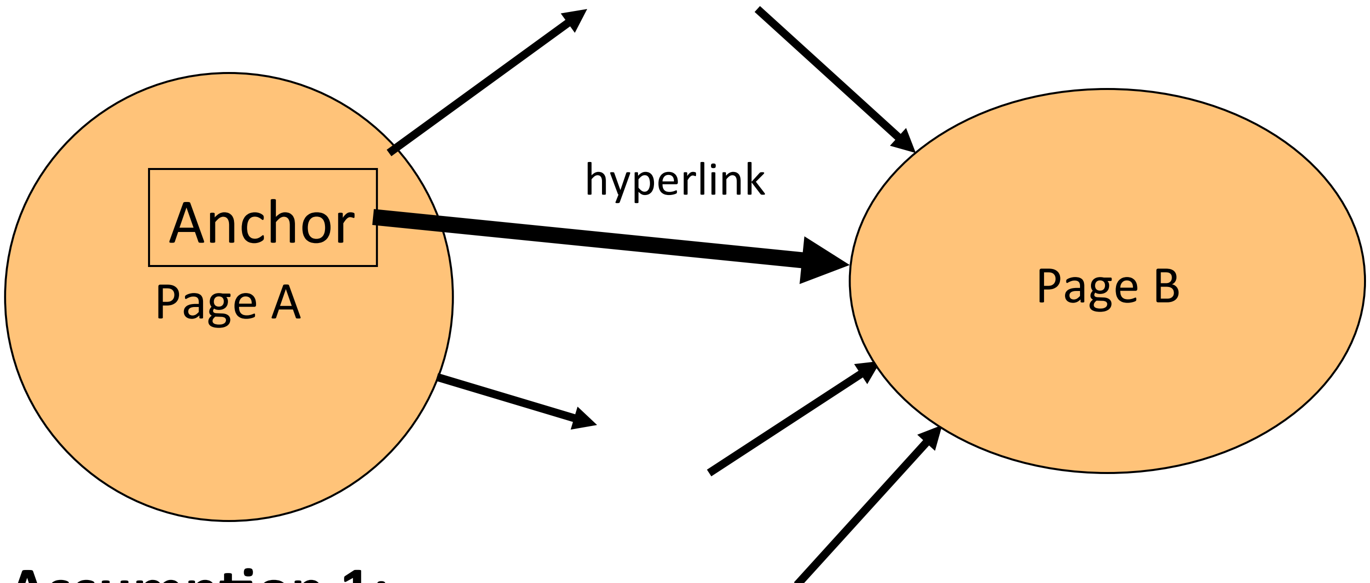
Networks and Link Analysis

Web Anchor Text

The Web as a Directed Graph

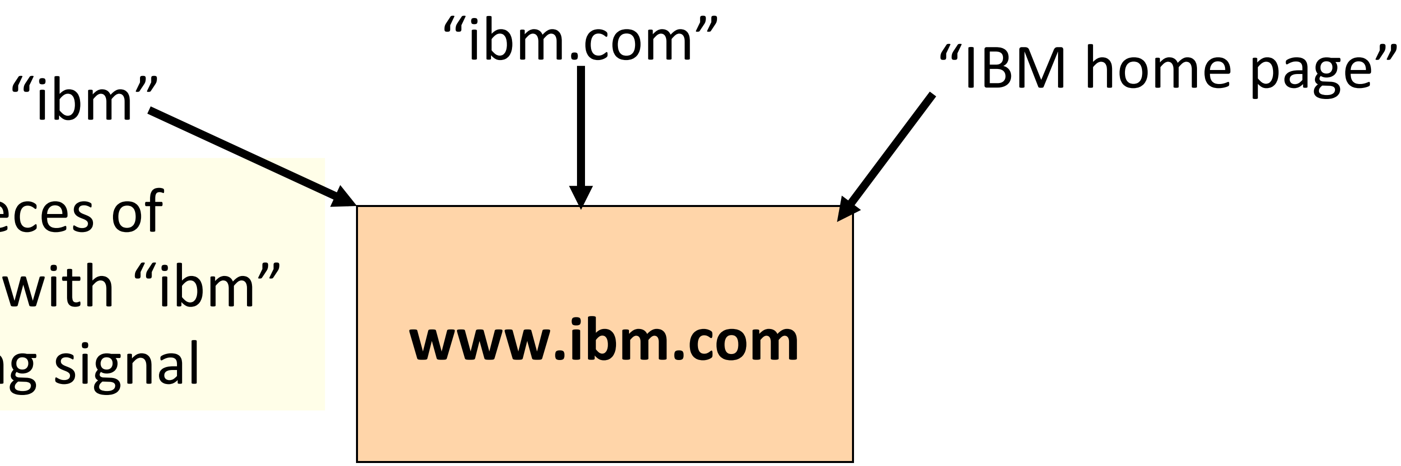


Assumption 1:
A hyperlink between pages denotes author perceived relevance (quality signal)

Assumption 2:
The anchor of the hyperlink describes the target page (textual context)

Anchor Text

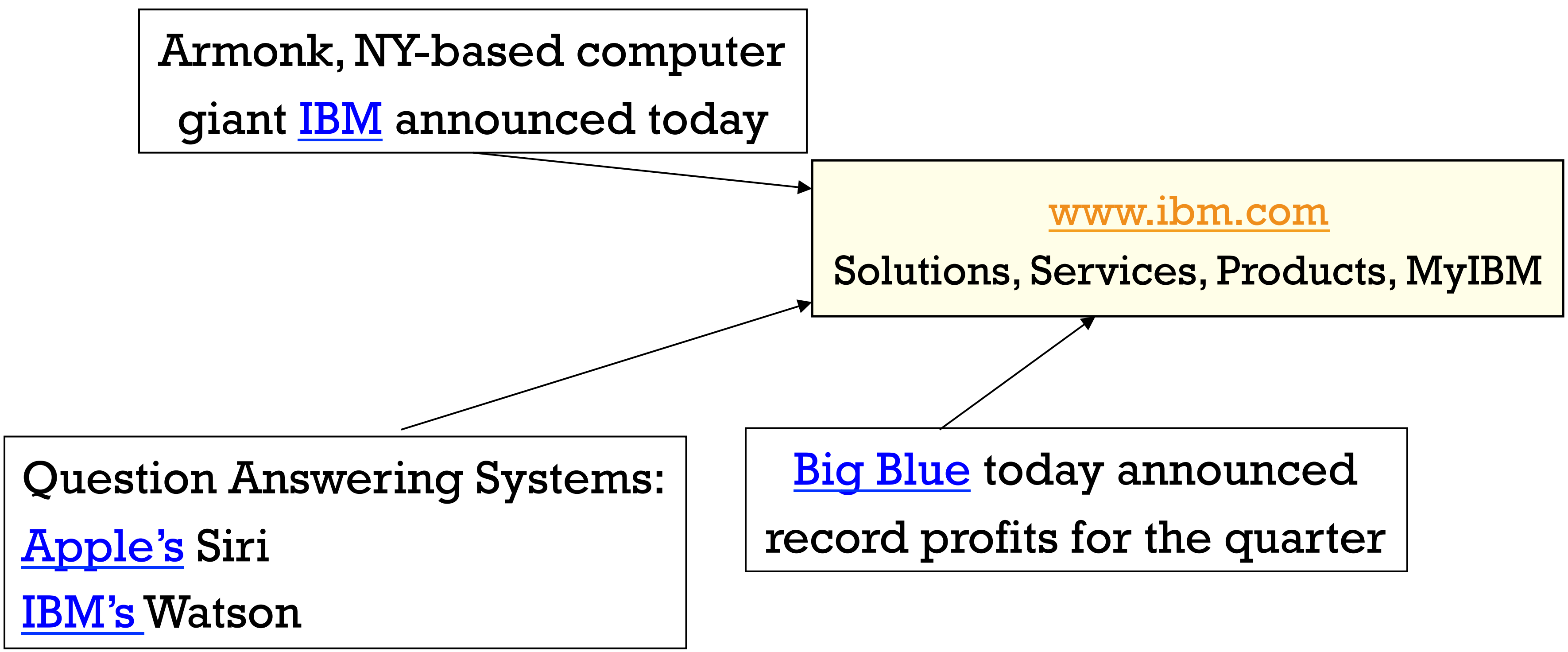
- For ***ibm*** how to distinguish between:
  - IBM's home page (mostly graphical)
  - IBM's copyright page (high term frequency for "ibm")
  - Rival's spam page (arbitrarily high term frequency)



A million pieces of anchor text with "ibm" send a strong signal

Indexing anchor text

When indexing a document *D*, include anchor text from links pointing to *D*.



Indexing anchor text

- Can sometimes have unexpected side effects –
  - Google bombing
- Can score anchor text with weight depending on the authority of the anchor page's website
  - E.g., if we were to assume that content from cnn.com or yahoo.com is authoritative, then trust the anchor text from them

Many NLP Applications of Anchor Text

- Finding synonyms
  - Federal Reserve: “Fed”, “U.S. Federal Reserve Board", “U.S. Federal Reserve System”, “Federal Reserve Bank”
- Finding translations of named entities
- Providing constituent boundaries for parsers

Networks and Link Analysis

Web Anchor Text

Networks and Link Analysis

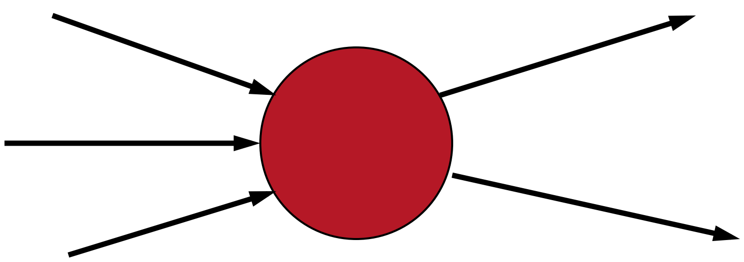
PageRank: Overview and Markov Chains

Combining link structure with text

- A good search result looks at more than just query-document text overlap
- One factor: page **popularity**.
  - Pages that are pointed to by lots of other pages are popular.
  - We can use link counts as a measure of static goodness,
  - Combine link counts with the text match score

Using link structure to measure page importance

- Simplest: use link counts as popularity measure
  - **Undirected popularity**:
    - Page score= **degree**: the number of in-links plus the number of out-links (3+2=5).
  - **Directed popularity**:
    - Page score = number of in-links (3).



Spamming simple popularity

- Simple popularity heuristics can be spammed to give your page a high score, whether it's:
  - the number of in-links plus the number of out-links
  - number of in-links

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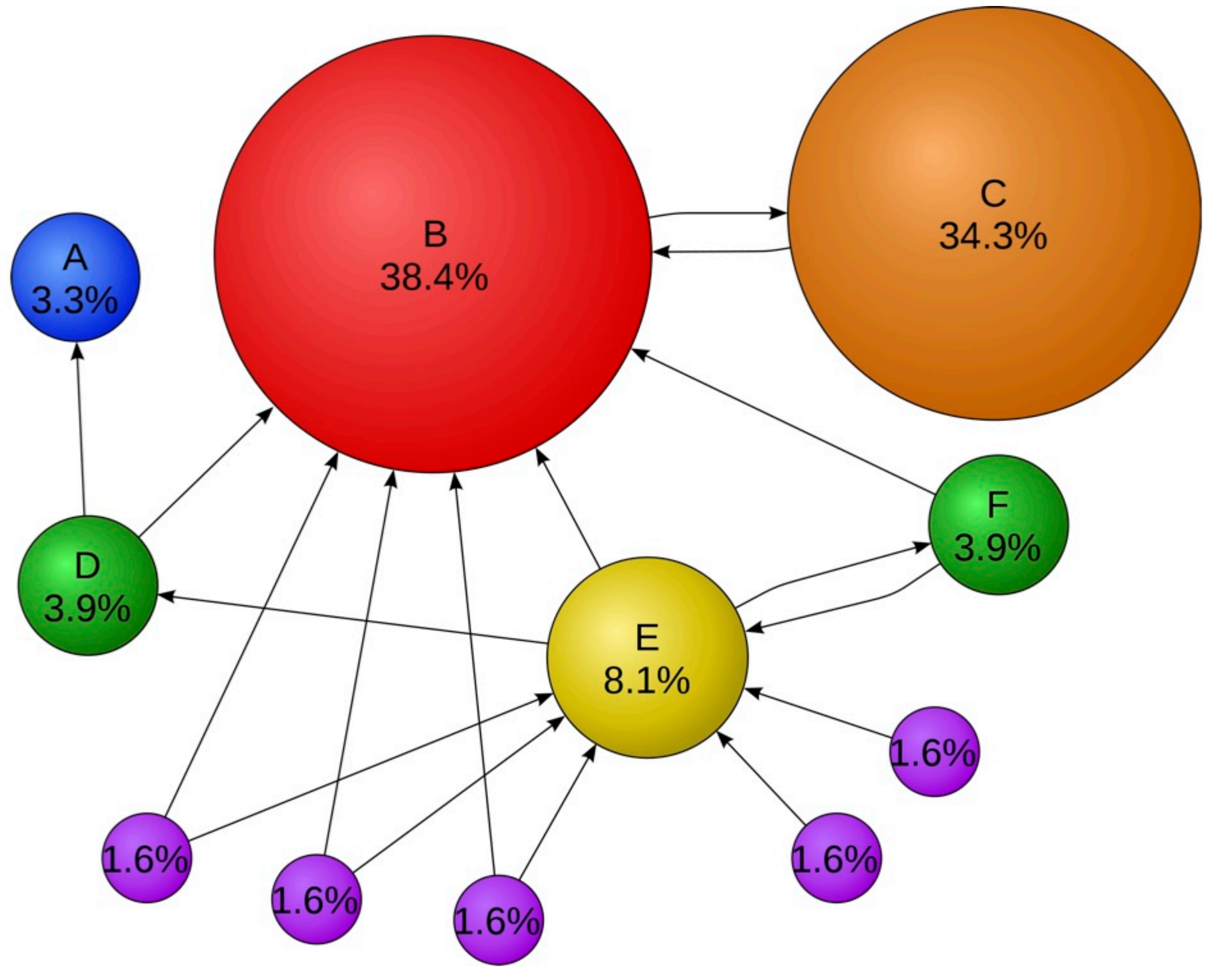
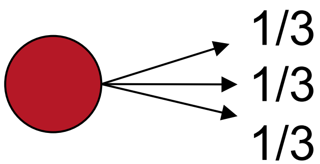
Intuition of PageRank



C has higher PageRank than E, even though E has more inlinks

PageRank scoring

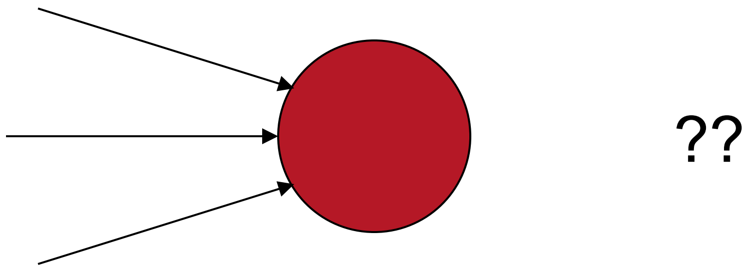
- Imagine a browser doing a random walk on web pages:
  - Start at a random page
  - At each step, go out of the current page along one of the links on that page, equiprobably



- "In the steady state" each page has a long-term visit rate - use this as the page's score.

Not quite enough

- The web is full of dead-ends.
  - Random walk can get stuck in dead-ends.
  - Makes no sense to talk about long-term visit rates.



Teleporting

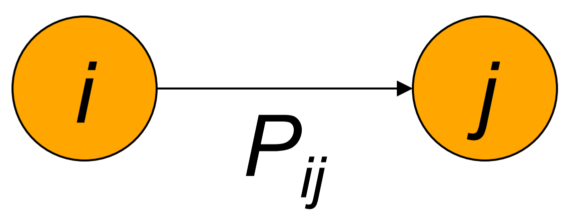
- At a dead end, jump to a random web page.
- At any non-dead end, with probability 10%, jump to a random web page.
  - With remaining probability (90%), go out on a random link.
  - 10% - a parameter.

Result of teleporting

- Now cannot get stuck locally.
- There is a long-term rate, the Pagerank, at which any page is visited

Markov chains

- A Markov chain:
  - *N* states,
  - *An N×N* transition probability matrix **P**.
- At each step, we are in exactly one of the states.
- For $1 \leq i,j \leq n$, the matrix entry P_{ij} tells us the probability of *j* being the next state, given we are currently in state *i*.

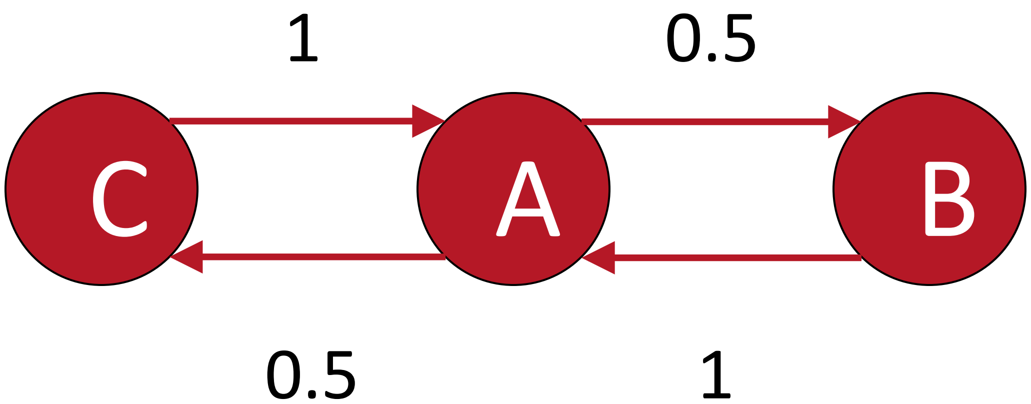


- For all i,

$$\sum_{j=1}^{n} P_{ij} = 1.$$

Dan Jurafsky

Markov chains



- Transition probability matrix P

$$P = \begin{pmatrix} 0 & 0.5 & 0.5 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Random Surfers and Markov chains

- Markov chains are abstractions of random walks.
- Each state
  - represents one web page
- Each transition probability
  - represents the probability of moving from one page to another
- We can derive the transition probability P from the adjacency matrix A of the web graph.

Teleporting, more formally

- If a node has no out-links, the random surfer teleports:
  - the transition probability to each node in the N-node graph is 1/N
- If a node has K>0 outgoing links:
  - with probability $0 < \alpha < 1$ the surfer teleports to a random node
    - probability is α/N
  - with probability $1 - \alpha$ the surfer takes a normal random walk
    - probability is $(1 - \alpha)/K$

Deriving transition probability matrix P from adjacency matrix A

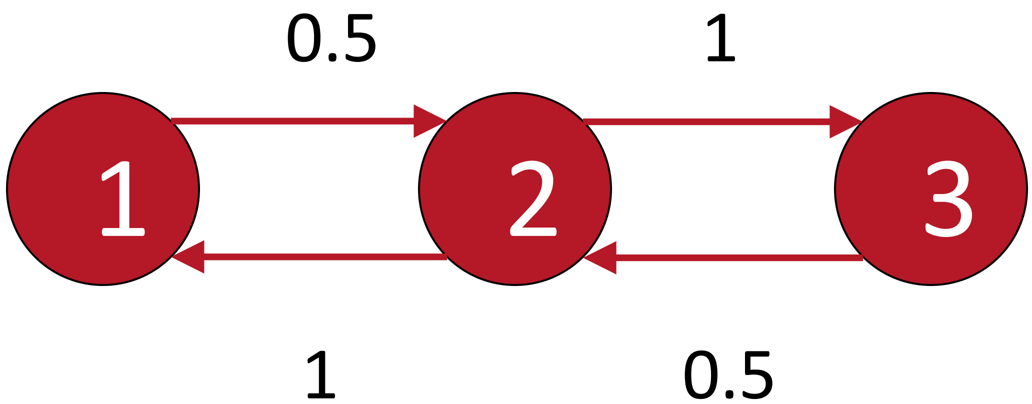
- A is the adjacency matrix of the web graph
  - A_{ij} is 1 if there is a hyperlink from page *i* to page *j*

- If a row of A has no 1's, then replace each element by 1/N.

For all other rows proceed as follows.

- Divide each 1 in A by the number of 1's in its row. Thus, if there is a row with three 1's, then each of them is replaced by 1/3
- Multiply the resulting matrix by $(1-\alpha)$
- Add α/N to every entry of the resulting matrix, to obtain P.

Computing P with teleportation



$$A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

P[1,*] = (1-α) (0 1 0) + α (1/N 1/N 1/N)

P[1,*] = 0.5 (0 1 0) + 0.5(1/3 1/3 1/3)

$$P_{\alpha=0} = \begin{pmatrix} 0 & 1 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 1 & 0 \end{pmatrix}$$

$$P_{\alpha=0.5} = \begin{pmatrix} 1/6 & 2/3 & 1/6 \\ 5/12 & 1/6 & 5/12 \\ 1/6 & 2/3 & 1/6 \end{pmatrix}$$

Networks and Link Analysis

PageRank: Overview and Markov Chains

Networks and Link Analysis

PageRank: Computation

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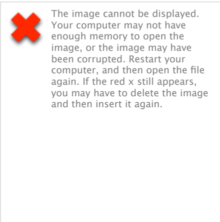
Computing PageRank: The probability of being in a state

- A probability (row) vector $\mathbf{x} = (x_1, \ldots x_n)$ tells us where the walk is at any point.
- E.g., $(\underset{1}{0}00\ldots\underset{i}{1}\ldots00\underset{n}{0})$ means we're in state *i*.

More generally, the vector $\mathbf{x} = (x_1, \ldots x_n)$ means the walk is in state *i* with probability x_i.

$$\sum_{i=1}^{n} x_i = 1$$

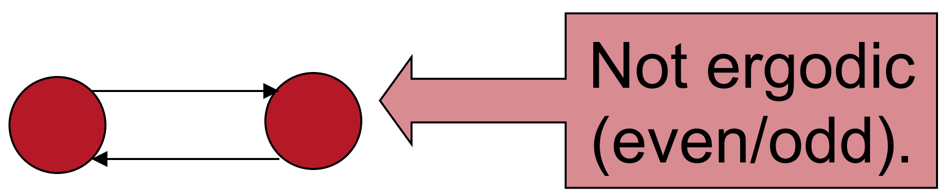


Computing PageRank: Change in probability vector

- If the probability vector is **x** $= (x_1, \ldots x_n)$ at this step, what is it at the next step?
- Recall that row *i* of transition matrix P tells us where we go next from state *i*.
- So from **x**, our next state is distributed as **xP**.

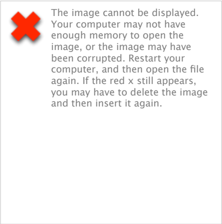
Ergodic Markov chains

- A Markov chain is ergodic if
  - you have a path from any state to any other
  - For any start state, after a finite transient time T_0, the probability of being in any state at a fixed time $T>T_0$ is nonzero.



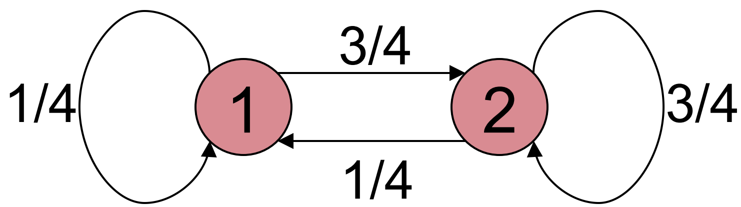
Ergodic Markov chains

- For any ergodic Markov chain, there is a unique long-term visit rate for each state.
  - A steady-state probability distribution $\boldsymbol{\pi} = (\boldsymbol{\pi}_1, \ldots \boldsymbol{\pi}_n)$.
  - Over a long time-period, we visit each state in proportion to this rate.
  - Thus $\boldsymbol{\pi}_i$ is the PageRank of state i.
- It doesn't matter where we start.

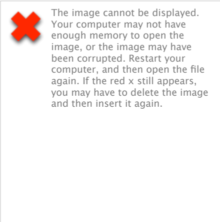


Steady state example

- The steady state looks like a vector of probabilities $\boldsymbol{\pi} = (\boldsymbol{\pi}_1, \ldots \boldsymbol{\pi}_n)$:
  - $\boldsymbol{\pi}_i$ is the probability that we are in state *i*.

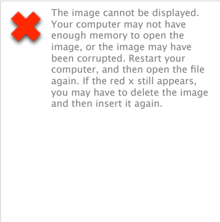


For this example, $\boldsymbol{\pi}_1 = 1/4$ and $\boldsymbol{\pi}_2 = 3/4$.



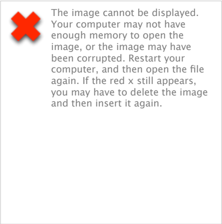
How do we compute this vector?

- Let $\boldsymbol{\pi} = (\boldsymbol{\pi}_1, \ldots \boldsymbol{\pi}_n)$ denote the row vector of steady-state probabilities
- If our current position is described by **π**, then the next step is distributed as **πP**.
- But **π** is the steady state, so **π=πP**.
- Solving this matrix equation gives us **π**.
  - So **π** is the (left) eigenvector for **P**.
  - (Corresponds to the "principal" eigenvector of **P** with the largest eigenvalue.)
  - Transition probability matrices always have largest eigenvalue 1.



The power iteration method of computing π

- Recall, regardless of where we start, we eventually reach the steady state $\boldsymbol{\pi}$.
- Start with any distribution (say **x**=(*10…0*)).
- After one step, we're at **xP**;
- after two steps at $\mathbf{xP}^2$, then $\mathbf{xP}^3$ and so on.
- "Eventually" means for "large" k, $\mathbf{xP}^k = \boldsymbol{\pi}$.
- Algorithm: multiply **x** by increasing powers of **P** until the product looks stable.



Example of power iteration

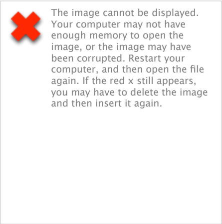
$$P_{\alpha=0.5} = \begin{pmatrix} 1/6 & 2/3 & 1/6 \\ 5/12 & 1/6 & 5/12 \\ 1/6 & 2/3 & 1/6 \end{pmatrix}$$

- Let's say surfer starts in state 1:

$$\vec{x}_0 = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$$

$$\vec{x}_1 = \vec{x}_0 P = \begin{pmatrix} 1/6 & 2/3 & 1/6 \end{pmatrix}$$

$$\vec{x}_2 = \vec{x}_1 P = \begin{pmatrix} 1/6 & 2/3 & 1/6 \end{pmatrix} \begin{pmatrix} 1/6 & 2/3 & 1/6 \\ 5/12 & 1/6 & 5/12 \\ 1/6 & 2/3 & 1/6 \end{pmatrix} = \begin{pmatrix} 1/3 & 1/3 & 1/3 \end{pmatrix}$$



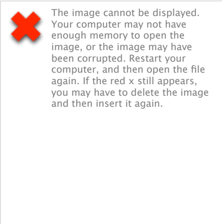
Power iteration example (continued)

$\vec{x}_0$	1	0	0
$\vec{x}_1$	1/6	2/3	1/6
$\vec{x}_2$	1/3	1/3	1/3
$\vec{x}_3$	1/4	1/2	1/4
$\vec{x}_4$	7/24	5/12	7/24
…	…	…	…
$\vec{x} = \vec{\pi}$	5/18	4/9	5/18

Node 1 PageRank

Node 2 PageRank

Node 3 PageRank



PageRank summary

- Preprocessing:
  - Given graph of links, build matrix **P**.
  - From it compute the PageRank vector $\boldsymbol{\pi}$.
  - The PageRank of page i, $\boldsymbol{\pi}_i$ is between 0 and 1
- Query processing:
  - Retrieve pages meeting query.
  - Rank them by their PageRank.
  - Order is query-*independent*.

Networks and Link Analysis

PageRank: Computation