1. Introduction

Methods:

The system is analyzed over the time span of [0, 20], and following parameters are fixed: . Setting the differential equations equal to 0, we can solve the equilibrium points as followed:

If then:

If :

We have, the system has two stable state solutions: and .

For part (a) of this task, the dynamics of the numerical solution for different values of delay were analyzed by running the difference scheme equation: for 2000 steps. The initial value is defined on , meaning all with were set to . The *PopGrowDelay.m* function runs the delay growth model over 2000 steps. It takes following parameters: Initial value , Growth rate , Step size , Carrying capacity , delay , and Number of steps. The results were then visualized and interpreted regarding the population size, the relationship between oscillatory peaks, period and the delay term.

The next three parts analyzed the differential equations of the 2D representation of the parasite/food system. The systems is defined on the time span [0, 20] with fixed birth rate = 1, and death rate . We performed multiple parameter sweep processes to analyze the effect of rate of food growth , food decay , and food consumption :

Parameters that do not satisfy following conditions will be ignored:

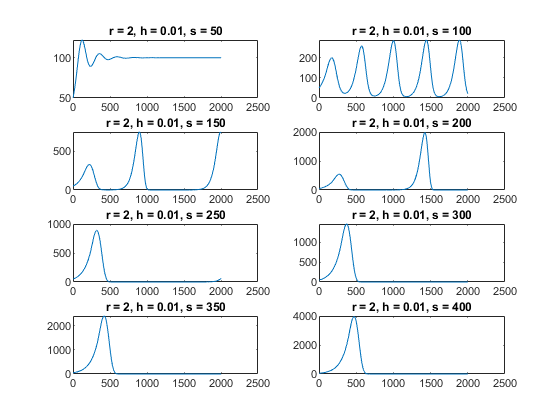
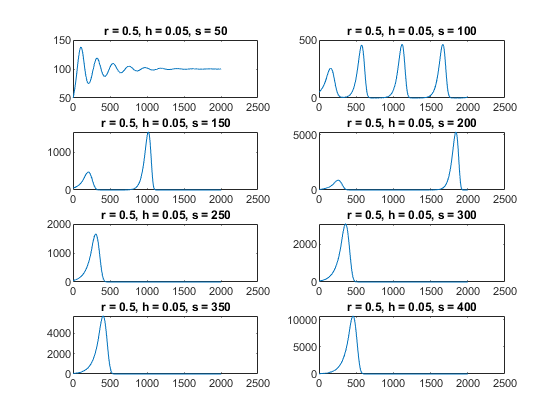
* The population is non-negative at all time.
* The system tend to enter a stable state of either or with and .

MATLAB ODE45 built-in function was used with the *ParasiteGrowthModel.m* function to solve the differential equations of and . The results then are then applied through the above conditions to determine successful parameters. Three parameter sweeps function *SweepK3.m, SweepK3K4.m,* and *SweepK4K5.m* are used for task b, c, and d respectively. We used the increment of 0.5 for all sweeping functions.

Results:

a) Delay Growth Model

After introduce the delay term into the equation, we plotted the population size against steps to see the dynamics of the system [**Figure 1**].



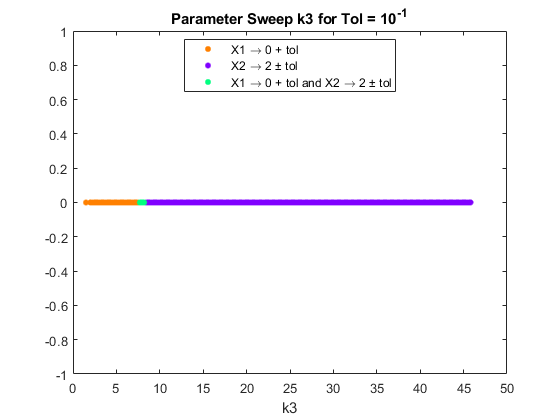
**Figure 1**: Population growth with delay term (s)

For both cases of growth rate *r* and stepsize *h*, with a small delay term (, the population temporarily oscillates and approach the stable point of carrying capacity . As s increases into 100, we observed permanent oscillations around the carrying capacity with a larger period. However, after reaching some delay threshold, the population goes extinct because the degree of the delay is so violent. Even though we see the oscillation peeks are higher, in reality, the population is extinct after it went down to 0.

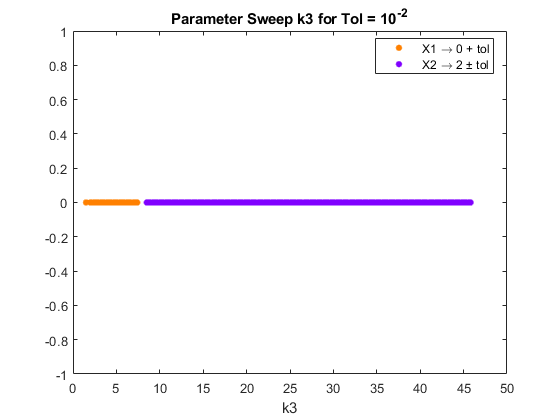
b) Parameter Sweep: Rate of Food Growth

Setting and , we perform parameter sweep for . **Figure 2** shows the results of successful that satisfied following conditions:

* The population is non-negative at all time.
* The system tend to enter a stable state of either or with and .



