HUST

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HANOI UNIVERSITY OF SCIENCE AND TECHNOLOGY

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Introduction to Constraint Programming

Almost slides from the course CP given by
Pascal Van Hentenryck & Yves Deville

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Content

- A first challenge
- Constraint Satisfaction Problem
- Constraint Programming
- Examples with Or-Tools

Sudoku

- Fill the empty cells with numbers in {1,..,9}
- **Constraint** : each number only appears once in each
  - Row
  - Column
  - Region

easy

		1				8	9	
	2	7			9		5	
		4		8	2			
	6		9	2		1	4	
				5				
	9	8		6	1		3	
			2	1		4		
	1		7			3	6	
	7	9				2		

hard

5	9				7		8	
			5					7
4	1		8			5		6
		1	3					4
		5				9		
8					4	2		
9		2			3		4	5
1					5			
	5		4				6	9

Sudoku

easy

		1				8	9	
	2	7			9		5	
		4		8	2			
	6		9	2		1	4	
				5				
	9	8		6	1		3	
			2	1		4		
	1		7			3	6	
	7	9				2		

6	3	1	5	7	4	8	9	2
8	2	7	1	3	9	6	5	4
9	5	4	6	8	2	7	1	3
7	6	5	9	2	3	1	4	8
1	4	3	8	5	7	9	2	6
2	9	8	4	6	1	5	3	7
3	8	6	2	1	5	4	7	9
4	1	2	7	9	8	3	6	5
5	7	9	3	4	6	2	8	1

hard

5	9				7		8	
			5					7
4	1		8			5		6
		1	3					4
		5				9		
8					4	2		
9		2			3		4	5
1					5			
	5		4				6	9

5	9	6	1	4	7	3	8	2
3	2	8	5	9	6	4	1	7
4	1	7	8	3	2	5	9	6
2	7	1	3	8	9	6	5	4
6	4	5	2	7	1	9	3	8
8	3	9	6	5	4	2	7	1
9	6	2	7	1	3	8	4	5
1	8	4	9	6	5	7	2	3
7	5	3	4	2	8	1	6	9

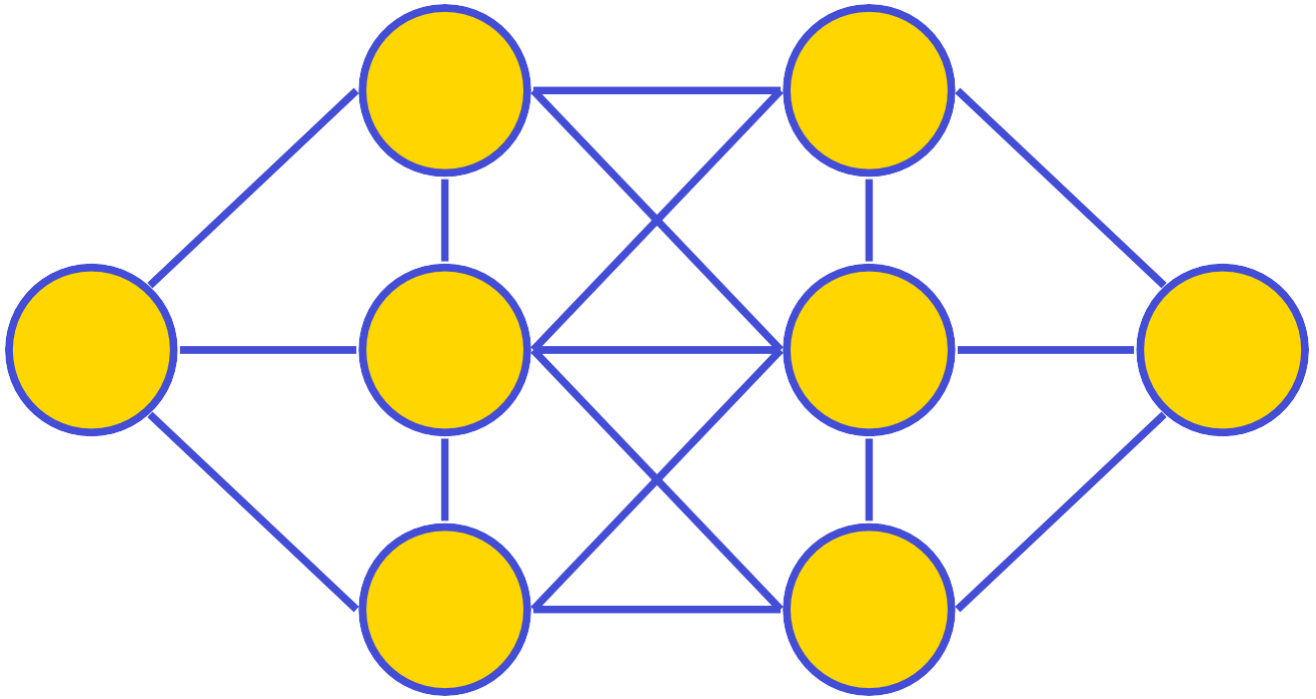
Sudoku

- Sudoku is a (very) simple example of Constraint Satisfaction Problem (CSP)
  - A set of variables, defined over domains
  - A set of constraints over the variables
- What is a solution?
  - A solution is an assignment of values to the variables
  - does not violate any constraint

A challenge from T. Walsh

- Place numbers 1 through 8 on nodes
  - Each number appears exactly once
  - No connected nodes have consecutive number

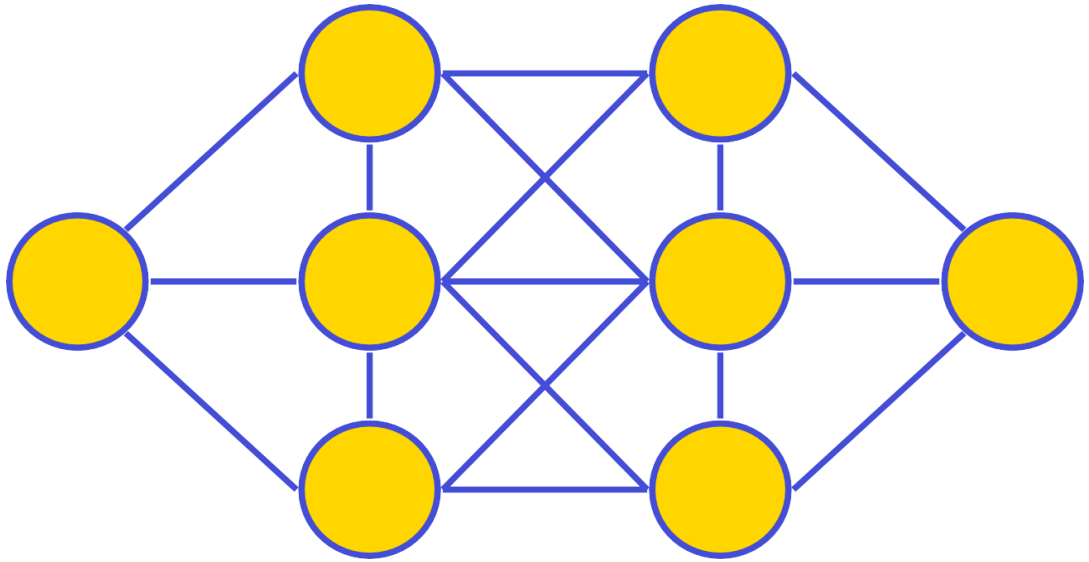
How long
will you take ?



- This is an example of a Constraint Satisfaction Problem (CSP)
  - A set of variables defined over domains
  - A set of constraints over the variables
- It can be solved by Constraint Programming

How did you solved this problem manually ?

Which techniques did you used ?



Content

- A first challenge
- **Constraint Satisfaction Problem**
- Constraint Programming
- Examples with Or-Tools

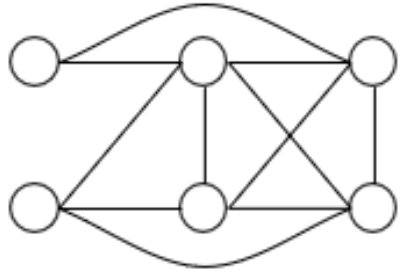
Constraint Optimization Problem

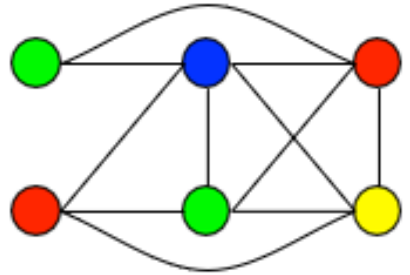
- Constraint Satisfaction Problem (CSP)
  - A set of variables, defined over finite domains
  - A set of constraints over the variables
- Solution
  - A solution is a assignment of values to the variables
  - No violation of constraint
- Constraint Optimization Problem (COP)
  - Add an objective function
  - Find a solution maximizing the objective function

Many CSP / COP are complex and challenging (NP-hard)

A simple example

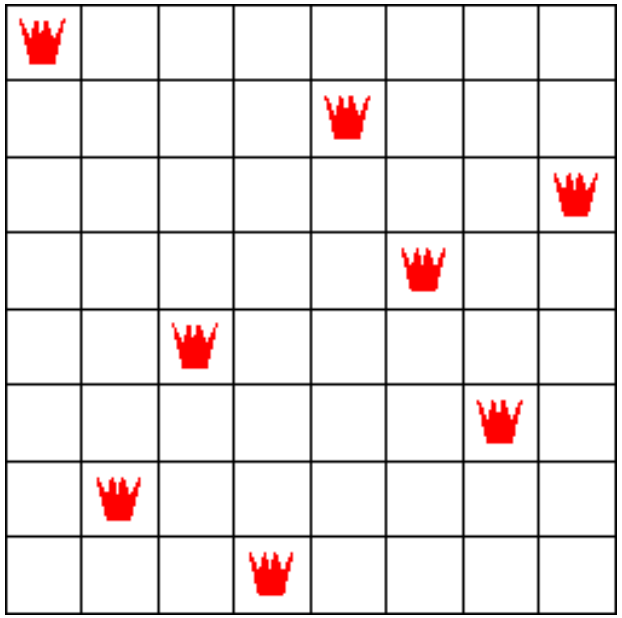
- **CSP** : Graph Coloring with 4 colors
  - Graph G=(V,E)
  - Decision variables
    - color[v] v < V
  - Domain of variables : {red,blue,green,yellow}
  - Constraints
    - 6(v,w) < E : **color[v] ≠ color[w]**
- **COP** : minimize the used colors

