In [1]:

```
import os, sys, copy
import math
import igraph as ig
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
from matplotlib import cm
from statistics import mode

sns.set_theme()
%matplotlib inline

# To export to PDF, run:
# jupyter nbconvert --to webpdf --allow-chromium-download assignment_02.ipynb
```

In [2]:

```
datadir = '../Datasets/'

## define edges color
cls_edges = 'gainsboro'

## we will consider 3 types of nodes with the following colors and sizes:
cls = ['silver','dimgray','black']
sz = [6,9,12]
```

In [3]:

```
cm_names = ['degree', 'pagerank', 'authority', 'hub', 'between', 'closeness']

## directed degree centrality
def degree_centrality(g, weights=None):
    n = g.vcount()
    if g.is_directed():
        dc = [sum(x)/(2*(n-1)) for x in zip(g.strength(mode='in', weights=weights),\
              g.strength(mode='out', weights=weights))]
    else:
        dc = [x/(n-1) for x in g.strength(weights=weights)]
    return dc

def generate_centrality_measure(G, node_type_name, with_weight=True):
    if with_weight:
        ## compute normalized weights
        mw = np.max(G.es['weight'])
        G.es()['normalized_weight'] = [w/mw for w in G.es()['weight']]
        normalized_weight = 'normalized_weight'
        weights = 'weight'
    else:
        normalized_weight = None
        weights = None

    ## compute several centrality measures for the CA subgraph G
    C = pd.DataFrame({node_type_name: G.vs()['name'],
                      'd': G.degree(),
                      'degree': degree_centrality(G,weights=normalized_weight),
                      'pagerank': G.pagerank(weights=weights),
                      'authority': G.authority_score(weights=weights),
                      'hub': G.hub_score(weights=weights),
                      'between': G.betweenness(),
                      'closeness': G.closeness()})

    ## normalize betweenness
```

```
    n = G.vcount()
    C['between'] = [2*x/((n-1)*(n-2)) for x in C['between']]

    return C
```

Problem 1

In [4]:
```
## read edges and build weighted directed graph
D = pd.read_csv(os.path.join(datadir, 'Airports/connections.csv'))
g = ig.Graph.TupleList([tuple(x) for x in D.values], directed=True, edge_attrs=['weight'

## read vertex attributes and add to graph
A = pd.read_csv(datadir+'Airports/airports_loc.csv')
lookup = {k:v for v,k in enumerate(A['airport'])}
l = [lookup[x] for x in g.vs()['name']]
g.vs()['state'] = [A['state'][i] for i in l]
g.vs()['city'] = [A['city'][i] for i in l]
print('State:', A.state.unique())
print(g.vcount(),'nodes and',g.ecount(),'directed edges')
```

```
State: ['PA' 'TX' 'NM' 'SD' 'GA' 'CA' 'NJ' 'AK' 'LA' 'IN' 'NY' 'IA' 'WA' 'AL'
 'SC' 'FL' 'MI' 'OR' 'WI' 'ME' 'NC' 'AZ' 'CT' 'NE' 'MT' 'ND' 'MN' 'OH'
 'WV' 'IL' 'TN' 'MA' 'VT' 'KY' 'MD' 'UT' 'MO' 'VA' 'CO' 'WY' 'DC' 'KS'
 'DE' 'NV' 'AR' 'MS' 'HI' 'ID' 'OK' 'NH' 'RI']
464 nodes and 12000 directed edges
```

In [5]:
```
display(D.head())
display(A.head())
```

	orig_airport	dest_airport	total_passengers
0	SFO	LAX	1442105
1	LAX	SFO	1438639
2	MCO	ATL	1436625
3	ATL	MCO	1424069
4	LAX	JFK	1277731

	airport	lon	lat	state	city
0	ABE	-75.440804	40.652100	PA	Allentown
1	ABI	-99.681900	32.411301	TX	Abilene
2	ABQ	-106.609001	35.040199	NM	Albuquerque
3	ABR	-98.421799	45.449100	SD	Aberdeen
4	ABY	-84.194504	31.535500	GA	Albany

