

Computer Vision

Ch3.1: Image enhancement – Image filtering

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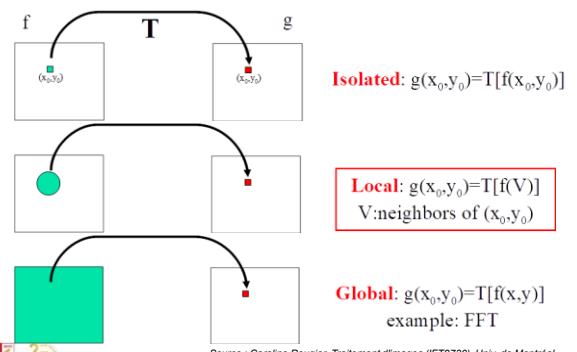
Content

- Remind: digital image
- Point operators: Contrast enhancement
- Other point operators
- Convolution and spatial filtering (local transformation)
 - Convolution
 - Linear spatial filtering
 - Other filters
- [Binary operator]

Computer Vision

Ch3.1: Image Enhancement – Image filtering

Pixel transformation



Convolution and spatial filtering

- Spatial filtering
 - affects directly to pixels in image
 - **Local transformation** in the spatial domain can be represented as: $g(x,y) = T(f[V(x,y)])$
 - T may affect the point (x,y) and its neighborhood (K) and output a new value
 - (K: Filter/Mask/Kernel/Window/Template Processing)
 - **Same function** applied at each position
 - Output and input image are typically **the same size**
- Convolution: Linear filtering, function is a weighted sum of pixel values

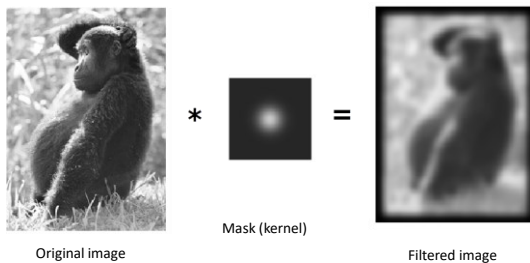
$$I' = I * K$$

Spatial Convolution

- Convolution: Linear filtering, function is a weighted sum/difference of pixel values

$$I'(n,m) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} I[k,l] \times K[n-k,m-l]$$

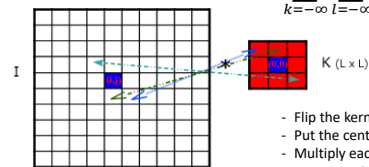
Spatial convolution



Spatial convolution

- Convolution: New value of a pixel(i,j) is a weighted sum of its neighbors

$$I'(n,m) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} I[k,l] \times K[n-k,m-l]$$

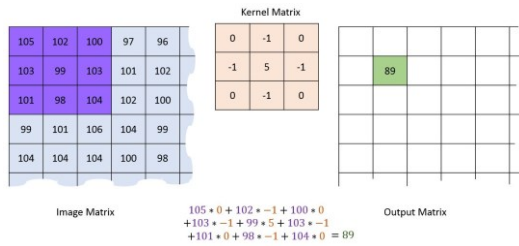


- Flip the kernel both horizontally and vertically.
- Put the center of kernel at each pixel (i,j).
- Multiply each element of the kernel with its corresponding element of the image matrix
- Sum up all product outputs

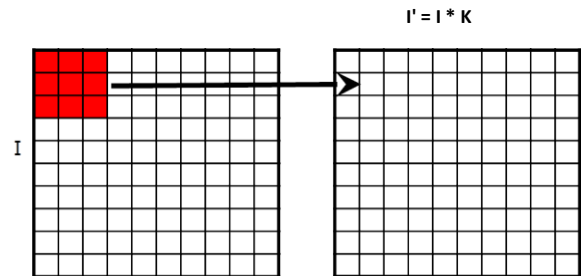
$$I'(i,j) = \sum_{u=-\lfloor L-1 \rfloor/2}^{\lfloor L-1 \rfloor/2} \sum_{v=-\lfloor L-1 \rfloor/2}^{\lfloor L-1 \rfloor/2} I(i-u, j-v) K(u,v)$$