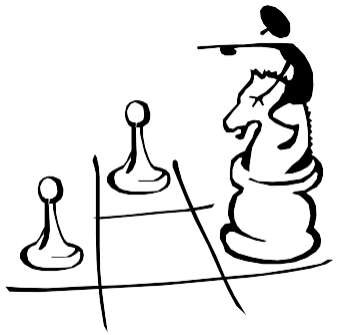




Example : N-queens

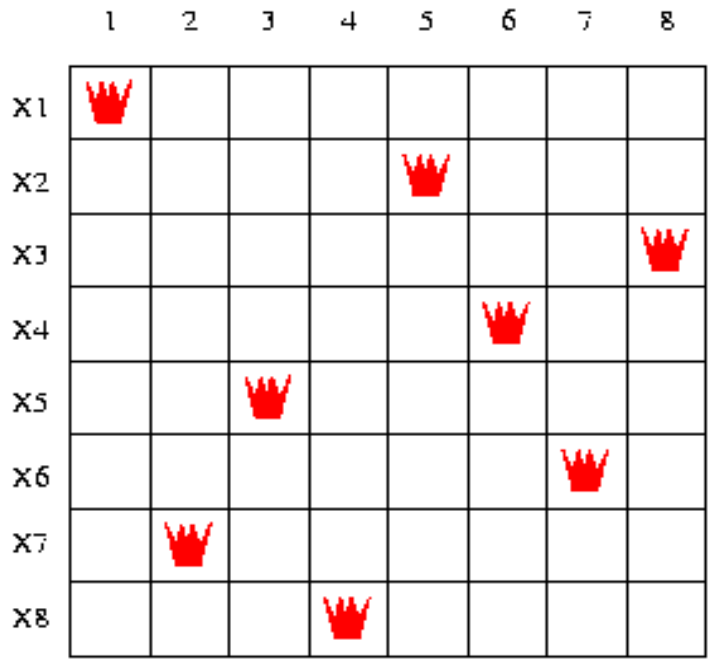
- How to place 8 queens on a 8x8 board such that they do not attack each other
- A solution





N-queens as a CSP

- **Variables**
  - X_i : position (column) of the queen on row i ($1 \leq i \leq 8$)
- **Domains**
  - Domain of X_i :
    - {1,2,3,4,5,6,7,8}



N-queens as a CSP

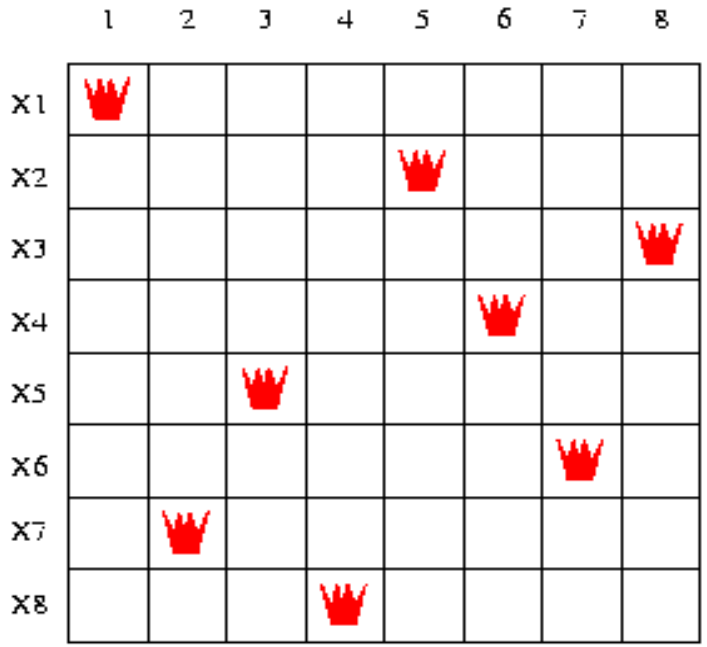
- **Constraints**
  - Two queens cannot be on the same column
    - $X_i \neq X_j$
  - Two queens cannot be on the same diagonal
    - $X_i - X_j \neq i - j$ $X_i - X_j \neq j - i$
- 8-queens
  - 8 variables
  - 84 contraints

X1≠X2, X1≠X3, X1≠X4, X1≠X5, X2≠X3, X2≠X4, X2≠X5, X3≠X4, X3≠X5, X4≠X5, …

X1-X2≠-1, X1-X2≠1, X1-X3≠-2, X1-X3≠2, X1-X4≠-3, X1-X4≠3, X1-X5≠-4, X1-X5≠4, X2-X3≠-1, X2-X3≠1, X2-X4≠-2, X2-X4≠2, X2-X5≠-3, X2-X5≠3, X3-X4≠-1, X3-X4≠1, X2-X5≠-2, X3-X5≠2, X4-X5≠-1, X4-X5≠1, …

N-queens as a CSP

- X_i : position of the queen on row i
  - $X_1 = 1$
  - $X_2 = 5$
  - $X_3 = 8$
  - $X_4 = 6$
  - $X_5 = 3$
  - $X_6 = 7$
  - $X_7 = 2$
  - $X_8 = 4$



Constraints

- Constraints as basic computational elements
  - $2X + 3Y \leq 17$ $\quad X^2 - Y^3 = 17$
- Constraints appear naturally in :
  - artificial intelligence
  - operational research
  - combinatorial optimization
  - graphics, visualisation
  - concurrency
  - databases
  - hardware design, verification
  - ...

Who does optimization ?

- Microsoft (1st largest software company)
- SAP (2nd largest software company)
- Oracle (3rd largest software company)
  - Supply chain management
- IBM (of course)
- Siebel, PeopleSoft
  - Personal management
- UPS
  - Dispatching
- Google, IBM, Carmen systems, Fair Isaac

Complexity of CSP

- Computational Complexity
  - NP-Hard or worse (at least exponential)
  - No reasonable approximation
- *Yet, it is important to solve them*
- The best specific algorithms
  - tough to design and implement
  - experimental

Solving Paradigms

- **Mathematical Programming** : *IP, MIP, column generation, SDP, ...*
- **Dynamic Programming**
- **Local Search** : tabu search, simulated annealing, genetic algorithms, evolutionary algorithms...
- Greedy algorithms, Ant Colony Optimization, greedy randomized, ...
- **Constraint Programming**
- ...

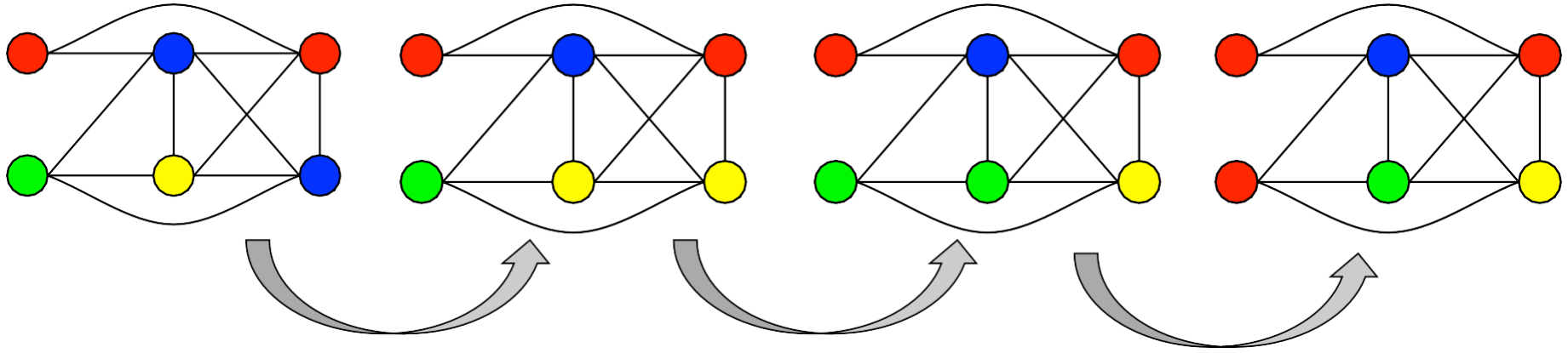
Solving Paradigms

Paradigms can usually be classified according to two criteria

- **Perturbative** vs **constructive**
- **Systematic** vs **incomplete**

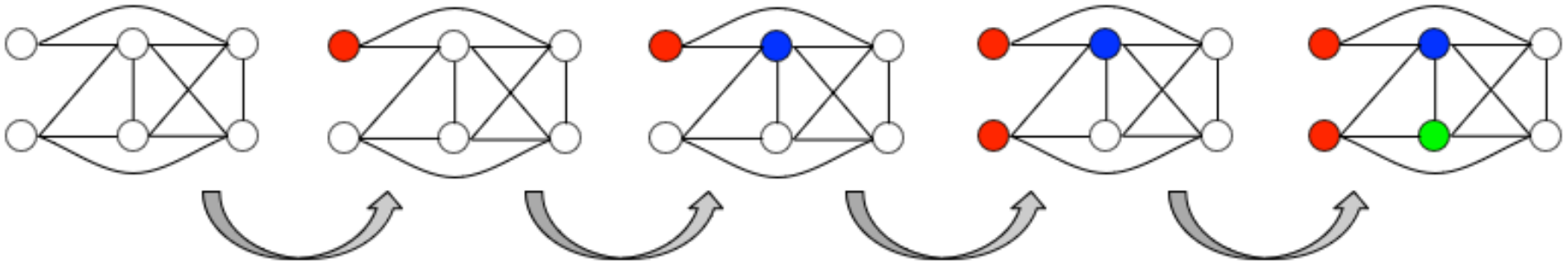
Perturbative vs Constructive

- **Perturbative** approaches
  - A candidate solution is a complete solution (a value is assigned to each variable)
  - Generation of a new solution by modifications, pertubation of an existing one
  - Search is performed on the succession of candidate solutions



Perturbative vs Constructive

- **Constructive** approaches
  - A candidate solutions is partial (some variables are not assigned)
  - A partial candidate solution is progressively extended, constructed



