VIETNAM NATIONAL UNIVERSITY, HO CHI MINH CITY
UNIVERSITY OF TECHNOLOGY
FACULTY OF APPLIED SCIENCE

ANALOG SIGNAL PROCESSING (MT1023)

Assignment

MATLAB PROJECT SEMESTER 223

Advisor: Ha Hoang Kha
Students: Luong Hoang Phuc 2010525

All LaTeX and source code are stored here: https://github.com/hoangphuchvcx02/ASP-PROJECT

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Contents

1 Question 1: Fourier Series

Consider the signal $x(t) = 120|sin(100\pi t)|$

1.1 Write a Matlab program to sketch the signal $x(t)$ for $-2T_0 \leq t \leq 2T_0$ where T_0 is the period of the signal $x(t)$.

Consider $x_f(t) = 120sin(100\pi t)$
We have $T_f = \frac{2\pi}{\omega} = \frac{2\pi}{100\pi} = 0.02(s)$ and $f_f = \frac{1}{T_f} = 50(Hz)$
Let consider $x(t) = 120|sin(100\pi t)|$
$T_0 = \frac{1}{2}T_f = 0.01(s)$

Listing 1: MATLAB code

```
% Given parameters
A = 120; % Amplitude
To = 0.01; % Period
T= 2*To
f= 1/T;

% Time vector
t = linspace(-2*To, 2*To, 1000); % Time range from -2To to 2To

% Signal x(t)
x_t = A * abs(sin(2*pi*f*t));

% Plot the signal x(t)
figure;
plot(t, x_t, 'b', 'LineWidth', 2);
title('Signal x(t) = 120|sin(100pit)|');
xlabel('Time (t)');
ylabel('Amplitude x(t)');
grid on;
```

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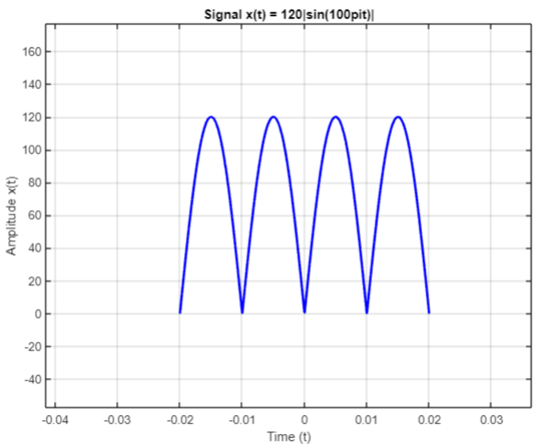

1.2 The signal $x(t)$ has the exponential Fourier series $x(t) = \sum_{-\infty}^{\infty} D_n e^{jn\omega_0 t}$ where $D_n = \frac{1}{T_0}\int_{T_0} x(t)e^{-jn\omega_0 t}dt$. Derive the expressions of D_n

The **complex Exponential Fourier Series** representation of a periodic signal x(t) with fundamental period To is given by,

$$x(t) = \sum_{-\infty}^{+\infty} D_n e^{jn\omega_0 t}$$

Consider $x(t) = Asin(\omega_0 t)$, where $\omega_0 = \frac{2\pi}{T_0}$ = fundamental frequency

Now, putting the value of n and expanding above series we get

$x(t) = ... + C_{-2}e^{-j2\omega_o t} + C_{-1}e^{-j\omega_o t} + C_0 + C_1 e^{2\omega_o t} + C_2 e^{-j2\omega_o t} + ...$

Consider the signal, $x(t) = A|sin(\omega_1 t)|$

The period of the rectified sinusoid is one half of this, $T = \frac{T_1}{2} = \frac{\pi}{\omega_1}$

Therefore, $\omega_1 = \frac{2\pi}{T} = \frac{\pi}{T} = \frac{\omega_0}{2}$ Thus, we can write $x(t) = A|sin(\frac{\omega_0 t}{2})|$

The Fourier series coefficients are:

$$\begin{aligned} D_n &= \frac{1}{T_0}\int_{T_0} x(t)e^{-jn\omega_0 t}dt \\ &= \frac{1}{T}\int_0^T A\left|\sin\left(\frac{\omega_0 t}{2}\right)\right| e^{-jn\omega_0 t}dt \end{aligned}$$

