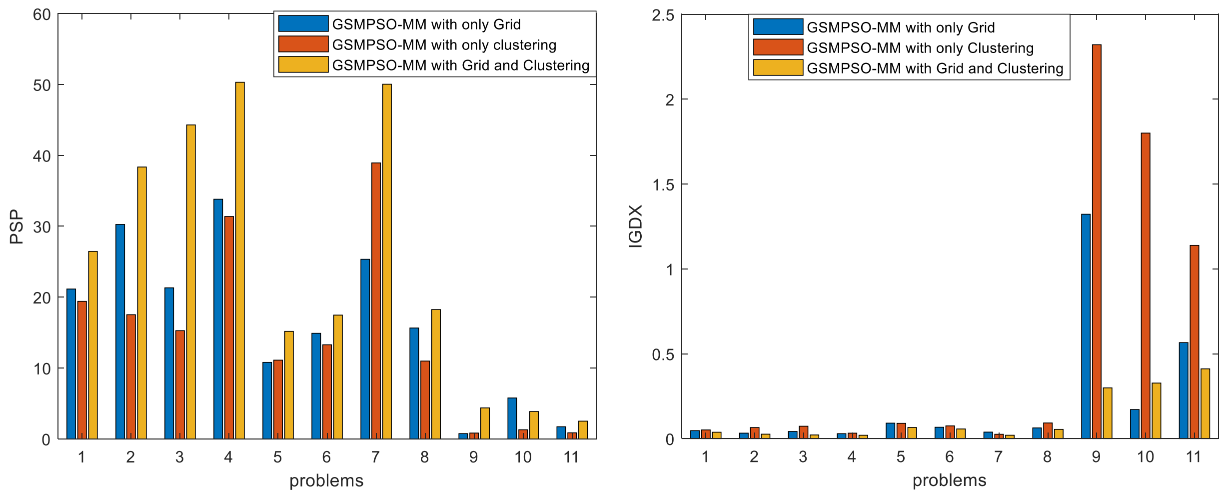
The result is formatted as the mean value (mean) and standard deviation (std) of IGDF on each problem. A1, A2, A3, A4, A5, A6, A7 and A8 indicate Omni-optimizer [21], DN-NSGAII [5], MO_Ring_PSO_SCD [22], TriMOEA-TA&R [29], MMO-CLRPSO [35], MMO-ClusteringPSO [43], DE-RLFR [36] and GSMPSO-MM respectively.

can be said that GSMPSO-MM with only clustering or grid cannot perform as well as GSMPSO-MM. Therefore, we can infer that both clustering and grid operations perform a critical contribution in locating more equivalent PS in decision space and improving the performance of GSMPSO-MM.

Furthermore, in an attempt to confirm the influence of different clustering methods on the performance of the proposed GSMPSO-MM, two additional means are introduced into the proposed GSMPSO-MM to demonstrate the effectiveness of the k-means clustering. One is the zoning search method in which the entire search space is divided into multiple identical subareas in decision space, the other is the special clustering method in MMO-CLRPSO, where the population is divided into multiple subpopulations according to one decision variable with the maximum standard deviation. For more detailed decision variables clustering methods, please refer to [35]. In this experiment, the decision space is divided into $2m$ identical subareas for the zoning search method, and the whole population is divided into an identical number of subpopulations for the decision variable clustering method and k-means clustering method. Three distinct clustering methodologies are performed on 11 problems, and the performance of the PSP and IGDX metrics for GSMPSO-MM is shown in Fig. 7. We can observe that the GSMPSO-MM using the zoning searching is worse than two other clustering methods for the PSP and IGDX metrics in most cases, and the GSMPSO-MM via k-means clustering method is better than or similar to the performance of the GSMPSO-MM using one decision variable with the maximum standard deviation clustering method. It reveals that the performance of the GSMPSO-MM using the k-means clustering method and other clustering methods has inconspicuous differences.

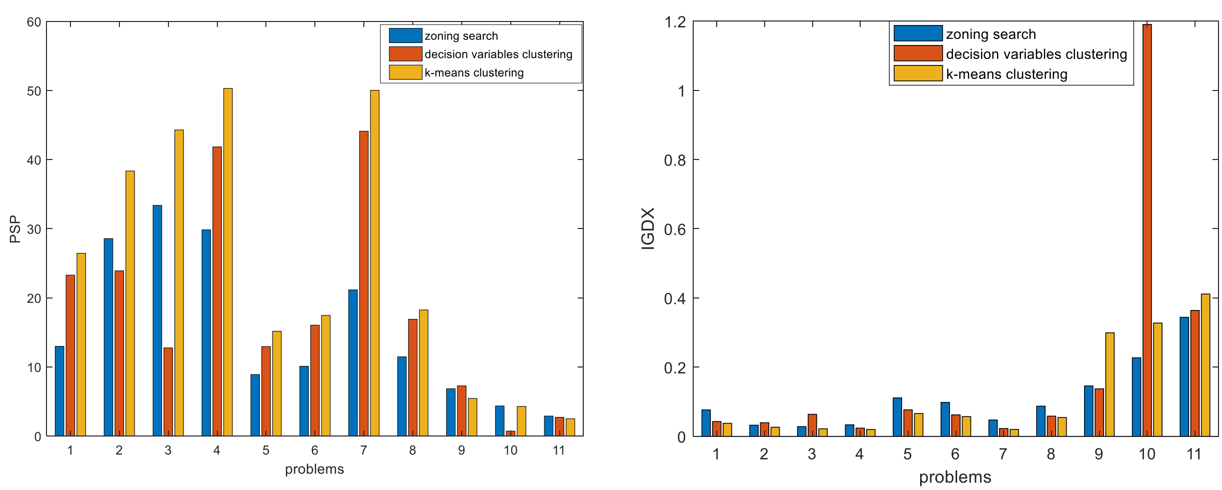
4.5. Effectiveness of the environmental selection strategy