Systematic vs Incomplete

- **Systematic** approaches
  - Systematic exploration of the search space
  - Completeness
    - CSP : find an *exact* solution
    - COP : find the *best* solution, (+ proof of optimality)
    - If no solution, proof of non existence
  - Computationally expensive

Systematic vs Incomplete

- **Incomplete** approaches
  - Exploration based on a neighborhood function
  - Incomplete
    - CSP: find an *approximation* of a solution
    - COP: find a *good* solution
    - No guarrantee to find the (best) solution
  - Computationally less expensive

Paradigms

	Systematic	Incomplete
Pertubative	▪ Simplex ▪ Systematic local search	▪ **Local search** ▪ Evolutionary algo.
Constructive	▪ Dynamic Programming ▪ Branch & Bound ▪ **Constraint Programming**	▪ Greedy ▪ Ant Colony Optimizatio ▪ GRASP

Content

- A first challenge
- Constraint Satisfaction Problem
- **Constraint Programming**
  - **What is CP ?**
  - Constraints and propagation
  - Modeling
  - Searching
  - Global constraint
  - Computation domain
- Examples with Or-Tools

Constraint programming

- A field at the intersection of
  - Artificial intelligence
  - Operational research
- A framework for solving CSP

Constraint-Based Modelling

Solution process
=
Model + Search

- The model
  - what the solutions are and their quality
- The search
  - how to find the solutions

CP and CBLS

- **Constraint Programming**
  - *Modeling with Constraints*
  - Systematic and constructive paradigm
  - Computation based on **branch & propagate**
- **Constraint-Based Local Search**
  - *Modeling with Constraints*
  - Incomplete and perturbative paradigm
  - Computation based on **neigborhood**

The CP computation model

- Iterations of two steps
- **Propagation**
  - reducing the domains of the variables
  - uses constraints to detect impossible values of variables
  - solve a relaxation of the problem
- **Branch** (also called **Search**)
  - Decompose into subproblems
  - nondeterministic choices (e.g., giving a value for a variable)
  - automatic support for backtracking

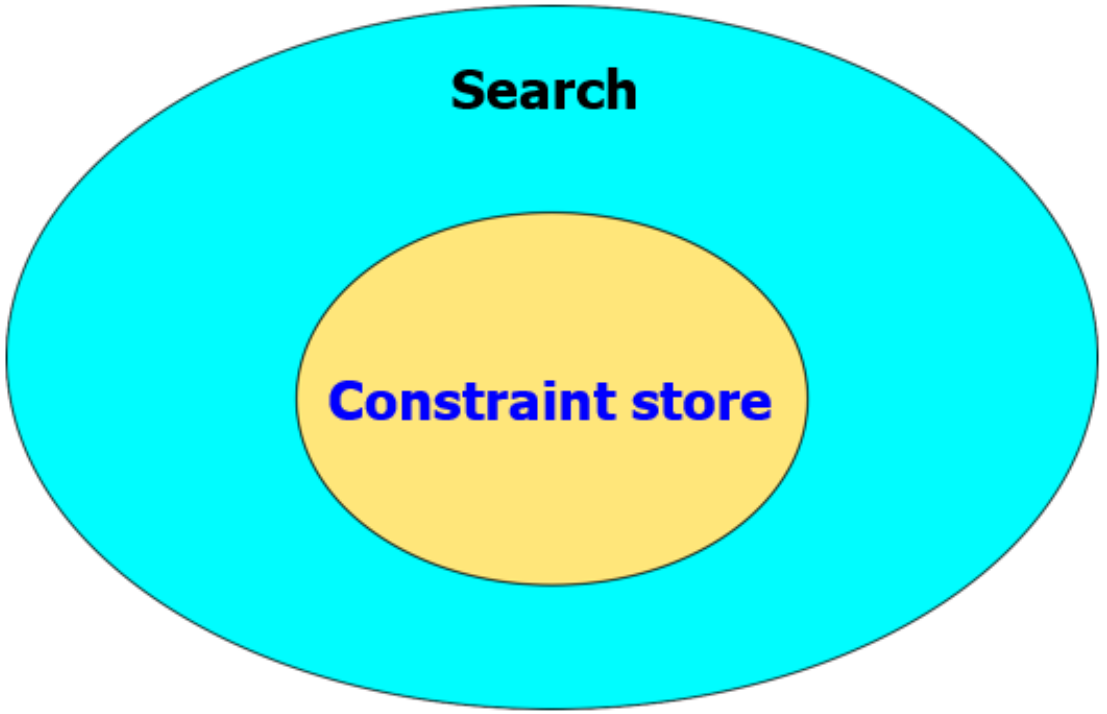
Constraint Programming

- Began in 1980s from AI world
  - Prolog III (Marseilles, France)
  - CLP(R) (Melbourne & IBM)
  - CHIP (ECRC, Germany)
- Active research area
  - Specialized conferences (CP, CP-AI-OR, …)
  - Journal (Constraints, CP Letters)
  - Commercial products

The CP Language

- A rich constraint language
  - Arithmetic, higher-order, logical constraints
  - Global constraints for natural substructures
- Specification of a search procedure
  - Definition of search tree to explore
  - Specification of exploration strategy
- Separation of concerns
  - Constraints and search are separated

Computational Model



Domains of computation

Constraint propagation techniques are related to specific **computation domains**

- Finite domains
- Finite sets
- Boolean
- Interval
- Reals
- Graphs
- …

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What is a constraint ?

- Numerical inequalities and equations
- Combinatorial / global constraints
  - natural subproblems arising of many applications
  - e.g. a set of activities A cannot overlap in time
- Logical/threshold combinations of these
- …

Constraints

- Powerful modelling tool
  - Numerical constraints
    - $x = y + 2z$, $x \neq y - 2$, $x \leq y - z$, $|x-y| \neq 2$, ...
  - Symbolic Constraints
    - $cost = \sum_{i<R} price[rack[i]]$
  - Composed constraints
    - $\sum_{1 \leq i \leq k} (x[i]=y) \leq k$ (at most k occurences of y in x[])
    - $load[j] = \sum_{i<Obj} (bin[i]=j).w[i]$ (load of bin j in bin packing)
  - Global constraints
    - alldifferent(x[1..n])
    - knapsack(x[1..n], weigth[1..n], cap)
    - Grammar constraints, scheduling constraints, ...

Propagation

Why propagation ?

- Exhaustive search of a solution: exponential time

Objectives of propagation

- Reduce the search space
- Find an equivalent CSP to the original one with *smaller domains* of variables

Techniques

- Consider the constraints *locally*
- **Consistency techniques**
  - Domain Consistency : consider each possible value in the domain
  - Bound Consistency : only considers the bounds of the domains
  - Other consistencies

Pruning

- Domain Consistency
  - The holy grail
- A constraint is **domain-consistent** if,
  - for every variable x and every value in the domain of x,
  - there exist values in the domains of the other variables
  - such that the constraint is satisfied for these values

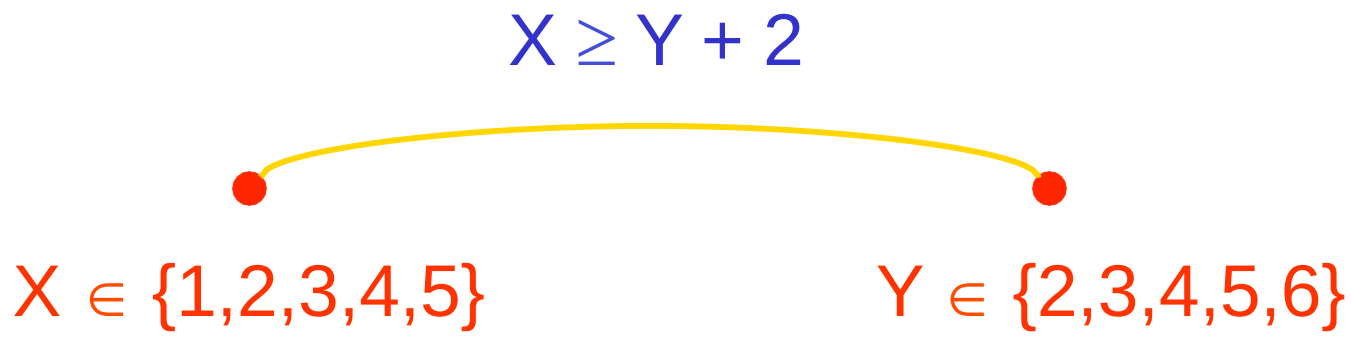
Domain consistency

$$X \geq Y + 2$$

$X \in \{1,2,3,4,5\}$ $Y \in \{2,3,4,5,6\}$

- remove any values from the domain of X for which there is no value in the domain of Y that satisfies the constraint (and vice versa)

Domain consistency



- remove any values from the domain of X for which there is no value in the domain of Y that satisfies the constraint (and vice versa)

Domain consistency

$$X \geq Y + 2$$

$X \in \{4,5\}$ $Y \in \{2,3\}$

- This constraint is now domain consistent
- Possible objective : make *all* constraints domain consistent
- Should be done *efficiently*

Consistency does not solve the CSP !

