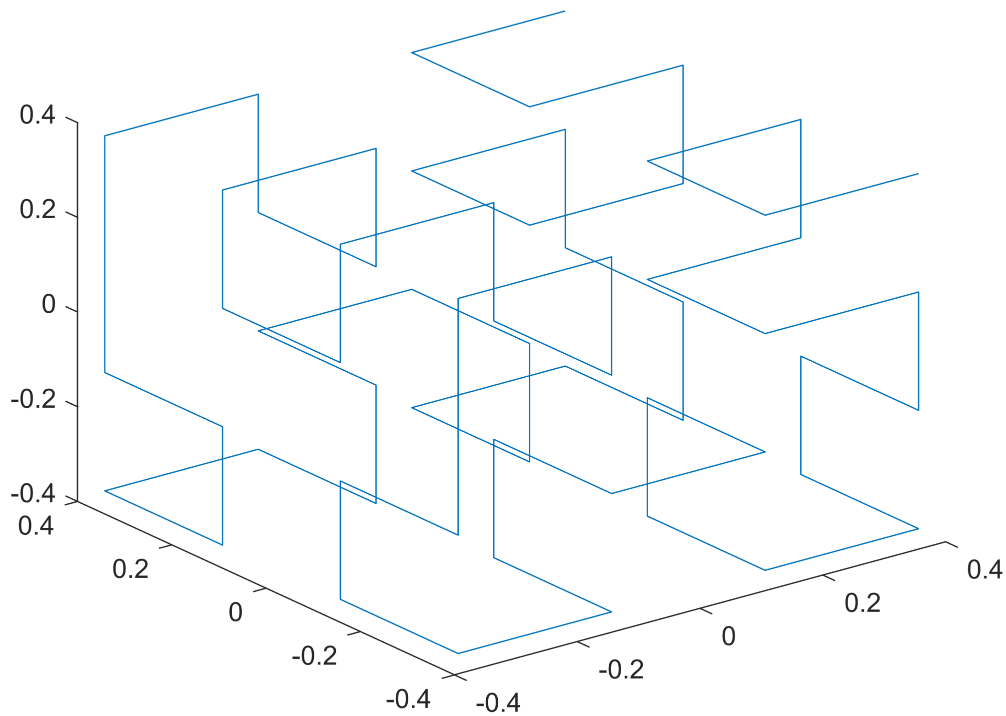
2.1 Some more on Hilbert Curve

The 3D version of Hilbert Curve

```
% Plot 3D curve with n = 2
[x, y, z] = hilbert3(2);
figure;
plot3(x, y, z)
```



Compute the number of turning points

```
% Number of turning points
nTurning_points = hilbert3_points(2)
```

```
nTurning_points = 62
```

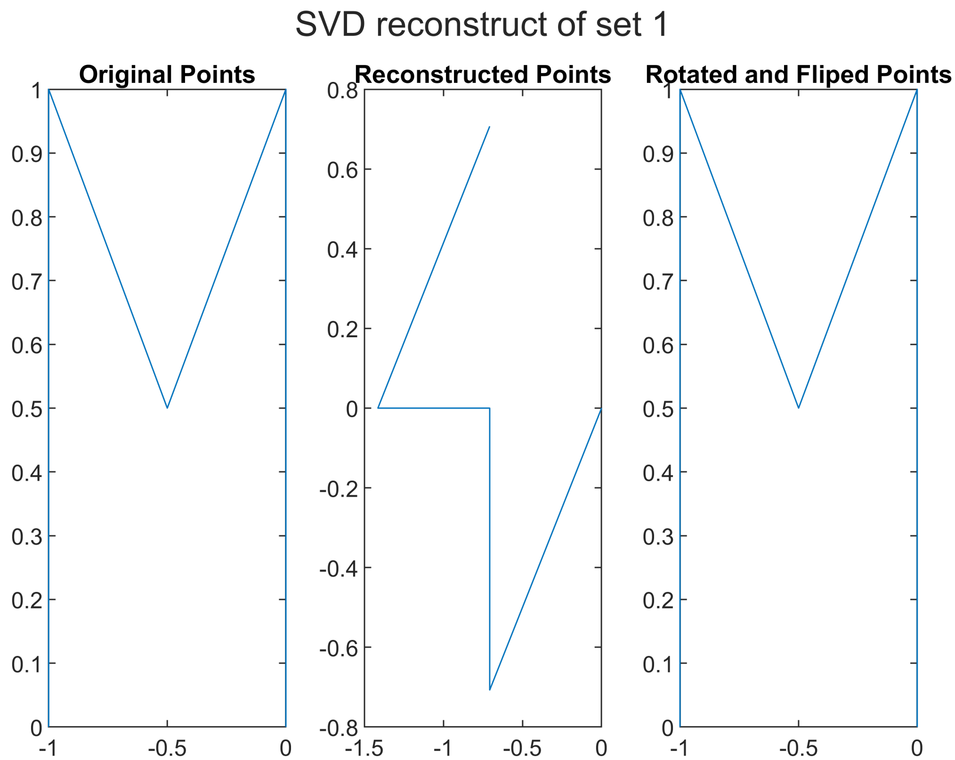
//TODO: Provide a written explanation as to how the 3D code works. What structure is being replicated and how is it moved and rotated?