In order to study the effectiveness of the environmental selection in GSMPSO-MM, a GSMPSO-MM variant without the environmental section operator is adopted to a competing algorithm, named as GSMPSO-MM-NG. The population number and the max fitness evaluations are set to $50 * m$ and $2500 * m$, separately. Four performance metrics of the GSMPSO-MM and its variant are shown in Table 6. It should be noted



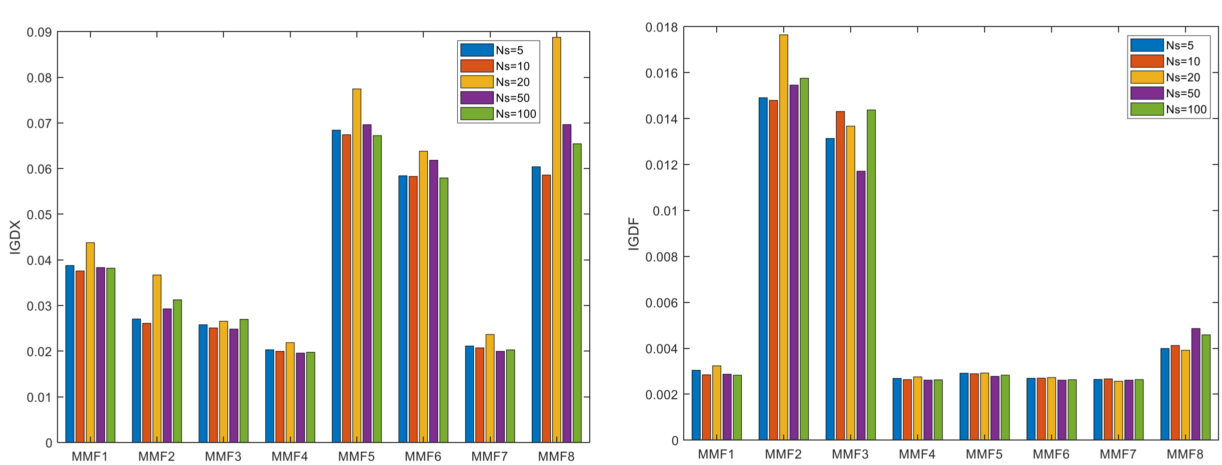
(a) The PSP values on 11 problems (b) The IGDX values on 11 problems

Fig. 6. The PSP and IGDX values of GSMPSO-MM, GSMPSO-MM with only clustering, and GSMPSO-MM with the only grid.The numbers on the horizontal axis denote the following problems: 1=MMF1, 2=MMF2, 3=MMF3, 4=MMF4, 5=MMF5, 6=MMF6, 7=MMF7, 8=MMF8, 9=SYM-PART simple, 10=SYM-PART rotated, and 11=Omni-test.



(a) The PSP values on 11 problems (b) The IGDX values on 11 problems

Fig. 7. The PSP and IGDX values of GSMPSO-MM with three distinct clustering methods.The numbers on the horizontal axis denote the following problems: 1=MMF1, 2=MMF2, 3=MMF3, 4=MMF4, 5=MMF5, 6=MMF6, 7=MMF7, 8=MMF8, 9=SYM-PART simple, 10=SYM-PART rotated, and 11=Omni-test.



(a). The IGDX value on MMF1-8 (b).The IGDF value on MMF1-8

Fig. 8. The IGDX and IGDF indicator of GSMPSO-MM on 8 problems.

Table 6
The performance of GSMPSO-MM and GSMPSO-MM-NG for four indicators.

