

# Time Series Data Analysis

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## Learning Objectives

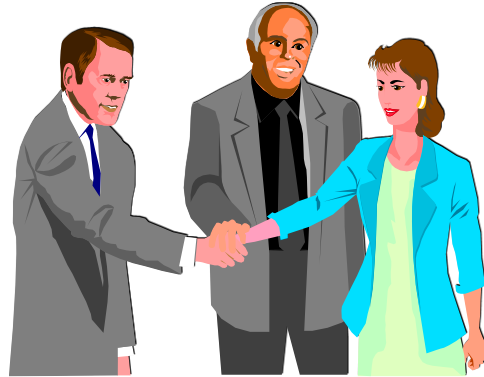
- Describe what forecasting is
- Explain time series & its components
- Smooth a data series
  - Moving average
  - Exponential smoothing
- Forecast using trend models
  - Linear Regression
  - regressive

Simple  
Auto-

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# What Is Forecasting?

- Process of predicting a future event
- Underlying basis of all business decisions
  - Production
  - Inventory
  - Personnel
  - Facilities



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## Forecasting Approaches

### Qualitative Methods

- Used when situation is vague & little data exist
  - New products
  - New technology
- Involve intuition, experience
- e.g., forecasting sales on Internet

### Quantitative Methods

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# Forecasting Approaches

## Qualitative Methods

- Used when situation is vague & little data exist
  - New products
  - New technology
- Involve intuition, experience
- e.g., forecasting sales on Internet

## Quantitative Methods

- Used when situation is 'stable' & historical data exist
  - Existing products
  - Current technology
- Involve mathematical techniques
- e.g., forecasting sales of color televisions

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## Quantitative Forecasting

- Select several forecasting methods
- 'Forecast' the past
- Evaluate forecasts
- Select best method
- Forecast the future
- Monitor continuously forecast accuracy

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# Quantitative Forecasting Methods

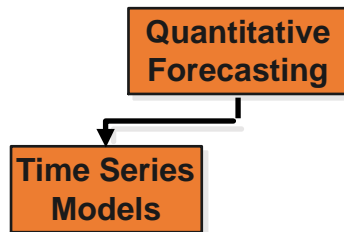
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# Quantitative Forecasting Methods

**Quantitative  
Forecasting**

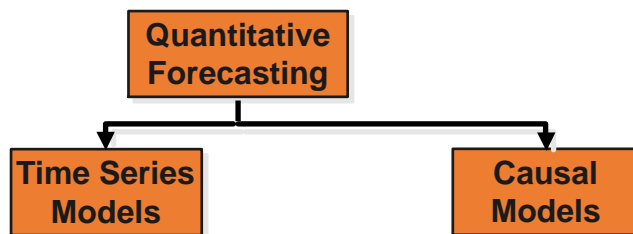
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## Quantitative Forecasting Methods



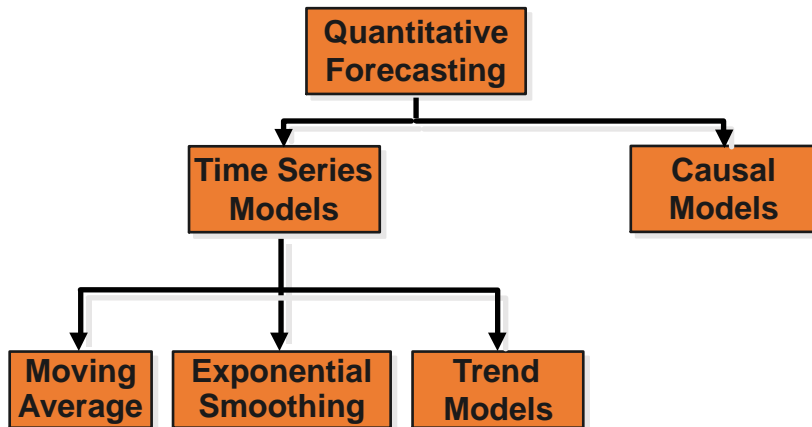
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## Quantitative Forecasting Methods



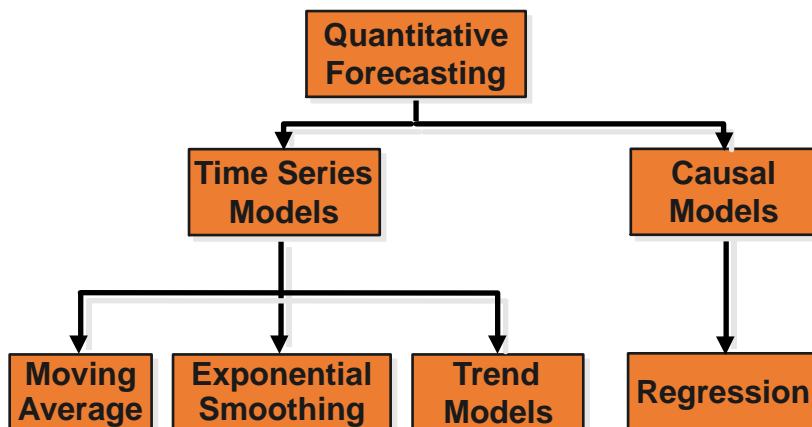
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## Quantitative Forecasting Methods



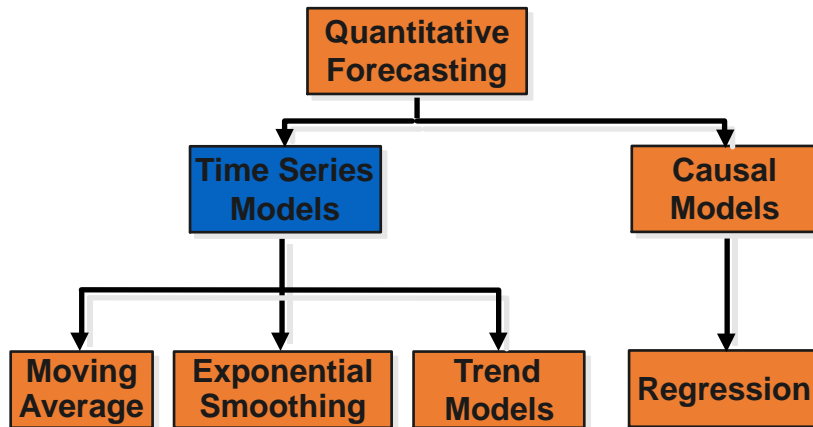
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## Quantitative Forecasting Methods



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# Quantitative Forecasting Methods



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## What is a Time Series?