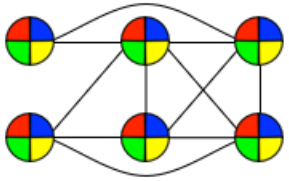
Bound Consistency

- Achieving domain consistency can be computationally expensive
- Bound consistency is a weaker form of consistency
  - a domain is approximated by an interval [min D(x), max D(x)]
  - only verify that the bounds of the domain are in a solution of the constraint

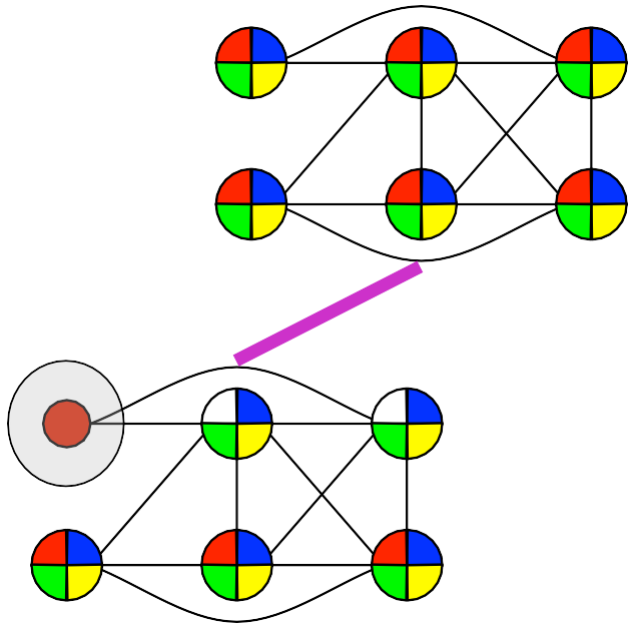
Bound Consistency

- Specialized to each constraint type
  - x + y + z = 10
  - x in {0,1,2,3,4}, y in {1,2,4}, z in {1,2,4}
- Domain consistency (AC) gives
  - x in {2,4}, y in {2,4}, z in {2,4}
  - expensive to compute ($O(d^3)$)
- Bound consistency (BC) gives
  - x in in {2,3,4}, y in {2,4}, z in {2,4}
  - can be computed very efficiently ($O(1)$)

Graph Coloring CP



Graph Coloring CP



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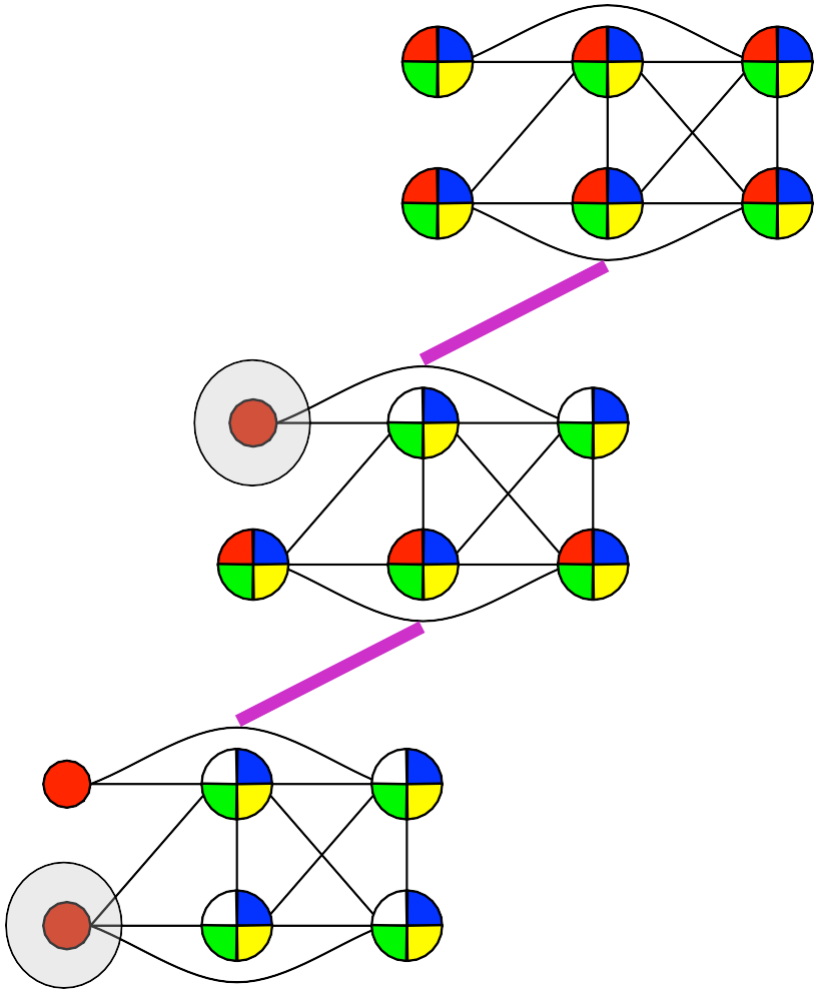

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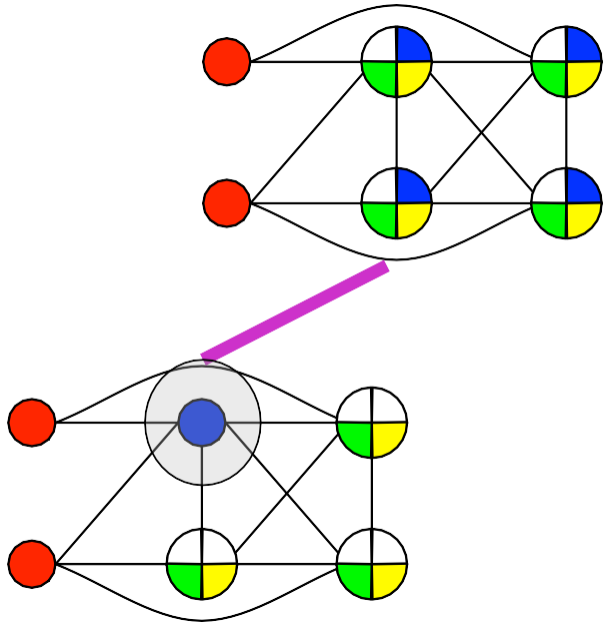

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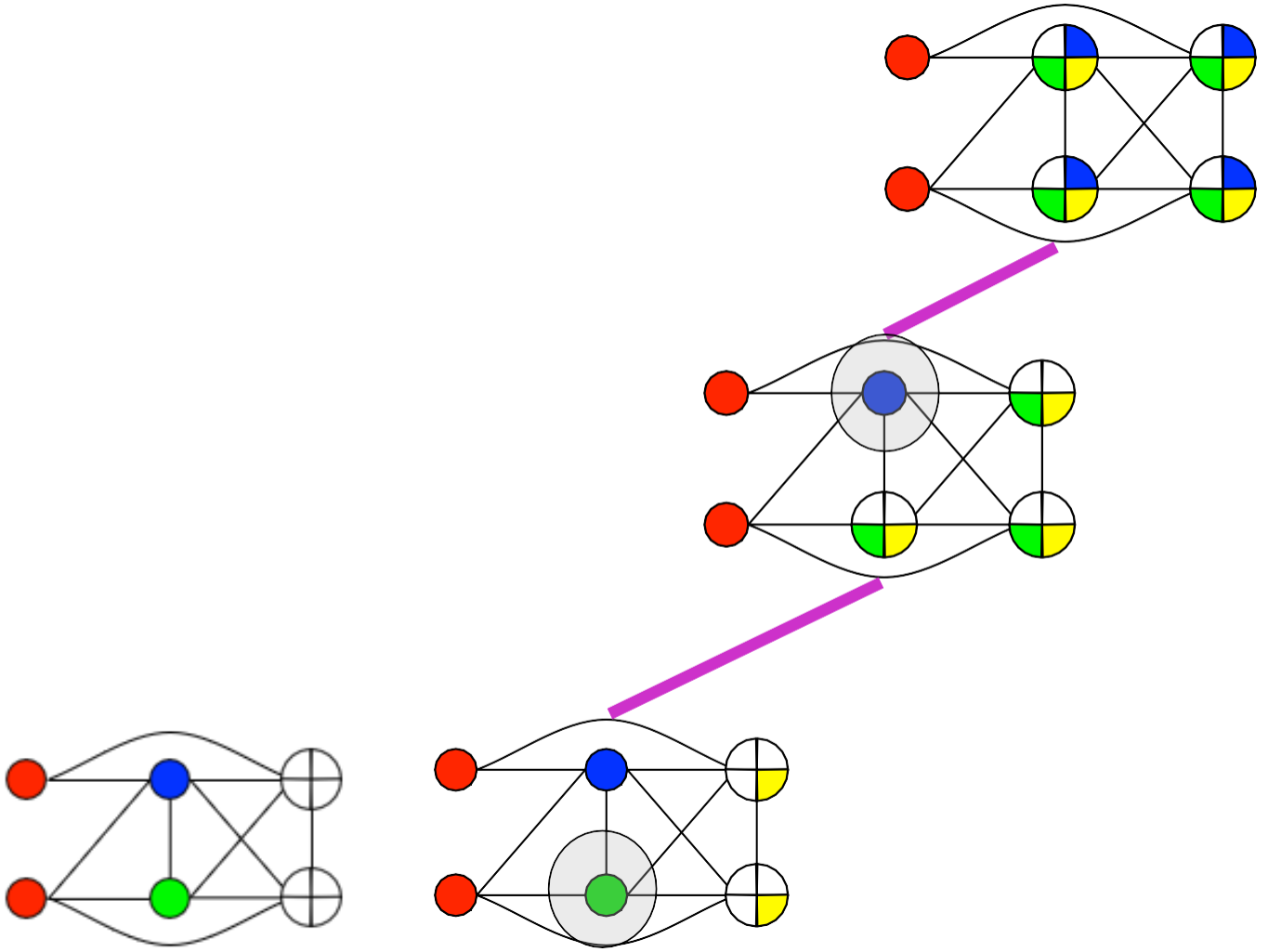