From 0 to T, $\left|\sin\left(\frac{\omega_0 t}{2}\right)\right| = \sin\left(\frac{\omega_0 t}{2}\right)$. Also, replace ω_0 by $\frac{2\pi}{T}$

The exponential Fourier series of a periodic signal x(t) is a representation that is the sum of the complex exponentials at positive and negative harmonic frequencies. By substituting the following Euler's identities

$$cosx = \frac{e^{ix} + e^{-ix}}{2}$$
$$sinx = \frac{e^{ix} - e^{-ix}}{2i}$$

$$\begin{aligned} D_n &= \frac{1}{T}\int_0^T A\sin\left(\frac{\pi t}{T}\right)e^{-\frac{j2\pi nt}{T}}dt \\ &= \frac{A}{T}\int_0^T \left(\frac{e^{\frac{j\pi t}{T}} - e^{-\frac{j\pi t}{T}}}{j2}\right)e^{-\frac{j2\pi nt}{T}}dt \\ &= \frac{A}{j2T}\int_0^T \left(e^{\frac{j\pi t(1-2n)}{T}} - e^{-\frac{j\pi t(1+2n)}{T}}\right)dt \\ &= \frac{A}{j2T}\left[\frac{e^{\frac{j\pi t(1-2n)}{T}}}{j\pi(1-2n)/T} - \frac{e^{-\frac{j\pi t(1+2n)}{T}}}{j\pi(1+2n)/T}\right]_0^T \\ &= -\frac{A}{2T}\left[\frac{e^{j\pi(1-2n)}}{\pi(1-2n)/T} + \frac{e^{-j\pi(1+2n)}}{\pi(1+2n)/T} - \frac{1}{\pi(1-2n)/T} - \frac{1}{\pi(1+2n)/T}\right] \end{aligned}$$

But $e^{j\pi(1-2n)} = e^{j\pi}e^{-j2\pi n} = -1$ and $e^{-j\pi(1+2n)} = -1$

$$\begin{aligned}
D_n &= -\frac{A}{2T}\left[\frac{-1}{\pi\left(1-2n\right)/T} + \frac{-1}{\pi\left(1+2n\right)/T} - \frac{1}{\pi\left(1-2n\right)/T} - \frac{1}{\pi\left(1+2n\right)/T}\right] \\
&= -\frac{A}{2}\left[-\frac{2}{\pi\left(1-2n\right)} - \frac{2}{\pi\left(1+2n\right)}\right] \\
&= \frac{A}{\pi}\left[\frac{1}{1-2n} + \frac{1}{1+2n}\right] \\
&= \frac{2A}{\pi}\left(\frac{1}{1-4n^2}\right)
\end{aligned}$$

Exponential Fourier series, $x(t) = \sum\limits_{-\infty}^{\infty} D_n e^{jn\omega_0 t}$

$\Rightarrow x\left(t\right) = -\frac{2A}{\pi}\sum\limits_{n=-\infty}^{\infty} \frac{1}{4n^2-1}e^{jn2\pi t}$

The given function is $x(t) = A|sin(\omega_1 t)| = 120|sin(100\pi t)|$

$$D_n = \frac{240}{\pi}\left(\frac{1}{1-4n^2}\right)$$

n	-2	-1	0	1	2
D_n	-5.092958	-25.46479	76.39437	-25.46479	-5.092958

Notably: $-A = Ae^{\pm j180^o}$ with A > 0

n	-2	-1	0	1	2
$\lvert D_n\rvert$	5.092958	25.46479	76.39437	25.46479	5.092958
Phase $\angle$ (degree)	$\pm180^o$	$\pm180^o$	0	$\pm180^o$	$\pm180^o$