- Set of evenly spaced numerical data
  - Obtained by observing response variable at regular time periods
- Forecast based only on past values
  - Assumes that factors influencing past, present, & future will continue
- Example
 

• Year:	1995	1996	1997	1998	1999
• Sales:	78.7	63.5	89.7	93.2	92.1

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## Time Series vs. Cross Sectional Data

Time series data is a sequence of observations

- collected from a **process**
- with **equally spaced** periods of time.

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## Time Series vs. Cross Sectional Data

Contrary to restrictions placed on cross-sectional data, the major purpose of forecasting with time series is to extrapolate beyond the range of the explanatory variables

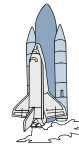


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## Time Series vs. Cross Sectional Data

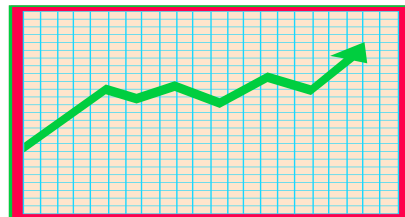
Time series is  
**dynamic**, it does  
change over time.



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## Time Series vs. Cross Sectional Data

When working with time series data, it  
is paramount that the data is plotted so  
the researcher can view the data.

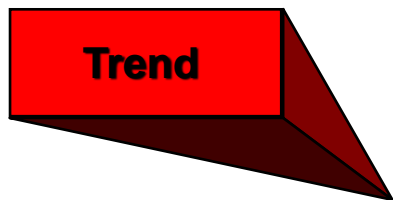


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# Time Series Components

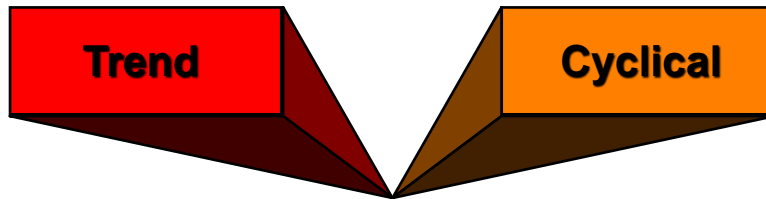
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# Time Series Components



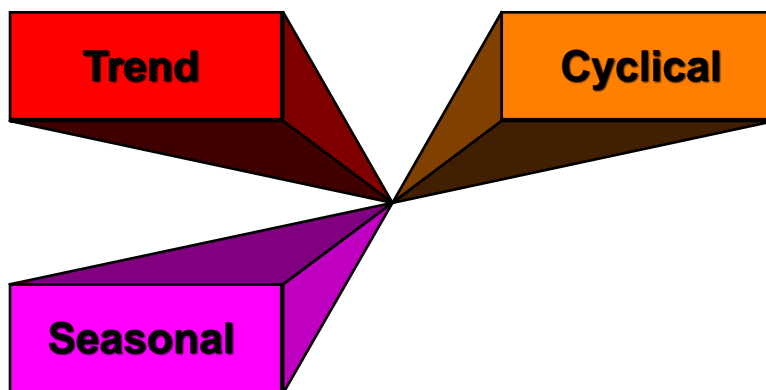
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# Time Series Components



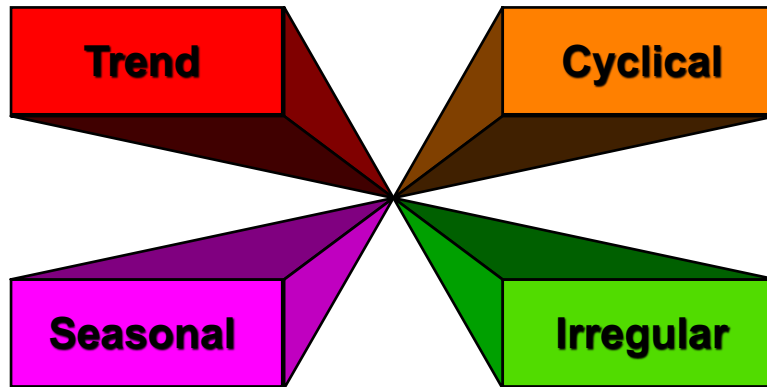
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# Time Series Components



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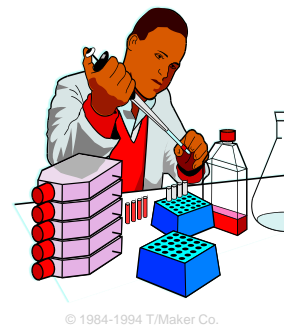
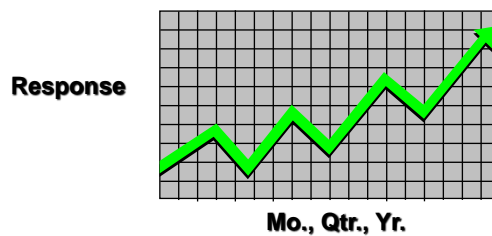
# Time Series Components



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## Trend Component

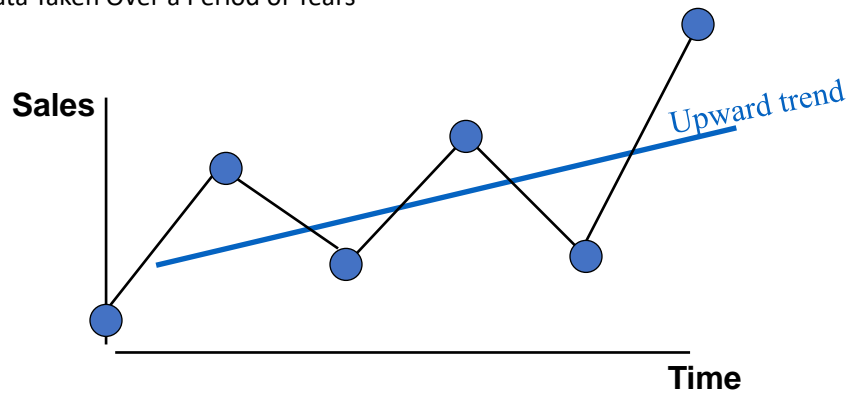
- Persistent, overall upward or downward pattern
- Due to population, technology etc.
- Several years duration



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# Trend Component

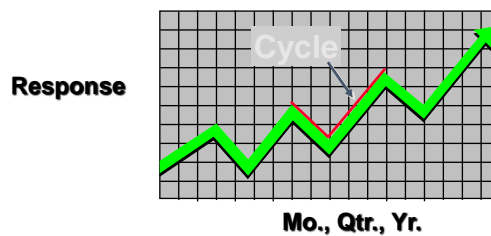
- Overall Upward or Downward Movement
- Data Taken Over a Period of Years