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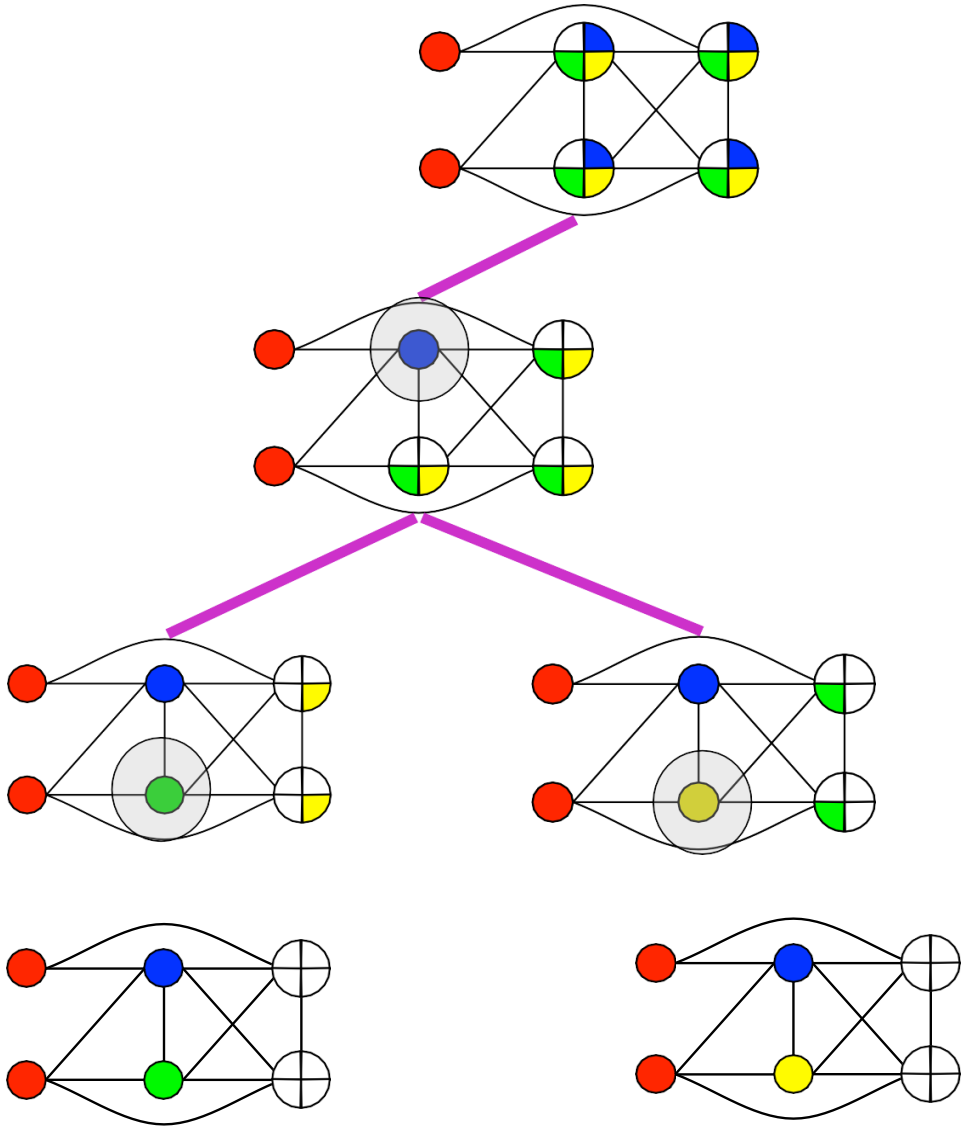

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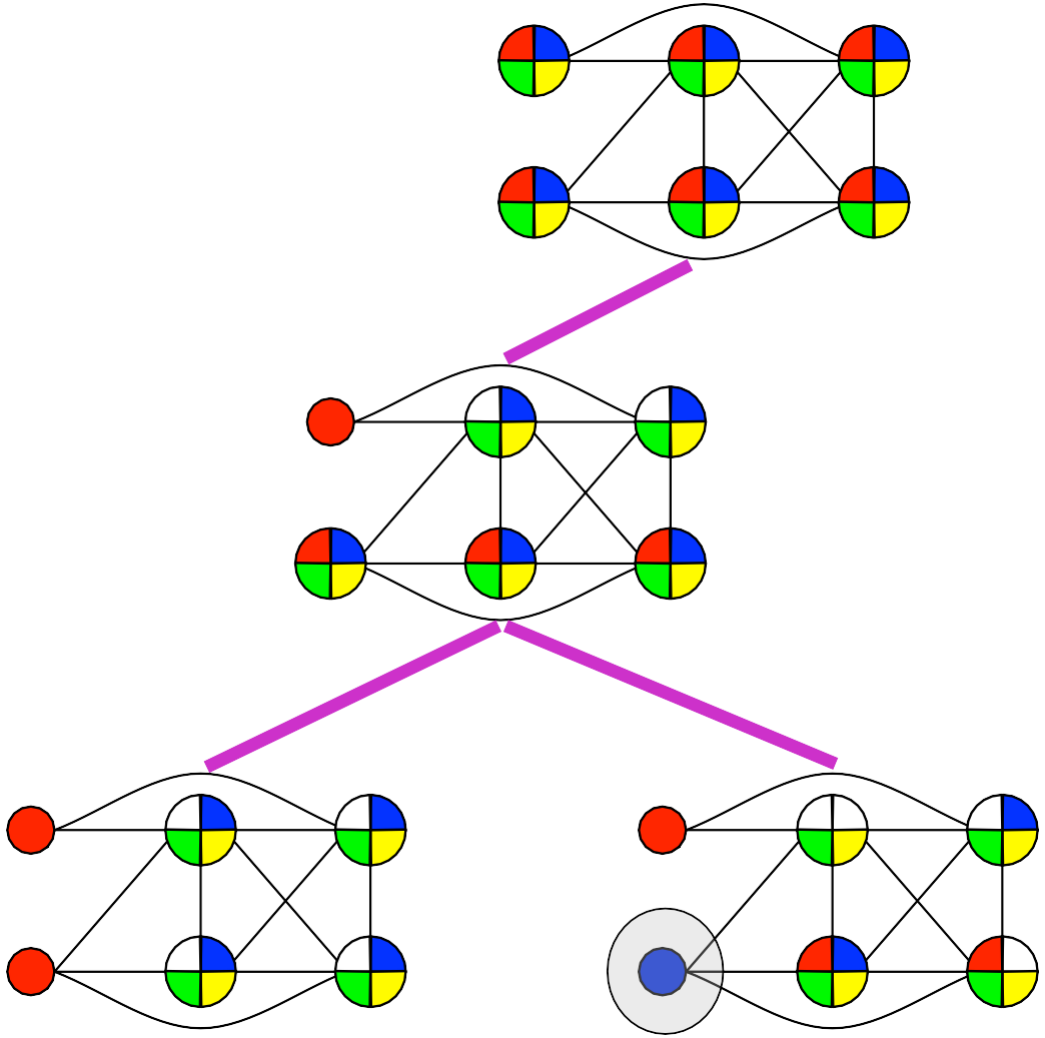

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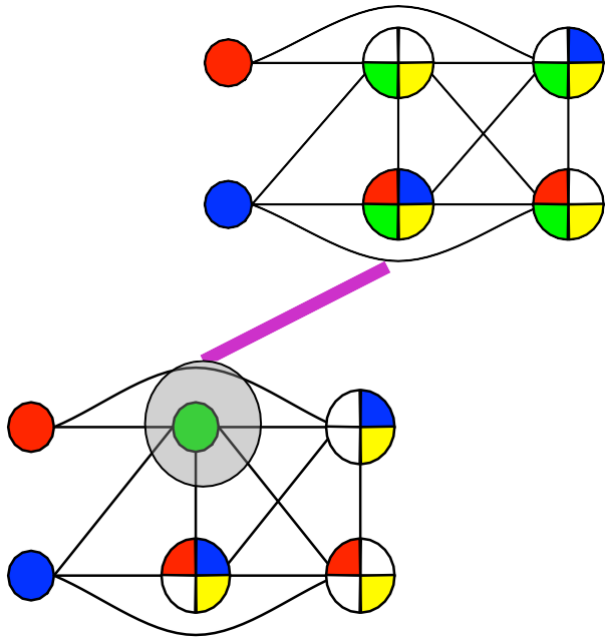