Problem	PSP mean(std)		HV mean(std)		IGDX mean(std)		IGDF mean(std)	
	Case 1	Case 2	Case 1	Case 2	Case 1	Case 2	Case 1	Case 2
MMF1	**15.68042**	1.90340	**0.86814**	0.84241	**0.06334**	0.36046	**0.00532**	0.01661
	(0.47672)	(0.28136)	**(0.00048)**	(0.02364)	**(0.00187)**	(0.04334)	**(0.00027)**	(0.01045)
MMF2	22.54003	**22.93864**	**0.83857**	0.83363	0.04968	**0.04293**	0.02212	**0.02190**
	(7.40955)	**(4.81091)**	**(0.00653)**	(0.00761)	(0.02822)	**(0.00831)**	(0.00318)	**(0.00347)**
MMF3	26.14475	**26.85754**	**0.84531**	0.84054	0.03668	**0.03664**	0.01924	**0.01908**
	(4.3056)	**(5.36247)**	**(0.00417)**	(0.00679)	(0.00654)	**(0.0078)**	(0.00275)	**(0.0027)**
MMF4	**29.02269**	0.65626	**0.53489**	0.40485	**0.03422**	0.68867	**0.00514**	0.23627
	(1.13552)	(0.20703)	**(0.00055)**	(0.05000)	**(0.00137)**	(0.08743)	**(0.00023)**	(0.10252)
MMF5	**9.63686**	1.91820	**0.86834**	0.85294	**0.10337**	0.36175	**0.00530**	0.01172
	(0.4701)	(0.21467)	**(0.00045)**	(0.01423)	**(0.00517)**	(0.02898)	**(0.00025)**	(0.0057)
MMF6	**11.43101**	1.59295	**0.86876**	0.80304	**0.08707**	0.41693	**0.00505**	0.03725
	(0.43069)	(0.35052)	**(0.00034)**	(0.06039)	**(0.00319)**	(0.07134)	**(0.00016)**	(0.03378)
MMF7	**28.56715**	1.16718	**0.86819**	0.68492	**0.03486**	0.49830	**0.00528**	0.10980
	(1.13614)	(0.33149)	**(0.00043)**	(0.09267)	**(0.00144)**	(0.09444)	**(0.00019)**	(0.0636)
MMF8	**10.5473**	1.82531	**0.41564**	0.34107	**0.09496**	0.64212	**0.00662**	0.00864
	(1.32951)	(1.22952)	**(0.00094)**	(0.19954)	**(0.01352)**	(0.33754)	**(0.00066)**	(0.00162)
MMF9	**84.28803**	1.43619	**9.65522**	7.99364	**0.01191**	0.32105	**0.02149**	1.47003
	(6.07016)	(0.33284)	**(0.00378)**	(0.49252)	**(0.00095)**	(0.03658)	**(0.00165)**	(0.43396)
MMF10	**6.97822**	6.87910	**12.46068**	12.45527	**0.14747**	0.15426	**0.20572**	0.22026
	(1.90379)	(3.05036)	**(0.13995)**	(0.18161)	**(0.03097)**	(0.03728)	**(0.02065)**	(0.04967)
MMF11	**2.21862**	1.49588	**14.47909**	13.63607	**0.23083**	0.35543	**0.08948**	0.67300
	(1.96745)	(0.90633)	**(0.00512)**	(0.55937)	**(0.0267)**	(0.07347)	**(0.00524)**	(0.43957)
MMF12	0.89172	**4.11110**	**1.56304**	1.55993	0.24184	**0.18882**	0.07635	**0.06889**
	(0.17464)	**(2.44512)**	**(0.00335)**	(0.00639)	(0.00229)	**(0.04017)**	(0.01439)	**(0.01246)**
MMF13	**1.80771**	1.76673	**18.41033**	17.66079	**0.26044**	0.32087	**0.13869**	0.34386
	(0.03756)	(0.65927)	**(0.00432)**	(0.50279)	**(0.00178)**	(0.05798)	**(0.01394)**	(0.24091)
MMF14	**15.75605**	0.99826	**2.90706**	1.54674	**0.06348**	0.43594	**0.09703**	1.29151
	(0.30868)	(0.10961)	**(0.19828)**	(0.27987)	**(0.00125)**	(0.01544)	**(0.00189)**	(0.0462)
MMF15	**6.27373**	1.09141	**4.15576**	2.75303	**0.16070**	0.45630	**0.18349**	1.28333
	(0.55478)	(0.14719)	**(0.20398)**	(0.14689)	**(0.01448)**	(0.02317)	**(0.00437)**	(0.07679)
MMF1_z	**20.6551**	1.83157	**0.86848**	0.83708	**0.04823**	0.36968	0.00518	0.01814
	(1.19936)	(0.24433)	**(0.00051)**	(0.02186)	**(0.00288)**	(0.03896)	(0.00023)	(0.00988)
MMF1_e	1.41313	**1.55247**	**0.84452**	0.76698	0.66744	**0.64347**	0.01877	**0.01765**
	(0.4017)	**(0.5462)**	**(0.00489)**	(0.10196)	(0.17301)	**(0.24282)**	(0.00296)	**(0.00317)**
MMF14_a	**10.72124**	1.03229	**3.00589**	1.56942	**0.09320**	0.43496	**0.09500**	1.27082
	(0.27179)	(0.08711)	**(0.2848)**	(0.27757)	**(0.00236)**	(0.01307)	**(0.00261)**	(0.03752)
MMF15_a	**9.72459**	1.19022	**4.30016**	2.75734	**0.10260**	0.42577	**0.19029**	1.25893
	(0.29695)	(0.12096)	**(0.17699)**	(0.14737)	**(0.00314)**	(0.01864)	**(0.00398)**	(0.0613)
SYM-PART	2.50975	**2.80124**	**16.3728**	16.36024	0.728140	**0.50937**	**0.07862**	0.07936
simple	(1.38597)	**(1.08172)**	**(0.0479)**	(0.05083)	(0.64006)	**(0.39592)**	**(0.01403)**	(0.01002)
SYM-PART	**2.39114**	2.06933	**16.35684**	16.33442	**0.72763**	0.75148	0.08316	**0.08283**
rotated	**(1.19494)**	(1.04767)	**(0.05343)**	(0.10776)	**(0.71388)**	(0.59167)	(0.01726)	**(0.01946)**
Omni-test	1.32406	**1.40050**	**52.51972**	52.51865	0.77625	**0.73077**	0.04812	**0.04519**
	(0.29731)	**(0.30813)**	**(0.05403)**	(0.06908)	(0.15135)	**(0.15623)**	(0.00453)	**(0.00438)**