Other method:

$$\begin{aligned}
D_n &= \frac{1}{T_0}\int_{T_0} x(t)e^{-jn\omega_0 t}dt \\
&= \frac{1}{T}\int\limits_0^T A\left|\sin\left(\frac{\omega_0 t}{2}\right)\right| e^{-jn\omega_0 t}dt \\
&= \frac{1}{0.01}\int\limits_0^{0.01} 120|sin(100\pi t)|e^{-jn200\pi t}dt \\
&= \frac{1}{0.01}\int\limits_0^{0.01} 120sin(100\pi t)[cos(n200\pi t) - jsin(n200\pi t)]dt \\
&= 100\int\limits_0^{0.01} 120sin(100\pi t)cos(n200\pi t) - j100\int\limits_0^{0.01} 120sin(100\pi t)sin(200\pi nt)dt \\
&= A - jB
\end{aligned}$$

n	-2	-1	0	1	2
A	-5.09296	-25.4647	76.39437	-25.46479	-5.092958
B	$1.977 * 10^{-15}$	$2.4999 * 10^{-16}$	0	$-2.4999 * 10^{-16}$	$-1.977 * 10^{-15}$
D_n	-5.092958	-25.46479	76.39437	-25.46479	-5.092958
$\|D_n\|$	5.092958	25.46479	76.39437	25.46479	5.092958
Phase $\angle$ (degree)	$\pm 180^o$	$\pm 180^o$	0	$\pm 180^o$	$\pm 180^o$

1.3 Write a Matlab program to determine D_n . Then, sketch the magnitude and phase spectrum in Matlab.

Listing 2: Matlab code for magnitude and phase spectrum

```
n = input('Enter the value of n: ');
A = input('Enter the value of A: ');
T = input('Enter the value of T: ');

Dn_expression = @(n, A, T) (1 / T) * integral(@(t) A * abs(sin(pi * t / T)) .* exp(-1j * 2 * pi * n * t / T), 0, T);
Dn_value = Dn_expression(n, A, T);

magnitude_spectrum = Dn_value;
phase_spectrum_deg = rad2deg(angle(Dn_value));

figure;
subplot(2, 1, 1);
stem(n, real(magnitude_spectrum)); % Use real() to display the real part
title('Magnitude Spectrum');
xlabel('n');
ylabel('D_n');

subplot(2, 1, 2);
stem(n, phase_spectrum_deg);
title('Phase Spectrum');
xlabel('n');
ylabel('Phase (degrees)');
```

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Set the input as follows: $A = 120, T = 0.02$

Let check some value

n	-5	-2	-1	0	1	2	5
D_n	-0.77166	-5.09296	-25.4648	76.3944	-25.4648	-5.09296	-0.77176
Phase $\angle$ (degree)	180	-180	180	0	-180	180	-180

Listing 3: The magnitude and phase spectrum between -n and n

```
n = input('Enter the value of n: ');
A = input('Enter the value of A: ');
T = input('Enter the value of T: ');

n_values = -n:n;
Dn_expression = @(n, A, T) (1 / T) * integral(@(t) A * abs(sin(pi * t / T)) .* exp(-1j * 2 * pi * n * t / T), 0, T);

Dn_values = arrayfun(@(n) Dn_expression(n, A, T), n_values);

```

```
magnitude_spectrum = Dn_values;
phase_spectrum_deg = rad2deg(angle(Dn_values));

figure;
subplot(2, 1, 1);
stem(n_values, real(magnitude_spectrum));
title('Magnitude Spectrum');
xlabel('n');
ylabel('D_n');

subplot(2, 1, 2);
stem(n_values, phase_spectrum_deg);
title('Phase Spectrum');
xlabel('n');
ylabel('Phase (degrees)');
```

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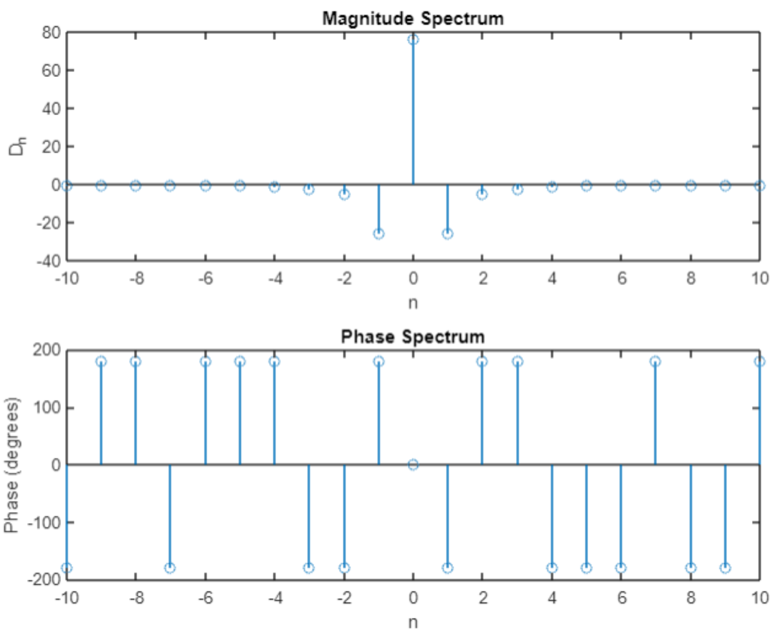