In [6]:
```
## Build smaller subgraph 'G' for New York
G = g.subgraph([v for v in g.vs() if v['state'] == 'NY'])

## drop isolated vertices (i.e. without in-state connections)
G = G.subgraph([v for v in G.vs() if v.degree()>0])

## remove loops if any
G = G.simplify(multiple=False)
print(G.vcount(),'nodes and',G.ecount(),'directed edges')
```

13 nodes and 50 directed edges

In [7]:
```python
## directed degree centrality
def degree_centrality(g, weights=None):
    n = g.vcount()
    if g.is_directed():
        dc = [sum(x)/(2*(n-1)) for x in zip(g.strength(mode='in',weights=weights),\
              g.strength(mode='out',weights=weights))]
    else:
        dc = [x/(n-1) for x in g.strength(weights=weights)]
    return dc

def generate_centrality_measure(G, node_type_name, with_weight=True):
    if with_weight:
        ## compute normalized weights
        mw = np.max(G.es['weight'])
        G.es()['normalized_weight'] = [w/mw for w in G.es()['weight']]
        normalized_weight = 'normalized_weight'
        weights = 'weight'
    else:
        normalized_weight = None
        weights = None

    ## compute several centrality measures for the CA subgraph G
    C = pd.DataFrame({node_type_name: G.vs()['name'],
                      'd': G.degree(),
                      'degree': degree_centrality(G,weights=normalized_weight),
                      'pagerank': G.pagerank(weights=weights),
                      'authority': G.authority_score(weights=weights),
                      'hub': G.hub_score(weights=weights),
                      'between': G.betweenness(),
                      'closeness': G.closeness()})

    ## normalize betweenness
    n = G.vcount()
    C['between'] = [2*x/((n-1)*(n-2)) for x in C['between']]

    return C
```

In [8]:
```python
C = generate_centrality_measure(G, node_type_name='airport')

## sort w.r.t. degree centrality, look at top airports
Cs = C.sort_values(by='degree', ascending=False)
Cs.head()
```

Out[8]:

	airport	d	degree	pagerank	authority	hub	between	closeness
0	JFK	14	0.173125	0.342720	1.000000	0.373760	0.647727	0.705882
2	BUF	12	0.101629	0.195702	0.261299	1.000000	0.167929	0.705882
4	ROC	9	0.054908	0.109433	0.141841	0.546989	0.000000	0.631579
5	SYR	14	0.044959	0.097118	0.113975	0.459400	0.243687	0.750000
1	LGA	14	0.040011	0.108312	0.213367	0.078989	0.286616	0.750000

Problem 2

In [9]:
```python
## fast Chung-Lu: generate m edges w.r.t. distribution d
```

```python
def fastCL(d, m):
    n = len(d)
    s = np.sum(d)
    p = [i/s for i in d]
    target = m
    tples = []
    ## generate edges (tuples), drop collisions, until m edges are obtained.
    while len(tples) < target:
        s = target - len(tples)
        e0 = np.random.choice(n, size=s, replace=True, p=p)
        e1 = np.random.choice(n, size=s, replace=True, p=p)
        tples.extend([(min(e0[i],e1[i]),max(e0[i],e1[i])) for i in range(len(e0)) if e0[
        tples = list(set(tples)) ## drop collisions
    return tples
```

In [10]:

```python
## power law graph
gamma = 2.5
n = 10000

## min and max degrees
delta = 1
Delta = np.sqrt(n)

## generate degrees
W = []
for i in np.arange(1,n+1):
    W.append(delta * (n/(i-1+n/(Delta/delta)**(gamma-1)))**(1/(gamma-1)))

# deg = [int(np.round(w)) for w in W] ## to enforce integer weights, not an obligation
deg = W

## generate graph with Chung-Lu model
m = int(np.mean(deg)*n/2)
tpl = fastCL(deg,m)
g1 = ig.Graph.TupleList(tpl)

## number of isolated nodes (no edges)
iso = n - g1.vcount()
print('isolates:',iso)
```

```
isolates: 1872
```