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# Cyclical Component

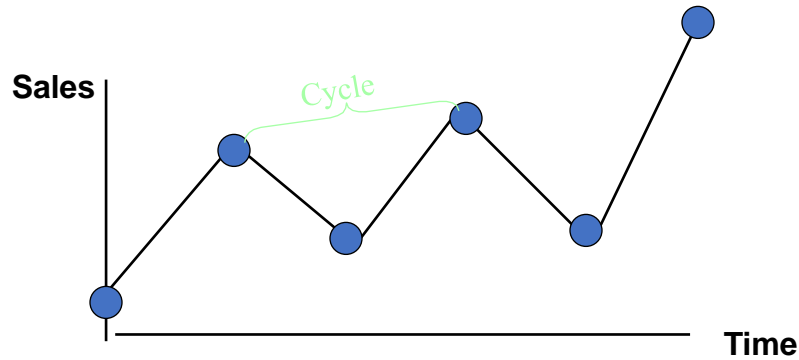
- Repeating up & down movements
- Due to interactions of factors influencing economy
- Usually 2-10 years duration



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# Cyclical Component

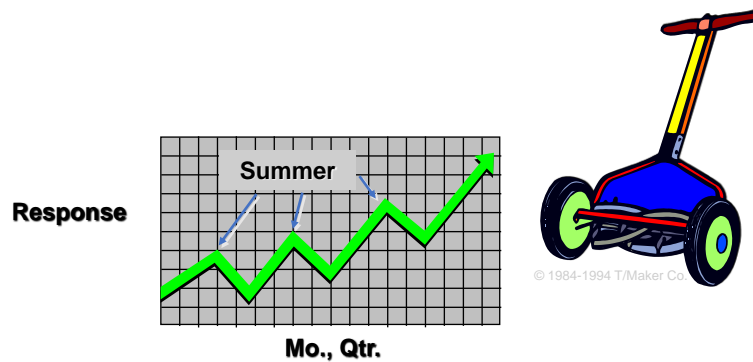
- Upward or Downward Swings
- May Vary in Length
- Usually Lasts 2 - 10 Years



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# Seasonal Component

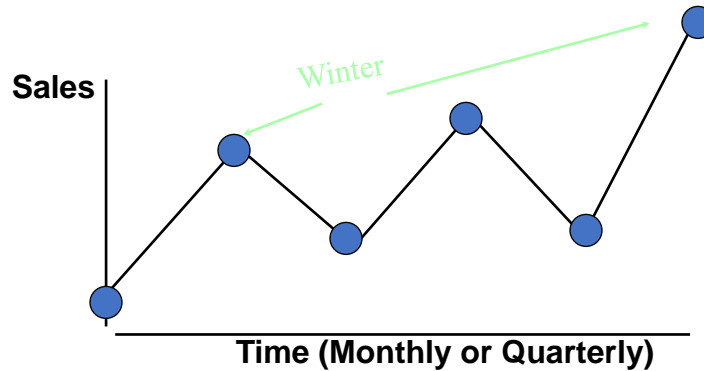
- Regular pattern of up & down fluctuations
- Due to weather, customs etc.
- Occurs within one year



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# Seasonal Component

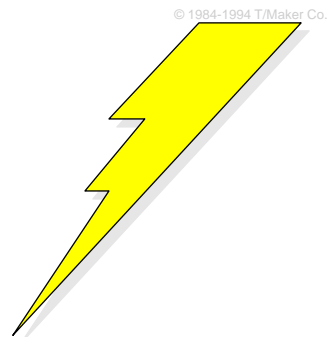
- Upward or Downward Swings
- Regular Patterns
- Observed Within One Year



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# Irregular Component

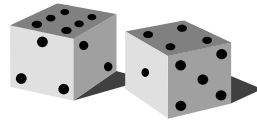
- Erratic, unsystematic, 'residual' fluctuations
- Due to random variation or unforeseen events
  - Union strike
  - War
- Short duration & nonrepeating



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# Random or Irregular Component

- Erratic, Nonsystematic, Random, 'Residual' Fluctuations
- Due to Random Variations of
  - Nature
  - Accidents
- Short Duration and Non-repeating



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# Time Series Forecasting

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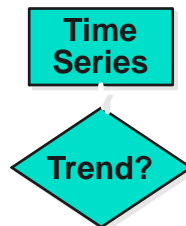


# Time Series Forecasting

Time  
Series

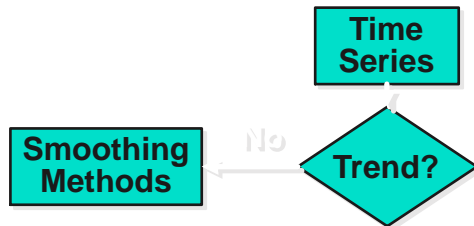
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# Time Series Forecasting



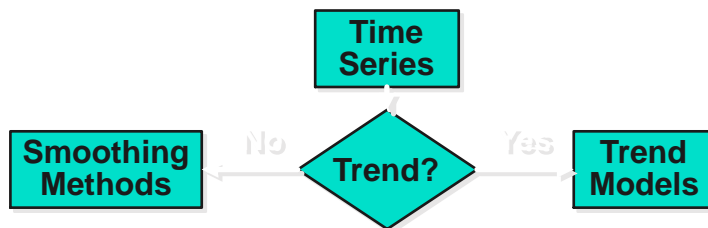
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# Time Series Forecasting



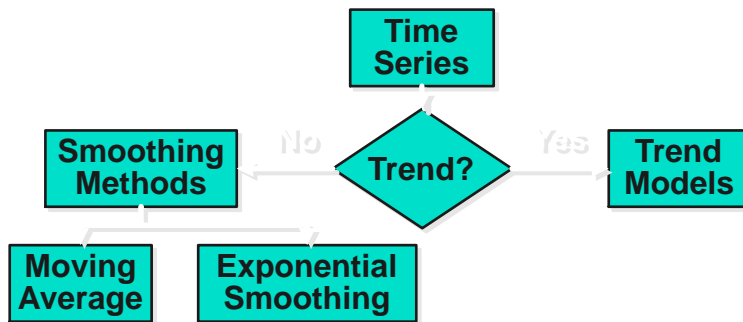
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# Time Series Forecasting



