ECE2810J

Data Structures and Algorithms

Non-comparison Sort

Learning Objective:

- Understand three non-comparison sorts, counting sort, bucket sort, and radix sort

Outline

- Non-comparison Sort
  - Counting Sort
  - Bucket Sort
  - Radix Sort

Counting Sort

A Simple Version

- Sort an array A of **integers** in the range $[0, k]$, where k is known.

1. Allocate an array **`count[k+1]`**.
2. Scan array A. For i**`=1`** to **`N`**, increment **`count[A[i]]`**.
3. Scan array **`count`**. For i**`=0`** to **`k`**, print **`i`** for **`count[i]`** times.

- Time complexity: $O(N + k)$.

- The algorithm can be converted to sort integers in some other known range $[a, b]$.
  - Minus each number by a, converting the range to $[0, b - a]$.

Counting Sort

A General Version

- In the previous version, we print **`i`** for **`count[i]`** times.
  - Simple but only works when sorting integer keys alone.
  - How to sort items when there is "additional" information with each key? Furthermore, how to guarantee the stability?
- A general version:

1. Allocate an array **`C[k+1]`**.
2. Scan array A. For i**`=1`** to **`N`**, increment **`C[A[i]]`**.
3. For **`i=1`** to **`k`**, **`C[i]=C[i-1]+C[i]`**
   - **`C[i]`** now contains number of items less than or equal to **`i`**.
4. For **`i=N`** downto **`1`**, put **`A[i]`** in new position **`C[A[i]]`** and decrement **`C[A[i]]`**.

Counting Sort

Example

1. Allocate an array **`C[k+1]`**.

2. Scan array A. For i**`=1`** to **`N`**, increment **`C[A[i]]`**.

3. For **`i=1`** to **`k`**, **`C[i]=C[i-1]+C[i]`**

4. For **`i=N`** downto **`1`**, put **`A[i]`** in new position **`C[A[i]]`** and decrement **`C[A[i]]`**.

k=5

	1	2	3	4	5	6	7	8
A	2	5	3	0	2	3	0	3

	0	1	2	3	4	5
C	2	0	2	3	0	1

Counting Sort

Example

k=5

1. Allocate an array **`C[k+1]`**.
2. Scan array A. For **`i=1`** to **`N`**, increment **`C[A[i]]`**.
3. For **`i=1`** to **`k`**, **`C[i]=C[i-1]+C[i]`**
4. For **`i=N`** downto **`1`**, put **`A[i]`** in new position **`C[A[i]]`** and decrement **`C[A[i]]`**.

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	0	1	2	3	4	5
C	2	0	2	3	0	1

	0	1	2	3	4	5
C	2	2	4	7	7	8

Counting Sort

Example

1. Allocate an array `C[k+1]`.

2. Scan array A. For `i=1` to `N`, increment `C[A[i]]`.

3. For `i=1` to `k`, `C[i]=C[i-1]+C[i]`

4. For `i=N` downto `1`, put `A[i]` in new position `C[A[i]]` and decrement `C[A[i]]`.

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	0	1	2	3	4	5
C	2	2	4	7	7	8

1	2	3	4	5	6	7	8
						3	

	0	1	2	3	4	5
C	2	2	4	6	7	8

Why putting 3 at location 7 is correct?

Counting Sort
Example

1. Allocate an array `C[k+1]`.
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C	2	2	4	6	7	8

1	2	3	4	5	6	7	8
	0					3	

	0	1	2	3	4	5
C	1	2	4	6	7	8

Counting Sort
Example

1. Allocate an array `C[k+1]`.
2. Scan array A. For `i=1` to `N`, increment `C[A[i]]`.
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C	1	2	4	6	7	8

1	2	3	4	5	6	7	8
	0				3	3	

	0	1	2	3	4	5
C	1	2	4	5	7	8

Counting Sort

Example

1. Allocate an array `C[k+1]`.
2. Scan array A. For `i=1` to `N`, increment `C[A[i]]`.
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	1	2	3	4	5	6	7	8
A	2	5	3	0	2	3	0	3

	0	1	2	3	4	5
C	1	2	4	5	7	8

1	2	3	4	5	6	7	8
	0		2		3	3	

	0	1	2	3	4	5
C	1	2	3	5	7	8

Counting Sort

Example

1. Allocate an array `C[k+1]`.
2. Scan array A. For `i=1` to `N`, increment `C[A[i]]`.
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k=5

	1	2	3	4	5	6	7	8
A	2	5	3	0	2	3	0	3

	0	1	2	3	4	5
C	1	2	3	5	7	8

1	2	3	4	5	6	7	8
0	0		2		3	3	

	0	1	2	3	4	5
C	0	2	3	5	7	8

Counting Sort
Example

1. Allocate an array `C[k+1]`.
2. Scan array A. For `i=1` to `N`, increment `C[A[i]]`.
3. For `i=1` to `k`, `C[i]=C[i-1]+C[i]`
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1	2	3	4	5	6	7	8
0	0		2	3	3	3	

	0	1	2	3	4	5
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Counting Sort

Example

1. Allocate an array `C[k+1]`.

2. Scan array A. For `i=1` to `N`, increment `C[A[i]]`.

3. For `i=1` to `k`, `C[i]=C[i-1]+C[i]`

4. For `i=N` downto `1`, put `A[i]` in new position `C[A[i]]` and decrement `C[A[i]]`.

k=5

	1	2	3	4	5	6	7	8
A	2	5	3	0	2	3	0	3

	0	1	2	3	4	5
C	0	2	3	4	7	8

1	2	3	4	5	6	7	8
0	0		2	3	3	3	5

	0	1	2	3	4	5
C	0	2	3	4	7	7

Counting Sort

Example

1. Allocate an array **`C[k+1]`**.
2. Scan array A. For **`i=1`** to **`N`**, increment **`C[A[i]]`**.
3. For **`i=1`** to **`k`**, **`C[i]=C[i-1]+C[i]`**
4. For **`i=N`** downto **`1`**, put **`A[i]`** in new position **`C[A[i]]`** and decrement **`C[A[i]]`**.

k=5

	1	2	3	4	5	6	7	8
A	2	5	3	0	2	3	0	3

	0	1	2	3	4	5
C	0	2	3	4	7	7

1	2	3	4	5	6	7	8
0	0	2	2	3	3	3	5

	0	1	2	3	4	5
C	0	2	2	4	7	7

Done!

Is counting sort stable?

Yes!

Outline

- Non-comparison Sort
  - Counting Sort
  - Bucket Sort
  - Radix Sort

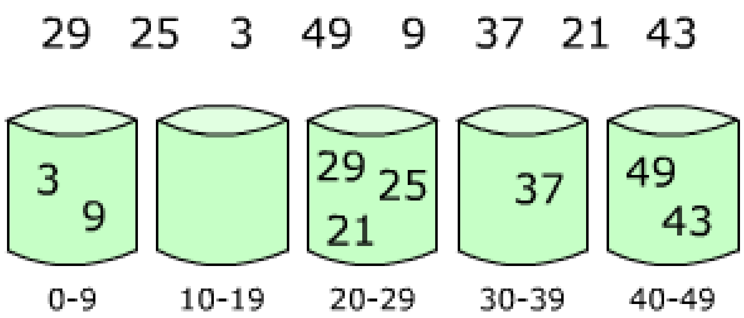
Bucket Sort

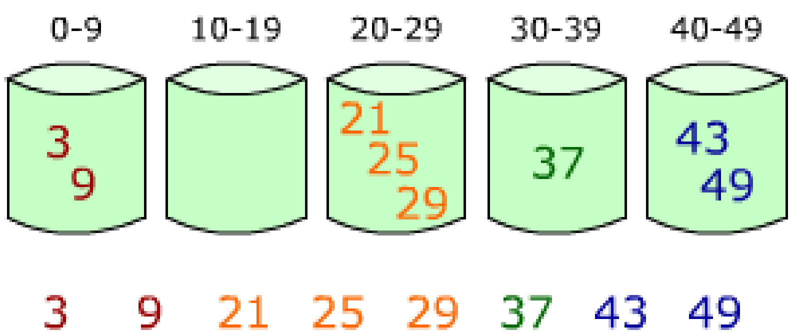
- Instead of simple integer, each key can be a complicated record, such as a real value.
- Then instead of incrementing the count of each bucket, **distribute** the records **by their keys** into appropriate buckets.
- Algorithm:

1. Set up an array of initially empty "buckets".
2. Scatter: Go over the original array, putting each object in its bucket.
3. Sort each non-empty bucket <u>by a comparison sort</u>.
4. Gather: Visit the buckets in order and put all elements back into the original array.

Bucket Sort

- Example





- Time complexity
  - Suppose we are sorting cN items and we divide the entire range into N buckets.
  - Assume that the items are uniformly distributed in the entire range.
  - The average case time complexity is $O(N)$.

Outline

- Non-comparison Sort
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  - Radix Sort

Radix Sort

- **Radix sort** sorts integers by looking at one digit at a time.
- Procedure: Given an array of integers, from the least significant bit (LSB) to the most significant bit (MSB), repeatedly do **stable** bucket sort according to the current bit.

- For sorting base-b numbers, bucket sort needs b buckets.
  - For example, for sorting decimal numbers, bucket sort needs 10 buckets.

Radix Sort
Example

- Sort 815, 906, 127, 913, 098, 632, 278.
- Bucket sort 815, 906, 127, 913, 098, 632, 278 according to the least significant bit:

0	1	2	3	4	5	6	7	8	9
		632	913		815	906	127	098 278	

- Bucket sort 632, 913, 815, 906, 127, 098, 278 according to the second bit.

Radix Sort

Example

- Bucket sort 632, 913, 815, 906, 127, 098, 278 according to the second bit.

0	1	2	3	4	5	6	7	8	9
906	913 815	127	632				278		098

- Bucket sort 906, 913, 815, 127, 632, 278, 098 according to the most significant bit.

Radix Sort

Example

- Bucket sort 906, 913, 815, 127, 632, 278, 098 according to the most significant bit.

0	1	2	3	4	5	6	7	8	9
098	127	278				632		815	906 913

- The final sorted order is: 098, 127, 278, 632, 815, 906, 913.

Radix Sort: Correctness

- <u>Claim</u>: after bucket sorting the i-th LSB, the numbers are sorted according to their last i digits

- Proof by mathematical induction
- Base case is obviously true
- Inductive step
  - Assume that according to the last i digits, order is $a_1 < \cdots < a_n$
  - For two adjacent numbers a_k and a_{k+1} if they are not in the same bucket, they are sorted according to their last i digits
  - If they are in the same bucket, then $a_k < a_{k+1}$ for the last $(i - 1)$ bits. They are also sorted due to stability of bucket sort

Radix Sort

Time Complexity

- Let k be the maximum number of digits in the keys and N be the number of keys.
- We need to repeat bucket sort k times.
  - Time complexity for the bucket sort is $O(N)$.

- The total time complexity is $O(kN)$.

Radix Sort

- Radix sort can be applied to sort keys that are built on **positional notation**.
  - **Positional notation**: all positions uses the same set of symbols, but different positions have different weight.
  - Decimal representation and binary representation are examples of positional notation.
  - Strings can also be viewed as a type of positional notation. Thus, radix sort can be used to sort strings.
- We can also apply radix sort to sort records that contain multiple keys.
  - For example, sort records (year, month, day).