2.2 Reconstructing the curve

```
% Three sample sets
set_1 = [ -1 0; -1 1; -1/2 1/2; 0 1; 0 0];
set_2 = [ -1 0; -1 1; -1/2 1/2; 0 1; -1/4 1/4];
set_3 = [ 0 0 1; 0 1 1; -1/2 1/2 1; -1 1 1; -1 0 1; -1 0 0; -1 1 0; -1/2 1/2 0; 0 1 0; 0 0 0];
```

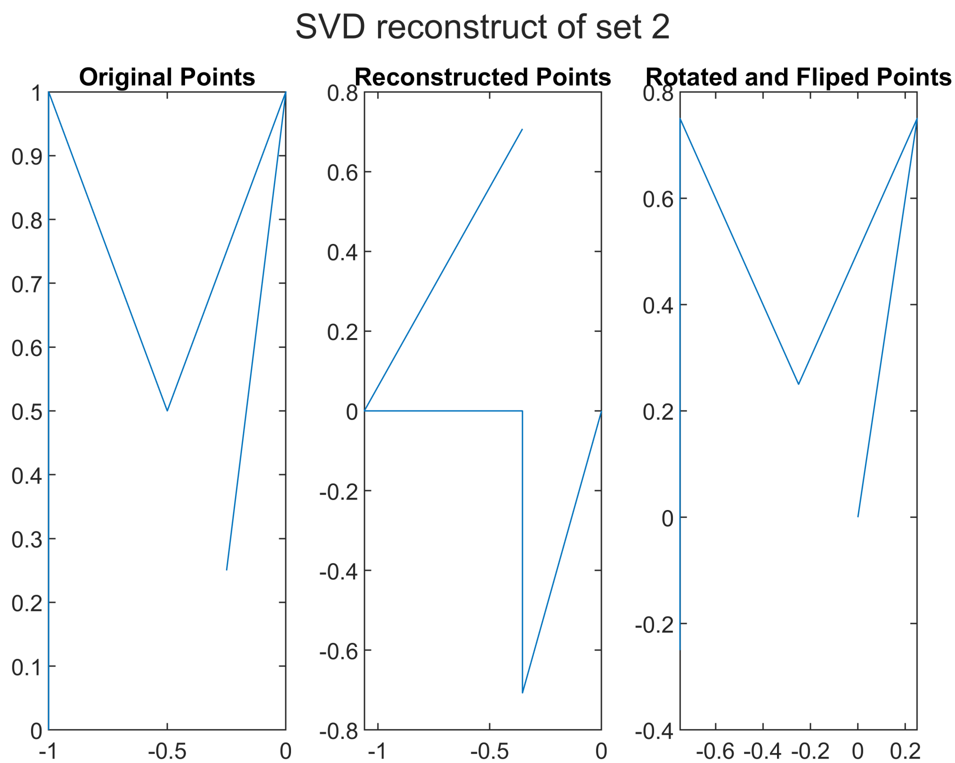
Solve the Distance Geometry problem using SVD:

```
% Create rotation matrix
theta = 135; % to rotate 135 counterclockwise
R = [cosd(theta) -sind(theta); sind(theta) cosd(theta)];


% The distance matrix of set 1
d = distance_matrix(set_1);
X = svd_reconstruct(d);


% Plots
figure;
t = tiledlayout(1,3);
title(t, "SVD reconstruct of set 1");

ax1 = nexttile;
plot(set_1(:,1), set_1(:, 2));
title(ax1,'Original Points');

ax2 = nexttile;
plot(X(:,1), X(:, 2));
title(ax2,'Reconstructed Points');

ax3 = nexttile;
for i = 1:length(X)
    X(i,:) =  X(i,:) * R;
    X(i, 1) = -X(i, 1);
end
plot(X(:,1), X(:, 2));
title(ax3,'Rotated and Fliped Points');
```




```
% The distance matrix of set 2
d = distance_matrix(set_2);
X = svd_reconstruct(d);

% Create rotation matrix
theta = 135; % to rotate 135 counterclockwise
R = [cosd(theta) -sind(theta); sind(theta) cosd(theta)];

% Plots
figure;
t = tiledlayout(1,3);
title(t, "SVD reconstruct of set 2");

ax1 = nexttile;
plot(set_2(:,1), set_2(:, 2));
title(ax1,'Original Points');

ax2 = nexttile;
plot(X(:,1), X(:, 2));
title(ax2,'Reconstructed Points');

ax3 = nexttile;
for i = 1:length(X)
    X(i,:) =  X(i,:) * R;
    X(i, 1) = -X(i, 1);
end
```

```
plot(X(:,1), X(:, 2));
title(ax3,'Rotated and Fliped Points');
```



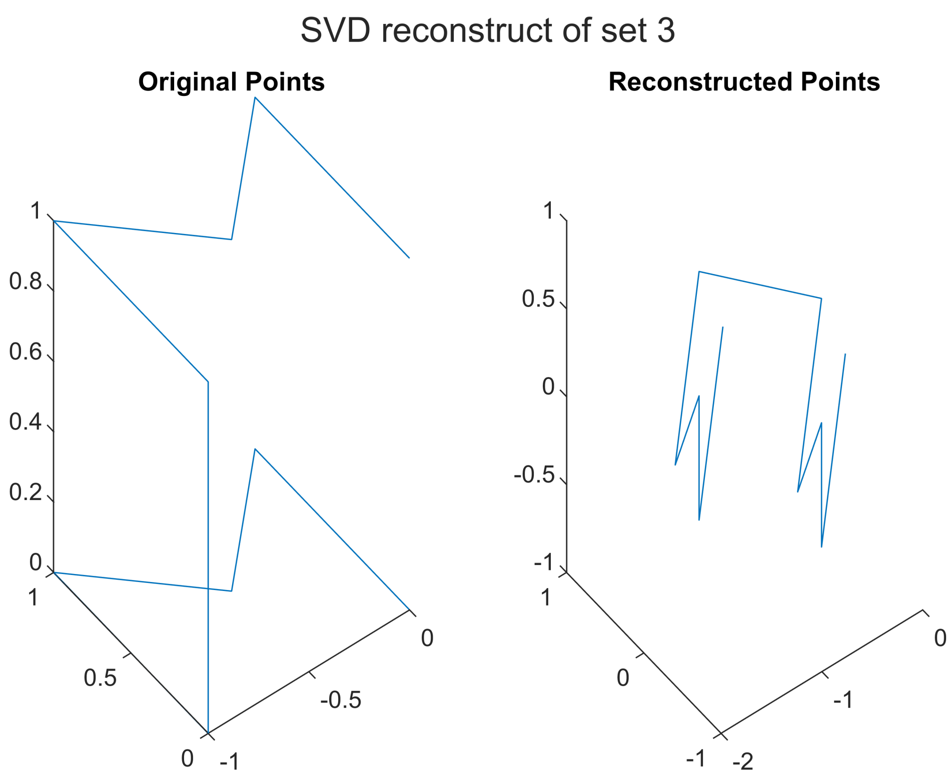

```
% The distance matrix of set 3
d = distance_matrix(set_3)
```

```
d = 10×10
         0    1.0000    0.7071    1.4142    1.0000    1.4142    1.7321    1.2247 ···
    1.0000         0    0.7071    1.0000    1.4142    1.7321    1.4142    1.2247
    0.7071    0.7071         0    0.7071    0.7071    1.2247    1.2247    1.0000
    1.4142    1.0000    0.7071         0    1.0000    1.4142    1.0000    1.2247
    1.0000    1.4142    0.7071    1.0000         0    1.0000    1.4142    1.2247
    1.4142    1.7321    1.2247    1.4142    1.0000         0    1.0000    0.7071
    1.7321    1.4142    1.2247    1.0000    1.4142    1.0000         0    0.7071
    1.2247    1.2247    1.0000    1.2247    1.2247    0.7071    0.7071         0
    1.4142    1.0000    1.2247    1.4142    1.7321    1.4142    1.0000    0.7071
    1.0000    1.4142    1.2247    1.7321    1.4142    1.0000    1.4142    0.7071
```

```
X = svd_reconstruct(d)
```

```
X = 10×3
   -0.6033    0.7975   -0.0000
   -1.1672    0.3710   -0.7071
   -1.1672    0.3710    0.0000
   -1.7312   -0.0556    0.0000
   -1.1672    0.3710    0.7071
   -0.5640   -0.4266    0.7071
   -1.1279   -0.8531    0.0000
   -0.5640   -0.4266    0.0000
   -0.5640   -0.4266   -0.7071
         0         0         0
```

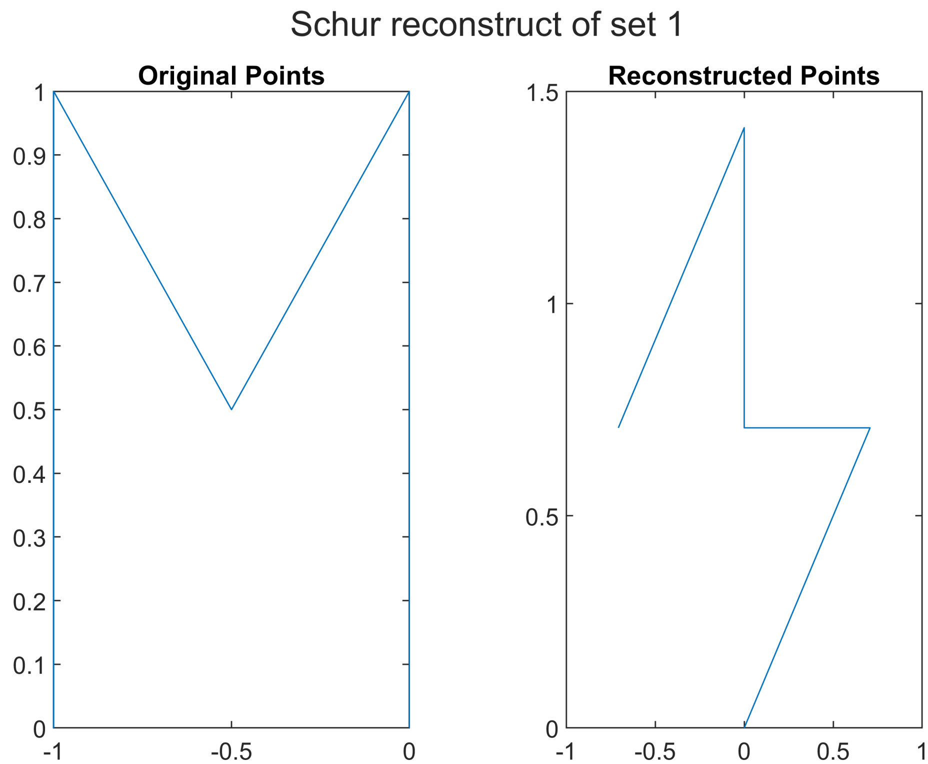
```
% Plots
figure;
t = tiledlayout(1,2);
title(t, "SVD reconstruct of set 3");

ax1 = nexttile;
plot3(set_3(:,1), set_3(:, 2), set_3(:,3));
title(ax1,'Original Points');

ax2 = nexttile;
plot3(X(:,1), X(:, 2), X(:, 3));
title(ax2,'Reconstructed Points');
```



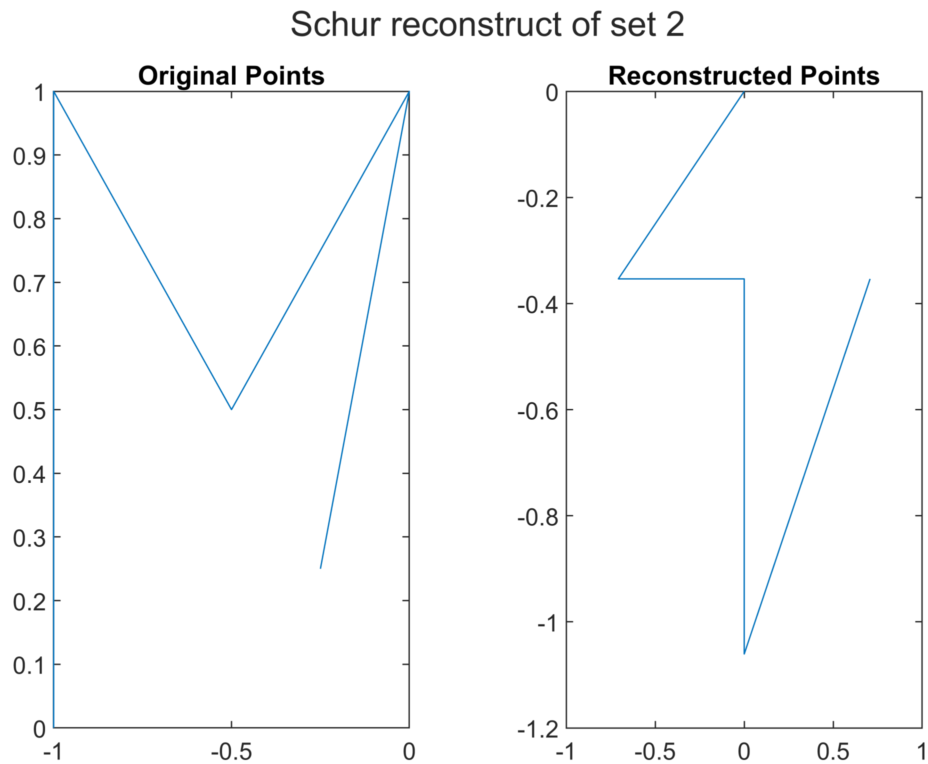
Solve the Distance Geometry problem using Schur

```
% The distance matrix of set 1
d = distance_matrix(set_1);
X = schur_reconstruct(d);

% Plots
figure;
t = tiledlayout(1,2);
title(t, "Schur reconstruct of set 1");

ax1 = nexttile;
plot(set_1(:,1), set_1(:, 2));
title(ax1,'Original Points');
```

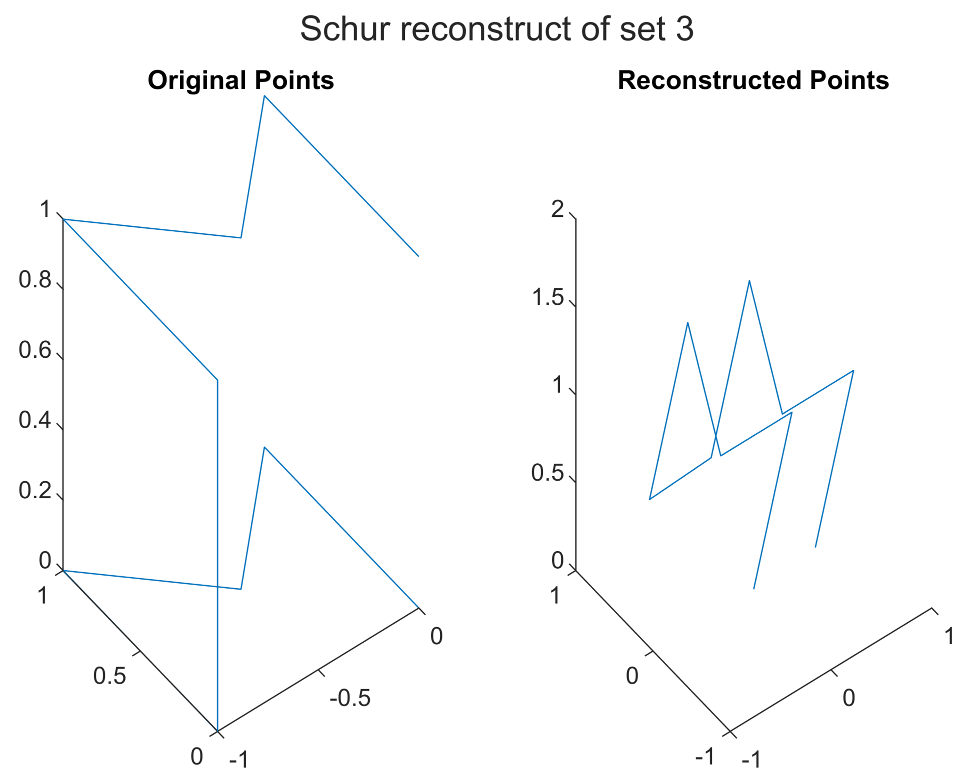
```
ax2 = nexttile;
plot(X(:,1), X(:, 2));
title(ax2,'Reconstructed Points');
```




```
% The distance matrix of set 2
d = distance_matrix(set_2);
X = schur_reconstruct(d);

% Plots
figure;
t = tiledlayout(1,2);
title(t, "Schur reconstruct of set 2");

ax1 = nexttile;
plot(set_2(:,1), set_2(:, 2));
title(ax1,'Original Points');

ax2 = nexttile;
plot(X(:,1), X(:, 2));
title(ax2,'Reconstructed Points');
```




```
% The distance matrix of set 3
d = distance_matrix(set_3);
X = schur_reconstruct(d);

% Plots
figure;
t = tiledlayout(1,2);
title(t, "Schur reconstruct of set 3");

ax1 = nexttile;
plot3(set_3(:,1), set_3(:, 2), set_3(:,3));
title(ax1,'Original Points');

ax2 = nexttile;
plot3(X(:,1), X(:, 2), X(:, 3));
title(ax2,'Reconstructed Points');
```

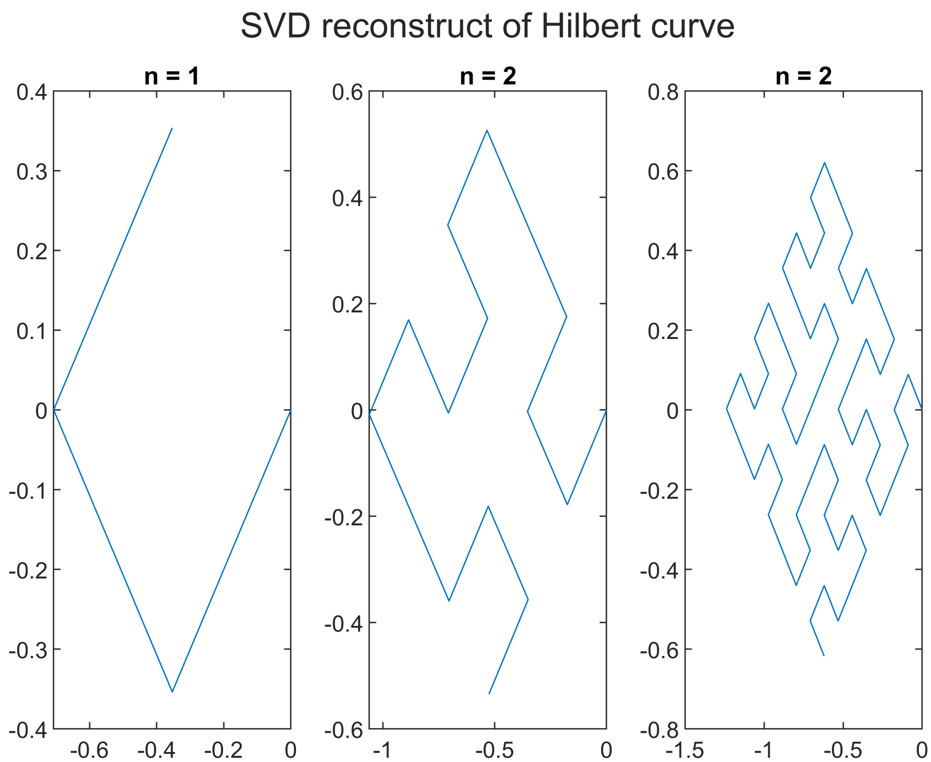


//TODO: Consider rotation

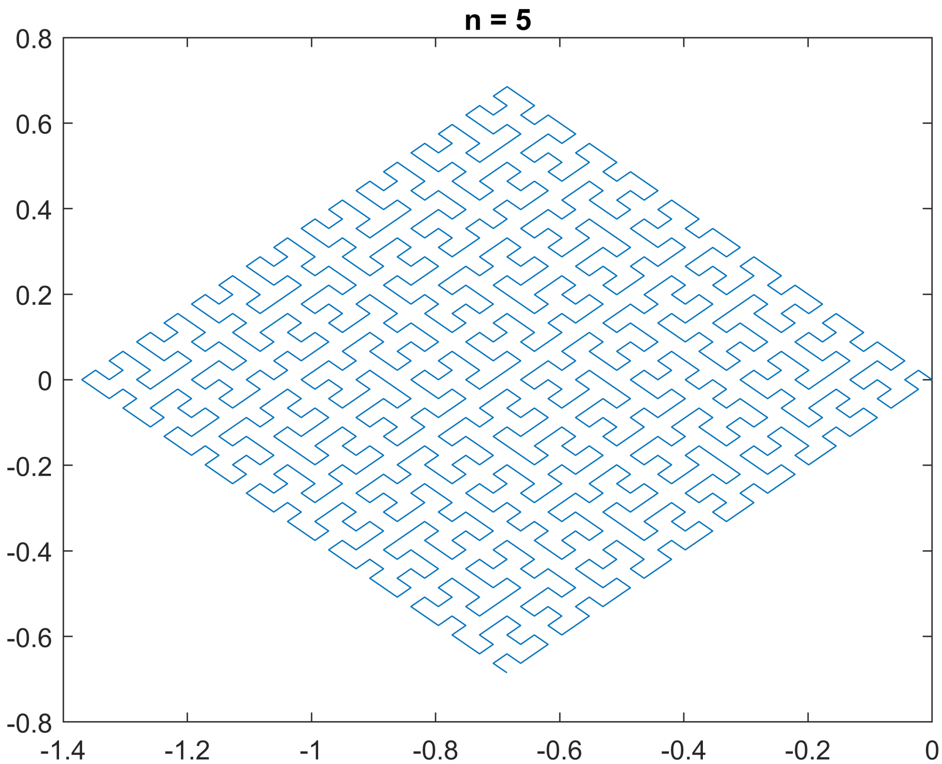
Reconstruct the 2D Hilbert Curve

```
% Plots
figure;
t = tiledlayout(1,3);
title(t, "SVD reconstruct of Hilbert curve");

% Hilbert Curve n = 1
d = distance_matrix_hilbert(1, 2);
X = svd_reconstruct(d);

ax1 = nexttile;
plot(X(:,1), X(:, 2));
title(ax1,'n = 1');

% Hilbert Curve n = 2
d = distance_matrix_hilbert(2, 2);
X = svd_reconstruct(d);

ax2 = nexttile;
plot(X(:,1), X(:, 2));
title(ax2,'n = 2');

% Hilbert Curve n = 3
d = distance_matrix_hilbert(3, 2);
```

```
X = svd_reconstruct(d);

ax3 = nexttile;
plot(X(:,1), X(:, 2));
title(ax3,'n = 2');
```

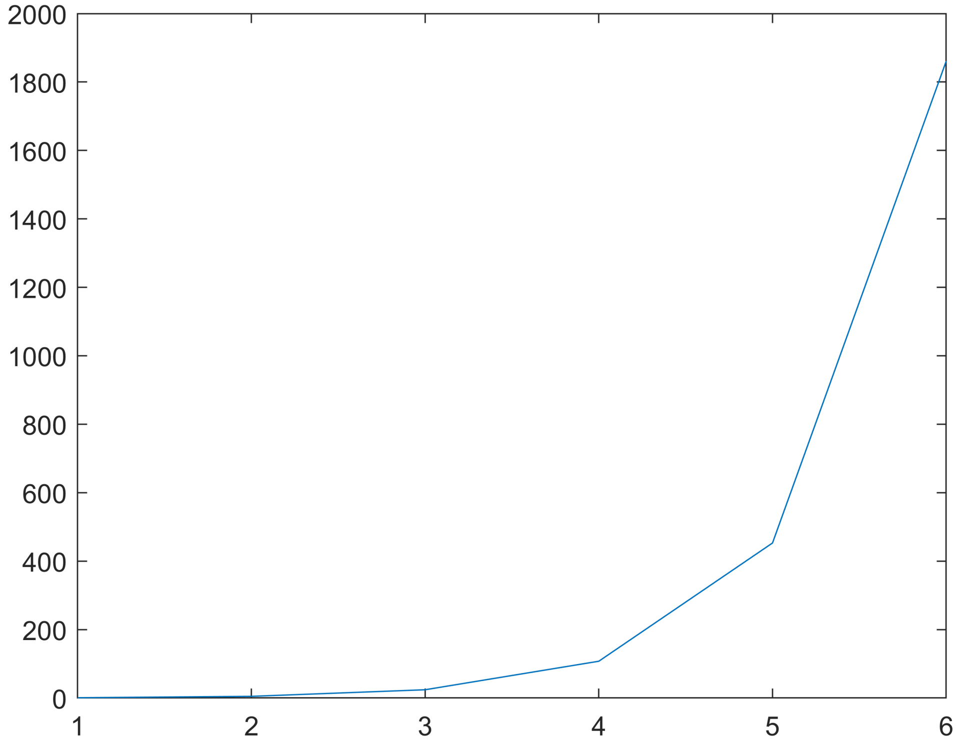



```
% Hilbert Curve n = 5
figure;
d = distance_matrix_hilbert(5, 2);
X = svd_reconstruct(d);


plot(X(:,1), X(:, 2));
title('n = 5');
```

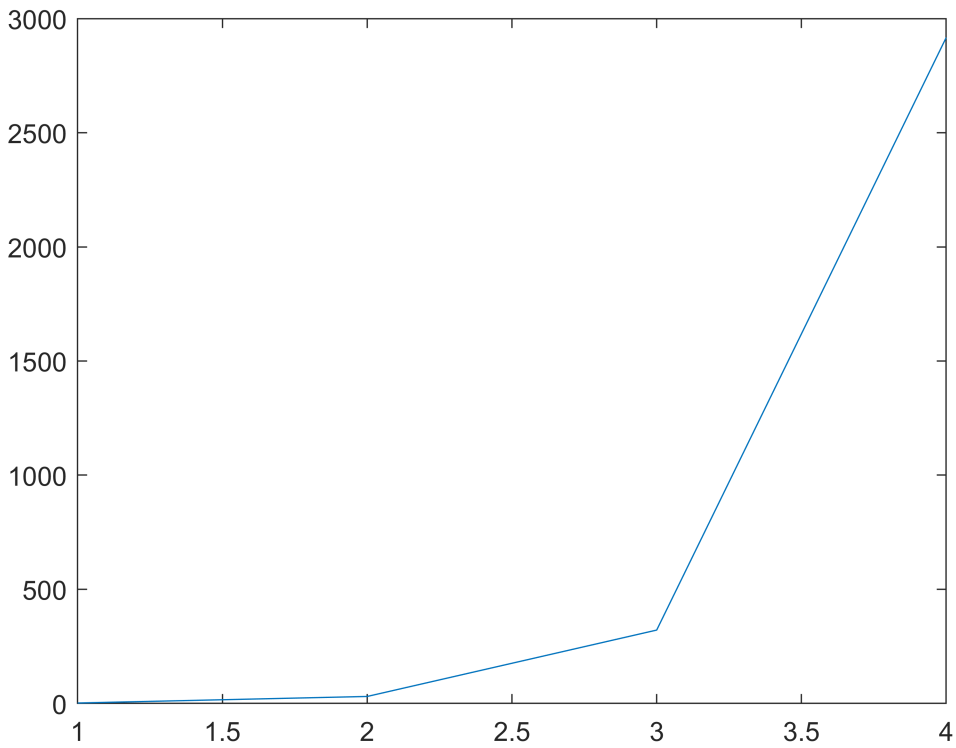


Plot singular value of SVD

```
s_values = [];
iteration = 1:6;
for i = 1:length(iteration)
    d = distance_matrix_hilbert(i, 2);
    % Length of  distance matrix
    n = length(d);

    % Calculate M = X * transpose(X)
    % M = (m(i,j) for i,j = 1, ... n-1))
    M = zeros(n, n);

    % SVD method to reconstruct coordinates
    for i = 1:n
        for j = 1:n
            M(i, j) = (d(i, n) ^ 2 - d(i, j) ^ 2 + d(j, n) ^ 2)/2;
        end
    end

    s = svd(M);

    s_values = [s_values max(s)];
end

figure;
plot(iteration, s_values)
```



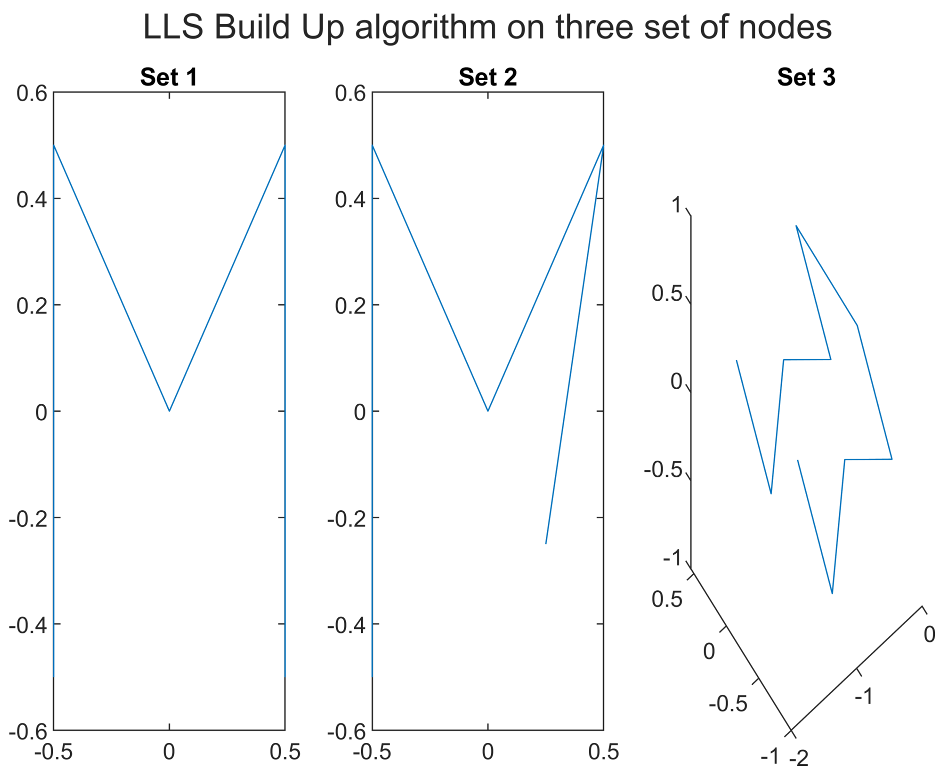

```
s_values = [];
iteration = 1:4;
for i = 1:length(iteration)
    d = distance_matrix_hilbert(i, 3);
    % Length of  distance matrix
    n = length(d);

    % Calculate M = X * transpose(X)
    % M = (m(i,j) for i,j = 1, ... n-1))
    M = zeros(n, n);

    % SVD method to reconstruct coordinates
    for i = 1:n
        for j = 1:n
            M(i, j) = (d(i, n) ^ 2 - d(i, j) ^ 2 + d(j, n) ^ 2)/2;
        end
    end

    s = svd(M);

    s_values = [s_values max(s)];
end

figure;
plot(iteration, s_values)
```



LLS Build Up algorithm on three set of nodes

```
% Plots
figure;
t = tiledlayout(1,3);
title(t, "LLS Build Up algorithm on three set of nodes");

% Set 1
d = distance_matrix(set_1);
X = build_up_LLS(d, 2);

ax1 = nexttile;
plot(X(:,1), X(:, 2));
title(ax1,'Set 1');

% Set 2
d = distance_matrix(set_2);
X = build_up_LLS(d, 2);

ax2 = nexttile;
plot(X(:,1), X(:, 2));
title(ax2,'Set 2');

% Set 3
d = distance_matrix(set_3);
X = build_up_LLS(d, 3);

ax3 = nexttile;
```

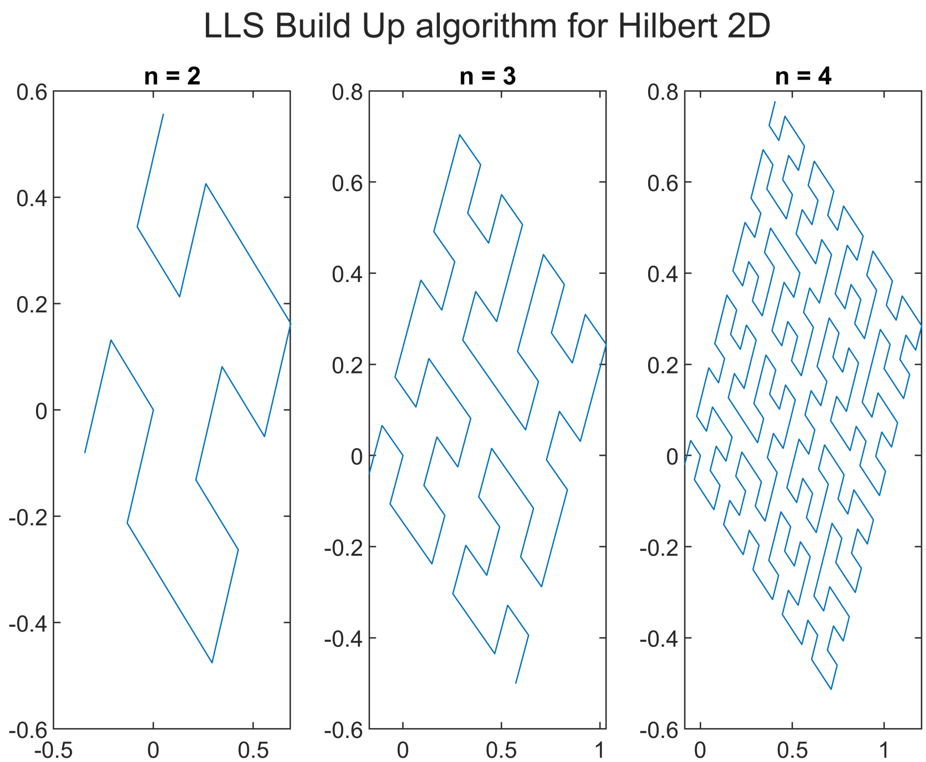
```
plot3(X(:,1), X(:, 2), X(:, 3));
title(ax3,'Set 3');
```



LLS Build Up algorithm on Hilbert 2D and 3D with n = 2, 3, 4

```
% Plots
figure;
t = tiledlayout(1,3);
title(t, "LLS Build Up algorithm for Hilbert 2D");

% 2D with n = 2
d = distance_matrix_hilbert(2, 2);
X = build_up_LLS(d, 2);

ax1 = nexttile;
plot(X(:,1), X(:, 2));
title(ax1,'n = 2');

% 2D with n = 3
d = distance_matrix_hilbert(3, 2);
X = build_up_LLS(d, 2);

ax2 = nexttile;
plot(X(:,1), X(:, 2));
title(ax2,'n = 3');

% 2D with n = 4
d = distance_matrix_hilbert(4, 2);
```

```
X = build_up_LLS(d, 2);

ax3 = nexttile;
plot(X(:,1), X(:, 2));
title(ax3,'n = 4');
```




```
% Plots
figure;
t = tiledlayout(1,3);
title(t, "LLS Build Up algorithm for Hilbert 3D");

% 3D with n = 2
d = distance_matrix_hilbert(2, 3);
X = build_up_LLS(d, 3);

ax1 = nexttile;
plot3(X(:,1), X(:, 2), X(:, 3));
title(ax1,'n = 2');

% 3D with n = 3
d = distance_matrix_hilbert(3, 3);
X = build_up_LLS(d, 3);

ax2 = nexttile;
plot3(X(:,1), X(:, 2), X(:, 3));
title(ax2,'n = 3');
```

```
% 3D with n = 4
d = distance_matrix_hilbert(4, 3);
X = build_up_LLS(d, 3);

ax3 = nexttile;
plot3(X(:,1), X(:, 2), X(:, 3));
title(ax3,'n = 4');
```