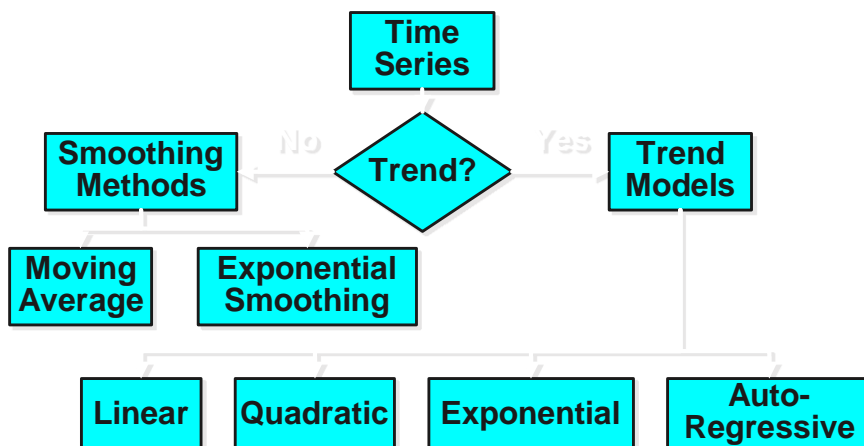
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# Time Series Forecasting



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# Time Series Forecasting



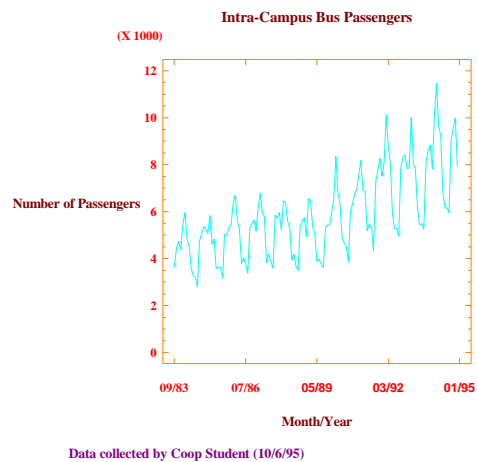
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# Time Series Analysis



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## Plotting Time Series Data

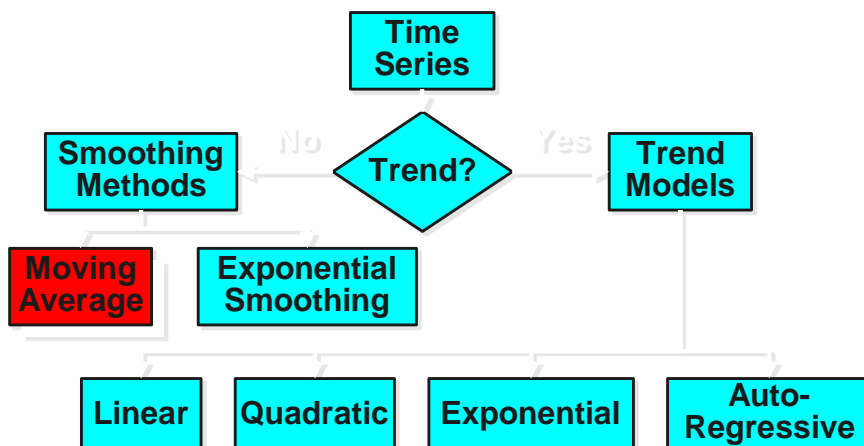


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# Moving Average Method

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## Time Series Forecasting



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# Moving Average Method

- Series of arithmetic means
- Used only for smoothing
  - Provides overall impression of data over time

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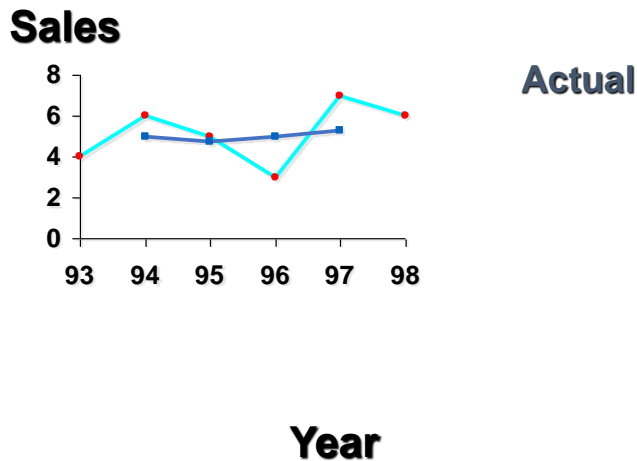
# Moving Average Method

- Series of arithmetic means
- Used only for smoothing
  - Provides overall impression of data over time

**Used for elementary forecasting**

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# Moving Average Graph



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## Moving Average [An Example]

You work for Firestone Tire. You want to smooth random fluctuations using a 3-period moving average.

1995	20,000
1996	24,000
1997	22,000
1998	26,000
1999	25,000



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# Moving Average

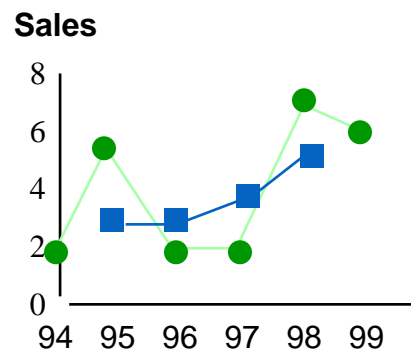
## [Solution]

Year	Sales	MA(3) in 1,000
1995	20,000	NA
1996	24,000	$(20+24+22)/3 = 22$
1997	22,000	$(24+22+26)/3 = 24$
1998	26,000	$(22+26+25)/3 = 24$
1999	25,000	NA

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# Moving Average

Year	Response ●	Moving Ave ■
1994	2	NA
1995	5	3
1996	2	3
1997	2	3.67
1998	7	5
1999	6	NA



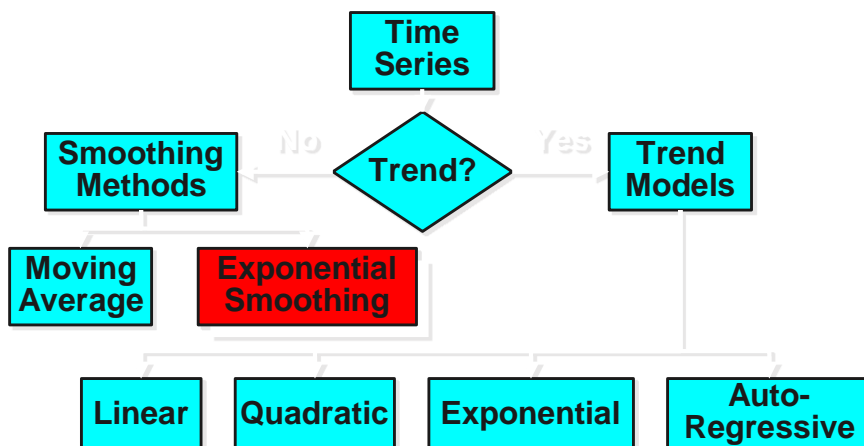
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# Exponential Smoothing Method