When comparing two tables of results calculated by traditional methods in part 1.2 and Matlab in part 1.3, we can see that the value of D_n in the two calculations is the same. However, Why is their phase slightly different? As mentioned above, based on the expression $-A = Ae^{\pm j180^o}$ (with A>0) we can derive:

Listing 4: Matlab code

```
%Copy line 1-8 from listing 3

magnitude_spectrum = abs(Dn_values);
phase_spectrum_rad = angle(Dn_values);
phase_spectrum_deg = rad2deg(phase_spectrum_rad);
```

```
% Concatenate phase values for both positive and negative n
n_all = [-n_values(end:-1:2), n_values];
phase_spectrum_all = [phase_spectrum_deg(end:-1:2),
    phase_spectrum_deg];

figure;
subplot(2, 1, 1);
stem(n_values, real(magnitude_spectrum));
title('Magnitude Spectrum');
xlabel('n');
ylabel('|D_n|');

subplot(2, 1, 2);
stem(n_all, phase_spectrum_all);
title('Phase Spectrum');
xlabel('n');
ylabel('Phase (degrees)');
```

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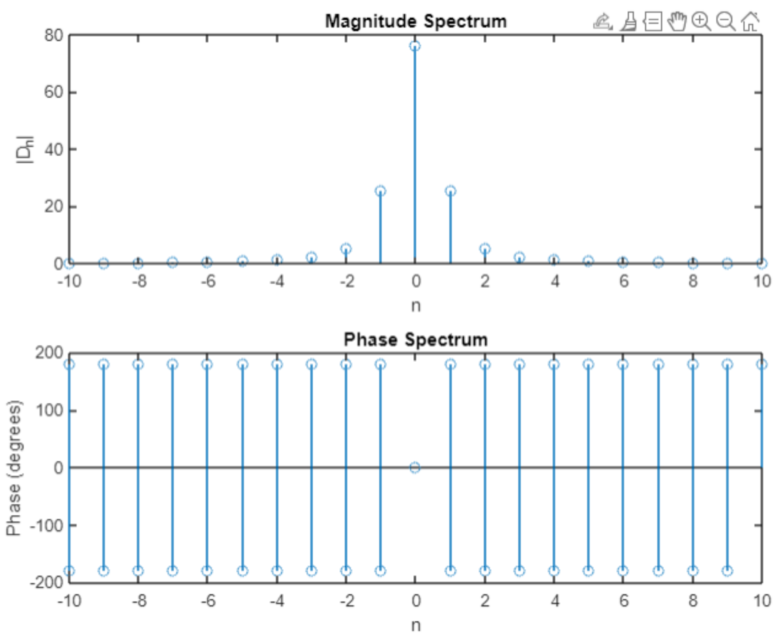
With this replacement code we can find the exact phase we want to find

n	-2	-1	0	1	2
$\lvert D_n \rvert$	5.092958	25.46479	76.39437	25.46479	5.092958
Phase $\angle$ (degree)	$\pm 180^o$	$\pm 180^o$	0	$\pm 180^o$	$\pm 180^o$



2 Question 2: Fourier Transform

Consider the following signal: $x(t) = A_1cos(\omega_1 t) + A_2cos(\omega_2 t)$ The modulated signal y(t) is defined by $y(t) = x(t)cos(\omega_c t)$

