Methods

This task uses Latin hypercube sampling in order to model the given system of equations in three-dimensional space. For this task, the parameters (birth rate) and (death rate) are set to fixed values. The following parameters are also fixed. The values of the parameters are: and . Additionally, the tolerance selected for this task was chosen as . After sampling. The values generated must fit the characteristics specified. The characteristics are defined as and . The populations must also be greater than zero (

Latin hypercube sampling was done in three dimensions to generate the samples for the values of . These parameters correspond to food growth, food decay and food consumption. The values for each of the parameters were generated in the range of [0 50], as specified in the task description.

Samples are generated using the Latin hypercube function defined in the script “lhs\_impl.m”. The implementation takes three arguments: the number of samples, the number of dimensions and the range of the samples being generated. The function generates the stratifications before generating the samples. The stratifications are generated by taking the number of samples to be generated and the upper bound of the range of values and dividing them. This gives the minimum and maximum value of each cell.

Each sample was then generated by selecting a random stratum and then generating a uniform random value between the lower and upper bound of the strata boundaries. This was done d times as specified in the number of dimensions for the function. These were then stored in the output matrix which has size of (n, d).

To use the samples to compute the behaviour of the parasite model, it needed to fulfill a set of defined characteristics. The inputs generated using the Latin hypercube sampling method had to be used as inputs to the ordinary differential equation function to compute the values for and This is done in the function “lhs\_system” which has been defined in the script “lhs\_system.m”. This function takes the tolerance, time, a row vector containing the values for , and the matrix containing the samples generated from the Latin hypercube function.

This function then iterates over the number of samples located in the LHS matrix. It calls ode45 using a function containing the equations for the parasite model. The function used is the “model” function defined in the script “lhs\_system.m”. This function takes the arguments tspan, , and and This will then generate a column vector with the samples for and .

The values generated for and then need to be verified to be determined if the samples are successful or unsuccessful. First all the samples generated need to be checked to ensure that the values were greater than zero. Then the values are checked to sure that they fulfill the characteristics defined. If the values fit the characteristics, they are deemed successful. Success is represented in MATLAB as being a value of ‘1’ or ‘2’ whereas an unsuccessful sample is ‘0’. ‘1’ represents a sample that satisfies the characteristic ‘x1’ and ‘2’ represents a sample that satisfies the characteristic ‘x2’. The success of a sample is stored in a column vector as output.

Results and Discussion

To plot the data generated, 100 samples were generated and concatenated with the column vector from the “lhs\_system” function. This was then used with the scatter3 function to generate visualisations of the data. In the first figure, the behaviour of shows a high concentration of points between the groups of and samples. Removing the unsuccessful populations, shows a distinct boundary between and . The number of successful populations is also skewed to the right. This is consistent with expectations as food growth increases, the number of successful populations also increases.

Chart, scatter chart

Description automatically generated

Scatter chart

Description automatically generated

Figure 2. Plot of x1 and x2

Figure 1. Plot of x1, x2 and failed populations

In the third figure which is the same visualisation with a side on view, the relationship between the variables and is made obvious. As increases, there are more successful populations. In contrast, when increases there is an increase in the number of unsuccessful populations. A clear divide can be seen between the two areas of successful and unsuccessful populations. This divide represents the equilibrium between food growth rate and food consumption.

Chart, scatter chart, qr code

Description automatically generated

Figure 3. k3 vs k5

In the orthogonal perspective, it can be concluded that the primary trends that exist in the given system are between the food growth rate and the food consumption rate. As expected, when food decay rate increases, the number of unsuccessful populations also increases. This is also true for the rate of food decay.

Chart, scatter chart

Description automatically generated

In comparison to task 1, the results of the Latin hypercube modelling correspond with those of task 1. The relationships between food growth, food decay and food consumption. In task 1, the relationship between food decay and food growth is the same as the results in task 2. When food decay increases, the number of unsuccessful populations decreases.

This is also true for increases in food consumption. This can be observed in the k3, k5 Latin hypercube figure where a defined boundary is shown between the successful and unsuccessful populations. As such the results of task 2 confirm the results and the expected behaviour of the system in task 1.