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## Time Series Forecasting



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# Exponential Smoothing Method

- Form of weighted moving average
  - Weights decline exponentially
  - Most recent data weighted most
- Requires smoothing constant ( $W$ )
  - Ranges from 0 to 1
  - Subjectively chosen
- Involves little record keeping of past data

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## Exponential Smoothing [An Example]

You're organizing a Kwanzaa meeting. You want to forecast attendance for 1998 using exponential smoothing

( $\alpha = .20$ ). Past attendance (00) is:

1995	4
1996	6
1997	5
1998	3
1999	7



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# Exponential Smoothing

$$E_i = W \cdot Y_i + (1 - W) \cdot E_{i-1}$$

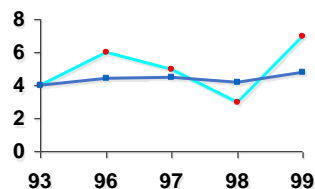
Time	$Y_i$	Smoothed Value, $E_i$ ( $W = .2$ )	Forecast $\hat{Y}_{i+1}$
1995	4	4.0	NA
1996	6	$(.2)(6) + (1-.2)(4.0) = 4.4$	4.0
1997	5	$(.2)(5) + (1-.2)(4.4) = 4.5$	4.4
1998	3	$(.2)(3) + (1-.2)(4.5) = 4.2$	4.5
1999	7	$(.2)(7) + (1-.2)(4.2) = 4.3$	4.2
2000	NA	NA	4.3

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## Exponential Smoothing [Graph]

**Attendance**

**Actual**



**Year**

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## Forecast Effect of Smoothing Coefficient (W)

$$\hat{Y}_{i+1} = W \cdot Y_i + W \cdot (1-W) \cdot Y_{i-1} + W \cdot (1-W)^2 \cdot Y_{i-2} + \dots$$

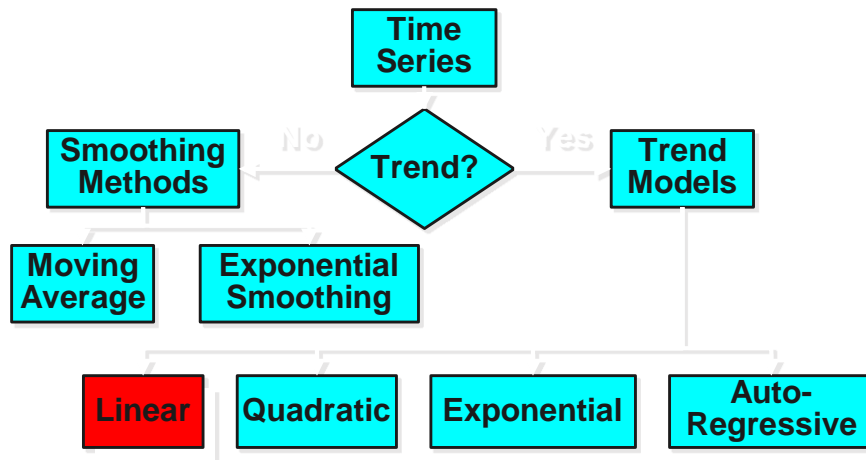
W is...	Prior Period	Weight 2 Periods Ago	3 Periods Ago
	$W$	$W(1-W)$	$W(1-W)^2$
0.10	10%	9%	8.1%
0.90	90%	9%	0.9%

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## Linear Time-Series Forecasting Model

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# Time Series Forecasting



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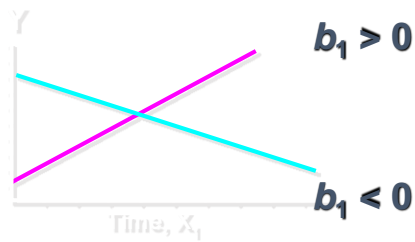
## Linear Time-Series Forecasting Model

- Used for forecasting trend
- Relationship between response variable  $Y$  & time  $X$  is a linear function
- Coded  $X$  values used often
  - Year  $X$ : 1995 1996 1997 1998 1999
  - Coded year: 0 1 2 3 4
  - Sales  $Y$ : 78.7 63.5 89.7 93.2 92.1

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# Linear Time-Series Model

$$\hat{Y}_j = b_0 + b_1 X_{1j}$$



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## Linear Time-Series Model [An Example]

You're a marketing analyst for Hasbro Toys. Using coded years, you find  $Y_i = .6 + .7X_i$ .

1995	1
1996	1
1997	2
1998	2
1999	4

Forecast 2000 sales.



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## Linear Time-Series [Example]

<u>Year</u>	<u>Coded Year</u>	<u>Sales (Units)</u>
1995	0	1
1996	1	1
1997	2	2
1998	3	2
1999	4	4
2000	5	?

2000 forecast sales:  $Y_i = .6 + .7 \cdot (5) = 4.1$

The equation would be different if 'Year' used.

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## The Linear Trend Model

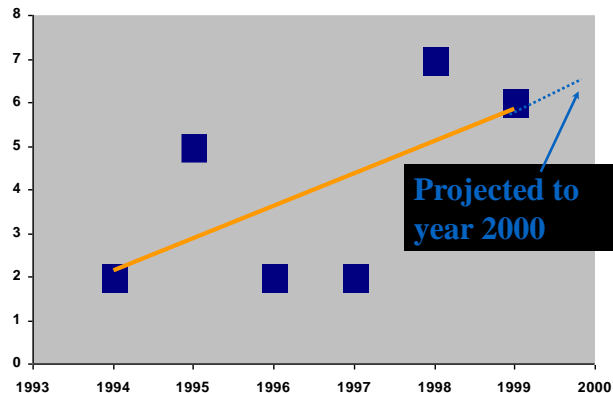
Year Coded Sales

94	0	2
95	1	5
96	2	2
97	3	2
98	4	7
99	5	6

Excel Output

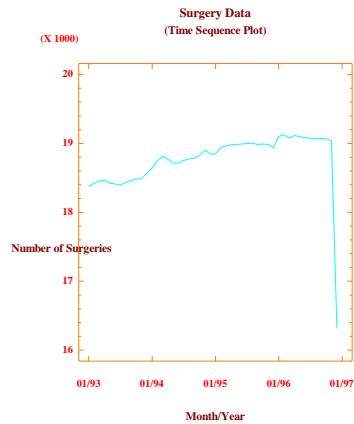
	<b>Coefficients</b>
Intercept	2.14285714
X Variable	0.74285714

$$\hat{Y}_i = b_0 + b_1 X_i = 2.143 + .743 X_i$$



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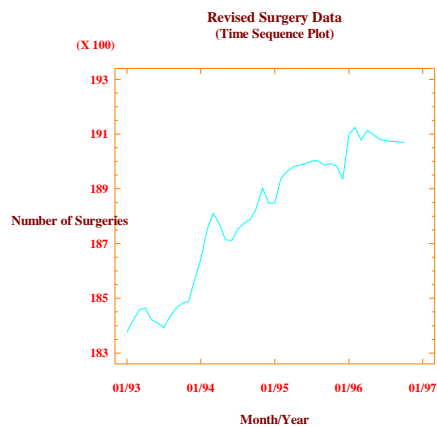
# Time Series Plot