2.1 Find $X(\omega)$ and the power of $x(t)$

$X(\omega) = A_1\pi\left(\delta(\omega-\omega_1)+\delta(\omega+\omega_1)\right) + A_2\pi\left(\delta(\omega-\omega_2)+\delta(\omega+\omega_2)\right)$

The power of signal $x(t)$: $P_x = \frac{{A_1}^2}{2} + \frac{{A_2}^2}{2}$ (W)

2.2 Find $Y(\omega)$ and the power of $y(t)$

$y(t) = x(t)cos(\omega_c t)$
$Y(\omega) = \frac{1}{2}X(\omega+\omega_c) + \frac{1}{2}X(\omega-\omega_c)$
$Y(\omega) = \frac{1}{2}\left[A_1\pi\left(\delta(\omega-\omega_1+\omega_c)+\delta(\omega+\omega_1+\omega_c)\right) + A_2\pi\left(\delta(\omega-\omega_2+\omega_c)+\delta(\omega+\omega_2+\omega_c)\right)\right] +$
$\frac{1}{2}\left[A_1\pi\left(\delta(\omega-\omega_1-\omega_c)+\delta(\omega+\omega_1-\omega_c)\right) + A_2\pi\left(\delta(\omega-\omega_2-\omega_c)+\delta(\omega+\omega_2-\omega_c)\right)\right]$

The power of signal $y(t)$: $P_y = \frac{(0.5*A_1)^2}{2} + \frac{(0.5*A_2)^2}{2} + \frac{(0.5*A_1)^2}{2} + \frac{(0.5*A_2)^2}{2}$ (W)

2.3 Write a Matlab program to input the values of $A_1, A_2, \omega_1, \omega_2, \omega_c$, and the sketch the waveform $x(t)$ and $y(t)$. Then, sketch the spectrum and $X(\omega)$ and $Y(\omega)$.

Listing 5: MATLAB code for sketch waveforms and spectrums

```
% Input parameters
A1 = input('Enter the value of A1: ');
A2 = input('Enter the value of A2: ');
w1 = input('Enter the value of W1 (in rad/s): ');
w2 = input('Enter the value of W2 (in rad/s): ');
wc = input('Enter the value of Wc (in rad/s): ');


% Time vector
% t1 and t2 display clear information for x(t) and y(t)
t1 = input('Enter the value of t1: ');
t2 = input('Enter the value of t2: ');
t = linspace(t1, t2, 1000); % Time range from t1 to t2

% Signal x(t)
x_t = A1 * cos(w1 * t) + A2 * cos(w2 * t);

% Modulated signal y(t)
y_t = x_t .* cos(wc * t);

% Plot waveforms x(t) and y(t)
figure;
subplot(2, 1, 1);
```

```
plot(t, x_t, 'b', 'LineWidth', 2);
title('Waveform x(t)');
xlabel('Time (t)');
ylabel('Amplitude');
grid on;


subplot(2, 1, 2);
plot(t, y_t, 'r', 'LineWidth', 2); %'b': This is the defined color of the line. 'b' represents the color blue. 'LineWidth', 2: This condition sets the line thickness to 2 units.
title('Waveform y(t)');
xlabel('Time (t)');
ylabel('Amplitude');
grid on;


% Calculate X(w) analytically
% Frequencies and amplitudes
frequencies = [-w2, -w1, w1, w2];
amplitudes = [A2* pi, A1* pi, A1* pi, A2* pi];

% Plot spectrum X(w)
figure;
subplot(2, 1, 1);
stem(frequencies, amplitudes, 'filled', 'MarkerSize', 10); %stem: used to create a "stem plot" chart. 'filled': The "stem plot" will be plotted with colored dot (or column) segments. 'MarkerSize', 10: Set the size of the dot or column mark on the chart to 10 units.
title('Spectrum of X(w)');
xlabel('Frequency (w)');
ylabel('Amplitude');
grid on;

% Set custom tick labels
xticks(frequencies);
xticklabels({['-w2 (' num2str(-w2) ')'], ['-w1 (' num2str(-w1) ')'], ['w1 (' num2str(w1) ')'], ['w2 (' num2str(w2) ')']});

% Adjust plot limits
axis([min(frequencies)-1 max(frequencies)+1 0 max(amplitudes)+1]);

% To calculate Y(w) we can do the same as calculating X(w) but in this case I won't show xtick labels
% Plot spectrum Y(w)
frequencies2 = [-w1 + wc, w1 + wc, -w2 + wc, w2 + wc, -w1 - wc, w1 - wc, -w2 - wc, w2 - wc];
amplitudes2 = [A1 * pi / 2, A1 * pi / 2, A2 * pi /2, A2 * pi/2, A1
```

```
    * pi / 2, A1 * pi / 2, A2 * pi / 2, A2 * pi / 2];
subplot(2, 1, 2);
stem(frequencies2, amplitudes2, 'filled', 'MarkerSize', 10);
title('Spectrum of Y(w) ');
xlabel('Frequency (w)');
ylabel('Amplitude');
grid on;
```