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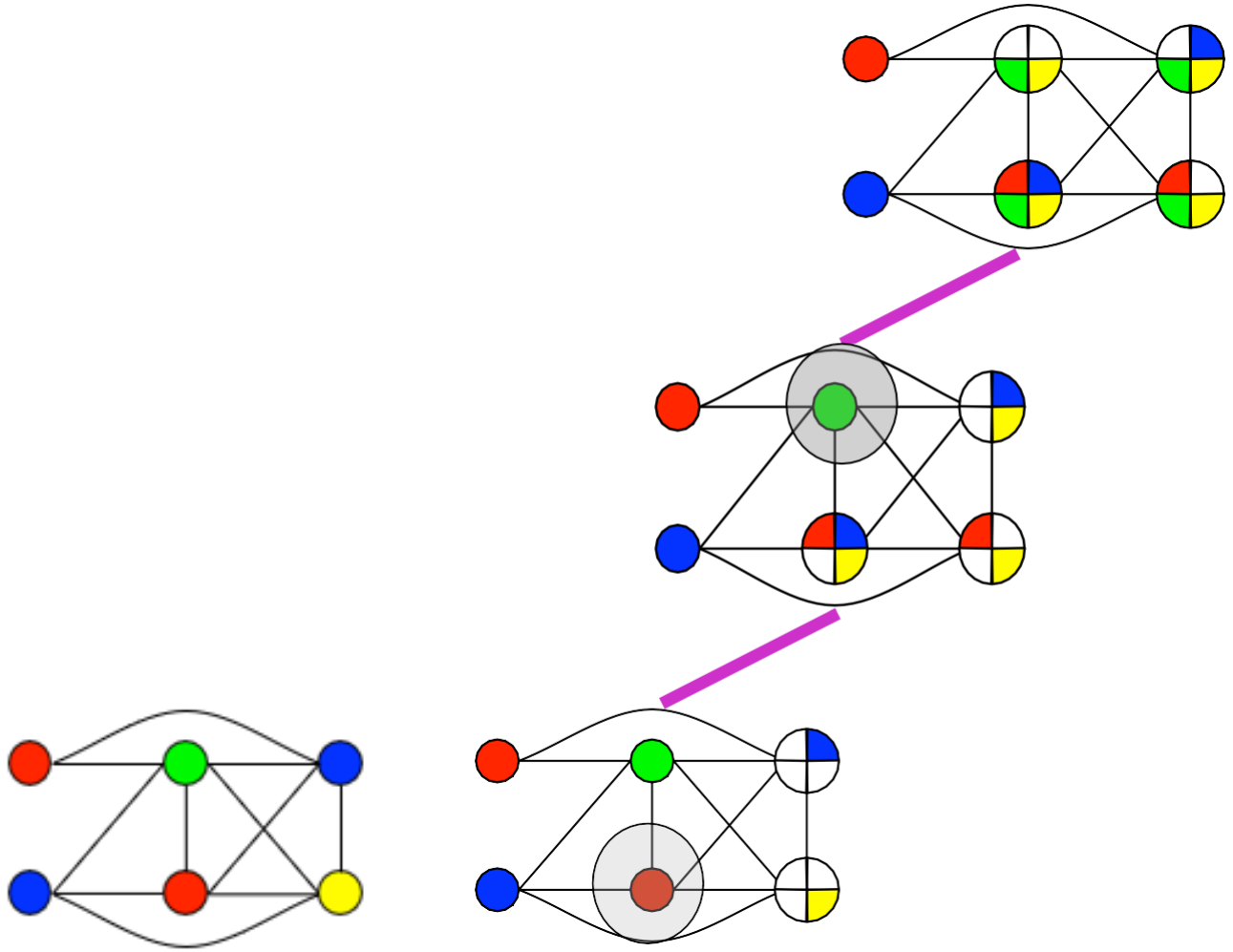
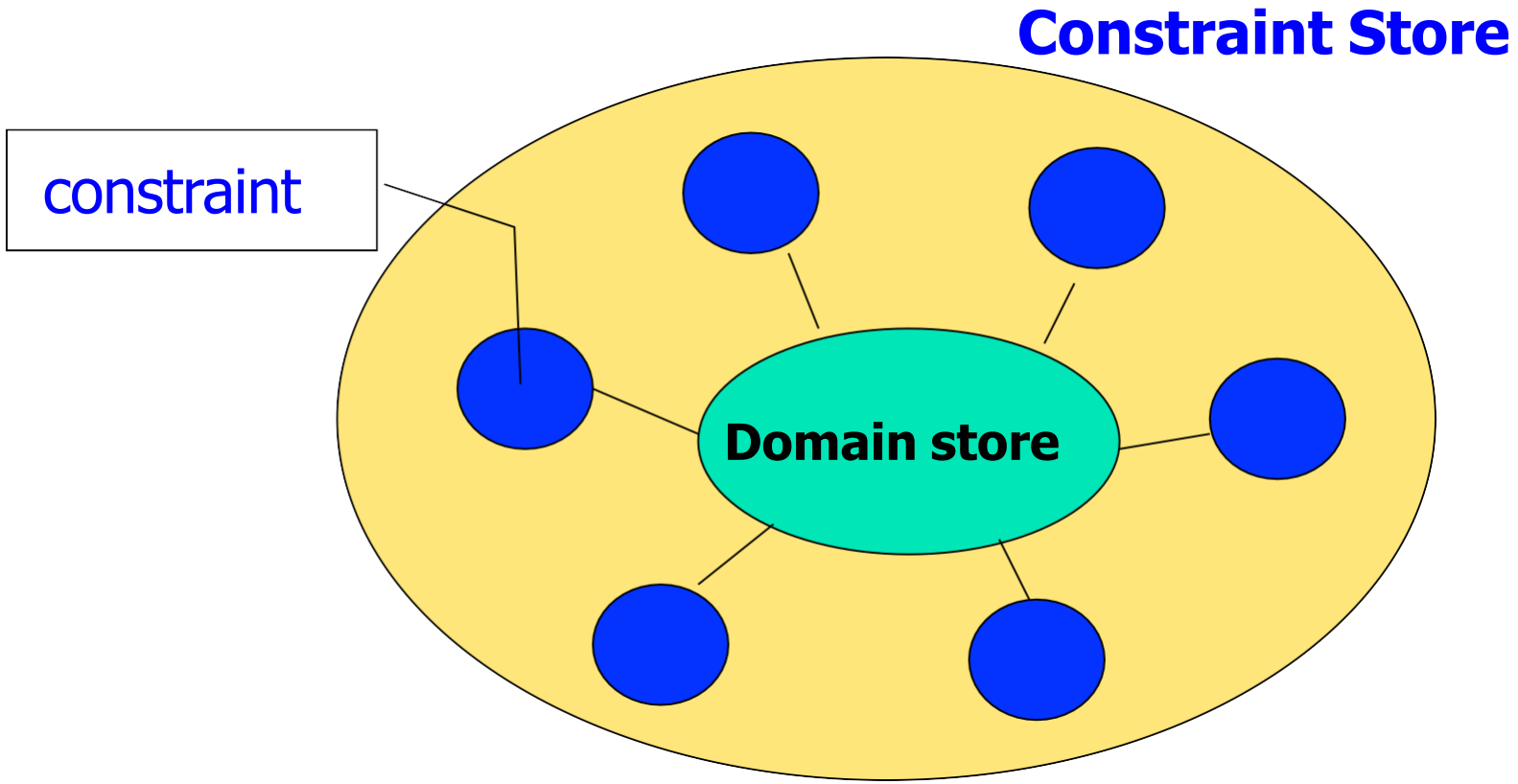


Computational Model



- Domain store
  - For each variable: what is the set of possible values?
  - If empty for any variable, then infeasible
  - If singleton for all variables, then solution
- Constraints
  - Capture interesting and well studied substructures
  - Need to
    - **Check Feasibility** : Determine if constraint is feasible wrt the domain store
    - **Prune** : remove "impossible" values from the domains

- General (fixpoint) algorithm is

```
repeat
   select a constraint c
   if c is infeasible wrt the domain store
      return infeasible
   else
      apply pruning algorithm of c
until no value can be removed
```

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Central theme

Solution process
=
Model + Search

- The model
  - what the solutions are and their quality
- The search
  - how to find the solutions

Example : SEND + MORE = MONEY

- Replace each letter by a different digit so that

```
  SEND
+ MORE
------
 MONEY
```

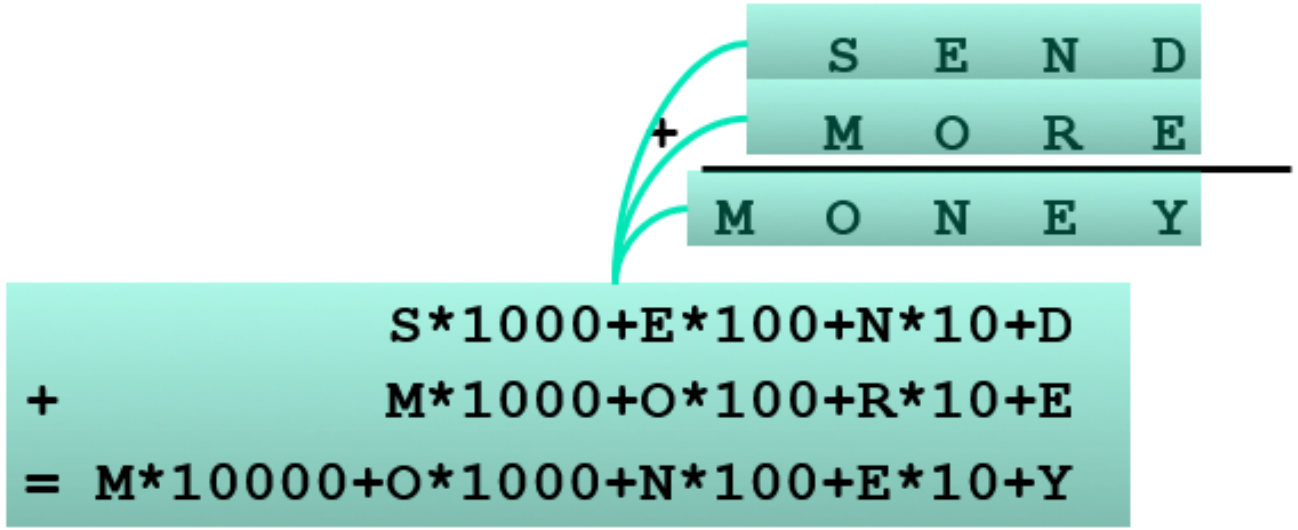
  is a correct sum.
  - Unique solution:

```
  9567
+ 1085
------
 10652
```

- Variables: S, E, N, D, M, O, R, Y
- Domains:
  - [1::9] for S, M,
  - [0::9] for E, N, D, O, R, Y .

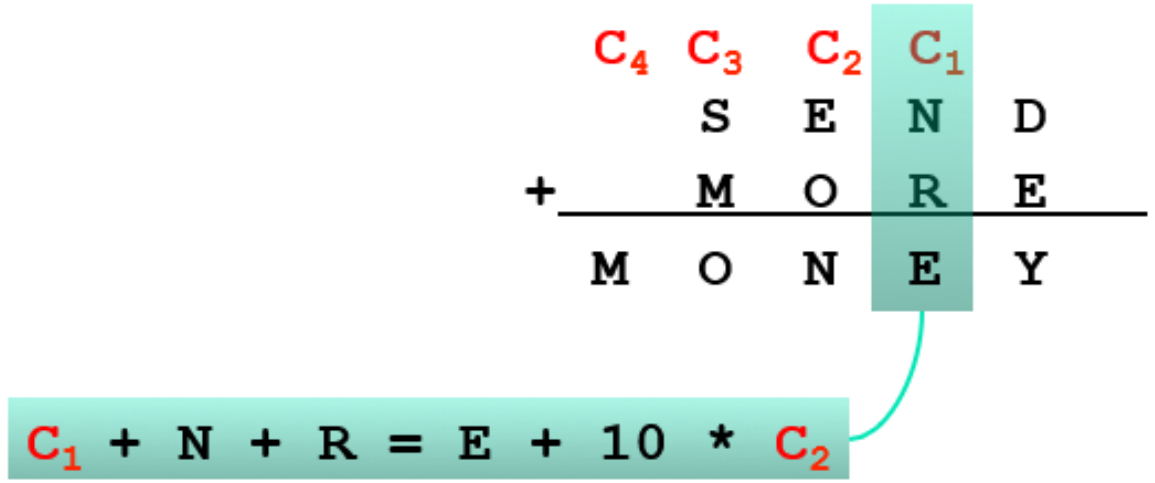
Example : SEND + MORE = MONEY

- 1 equality constraint
  - 1000 S + 100 E + 10 N + D +
    1000 M + 100 O + 10 R + E
    = 10000 M + 1000 O + 100 N + 10 E + Y



Example : SEND + MORE = MONEY

- **Or** 5 equality constraints.
  - Use carry variables C1, …, C4 ∈ [0..1]
  - D + E = 10 C1 + Y
  - C1 + N + R = 10 C2 + E
  - C2 + E + O = 10 C3 + N
  - C3 + S +M = 10 C4 + O
  - C4 = M



Example : SEND + MORE = MONEY

- 28 disequality constraints.
  - $x \neq y$ for $x, y \in \{S;E;N;D;M;O;R;Y\}$
- **Or** a single constraint for disequalities.
  - For variables x1,…, xn with domains D1,…,Dn : all_different(x1, …, xn) := {(d1,…,dn) | $d_i \neq d_j$ for $i \neq j$}
  - Use `all_different(S,E,N,D,M,O,R,Y)`
- **Or** modeling it as an IP problem.
  - For $x, y \in \{S;E;N;D;M;O;R;Y\}$ transform $x \neq y$ to
  - $x - y <= 10 - 11\ z_{xy}$
  - $y - x <= 11\ z_{xy} - 1$
- where $z_{xy} \in [0..1]$; $z_{xy}=1 \Rightarrow x<y$
  - Disadvantage: 28 new variables.