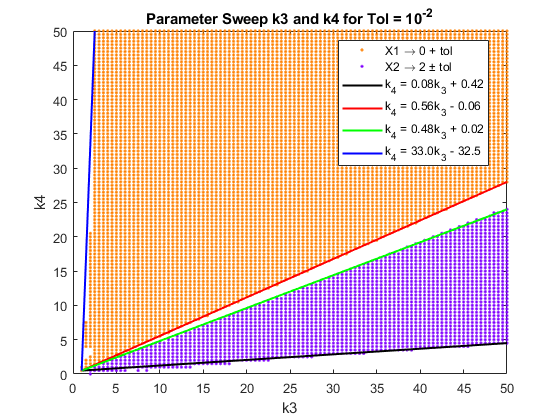
**Figure 2**: Parameter Sweep for k3



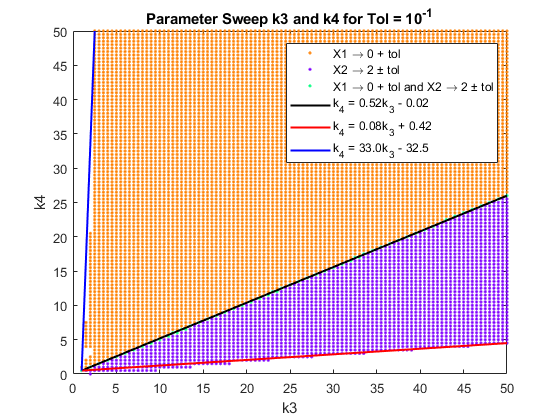
For , is in the range of [1.5, 45.5]. In order to satisfied both condition and , is found to be . For smaller *Tol* (), is still successful in the same but instead, there is no value of satisfied both conditions.

c) Parameter Sweep: Rate of Food Growth and Food Decay

Every parameter is fixed except and , we perform sweeping for both big and small value of .



**Figure 3**: Parameter Sweep for k3 and k4



For , successful parameters must be in the following range:

* for in the range of [2.5, 50].
* If , then the result satisfies both condition and .

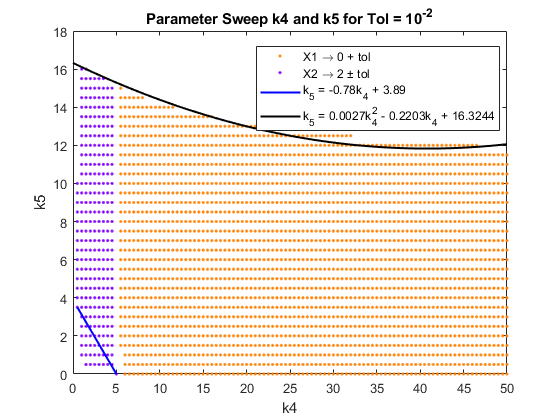
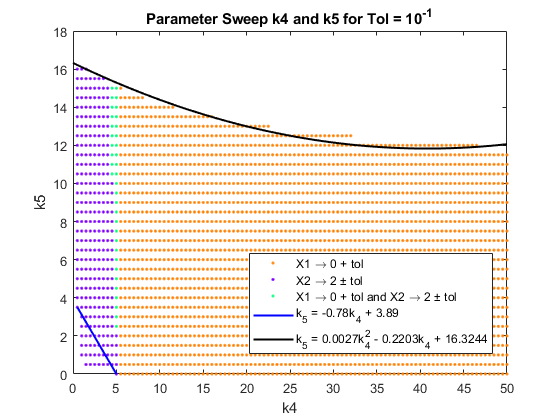
For , and are found to be:

* to satisfy .
* to satisfy .

d) Parameter Sweep: Food Decay and Food Consumption

As shown in **Figure 4**, for all cases, successful must be smaller than . For , needs to be bigger than . For , all lying under the black parabola satisfies the conditions. The result is the same for , except for , can only be equal to 0.

**Figure 4**: Parameter Sweep for k4 and k5



Part (d) set which is in the successful range in the result of part (b). With , and (from part b), the parameters lie in the first case of part (d), which and needs to be bigger than . Substituted into the quadratic function, we have the result of , which is smaller that . Therefore, this result is consistent with the result from part (b).

e) Discussion

As mentioned above, the system equilibrium solutions are and . In the case of , two stages are equal with . This result remained consistent in all parameter sweeps performed in task 1. In the first sweep with one parameter , we observed the merged point of two stages is in the range of , which is approximately equal to . The second sweep showed a linear line separating two states. And the final instance have the merged point at . It can be inferred from the results that:

* If , the rate of growth of the host is doubled the rate of its decay, the population tends to get stable at (0, 2).
* If , , the rate of growth of the host is less than double of the rate of its decay, the population will approach meaning the parasite will go extinct while the host get to .
* If , , the rate of growth of the host is more than double of the rate of its decay, the population will approach