Source: General Hospital, Metropolis

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# Time Series Plot [Revised]

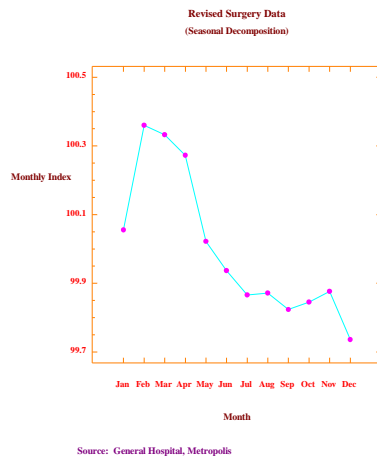


Source: General Hospital, Metropolis

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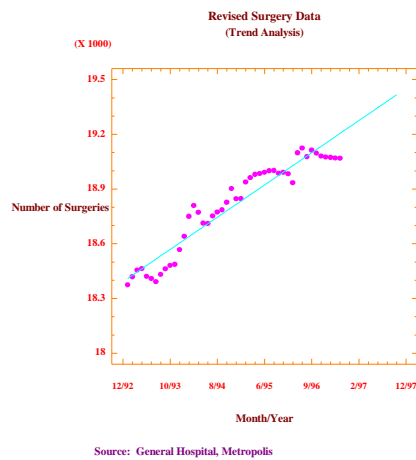


# Seasonality Plot



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# Trend Analysis

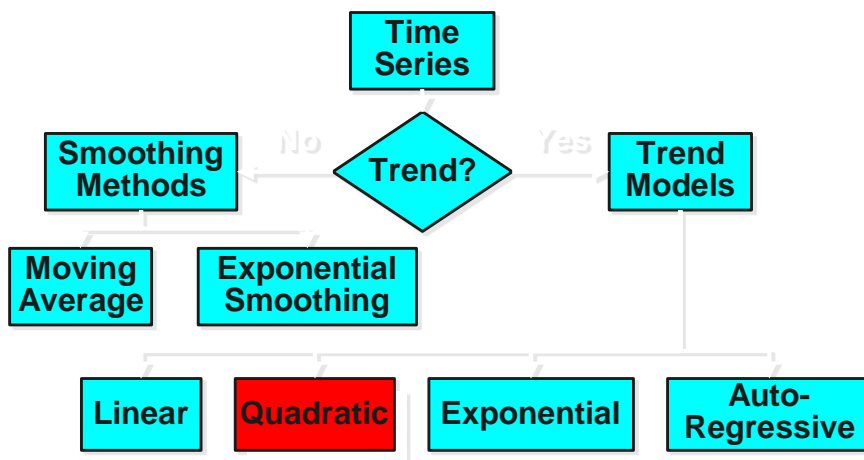


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# Quadratic Time-Series Forecasting Model

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## Time Series Forecasting



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# Quadratic Time-Series Forecasting Model

- Used for forecasting trend
- Relationship between response variable  $Y$  & time  $X$  is a quadratic function
- Coded years used

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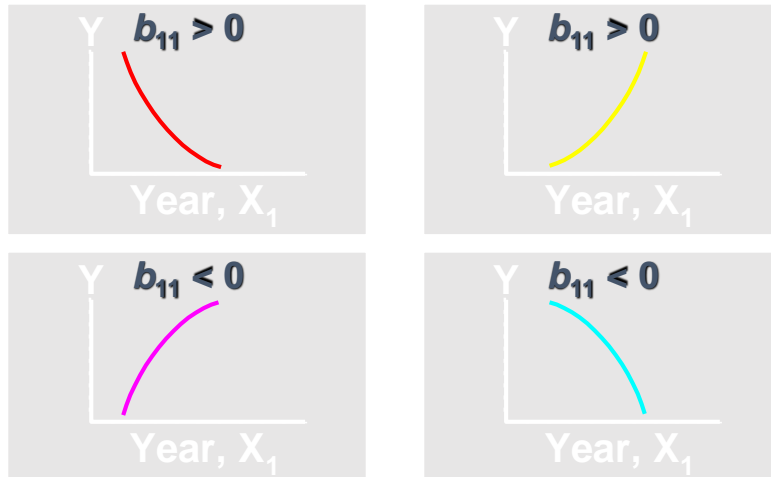
# Quadratic Time-Series Forecasting Model

- Used for forecasting trend
- Relationship between response variable  $Y$  & time  $X$  is a quadratic function
- Coded years used
- Quadratic model

$$\hat{Y}_t = b_0 + b_1 X_{1t} + b_{11} X_{1t}^2$$

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# Quadratic Time-Series Model Relationships



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## Quadratic Trend Model

Year	Coded	Sales	$\hat{Y}_i = b_0 + b_1 X_i + b_2 X_i^2$
94	0	2	
95	1	5	
96	2	2	
97	3	2	
98	4	7	
99	5	6	

Coefficients	
Intercept	2.85714286
X Variable 1	-0.3285714
X Variable 2	0.21428571

Excel Output

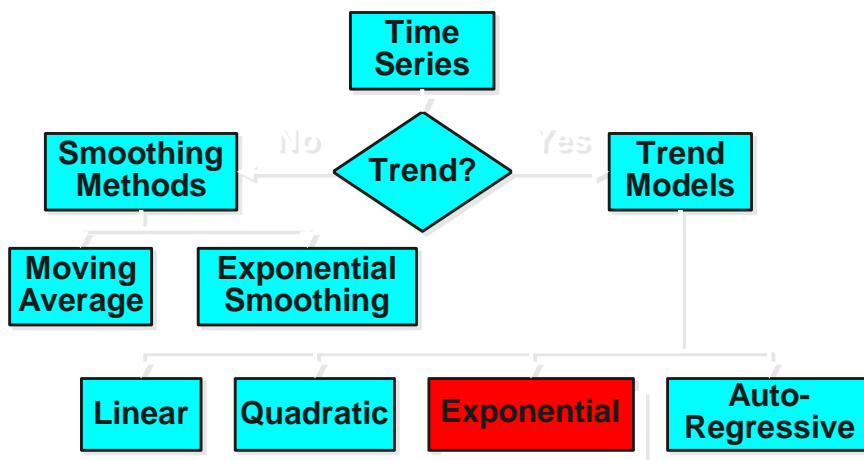
$$\hat{Y}_i = 2.857 - 0.33 X_i + .214 X_i^2$$

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# Exponential Time-Series Model

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## Time Series Forecasting



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# Exponential Time-Series Forecasting Model

- Used for forecasting trend
- Relationship is an exponential function
- Series increases (decreases) at increasing (decreasing) rate

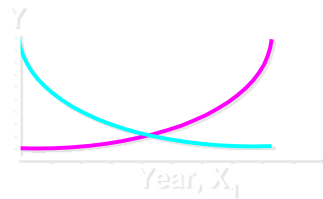
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# Exponential Time-Series Forecasting Model

- Used for forecasting trend
- Relationship is an exponential function
- Series increases (decreases) at increasing (decreasing) rate

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# Exponential Time-Series Model Relationships

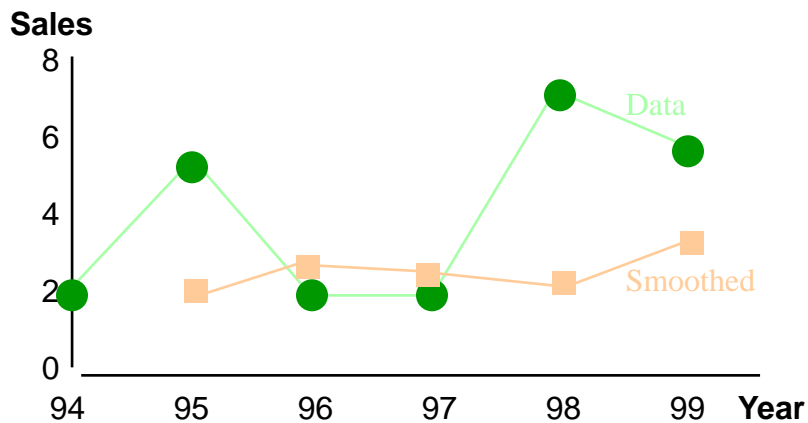


$$b_1 > 1$$

$$0 < b_1 < 1$$

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## Exponential Weight [Example Graph]



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# Exponential Trend Model

$$\hat{Y}_i = b_0 b_1^{X_i} \quad \text{or} \quad \log \hat{Y}_i = \log b_0 + X_i \log b_1$$

Year	Coded	Sales
94	0	2
95	1	5
96	2	2
97	3	2
98	4	7
99	5	6

	Coefficients
Intercept	0.33583795
X Variable	0.08068544

Excel Output of Values in logs

antilog(.33583795) =	2.17
antilog(.08068544) =	1.2

$$\hat{Y}_i = (2.17)(1.2)^{X_i}$$

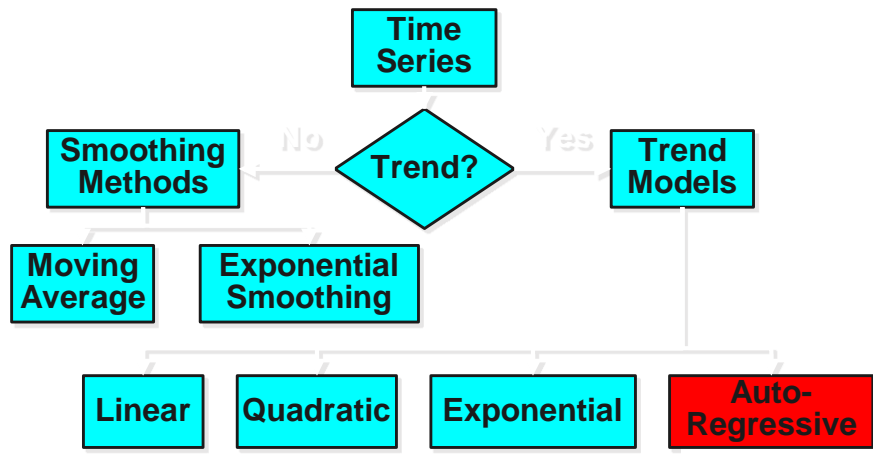
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# Autoregressive Modeling

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# Time Series Forecasting



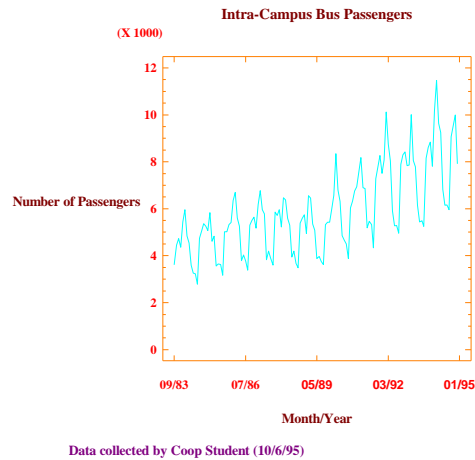
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## Autoregressive Modeling

- Used for forecasting trend
- Like regression model
  - Independent variables are lagged response variables  $Y_{t-1}$ ,  $Y_{t-2}$ ,  $Y_{t-3}$  etc.
- Assumes data are correlated with past data values
  - 1<sup>st</sup> Order: Correlated with prior period
- Estimate with ordinary least squares

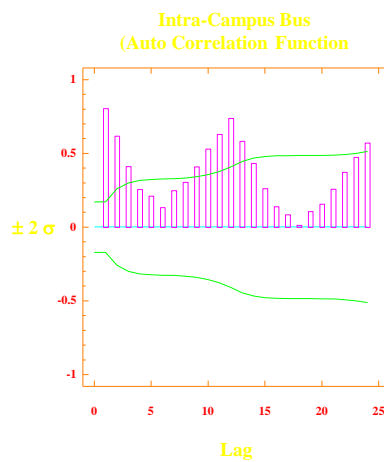
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# Time Series Data Plot



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# Auto-correlation Plot



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# Autoregressive Model [An Example]

The Office Concept Corp. has acquired a number of office units (in thousands of square feet) over the last 8 years.  
Develop the 2nd order Autoregressive models.