Content

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- Constraint Satisfaction Problem
- **Constraint Programming**
  - What is CP ?
  - Constraints and propagation
  - Modeling
  - **Searching**
  - Global constraint
  - Computation domain
- Examples with Or-Tools

Branching

- Once constraint propagation is done, apply the search method

Choose a variable x with non-singleton domain (d_1, d_2, … d_i)

For each **d** in (d_1, d_2, … d_i)

  try to solve the problem with the additional constraint **x=d**

Central theme

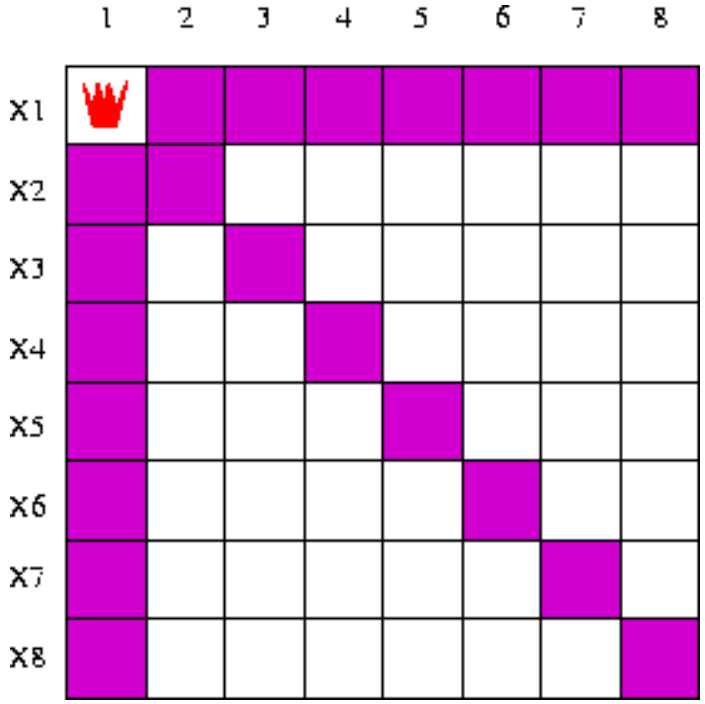
Solution process
=
Model + Search

- The search
  - nondeterministic program + exploration strategy
  - exploration strategy: DFS, LDS, BFS, …

The Queen Problem

Assuming domain consistency

Propagation of
queen[1] <> queen[2]
queen[1] <> queen[3]
…
queen[1] <> queen[8]

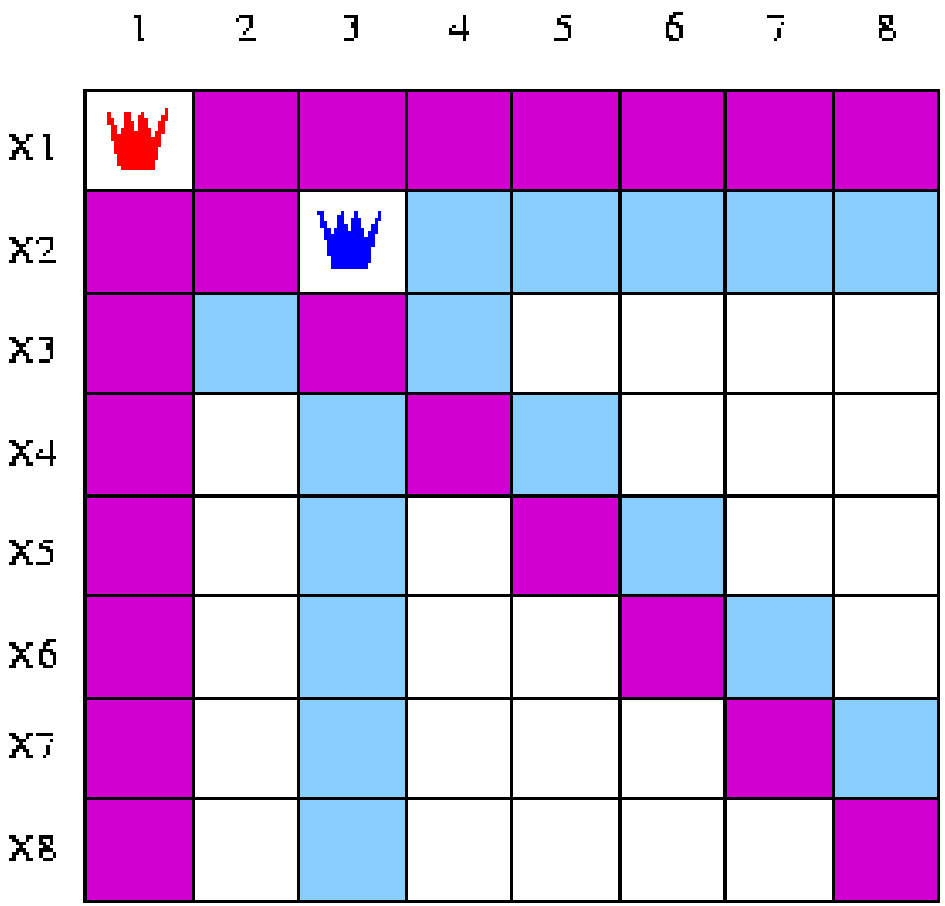


Consequence of
queen[1]=1

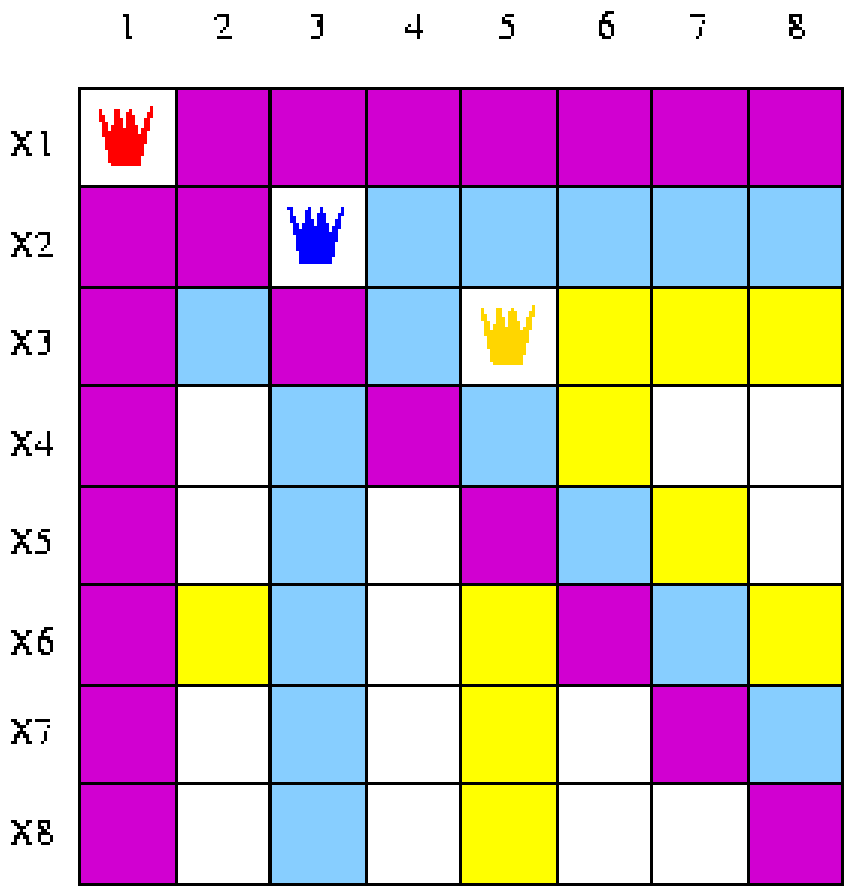
Propagation of
queen[1]+1<> queen[2]+2
queen[1]+1<> queen[3]+3
…
queen[1]+1 <> queen[8]+8

No more pruning
Place another queen
Questions
Which one ?
On which tile ?

The Queen Problem



The Queen Problem



The search

Search Procedure

=

Nondeterministic Program + Exploration Strategy

- Nondeterministic program
  - specify (implicitly) an **and-or tree**
- Exploration strategy
  - specify **how to explore the tree**

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Global Constraints

- Recognize some combinatorial substructures arising in many practical applications
  - alldifferent is a fundamental building block
  - many others (as we will see)
- Make modeling easier and more natural
  - Declarative
  - Compositionality

The alldifferent Constraint

- Most well-known global constraint.
  - `alldifferent(x,y,z)`
- states that x, y, and z take on different values.
  - x=2, y=1, z=3 would be ok,
  - but not x=1, y=3, z=1
- Why does it prune more than x≠y, y≠z, x≠z ?
- Very useful in many resource allocation, time tabling, sport scheduling problems

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Computation domains

- Propagators are associated to constraints and their underlying **computation domain**
- Basic computation domain : **finite domains**
- A set of constraints and their associated propagators (over a computation domain), is called a **constraint solver**
- Some possibilities to combine different computation domains (i.e. different solvers) within a CSP
- **Constraint programming engine** = constraint solvers + search

Computation domains

- Finite domains
- Linear real (rational) arthmetics
- Boolean
- Finite sets
- Terms
- Reals
- Intervals
- Graphs
- ...