In [11]:

```python
C = generate_centrality_measure(g1, node_type_name='node_id', with_weight=False)
C.head()
```

Out[11]:

	node_id	d	degree	pagerank	authority	hub	between	closeness
0	94	14	0.001723	0.000424	0.014150	0.014150	0.002622	0.207897
1	4219	1	0.000123	0.000044	0.001080	0.001080	0.000000	0.172119
2	123	21	0.002584	0.000605	0.105427	0.105427	0.007892	0.238184
3	3904	1	0.000123	0.000043	0.008045	0.008045	0.000000	0.192371
4	3951	3	0.000369	0.000138	0.000448	0.000448	0.000259	0.161217

In [12]:

```python
degrees = g1.degree()
min_degree = min(degrees)
max_degree = max(degrees)
ls_mean_centrality_measure = []
print(f'min_degree={min_degree} - max_degree={max_degree}')
for k in sorted(set(degrees)):
```

```
#     G1 = g1.subgraph([v for v in g1.vs() if v.degree() == k])
#     if len(G1.vs()) <= 0: continue
#     G1_vs_ids = [v['name'] for v in G1.vs()]
#     C1 = C[C['node_id'].isin(G1_vs_ids)]
    C1 = C[C['d']==k]
    mean_df = C1[cm_names].mean(axis=0)
    ls_mean_centrality_measure.append([k] + list(mean_df.values))
```

```
min_degree=1 - max_degree=103
```

In [13]:

```
df2 = pd.DataFrame(np.array(ls_mean_centrality_measure),
                   columns=['k'] + cm_names).fillna(0.0)
display(df2.head())
```

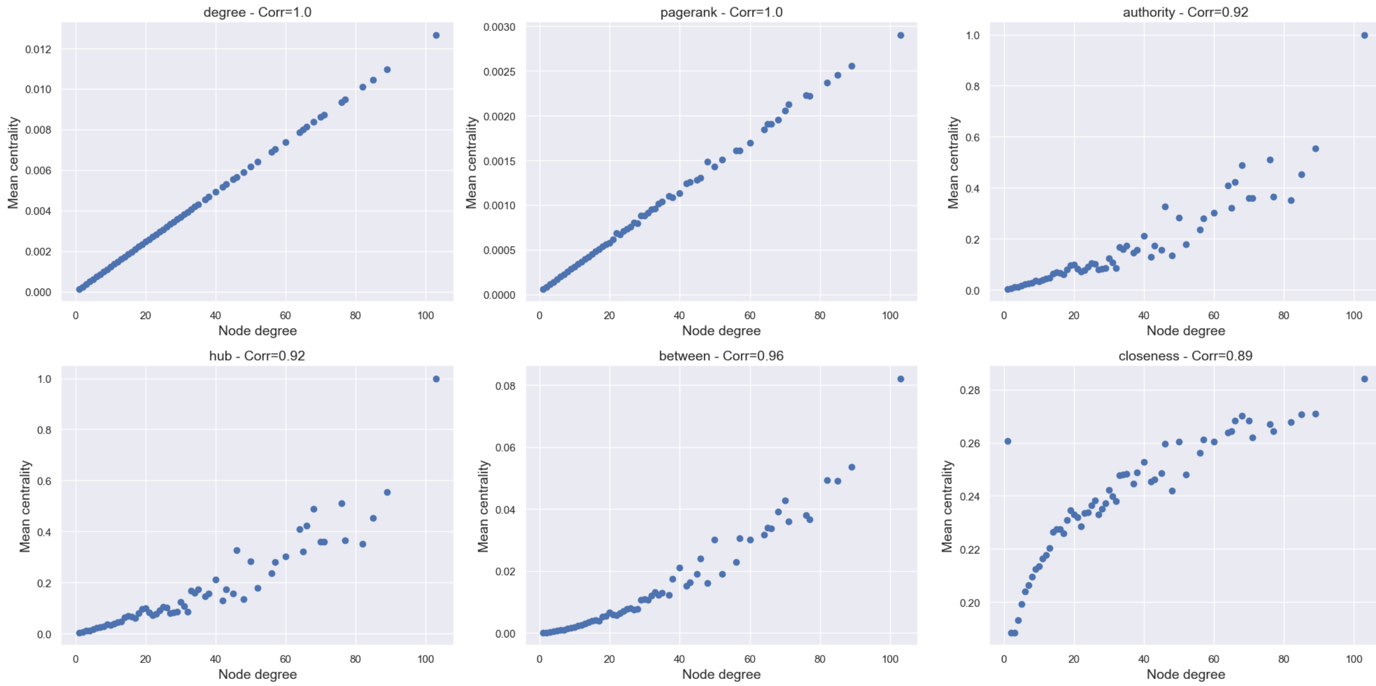
	k	degree	pagerank	authority	hub	between	closeness
0	1.0	0.000123	0.000059	0.003427	0.003427	0.000000	0.260585
1	2.0	0.000246	0.000085	0.006757	0.006757	0.000122	0.188432
2	3.0	0.000369	0.000112	0.011180	0.011180	0.000272	0.188496
3	4.0	0.000492	0.000141	0.012963	0.012963	0.000416	0.193171
4	5.0	0.000615	0.000168	0.018655	0.018655	0.000614	0.199173