Year	Units
92	4
93	3
94	2
95	3
96	2
97	2
98	4
99	6



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## Autoregressive Model [Example Solution]

- Develop the 2nd order table
- Use Excel to run a regression model

### Excel Output

	Coefficients
Intercept	3.5
X Variable 1	0.8125
X Variable 2	-0.9375

Year	$Y_i$	$Y_{i-1}$	$Y_{i-2}$
92	4	---	---
93	3	4	---
94	2	3	4
95	3	2	3
96	2	3	2
97	2	2	3
98	4	2	2
99	6	4	2

$$Y_i = 3.5 + .8125 Y_{i-1} - .9375 Y_{i-2}$$

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# Evaluating Forecasts

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## Quantitative Forecasting Steps

- Select several forecasting methods
- 'Forecast' the past

### • Evaluate forecasts



- Select best method
- Forecast the future
- Monitor continuously forecast accuracy

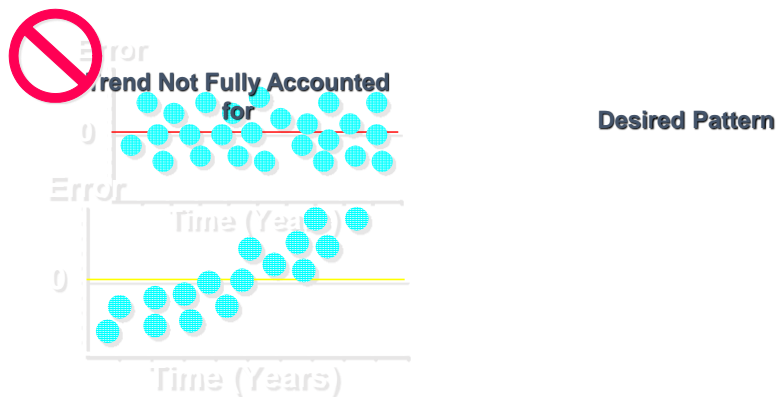
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# Forecasting Guidelines

- No pattern or direction in forecast error
  - $e_i = (\text{Actual } Y_i - \text{Forecast } \hat{Y}_i)$
  - Seen in plots of errors over time
- Smallest forecast error
  - Measured by mean absolute deviation
- Simplest model
  - Called principle of parsimony

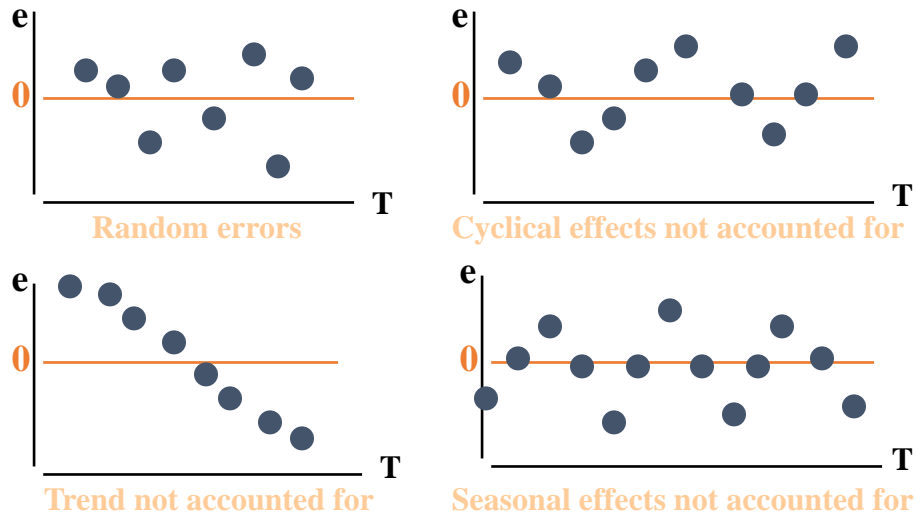
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## Pattern of Forecast Error



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# Residual Analysis



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# Principal of Parsimony

- Suppose two or more models provide good fit for data
- Select the Simplest Model
  - Simplest model types:
    - least-squares linear
    - least-square quadratic
    - 1st order autoregressive
  - More complex types:
    - 2nd and 3rd order autoregressive
    - least-squares exponential

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# Summary

- Described what forecasting is
- Explained time series & its components
- Smoothed a data series
  - Moving average
  - Exponential smoothing
- Forecasted using trend models

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# You and StatGraphics



- **Specification**  
[Know assumptions underlying various models.]
- **Estimation**  
[Know mechanics of StatGraphics Plus Win].
- **Diagnostic checking**

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# Questions?



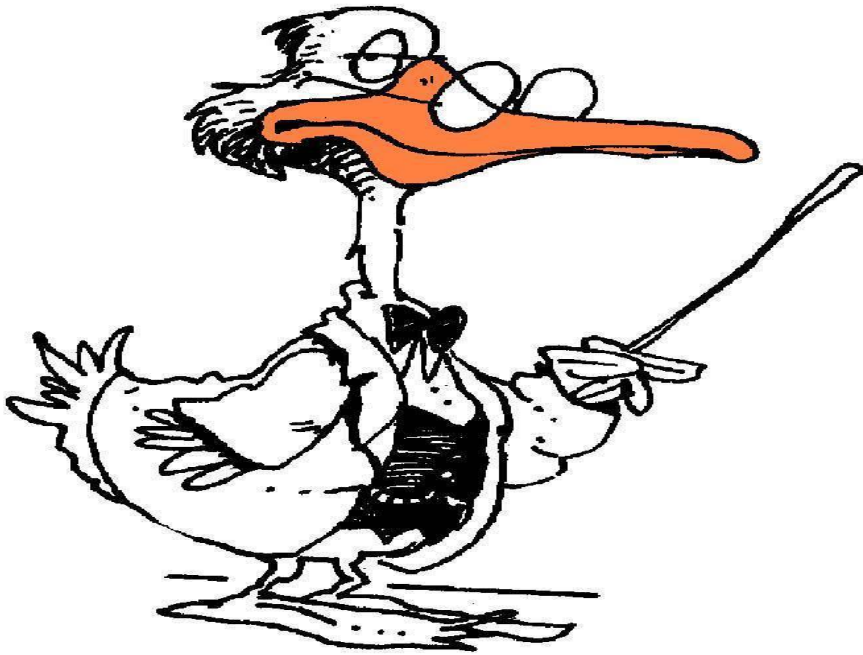
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## Source of Elaborate Slides

Prentice Hall, Inc  
Levine, et. all, First Edition

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End of Chapter

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