Content

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  - Constraints and propagation
  - Modeling
  - Searching
  - Global constraint
  - Computation domain
- **Examples with Or-Tools**

Example 1

- **Biến**
  - X = {X0, X1, X2, X3, X4}
- **Miền giá trị**
  - X0, X1, X2, X3, X4 ∈ {1,2,3,4,5}
- **Ràng buộc**
  - C1: X2 + 3 ≠ X1
  - C2: X3 ≤ X4
  - C3: X2 + X3 = X0 + 1
  - C4: X4 ≤ 3
  - C5: X1 + X4 = 7
  - C6: X2 = 1 => X4 ≠ 2

Solution

```python
from ortools.sat.python import cp_model

model = cp_model.CpModel()

x0 = model.NewIntVar(1, 5, 'x0')
x1 = model.NewIntVar(1, 5, 'x1')
x2 = model.NewIntVar(1, 5, 'x2')
x3 = model.NewIntVar(1, 5, 'x3')
x4 = model.NewIntVar(1, 5, 'x4')

model.Add(x2 + 3 != x1)
model.Add(x3 <= x4)
model.Add(x2 + x3 == x0 + 1)
model.Add(x4 <= 3)
model.Add(x1 + x4 == 7)

b = model.NewBoolVar('b')
model.Add(x2 == 1).OnlyEnforceIf(b)
model.Add(x2 != 1).OnlyEnforceIf(b.Not())
model.Add(x4 != 2).OnlyEnforceIf(b)

solver = cp_model.CpSolver()
status = solver.Solve(model)

if status == cp_model.OPTIMAL:
    print('Maximum of objective function: %i' % solver.ObjectiveValue())
    print()
    print('x0 value: ', solver.Value(x0))
    print('x1 value: ', solver.Value(x1))
    print('x2 value: ', solver.Value(x2))
    print('x3 value: ', solver.Value(x3))
    print('x4 value: ', solver.Value(x4))
```

Example 2: CP + IS + FUN = TRUE

```
def CPIsFunSat():
    """Solve the CP+IS+FUN==TRUE cryptarithm."""
    # Constraint programming engine
    model = cp_model.CpModel()

    base = 10

    c = model.NewIntVar(1, base - 1, 'C')
    p = model.NewIntVar(0, base - 1, 'P')
    i = model.NewIntVar(1, base - 1, 'I')
    s = model.NewIntVar(0, base - 1, 'S')
    f = model.NewIntVar(1, base - 1, 'F')
    u = model.NewIntVar(0, base - 1, 'U')
    n = model.NewIntVar(0, base - 1, 'N')
    t = model.NewIntVar(1, base - 1, 'T')
    r = model.NewIntVar(0, base - 1, 'R')
    e = model.NewIntVar(0, base - 1, 'E')

    # We need to group variables in a list to use the constraint AllDifferent.
    letters = [c, p, i, s, f, u, n, t, r, e]
```

Example 2: CP + IS + FUN = TRUE

```
# Verify that we have enough digits.
assert base >= len(letters)

# Define constraints.
model.AddAllDifferent(letters)

# CP + IS + FUN = TRUE
model.Add(c * base + p + i * base + s + f * base * base + u * base +
          n == t * base * base * base + r * base * base + u * base + e)

### Solve model.
solver = cp_model.CpSolver()
solution_printer = VarArraySolutionPrinter(letters)

# Enumerate all solutions.
solver.parameters.enumerate_all_solutions = True
# Solve.
status = solver.Solve(model, solution_printer)

print()
print('Statistics')
print('  - status          : %s' % solver.StatusName(status))
print('  - conflicts       : %i' % solver.NumConflicts())
print('  - branches        : %i' % solver.NumBranches())
print('  - wall time       : %f s' % solver.WallTime())
print('  - solutions found : %i' % solution_printer.solution_count())
```

Example 2: CP + IS + FUN = TRUE

```
class VarArraySolutionPrinter(cp_model.CpSolverSolutionCallback):
    """Print intermediate solutions."""

    def __init__(self, variables):
        cp_model.CpSolverSolutionCallback.__init__(self)
        self.__variables = variables
        self.__solution_count = 0

    def on_solution_callback(self):
        self.__solution_count += 1
        for v in self.__variables:
            print('%s=%i' % (v, self.Value(v)), end=' ')
        print()

    def solution_count(self):
        return self.__solution_count
```

Example 3: N-Queen problem

```
def main(board_size):
  model = cp_model.CpModel()
  # Creates the variables.
  # The array index is the column, and the value is the row.
  queens = [model.NewIntVar(0, board_size - 1, 'x%i' % i)
            for i in range(board_size)]
  # Creates the constraints.
  # The following sets the constraint that all queens are in different rows.
  model.AddAllDifferent(queens)

  # Note: all queens must be in different columns because the indices
  #of queens are all different.
```

Example 3: N-Queen problem

```
# The following sets the constraint that no two queens can be on
# the same diagonal.
for i in range(board_size):
  # Note: is not used in the inner loop.
  diag1 = []
  diag2 = []
  for j in range(board_size):
    # Create variable array for queens(j) + j.
    q1 = model.NewIntVar(0, 2 * board_size, 'diag1_%i' % i)
    diag1.append(q1)
    model.Add(q1 == queens[j] + j)
    # Create variable array for queens(j) - j.
    q2 = model.NewIntVar(-board_size, board_size, 'diag2_%i' % i)
    diag2.append(q2)
    model.Add(q2 == queens[j] - j)
  model.AddAllDifferent(diag1)
  model.AddAllDifferent(diag2)
```

Example 3: N-Queen problem

```
  ### Solve model.
  solver = cp_model.CpSolver()
  solution_printer = SolutionPrinter(queens)
  status = solver.SearchForAllSolutions(model, solution_printer)
  print()
  print('Solutions found : %i' % solution_printer.SolutionCount())


class SolutionPrinter(cp_model.CpSolverSolutionCallback):
  """Print intermediate solutions."""

  def __init__(self, variables):
    cp_model.CpSolverSolutionCallback.__init__(self)
    self.__variables = variables
    self.__solution_count = 0

  def OnSolutionCallback(self):
    self.__solution_count += 1
    for v in self.__variables:
      print('%s = %i' % (v, self.Value(v)), end = ' ')
    print()

  def SolutionCount(self):
    return self.__solution_count
```

Example 4: Bài toán Phân bổ môn học

- Có N môn học 1, 2, …, N cần được phân bổ vào P học kỳ 1, 2, …, P
- Mỗi môn học i có số tín chỉ là $credit(i)$
- $L = \{(i, j)\}$: tập các cặp môn học (i, j) trong điều kiện tiên quyết (môn i phải được xếp và học kỳ trước học kỳ của môn j)
- Cho trước các hằng số α, β, λ, γ. Hãy tìm cách xếp N môn học vào P học kỳ sao cho
  - Tổng số môn học trong mỗi học kỳ phải lớn hơn hoặc bằng α và nhỏ hơn hoặc bằng β
  - Tổng số tín chỉ các môn học trong mỗi học kỳ phải lớn hơn hoặc bằng λ và nhỏ hơn hoặc bằng γ

Example 4: Bài toán Phân bổ môn học

Môn	1	2	3	4	5	6	7	8	9	10	11	12
Số tín chỉ	2	1	2	1	3	2	1	3	2	3	1	3

$3 \leq$ Số môn học mỗi học kỳ ≤ 3

$5 \leq$ Số tín chỉ các môn học mỗi học kỳ ≤ 7

Phương án phân bổ

Học kỳ	1	2	3	4
Danh sách môn	2, 5, 3	1, 6,10	4,7,8	9,11,12

2	1
6	9
5	6
5	8
4	11
6	12
2	7
3	10
5	7
8	11
4	12

Example 4: Bài toán Phân bổ môn học

- Biến
  - X[p,i] = 1: môn i được phân vào học kỳ p
  - D(X[p,i])= {0,1}
- Ràng buộc
  - X[q,i] = 1 → X[p,j] = 0, (i,j) ∈ L, 1 ≤ p ≤ q ≤ P
  - $\sum_{p=1}^{P} X[p,i] = 1$, với mọi i = 1,2,…,N
  - $\alpha \leq \sum_{i=1}^{N} X[p,i] \leq \beta$, với mọi p 1,2,.., P
  - $\lambda \leq \sum_{i=1}^{N} X[p,i]credit(i) \leq \gamma$, với mọi p = 1,2,.., P

Example 4: Bài toán Phân bổ môn học

```
N = 12
P = 4
credits = [2, 1, 2, 1, 3, 2, 1, 3, 2, 3, 1, 3]

orderCourseLst = [(0, 1), (1, 2)]

alpha = 3
beta = 3
lamb = 5
gamma = 7

model = cp_model.CpModel()

# Khai báo biến x[i,j] = 1 nếu môn học j được phân vào kỳ p
x = []
for i in range(P):
    t = []
    for j in range(N):
        t.append(model.NewIntVar(0, 1, 'x['+ str(i) + "," + str(j) + "]"))
    x.append(t)
```

Example 4: Bài toán Phân bổ môn học

```
#Ràng buộc: Mỗi môn học chỉ được phân vào duy nhất 1 kỳ
for j in range(N):
    model.Add(sum(x[i][j] for i in range(P)) == 1)

#Ràng buộc: Số lượng môn học trong một kỳ phải nằm trong [alpha, beta]
for i in range(P):
    model.Add(sum(x[i][j] for j in range(N)) >= alpha)
    model.Add(sum(x[i][j] for j in range(N)) <= beta)

#Ràng buộc: Số tín chỉ trong một kỳ phải nằm trong [lambda, gamma]
for i in range(P):
    model.Add(sum(x[i][j]*credits[j] for j in range(N)) >= lamb)
    model.Add(sum(x[i][j]*credits[j] for j in range(N)) <= gamma)
```

Example 4: Bài toán Phân bổ môn học

```
#Ràng buộc: Thứ tự các môn học
for item in orderCourseLst:
    i = item[0]
    j = item[1]

    for q in range(P):
        for p in range(q+1):

            b = model.NewBoolVar('b')
            model.Add(x[q][i] == 1).OnlyEnforceIf(b)
            model.Add(x[q][i] != 1).OnlyEnforceIf(b.Not())
            model.Add(x[p][j] == 0).OnlyEnforceIf(b)
```

Example 4: Bài toán Phân bổ môn học

```
### Solve model.
solver = cp_model.CpSolver()
status = solver.Solve(model)

if status == cp_model.OPTIMAL or status == cp_model.FEASIBLE:
    print(f'Total cost = {solver.ObjectiveValue()}')
    print()
    for i in range(P):
        for j in range(N):
            if solver.BooleanValue(x[i][j]):
                print(
                    f'Môn học {j} được phân cho kỳ {i}')
else:
    print('No solution found.')
```

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THANK YOU !