In [14]:

```
ncols = 3
nrows = math.ceil(len(cm_names)/ncols)
fig, axes = plt.subplots(figsize=[20, 5*nrows], nrows=nrows, ncols=ncols)
xticks = range(min_degree, max_degree+1)

for i,var in enumerate(cm_names):
    corr = np.round(df2['k'].corr(df2[var]), 2)
    ax = axes[i//ncols, i%ncols]
    ax.scatter(x=df2['k'], y=df2[var])
    ax.grid(True)
    ax.set_title(f'{var} - Corr={corr}', size=14)
    ax.set_xlabel('Node degree', size=14)
    ax.set_ylabel('Mean centrality', size=14)

# fig.suptitle(, size=14)
plt.tight_layout()
plt.show()
```



Problem 3

In [15]:
```
## read edges and build weighted directed graph
D = pd.read_csv(os.path.join(datadir, 'Airports/connections.csv'))
g = ig.Graph.TupleList([tuple(x) for x in D.values], directed=True, edge_attrs=['weight'

## read vertex attributes and add to graph
A = pd.read_csv(datadir+'Airports/airports_loc.csv')
lookup = {k:v for v,k in enumerate(A['airport'])}
l = [lookup[x] for x in g.vs()['name']]
A.state.unique()
```

Out[15]:
```
array(['PA', 'TX', 'NM', 'SD', 'GA', 'CA', 'NJ', 'AK', 'LA', 'IN', 'NY',
       'IA', 'WA', 'AL', 'SC', 'FL', 'MI', 'OR', 'WI', 'ME', 'NC', 'AZ',
       'CT', 'NE', 'MT', 'ND', 'MN', 'OH', 'WV', 'IL', 'TN', 'MA', 'VT',
       'KY', 'MD', 'UT', 'MO', 'VA', 'CO', 'WY', 'DC', 'KS', 'DE', 'NV',
       'AR', 'MS', 'HI', 'ID', 'OK', 'NH', 'RI'], dtype=object)
```

In [16]:
```
# get big weakly connected component
G = g.connected_components(mode='weak').giant()
```