Spatial convolution

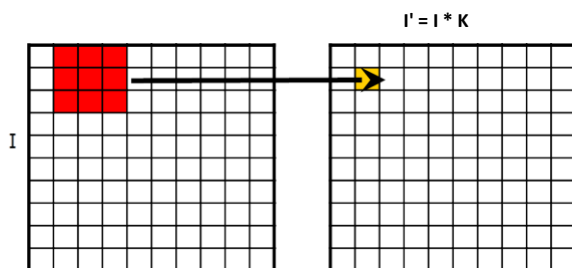
- New value of a pixel(i,j) is a weighted sum of its neighbors



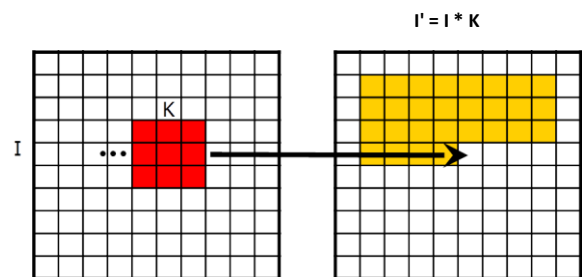
Spatial convolution



Spatial convolution

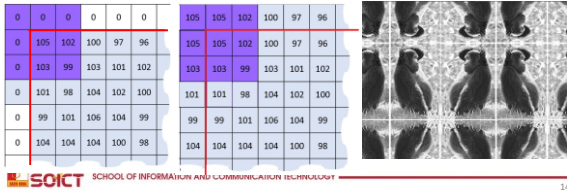
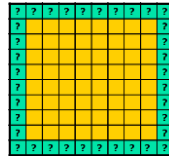


Spatial convolution



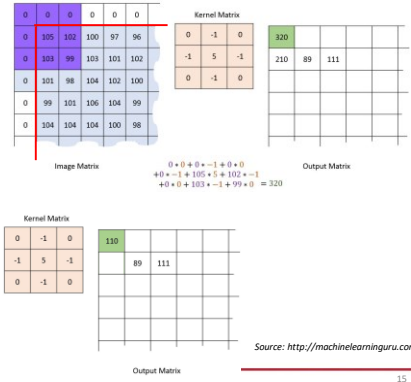
Spatial convolution

- Border problem?
 - Ignore, set to 0
 - Zero padding on the input matrix
 - reflect across edge:
 - $f(-x,y) = f(x,y)$
 - $f(x,-y) = f(x,y)$



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Spatial convolution

Source: <http://machinelearningguru.com>

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Spatial Correlation vs Convolution

Spatial Correlation (\star) and Convolution (\star)

$$w(x,y) \star f(x,y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s,t) f(x-s, y-t)$$

$$w(x,y) \star f(x,y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s,t) f(x+s, y+t)$$

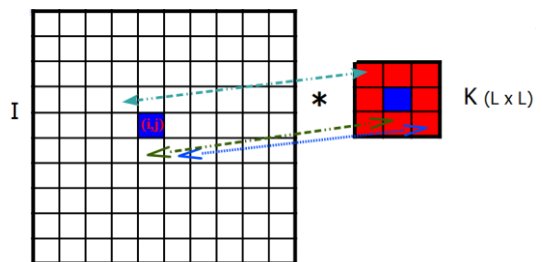
$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \Rightarrow \begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Convolution is correlation with the filter rotated 180 degrees.

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Spatial Correlation vs Convolution

- Correlation



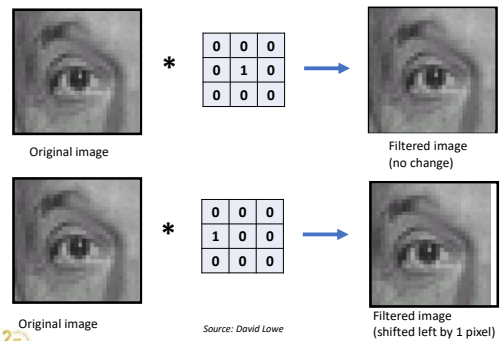
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Spatial Correlation vs Convolution

- If the **mask is symmetric** these two operations are identical
- Correlation:
 - Use to find which part in image match with a certain "template"
 - **Not associative** → if you are doing template matching, i. e. looking for a single template, correlation is sufficient
- Convolution:
 - Use for filtering image (noise removing, enhancement, ...)
 - Associative: allows you to "pre-convolve" the filters → useful when you need to use multiple filters in succession:

$$I * h * g = I * (h * g)$$
- In Matlab: correlation: `filter2`, convolution: `conv2`

Examples



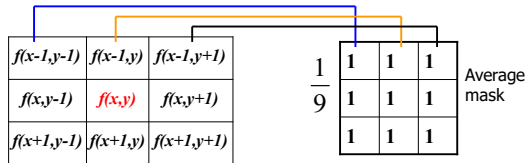
Some kernels

- 2D spatial convolution
 - is mostly used in image processing for feature extraction
 - And is also the core block of Convolutional Neural Networks (CNNs)
- Each kernel has its own effect and is useful for a specific task such as
 - blurring (noise removing),
 - sharpening,
 - edge detection,
 -

Smooth filtering

- "Low-pass" filters :
 - Used for noise removing/blurring,...
 - mean filters, Gaussian filters, ...
- Average filtering is the simplest of smooth filtering
- To avoid changing the image brightness, **the sum of the coefficients must be equal to 1**

Average (mean) filtering



$$g(x, y) = \frac{1}{9} [f(x-1, y-1) + f(x-1, y) + f(x-1, y+1) + f(x, y-1) + f(x, y) + f(x, y+1) + f(x+1, y-1) + f(x+1, y) + f(x+1, y+1)]$$

Average (mean) filtering

- Mask:
 - All elements of the mask are equal
- Results:
 - Replacing each pixel with an average of its neighborhood
 - Achieve smoothing effect

$$1/9 \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$



Original image Filtered image with box size 5x5 Filtered image with box size 11x11

Weighted Average filtering

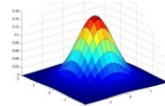
- The pixel corresponding to the center of the mask is more important than the other ones.

$$\frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

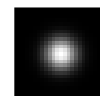
$$g(x, y) = \frac{1}{16} [f(x-1, y-1) + 2f(x-1, y) + f(x-1, y+1) + 2f(x, y-1) + 4f(x, y) + 2f(x, y+1) + f(x+1, y-1) + 2f(x+1, y) + f(x+1, y+1)]$$

Gaussian filtering

Gaussian filter



Gaussian function in 3D



Gaussian image

0.003	0.013	0.022	0.013	0.003
0.013	0.059	0.097	0.059	0.013
0.022	0.097	0.159	0.097	0.022
0.013	0.059	0.097	0.059	0.013
0.003	0.013	0.022	0.013	0.003

Gaussian filter with size 5 x 5, sigma = 1

$$G_{\sigma} = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

Rule for Gaussian filter:
set filter width to about 6σ or 8σ [+1]
(+1 if output is even number)

Gaussian filtering



Original image

Filtered image
with box size 5x5Filtered image
with box size 11x11

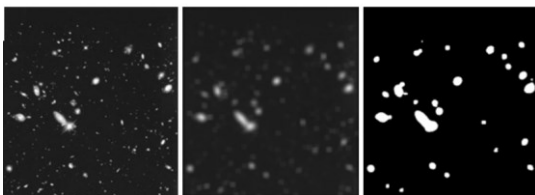
Gaussian filter

- Gaussian filter:
 - Low-pass filter:** Remove high-frequency component from the image
 - Image becomes more smooth
 - Better than mean filter
 - Convolution with itself is another Gaussian
 - Repeat the conv. With small-width kernel => get the same result as larger-width kernel would have.
 - Convolving **2 times** with Gaussian kernel of width σ is the same as convolving **once** with kernel of width $\sigma\sqrt{2}$: $I * G_{\sigma} * G_{\sigma} = I * G_{\sigma\sqrt{2}}$
 - Separable filter:** The 2D Gaussian can be expressed as the product of 2 functions: one function of x and a function of y :
 - $G_{\sigma}(x,y) = G_{\sigma}(x) \cdot G_{\sigma}(y)$

Smooth filtering

- Blurring usage: delete unwanted (small) subjects

Original image Average filtering: 15x15 Thresholding of blurring image



Sharpening filter

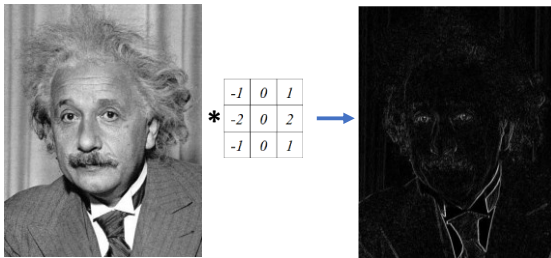
- Highlights Intensity Transitions
- Base on first and second order derivatives

$$\frac{\partial f}{\partial x} \approx \begin{cases} f(x+1,y) - f(x,y) \\ f(x,y) - f(x-1,y) \\ 0.5(f(x+1,y) - f(x-1,y)) \end{cases}$$

$$\frac{\partial^2 f}{\partial x^2} \approx f(x+1,y) - 2f(x,y) + f(x-1,y)$$

First derivatives

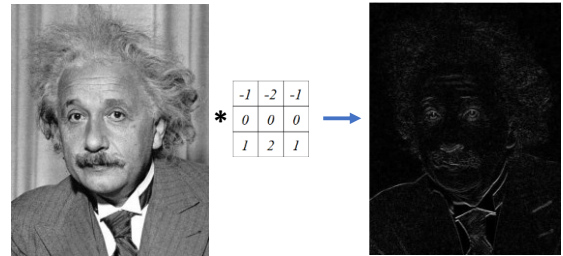
- Sobel



Vertical Edge
(absolute value)

First derivatives

- Sobel



Horizontal Edge
(absolute value)

2nd derivatives - Laplacian filtering

- The Laplacian operator is defined as:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

2nd derivatives - Laplacian filtering

- Can be computed as

$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x, y)$$

0	1	0
1	-4	1
0	1	0

- Or

$$\nabla^2 f = 4f(x, y) - [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)]$$

0	-1	0
-1	4	-1
0	-1	0

- 90° isotropic filter

2nd derivatives - Laplacian filtering

- Can be computed as

$$\nabla^2 f = [f(x+1, y+1) + f(x+1, y) + f(x+1, y-1) + f(x-1, y+1) + f(x-1, y) + f(x-1, y-1) + f(x, y+1) + f(x, y-1)] - 8f(x, y)$$

1	1	1
1	-8	1
1	1	1

- Or

$$\nabla^2 f = 8f(x, y) - [f(x+1, y+1) + f(x+1, y) + f(x+1, y-1) + f(x-1, y+1) + f(x-1, y) + f(x-1, y-1) + f(x, y+1) + f(x, y-1)]$$

-1	-1	-1
-1	8	-1
-1	-1	-1

- 45° isotropic filter

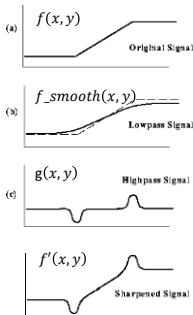


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Sharpening filter using first derivative

$$g(x, y) = f(x, y) - f_{smooth}(x, y)$$



$$f'(x, y) = f(x, y) + k * g(x, y) \quad (k = 0.2..0.7)$$



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Sharpening filter using first derivative

What does blurring take away?



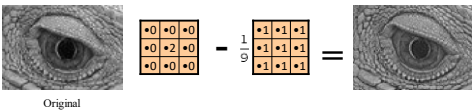
- Let's add it back:



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Sharpening filter



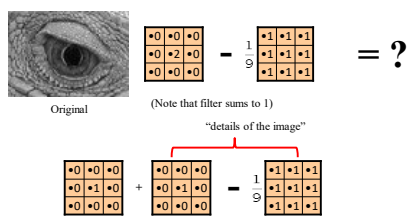
Sharpening filter: Accentuates differences with local average



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Juan Carlos Niebles and Ranjay Krishna

Sharpening filter



Sharpening filter using laplacian

Popular method: original image – scaled seconde derivative

$$g(x,y) = \begin{cases} f(x,y) - \nabla^2 f(x,y) & -sign \\ f(x,y) + \nabla^2 f(x,y) & +sign \end{cases}$$

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & +5 & -1 \\ 0 & -1 & 0 \end{bmatrix} \quad 90^\circ \text{ isotropic} \quad \begin{bmatrix} -1 & -1 & -1 \\ -1 & +9 & -1 \\ -1 & -1 & -1 \end{bmatrix} \quad 45^\circ \text{ isotropic}$$

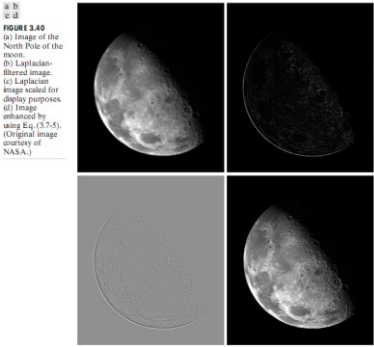
Note

$$g(x,y) = \begin{cases} f(x,y) - \nabla^2 f(x,y) \\ f(x,y) + \nabla^2 f(x,y) \end{cases}$$

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$
$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 9 & -1 \\ -1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Sharpening filter

- a) Original image
- b) Laplacian filtering
- c) Laplacian with scaling
- d) Adding original with Laplacian image



Sharpening filter

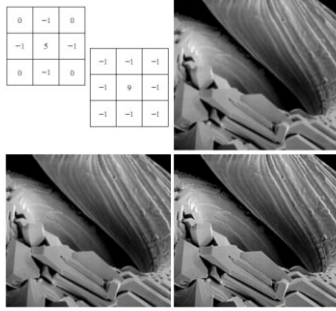
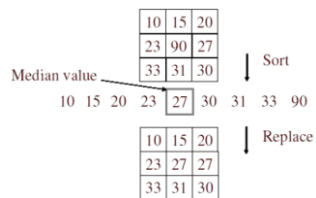


FIGURE 3.41 (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

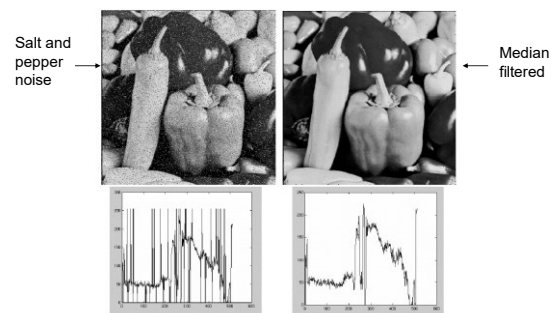
Other filters (non-linear filtering)

Median filter

- No new pixel values introduced
- Removes spikes:
 - good for impulse, salt & pepper noise
- Non-linear filter



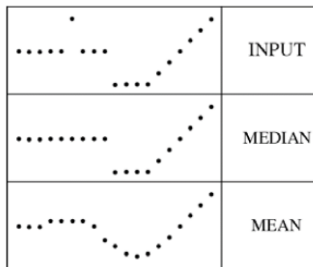
Median filter



Plots of a row of the image

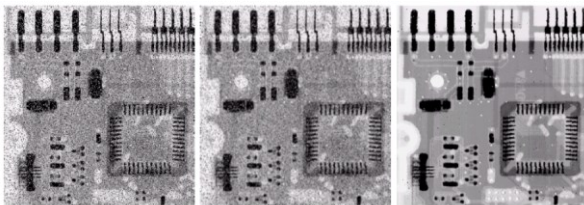
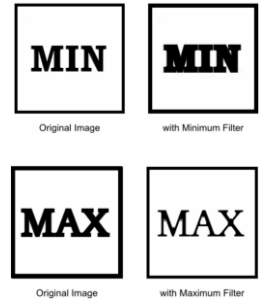
Median filter

- Median filter is edge preserving



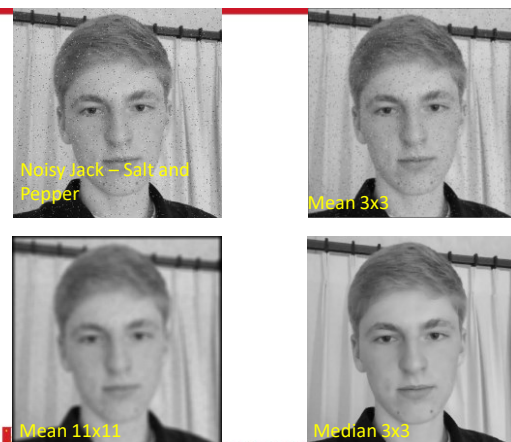
Max/ Min filters

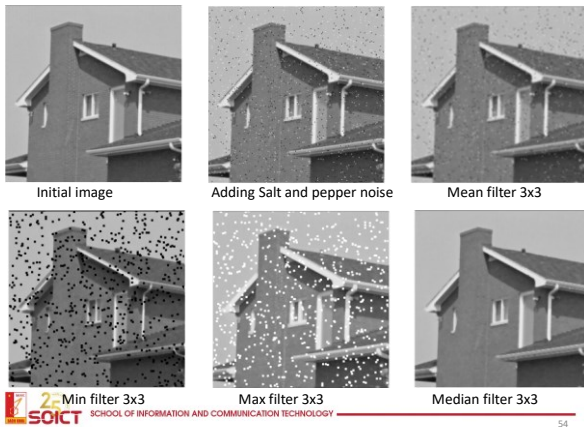
- The maximum and minimum filters are shift-invariant
 - The Minimum Filter: replaces the central pixel with the darkest one in the running window
 - The Maximum Filter: replaces the central pixel with the lightest one



a b c

FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

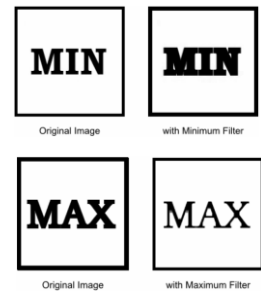




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Median/max/min: problem?

- Size filter !



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Exercise

- Given an 8-bit image – 8 x 8

52	55	61	66	70	61	64	73
63	59	55	90	109	85	69	72
62	59	68	113	144	104	66	73
63	58	71	122	154	106	70	69
67	61	68	104	126	88	68	70
79	65	60	70	77	68	58	75
85	71	64	59	55	61	65	83
87	79	69	68	65	76	78	94

- 1) Compute and show the histogram: 8 bins, 16 bins, 32 bins
- 2) Comment about the contrast of the image
- 3) Equalize the histogram for above image with 8-bins



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Practice (home work)

- Try to implement/use different filters to apply on images:
 - Mean, gaussian filters
 - First and second image derivative (sobel, prewitt, Laplacian filters)
 - Sharpening filters
 - Median/max/min filters
- Modify the kernel size to see its effet

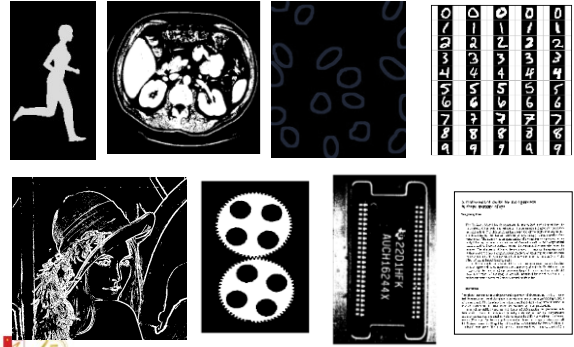


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Content

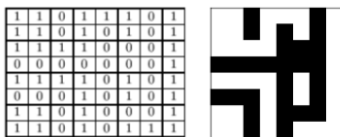
- Remind: digital image
- Point operators: Contrast enhancement
- Other point operators
- Convolution and spatial filtering (local transformation)
 - Convolution
 - Linear spatial filtering
 - Other filters
- Binary images

Binary images



Binary images

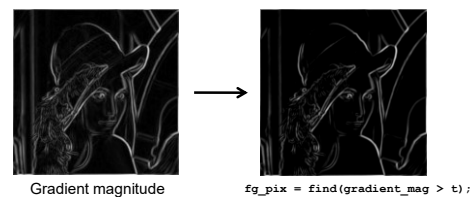
- Two pixel values: foreground (object, 1) and background (0)
- Be used
 - To mark region(s) of interest
 - As results of thresholding method



Thresholding

- Given a grayscale image or an intermediate matrix → threshold to create a binary output.

Example: edge detection

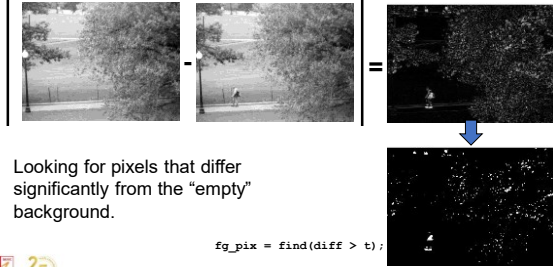


Looking for pixels where gradient is strong.

Thresholding

- Given a grayscale image or an intermediate matrix → threshold to create a binary output.

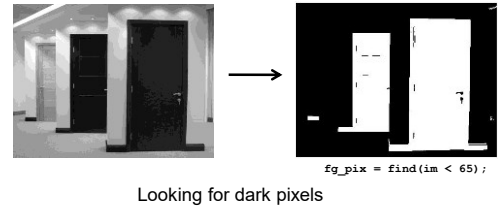
Example: background subtraction



Thresholding

- Given a grayscale image or an intermediate matrix → threshold to create a binary output.

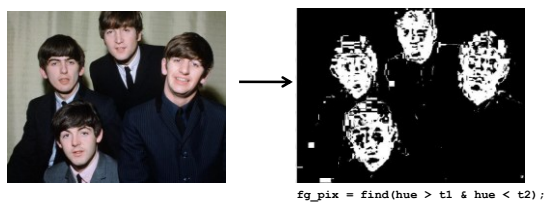
Example: intensity-based detection



Thresholding

- Given a grayscale image or an intermediate matrix → threshold to create a binary output.

Example: color-based detection

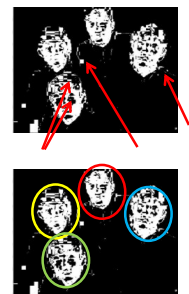


Looking for pixels within a certain hue range.

Slide credit: Kristen Grauman

Issues

- What to do with "noisy" binary outputs?
 - Holes
 - Extra small fragments
- How to demarcate multiple regions of interest?
 - Count objects
 - Compute further features per object



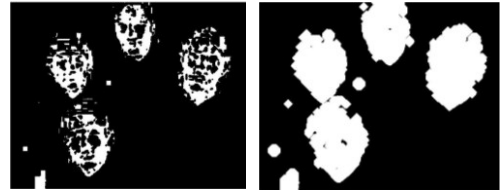
Slide credit: Kristen Grauman

Morphological operators

- Change the shape of the foreground regions via intersection/union operations between a scanning structuring element and binary image.
- Useful to clean up result from thresholding
- Main components
 - Structuring element
 - Operators:
 - Basic operators: Dilation, Erosion
 - Others: Opening, Closing, ...

Dilation

- Expands connected components
- Grow features
- Fill holes

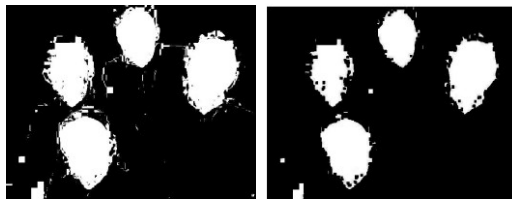


Before dilation

After dilation

Erosion

- Erode connected components
- Shrink features
- Remove bridges, branches, noise



Before erosion

After erosion

Structuring elements

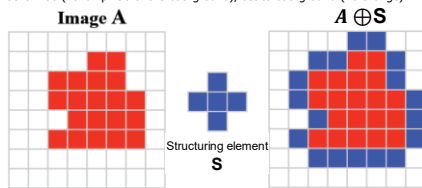
- **Masks** of varying shapes and sizes used to perform morphology, for example:



- Scan mask (structuring element) over the **object (foreground) borders (inside and outside)** and transform the binary image

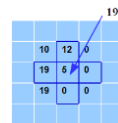
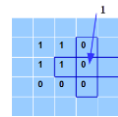
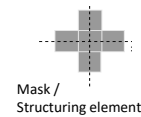
Dilation

- Moving S on each pixel of A
 - check if the intersection (pixels belonging to object) is not empty
 - If yes, the center of B belongs to the result image
- If a pixel of S is onto object pixels (A), then the central pixel belongs to object
 - Otherwise (i.e. all pixels are background), set to background (no change)



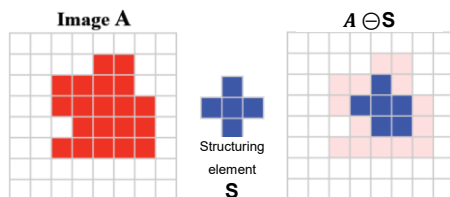
Dilation

- As max filter
- Can be applied both on
 - binary images
 - or grayscale images



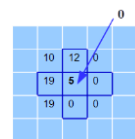
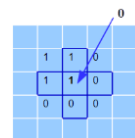
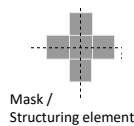
Erosion

- We put the element S on each pixel x of A
 - like convolution
- If all pixels of S are onto object pixels (A), then the central pixel belongs to object
 - Otherwise (i.e. a mask pixel is background), set to background



Erosion

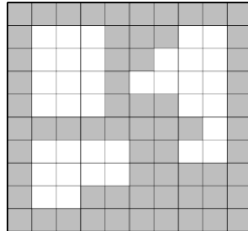
- As min filter
- Can be applied both on
 - binary images
 - or grayscale images



Connected component labeling

- We loop over all the image to give a **unique number (label)** for each **region**
- All pixels from the **same region** must have the **same number (label)**
- Objectifs:
 - Counting objects
 - Separating objects
 - Creating a mask for each object
 - ...


 ← Background
 ← Segmented objects

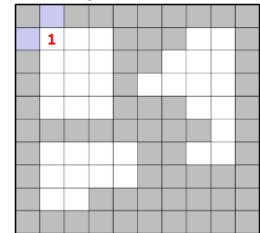


Connected component labeling

First loop over the image

- For each pixel in a region, we set
 - or the smallest label from its **top** or **left** neighbors
 - or a new label

Loop 
 Neighbors 

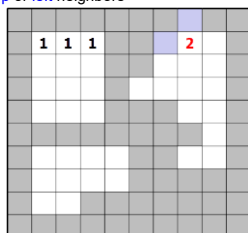


Connected component labeling

First loop over the image

- For each pixel in a region, we set
 - or the smallest label from its **top** or **left** neighbors
 - or a new label

Loop 
 Neighbors 

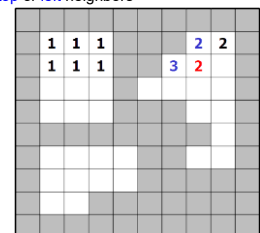


Connected component labeling

First loop over the image

- For each pixel in a region, we set
 - or the smallest label from its **top** or **left** neighbors
 - or a new label

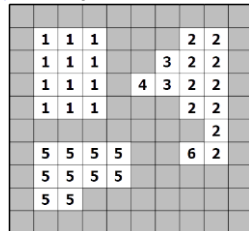
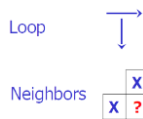
Loop 
 Neighbors 



Connected component labeling

First loop over the image

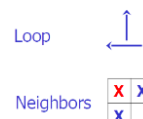
- For each pixel in a region, we set
 - or the smallest label from its **top** or **left** neighbors
 - or a new label



Connected component labeling

Second loop over the image

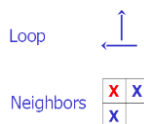
- For each pixel in a region, we set
 - the smallest from its **own label** and the labels from its **down** and **right** neighbors



Connected component labeling

Second loop over the image

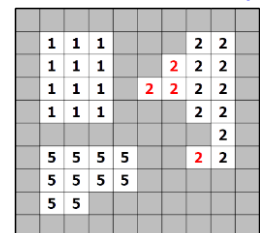
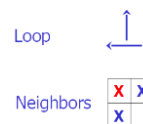
- For each pixel in a region, we set
 - the smallest from its **own label** and the labels from its **down** and **right** neighbors



Connected component labeling

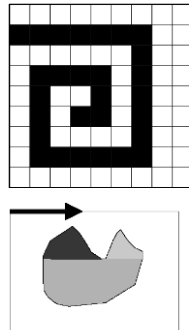
Second loop over the image

- For each pixel in a region, we set
 - the smallest from its **own label** and the labels from its **down** and **right** neighbors



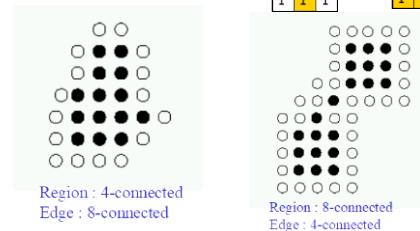
Connected component labeling

- Two loops are enough?
 - example: spiral region !
- Solutions
 - We continue, go and back two ways, until no new change in labels
 - It is possible to do only one loop: manage a table of equivalences when 2 different labels are neighbors



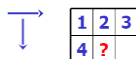
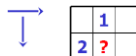
CC labeling: how many neighbors?

- Advice: Use different connectivity for edges and regions
 - 4-Connectivity for regions
 - 8-Connectivity for edges



CC labeling: how many neighbors?

- Regions labeling
 - We use 4-connectivity
 - Each loop, we compare 2 neighbors
- Edge labeling
 - 8-connectivity
 - Each loop, we compare 4 neighbors



Next lesson

- Rappel: digital image representation
- Point Processing
- Convolution and Linear filtering
- More neighborhood operators
- Image transforms
 - Frequency domain
 - Frequencies in images
 - Fourier transform
 - Frequential Processing (frequential filters)
 - PCA (additional reading)

References

- Lecture 2: CS131 - Juan Carlos Niebles and Ranjay Krishna, Stanford Vision and Learning Lab
- Vision par Ordinateur, Alain Boucher, IFI