The result is formatted as the mean value (mean) and standard deviation (std) of PSP, HV, IGDX, and IGDF on each problem. Case 1: GSMPSO-MM, Case 2: GSMPSO-MM without environmental selection operator (GSMPSO-MM-NG).

that Case 1 denotes GSMPSO-MM and Case 2 indicates GSMPSO-MM-NG in Table 6.

For PSP, it can be seen that the proposed GSMPSO-MM is superior to its competing operator in most problems in Table 6. GSMPSO-MM-NG obtains the best on MMF2, MMF3, MMF12, MMF1_e, SYM-PART simple, and Omni-test, but the performance of the GSMPSO-MM on others problems is close to that of GSMPSO-MM-NG. Similarly, the proposed GSMPSO-MM performs the best on the same problems for IGDX, and GSMPSO-MM and its variant perform similar results on other problems. Based on the performance of GSMPSO-MM and its variant on the PSP and IGDX metrics, it can be confirmed that the operator of updating double archives in GSMPSO-MM is suitable to find more equivalent PS and develop the diversity and convergence of solutions in decision space. Meanwhile, it demonstrates that the environmental selection operator in GSMPSO-MM conducives to obtain better performance in most cases.

Compared with the performance of GSMPSO-MM-NG in objective space, the proposed GSMPSO-MM with the removing boundary redundant solutions operator performs the highest HV values on all problems. The significant improvement for GSMPSO-MM on the HV metric declares that this operator is effective to the convergence of solutions. For IGDF, it is observed that GSMPSO-MM achieves the best on 16 out of 22, which makes an advance on most problems owing to a removed operator in the environmental selection process. The operator, removing redundant boundary solutions, is suitable to ensure the convergence of solutions in the future evolutionary process.

Compared with the PSP, HV, IGDX, and IGDF of GSMPSO-MM-NG, it is reasonable to estimate that the environmental selection, including the double archives operator and the removing redundant solutions operator, is helpful for GSMPSO-MM to maintain the diversity and convergence of solutions in both decision space and objective space.

4.6. Analysis of the sensitivity of the subpopulations number in GSMPSO-MM

In GSMPSO-MM, the performance of GSMPSO-MM is influenced by the number of subpopulations. Therefore, an extensive experiment is conducted on MMF1–8 to explore an appropriate number of subpopulations. The IGDX and IGDF value of GSMPSO-MM on MMF1–8 is provided in Fig. 8. There is no obvious fluctuation with the raise of the subpopulation number for IGDX and IGDF on MMF1–8, while the performance of GSMPSO-MM with various subpopulation numbers has subtle difference. It is observed that the subpopulation number $Ns = 20$ shows poor

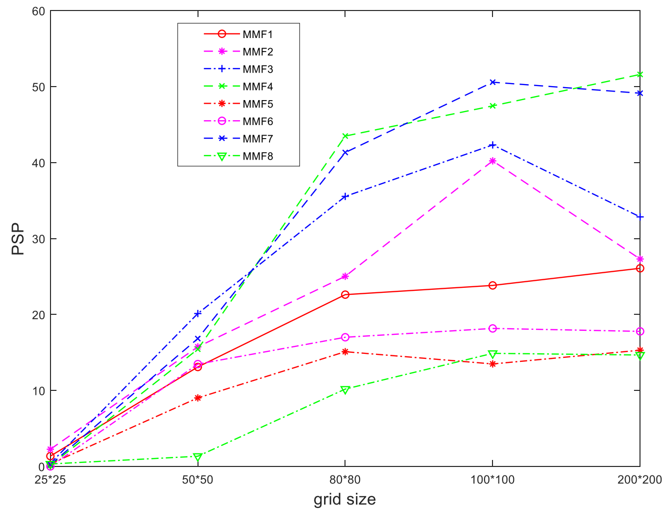


Fig. 9. The PSP value of GSMPSO-MM with different grid sizes.