In [17]:
```
# get centrality with weights
C = generate_centrality_measure(G, node_type_name='airport')
for cm_name in cm_names:
    print(f"\n\n=========== centrality_measure = {cm_name} ===========")
    for ascending in [True, False]:
        print(f"----------- remove_smallest = {ascending} ----------")
        G_i = copy.deepcopy(G)
        C = C.sort_values(by=cm_name, ascending=ascending)
        ls_airport_i = C['airport'].values.tolist()
        removed_airports = []
        while G_i.is_connected(mode='weak'):
            if (len(G_i.vs()) == 0) or (len(G_i.es()) == 0):
                break
            removed_one = ls_airport_i.pop()
            removed_airports.append(removed_one)
            vs = G_i.vs()
```

```
            G_i.delete_vertices([removed_one])
        print(f"remove {len(removed_airports)} airports")
```

```
=========== centrality_measure = degree ===========
----------- remove_smallest = True ----------
remove 2 airports
----------- remove_smallest = False ----------
remove 179 airports


=========== centrality_measure = pagerank ===========
----------- remove_smallest = True ----------
remove 2 airports
----------- remove_smallest = False ----------
remove 36 airports


=========== centrality_measure = authority ===========
----------- remove_smallest = True ----------
remove 2 airports
----------- remove_smallest = False ----------
remove 10 airports


=========== centrality_measure = hub ===========
----------- remove_smallest = True ----------
remove 2 airports
----------- remove_smallest = False ----------
remove 462 airports


=========== centrality_measure = between ===========
----------- remove_smallest = True ----------
remove 4 airports
----------- remove_smallest = False ----------
remove 462 airports


=========== centrality_measure = closeness ===========
----------- remove_smallest = True ----------
remove 1 airports
----------- remove_smallest = False ----------
remove 462 airports
```

In [18]:

```
# get centrality without weights
C = generate_centrality_measure(G, node_type_name='airport', with_weight=False)
for cm_name in cm_names:
    print(f"\n\n=========== centrality_measure = {cm_name} ===========")
    for ascending in [True, False]:
        print(f"----------- remove_smallest = {ascending} ----------")
        G_i = copy.deepcopy(G)
        C = C.sort_values(by=cm_name, ascending=ascending)
        ls_airport_i = C['airport'].values.tolist()
        removed_airports = []
        while G_i.is_connected(mode='weak'):
            if (len(G_i.vs()) == 0) or (len(G_i.es()) == 0):
                break
            removed_one = ls_airport_i.pop()
            removed_airports.append(removed_one)
            vs = G_i.vs()
```

```
            G_i.delete_vertices([removed_one])
        print(f"remove {len(removed_airports)} airports")
```

```
=========== centrality_measure = degree ===========
----------- remove_smallest = True ----------
remove 3 airports
----------- remove_smallest = False ----------
remove 462 airports


=========== centrality_measure = pagerank ===========
----------- remove_smallest = True ----------
remove 6 airports
----------- remove_smallest = False ----------
remove 189 airports


=========== centrality_measure = authority ===========
----------- remove_smallest = True ----------
remove 2 airports
----------- remove_smallest = False ----------
remove 10 airports


=========== centrality_measure = hub ===========
----------- remove_smallest = True ----------
remove 2 airports
----------- remove_smallest = False ----------
remove 20 airports


=========== centrality_measure = between ===========
----------- remove_smallest = True ----------
remove 4 airports
----------- remove_smallest = False ----------
remove 462 airports


=========== centrality_measure = closeness ===========
----------- remove_smallest = True ----------
remove 1 airports
----------- remove_smallest = False ----------
remove 462 airports
```

Observation: The results when centrality measures computed with weights and without weights are slightly different. However, in overall, we need to remove more nodes with low centrality value to make the graph disconnected compared to remove nodes with high centrality value.

Problem 5

In [19]:

```
## read edges and build weighted directed graph
D = pd.read_csv(os.path.join(datadir, 'Airports/connections.csv'))
g = ig.Graph.TupleList([tuple(x) for x in D.values], directed=True, edge_attrs=['weight'

## read vertex attributes and add to graph
A = pd.read_csv(datadir+'Airports/airports_loc.csv')
lookup = {k:v for v,k in enumerate(A['airport'])}
```

```
l = [lookup[x] for x in g.vs()['name']]
g.vs()['state'] = [A['state'][i] for i in l]
```

In [20]:

```
states = sorted(set([v['state'] for v in g.vs()]))
state_dict = {state:i for i, state in enumerate(states)}
mapping = [state_dict[v['state']] for v in g.vs()]
state_g = g.copy()
state_g.contract_vertices(mapping, combine_attrs='first')
state_g = state_g.simplify(loops=True, combine_edges={"weight": sum})
```