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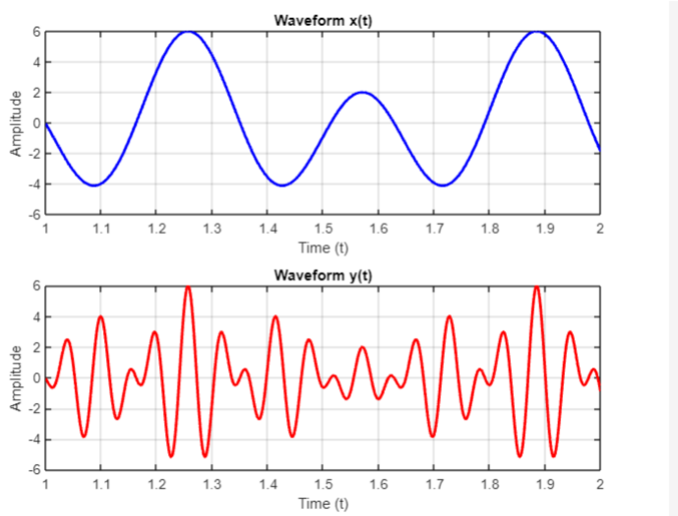
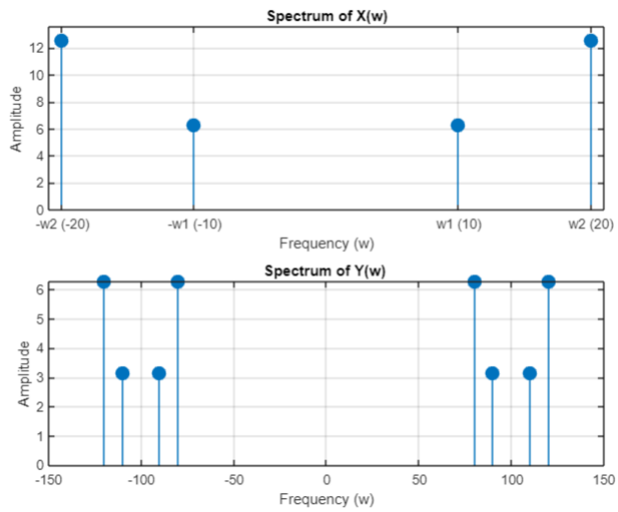
For example $X(t) = 2cos(10t) + 4cos(20t)$

We apply above solution in 2.1 and 2.2 to solve this:
$X(\omega) = 2\pi\,(\delta(\omega - 10) + \delta(\omega + 10)) + 4\pi\,(\delta(\omega - 20) + 4\pi(\delta(\omega + 20))$
$Y(\omega) = \pi\delta(\omega + 110) + \pi\delta(\omega + 90) + 2\pi\delta(\omega + 120) + 2\pi\delta(\omega + 80) + \pi\delta(\omega - 90) + \pi\delta(\omega - 110) + 2\pi\delta(\omega - 80) + 2\pi\delta(\omega - 120)$





References

[1] *Signals and Systems - A primer with Matlab - Matthew N.O.Sadiku, Warsame H.Ali*
Matthew N.O.Sadiku, Warsame H.Ali- Signals and Systems - A primer with Matlab

[2] *ANALOG SIGNAL PROCESSING (EXC) (EE2406) DTQ1 (LKQT HK223)*
MATLAB PROJECT

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