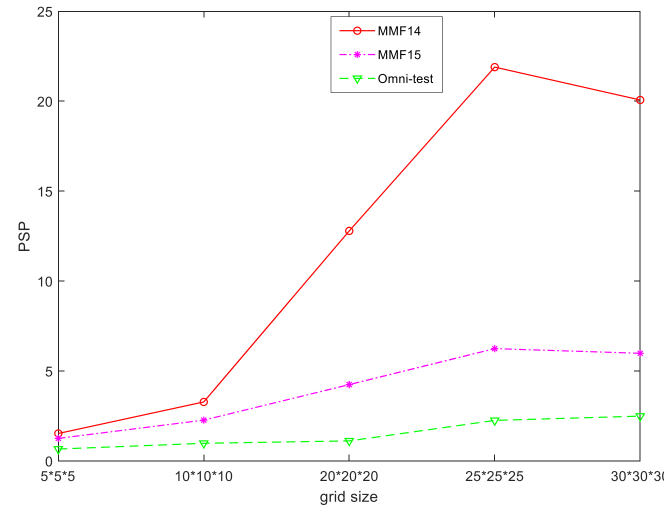
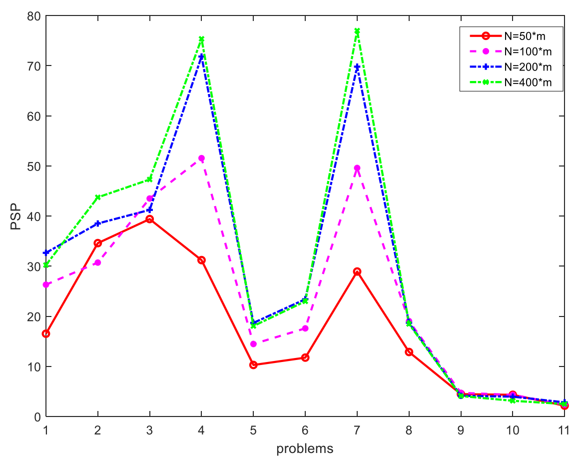
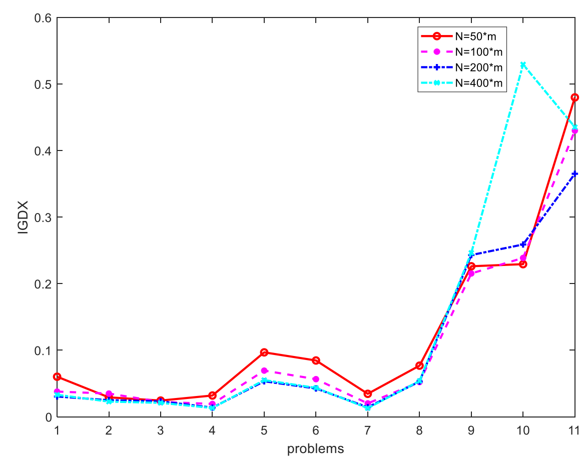


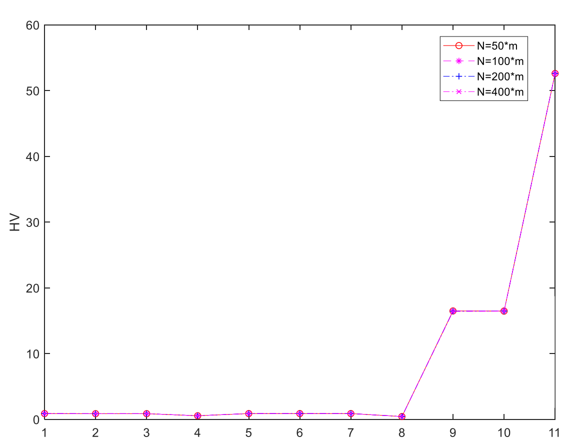
Fig. 10. The PSP value of GSMPSO-MM with different grid sizes for 3-dimensional MMOPs.



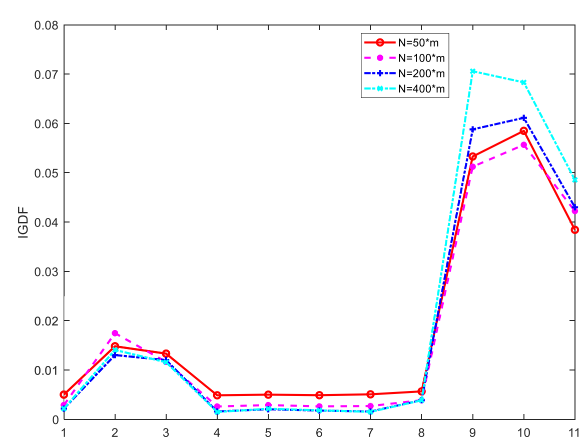
(a). The performance of the PSP metric of GSMPSO-MM with different population size



(b). The performance of the IGDX metric of GSMPSO-MM with different population size

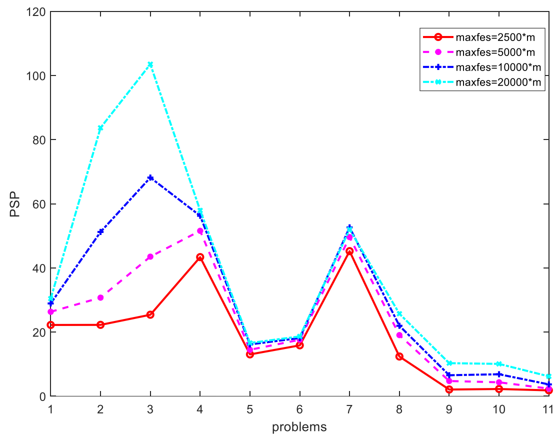


(c). The performance of the HV metric of GSMPSO-MM with different population size

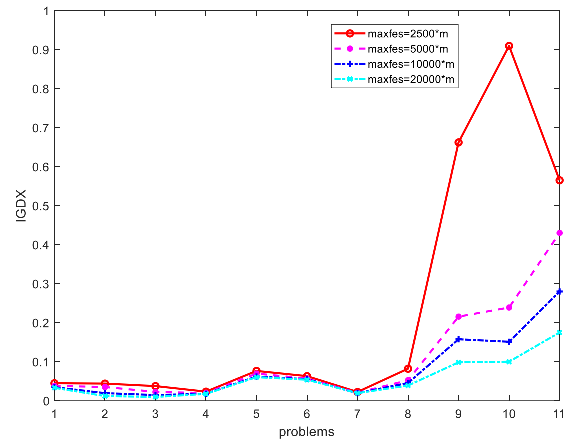


(d). The performance of the IGDF metric of GSMPSO-MM with different population size

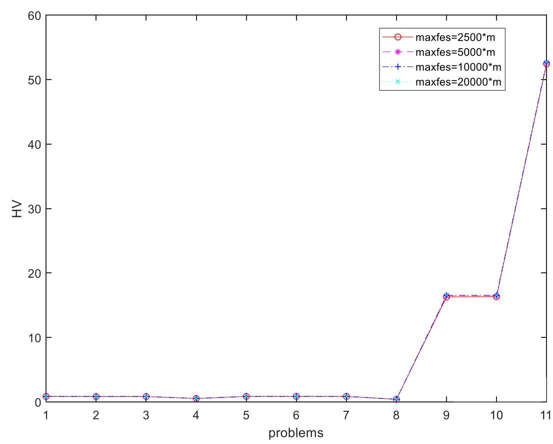
Fig. 11. The performance metrics of GSMPSO-MM with different population sizes and the same fitness evaluations. The numbers on the horizontal axis denote the following problems: 1=MMF1, 2=MMF2, 3=MMF3, 4=MMF4, 5=MMF5, 6=MMF6, 7=MMF7, 8=MMF8, 9=SYM-PART simple, 10=SYM-PART rotated, and 11=Omni-test.



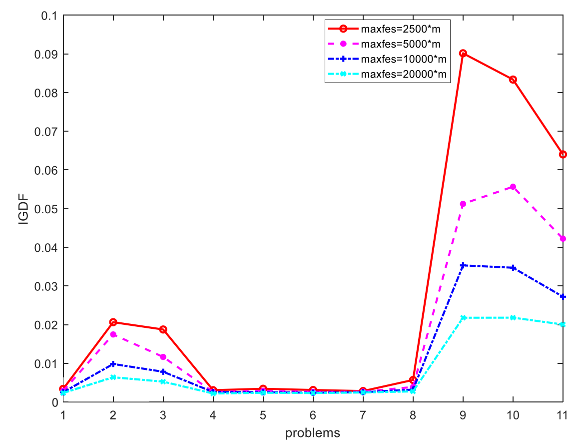
(a). The performance of the PSP metric of GSMPSO-MM with different fitness evaluations



(b). The performance of the IGDX metric of GSMPSO-MM with different fitness evaluations



(c). The performance of the HV metric of GSMPSO-MM with different fitness evaluations



(d). The performance of the IGDF metric of GSMPSO-MM with different fitness evaluations

Fig. 12. The performance metrics of GSMPSO-MM with different fitness evaluations and the same population size. The numbers on the horizontal axis denote the following problems: 1=MMF1, 2=MMF2, 3=MMF3, 4=MMF4, 5=MMF5, 6=MMF6, 7=MMF7, 8=MMF8, 9=SYM-PART simple, 10=SYM-PART rotated, and 11=Omni-test.

behavior and $Ns = 10$ achieves better result on IGDX, and there is no difference for IGDF on MMF1, MMF4, MMF5, MMF6, MMF7. In view of the number of subpopulations Ns is too large and the population is set to$N = 100 * m$, it results in that most subpopulations only have few particles. Thus, the number of subpopulations $Ns = 10$ is a reasonable choice for GSMPSO-MM.

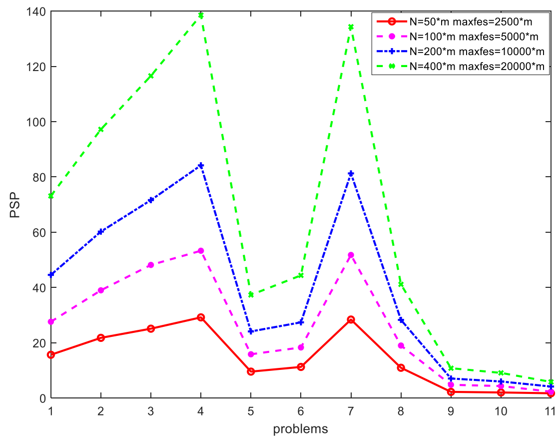
4.7. Analysis of the grid size in GSMPSO-MM

Here, to analyze the sensitivity of the grid size G in GSMPSO-MM, the 2-dimension decision space is divided into $G = 25 * 25$, $G = 50 * 50$, $G = 80 * 80$, $G = 100 * 100$ and $G = 200 * 200$, respectively. The 3-dimensional decision space is divided into $G = 5 * 5 * 5$, $G = 10 * 10 * 10$, $G = 20 * 20 * 20$, $G = 25 * 25 * 25$, and $G = 30 * 30 * 30$. The maximum fitness evaluations $maxfit$ are set to $5000 *m$ and the experiment is conducted on MMF1–8, MMF14, MMF15, and Omni-test. The PSP value of GSMPSO-MM with different grid sizes is shown in Figs. 9 and 10.

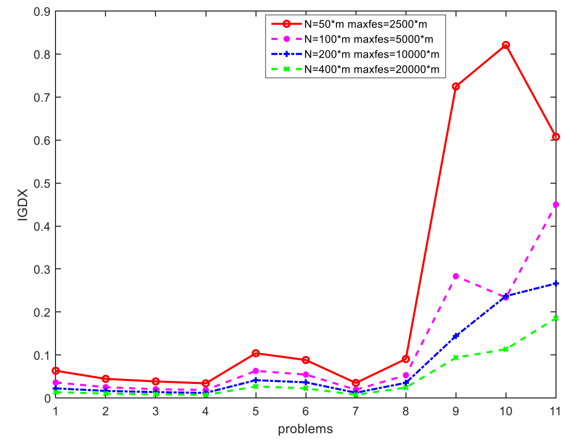
In Fig. 9, it is found that the PSP values take on increases from $G = 25 * 25$ to $G = 100 * 100$ at first and smoothness later in most cases, while MMF2, MMF3, and MMF7 appear a decreasing trend from $G = 100 * 100$ to $G = 200 * 200$. It is clear that the grid size $G = 100 * 100$ performs best on MMF2, MMF3, MMF6, and MMF7, and the performance of other problem on $G = 80 * 80$ and $G = 200 * 200$ is similar. The performance of GSMPSO-MM with different grid sizes implies that higher and lower grid sizes are undesirability and $G = 100 * 100$ is the best choice. Likewise, the proposed GSMPSO-MM gets a poor performance on 3-dimensional MMOPs with a smaller grid size in Fig. 10, and the grid size $G = 25 * 25 * 25$ is the best option. According to the analysis, it is deduced that dividing decision space into $G = t^m$gird in Section 3.2 is reasonable.

4.8. Analysis of the influence of the population size and maximum fitness evaluations for GSMPSO-MM

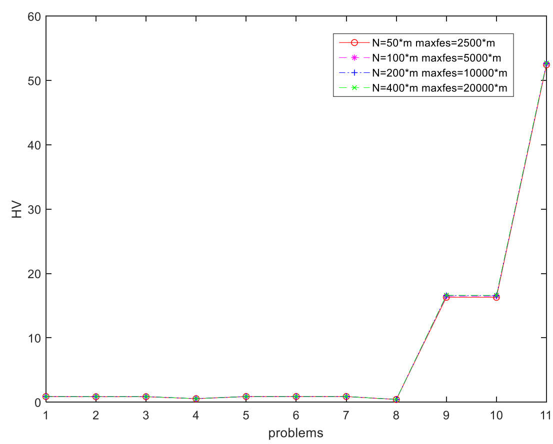
The performance of GSMPSO-MM is influenced by the population size and maximum fitness evaluations. Here, three associated experiments are performed to validate the influences on the performance of the proposed GSMPSO-MM algorithm. One is that the population size is increasing and the maximum fitness evaluations are fixed for GSMPSO-MM; Another is that the population size is fixed and the maximum fitness



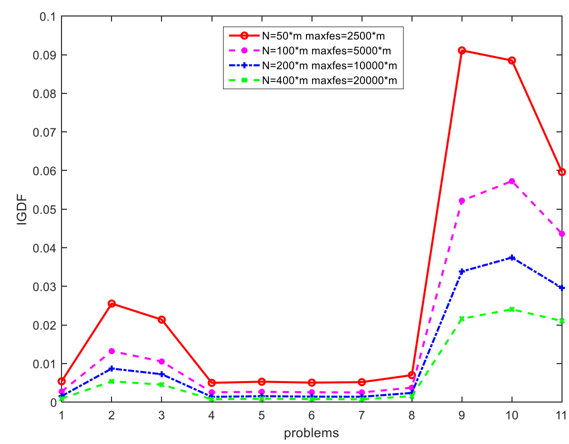
(a). The performance of the PSP metric of GSMPSO-MM with different population and fitness evaluations



(b). The performance of the IGDX metric of GSMPSO-MM with different population size and fitness evaluations



(c). The performance of the HV metric of GSMPSO-MM with different population and fitness evaluations



(d). The performance of the IGDF metric of GSMPSO-MM with different population size and fitness evaluations

Fig. 13. The performance metrics of GSMPSO-MM with different fitness evaluations and population size. The numbers on the horizontal axis denote the following problems: 1=MMF1, 2=MMF2, 3=MMF3, 4=MMF4, 5=MMF5, 6=MMF6, 7=MMF7, 8=MMF8, 9=SYM-PART simple, 10=SYM-PART rotated, and 11=Omni-test.

evaluations are dynamic for GSMPSO-MM; The third is the population size and the maximum fitness evaluations are all gradually increasing.

To demonstrate the influence of the population size on the performance of the proposed GSMPSO-MM, the population size N is set to $50 * m$, $100 *m$, $200 *m$, and $400 *m$, respectively, and the maximum fitness evaluations $maxfes$ is fixed as $5000 *m$. The experiment is performed on 11 problems and the four performance metrics are shown in Fig. 11. For the PSP and IGDX metrics, it is clear that the performance of the GSMPSO-MM algorithm is significantly improving with the increase in the population size in most cases. There is no obvious difference for the HV metric as population size increases, and the performance of the IGDX values is steadily being improved. Besides, we can observe that the four performance metrics of the GSMPSO-MM algorithms are similar in most cases when the population size is set to $200 *m$ and $400 *m$. It means that the population size is too large, resulting that fewer evolutionary iterations in GSMPSO-MM are conducted. Thereby, there is no significant improvement in the performance of the proposed algorithm.

To examine the impact of the maximum fitness evaluations on the performance of the proposed GSMPSO-MM, the maximum fitness evaluations $maxfes$ is denoted as 2500^*m, 5000^*m, $10{,}000^*m$, and $20{,}000^*m$, respectively, and the population size N is set to 100^*m. The performance of the proposed GSMPSO-MM is presented in Fig. 12. It can be seen that the performance of the proposed GSMPSO-MM for four indicators on MMF2, MMF3, SYM-PART simple, SYM-PART rotated, and Omni-test is a remarkable improvement with the growth of the maximum fitness evaluations, and there is no significant distinction in other matters. This is primarily for the reason that locating equivalent PS in decision space is difficult for the GSMPSO-MM in MMF2, MMF3, SYM-PART simple, SYM-PART rotated, and Omni-test. And the performance of the GSMPSO-MM algorithm is enhanced when more equivalent PS is found by increasing the maximum fitness evaluations. Furthermore, the size of the non-dominated solutions archive in GSMPSO-MM is equal to the population size 100^*m, and the number of diverse PS in decision space on the other cases is far greater than 100^*m. But 100^*m non-dominated solutions are only survived in the archives for MMF1, MMF4, MMF5, MMF6, MMF7, and MMF8. Therefore, there is no significant distinction in the performance of the GSMPSO-MM as the population size increases on MMF1, MMF4, MMF5, MMF6, MMF7, and MMF8.

Besides, the performance of the proposed GSMPSO-MM with the increase of the population size and the maximum fitness evaluations is investigated. The population size N is set to 2500^*m, 5000^*m, $10{,}000^*m$, and $20{,}000^*m$, respectively, and the corresponding max fitness evaluations $maxfes$ are fixed to 50*m, 100*m, 200*m, and 400*m, respectively. The experimental results are exhibited in Fig. 13. We can observe that the proposed GSMPSO-MM algorithm with larger population size and more maximum fitness evaluations is significantly better than the other states except for the HV metric. Combined with the analysis of two experimental results above, we can infer that increasing both the population size and the maximum fitness evaluations simultaneously will improve dramatically the performance of the proposed GSMPSO-MM algorithm.

5. Conclusion

In this paper, the GSMPSO-MM algorithm is proposed to address MMOPs. A grid search based on multi-population particle swarm optimization algorithm (GSMPSO-MM) is adopted to develop more equivalent PS in decision space. The niching technology, the k-means clustering method, is used to divide the population into multiple subpopulations for the purpose of locating diverse PS with identical PF, and a grid is constructed in decision space to assist in multi-population searching for more high-quality solutions in decision space. To survive elite non-dominated solutions, the environmental selection operator, including two critical operations (removing redundant solutions and surviving the non-dominating solution archive operator), is utilized to sustain a fine diversity of solutions in decision space and convergence of solution in objective space, simultaneously. To demonstrate the superior performance of the proposed GSMPSO-MM algorithm, a standard benchmark function that originated from the CEC 2019 competition on multimodal multiobjective optimization is applied to the proposed GSMPSO-MM algorithm and its competing algorithms. Compared with seven state-of-the-art algorithms, the experimental results show that the proposed GSMPSO-MM algorithm is capable of handling MMOPs properly. Beyond that, some relevant experiments are conducted to validate the effectiveness of some strategies and analyze the sensitivity of some parameters in GSMPSO-MM.

Our future research direction is to take into account adaptive grid mechanism for locating more local PS in decision space. Besides, some novel multimodal many-objective optimization problems should be developed.

Declaration of Competing Interest

None.

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CRediT authorship contribution statement

1. Guoqing Li: Conceptualization, Investigation, Methodology, Writing - review & editing
2. Wanliang Wang: Revising-review and editing
3. Weiwei Zhang: Software, Validation
4. Zheng Wang: Methodology
5. Hangyao Tu: Data Analysis
6. Wenbo You: Writing - revising & editing.

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