In [21]:

```
print('Number of original nodes:', len(g.vs()))
print('Number of state:', len(states))
print('Number of nodes in state-to-state graph:', len(state_g.vs()))
```

```
Number of original nodes: 464
Number of state: 51
Number of nodes in state-to-state graph: 51
```

Problem 5a

In [22]:

```
print(state_g.vcount())
state_g_biggest_strong_comp = state_g.connected_components(mode='STRONG').giant()
print(state_g_biggest_strong_comp.vcount())
state_g_biggest_weak_comp = state_g.connected_components(mode='WEAK').giant()
print(state_g_biggest_weak_comp.vcount())
# ---> Strong connected
```

```
51
51
51
```

Problem 5b

In [23]:

```
out_degree = state_g.strength(mode='out', weights='weight')
in_degree = state_g.strength(mode='in', weights='weight')
df = pd.DataFrame({'state': state_g.vs['state'], 'in_degree': in_degree, 'out_degree': o
max_in_degree = df['in_degree'].max()
max_out_degree = df['out_degree'].max()
print('State has the most incoming passengers:', df[df['in_degree']==max_in_degree]['sta
print('State has the most departing passengers:', df[df['out_degree']==max_out_degree]['
```

```
State has the most incoming passengers: ['CA']
State has the most departing passengers: ['CA']
```

In [24]:

```
# state_vs = state_g.vs()
# for v in state_vs:
#     print("-----------------")
#     out_es = state_g.es.select(_source=v)
#     out_weight = sum(out_e['weight'] for out_e in out_es)
#     print(f"passengers depart {v['state']}: {out_weight}")

#     in_es = state_g.es.select(_target=v)
#     in_weight = sum(in_e['weight'] for in_e in in_es)
#     print(f"passengers arrive {v['state']}: {in_weight}")
```

Problem 5c

In [25]:

```
state_vs = state_g.vs
```

```python
state_es = list(state_g.es())
state_es.sort(key=lambda x: x['weight'], reverse=True)
busiest_route = state_es[0]
source_vertex_id = busiest_route.source
target_vertex_id = busiest_route.target
source_vertex = state_vs[source_vertex_id]
target_vertex = state_vs[target_vertex_id]

print('Pair of airports have the most passengers travelling from x to y')
print('x:', source_vertex['state'], '- y:', target_vertex['state'], '- num_passengers:',
```

```
Pair of airports have the most passengers travelling from x to y
x: FL - y: GA - num_passengers: 6584065
```

Problem 5d

In [26]:
```python
state_g.vs['name'] = state_g.vs['state']
state_C = generate_centrality_measure(state_g, node_type_name='state')[['state']+cm_name

for cm in ['degree', 'between']:
    print(f"\nTop {cm} centrality")
    state_Cs = state_C.sort_values(by=cm, ascending=False)
    display(state_Cs.head())
```

```
Top degree centrality
```

	state	degree	pagerank	authority	hub	between	closeness
4	CA	0.135169	0.075945	0.969771	0.971337	0.021798	0.943396
9	FL	0.131269	0.073844	1.000000	1.000000	0.025327	0.943396
43	TX	0.130073	0.077688	0.925013	0.925349	0.028044	0.980392
10	GA	0.107684	0.064350	0.829178	0.827263	0.024180	0.961538
14	IL	0.104197	0.063423	0.743696	0.742924	0.037366	1.000000

```
Top between centrality
```

	state	degree	pagerank	authority	hub	between	closeness
14	IL	0.104197	0.063423	0.743696	0.742924	0.037366	1.000000
35	OH	0.047638	0.030171	0.362870	0.363212	0.030574	0.980392
43	TX	0.130073	0.077688	0.925013	0.925349	0.028044	0.980392
23	MN	0.042512	0.031069	0.284188	0.284230	0.026444	0.961538
7	DC	0.048661	0.030078	0.418859	0.418278	0.026146	0.980392

In [27]:
```python
print('State has highest degree centrality: CA')
print('State has highest between centrality: IL')
```

```
State has highest degree centrality: CA
State has highest between centrality: IL
```

In [ ]: