4

Current loop design

a) Find $P_{elec}(s) = \frac{V_s(s)}{V_c(s)}$ with the back emf voltage

we have.

$$V_s = (i_c \times R_s)\,\frac{25k}{5k} = 5(I_c \times R_s)$$

$$\begin{cases} V_s = I_c \quad (1) \\ V_a = K_f \dot{x} + (Z_{LC} + R_c + R_s) I_c \\ 5V_c = V_a \end{cases}$$

$$\Rightarrow \begin{cases} 5V_c = K_f \dot{x} + V_s (Z_{LC} + R_c + R_s) \quad (2) \\ 5V_c = V_a \\ V_s = I_c \end{cases}$$

Also from the mechanic system we have:

$$f - m_1\ddot{x} - K_1 x - b_1 \dot{x} = 0$$

$$\Rightarrow f = m_1\ddot{x} + K_1 x + b_1 \dot{x}$$

$$\Rightarrow K_f I_c(s) = m s^2 X(s) + K_1 X(s) + b_1 s X(s)$$

$$\Rightarrow X(s) = \frac{K_f i_c(s)}{m s^2 + K_1 + b_1 s} \quad (3)$$

(3) -> (2) we have.

$$5V_c = K_f s X(s) + (sL + R_c + R_s) I_c$$

$$\Rightarrow 5V_c = \frac{K_f s\, i_c K_f}{m_1 s^2 + K_1 + b_1 s} + V_s (sL + R_c + R_s)$$

$$\Rightarrow \boxed{\frac{V_s}{V_c} = \frac{5(m_1 s^2 + b_1 s + K_1)}{K_f^2 s + (sL + R_c + R_s)(m_1 s^2 + K_1 + b_1 s)}}$$

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(b) $P_{cl_c}(s) = \dfrac{V_s(s)}{V_c(s)}$ without emf.

$$5V_c = V_a.$$

$$\begin{cases} V_a = I_c\,(Z_L + R_c + R_s) \\ V_s = I_c \end{cases} \Rightarrow \boxed{\dfrac{V_s(s)}{V_c(s)} = \dfrac{5}{SL + R_c + R_s}}$$

(c) at high frequency we have.

$$\frac{V_s(s)}{V_c(s)} \approx \frac{5(m_1 s^2 + b_1 s + K_1)}{(SL + R_s + R_c)(m_1 s^2 + b_1 s + K_1)} = \boxed{\frac{5}{SL + R_c + R_s}}$$

which is equal to ~~the~~ each other

(d) Node Voltage !

$$\frac{0 - V_{set}}{R_1} + \frac{0 - I_c(Z_L + R_s + R_c)}{5R_{eq}} - \frac{0 - V_s}{R_2} = 0$$

$$\Rightarrow \frac{V_{set}}{R_1} = -I_c\left(\frac{Z_L + R_s + R_c}{5_{eq}} + \frac{1}{R_2}\right)$$

$$\Rightarrow \frac{I_c}{V_{set}} = \frac{-5R_{eq}R_2}{R_1R_2(Z_L + R_s + R_c) + 5R_1R_{eq}}$$

$$\Rightarrow \boxed{\frac{I_c}{V_{set}} = \frac{-5(R_3 S C_1 + 1)R_2}{R_1R_2(SL + R_s + R_c)S(SR_3C_1C_2 + C_2 + C_1) + 5R_1(R_3SC_1 + 1)}}$$

$$\Rightarrow \left.\frac{I_c}{V_{set}}\right|_{s=0} = -0.5 \Rightarrow -0.5 = \frac{-R_2}{R_1} \Rightarrow \boxed{R_2 = \frac{1}{2}R_1 = 5K}$$

$5 + \frac{1}{7}$

set $V_s = 0$

$$\Rightarrow \frac{0 - V_c}{r_{eq}} + \frac{0 - V_s}{R_2} \Rightarrow \frac{-V_c(s)}{V_s(s)} = \frac{r_{eq}}{R_2} = \frac{R_3 S C_2 + 1}{[S(R_3 S C_2 C_1 + C_1 + C_2)]R_2}$$

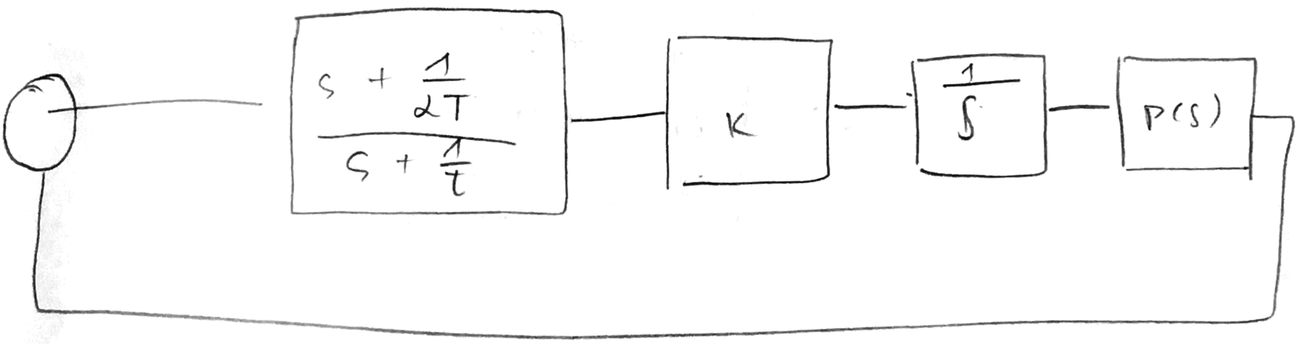
4

d) using lead compensator

$$C_{lead}(s) = K \frac{1+\alpha T s}{\alpha(1+Ts)}$$

$$\frac{-V_c(s)}{V_s(s)} = \frac{R_3C_2\left(s+\frac{1}{R_3C_2}\right)}{sR_2\left(R_3C_2C_1\left(s+\frac{C_1+C_2}{R_3C_2C_1}\right)\right)}$$

$$= \frac{R_3C_2\left(s+\frac{1}{R_3C_2}\right)}{sR_2(C_1+C_2)\left(\frac{R_3C_2C_1}{C_1+C_2}s+1\right)} = \frac{s+\frac{1}{R_3C_2}}{(sR_2)(C_1)\left(s+\frac{C_1+C_2}{R_3C_2C_1}\right)}$$



Plot and looking at the $\frac{1}{s}P(s)$ we have. Phase angle as $w = 6\times10^5$ is -17

$$\Rightarrow \phi_m = 60^\circ - (180-178^\circ) =$$

$$\Rightarrow \alpha = \frac{1+\sin 58^\circ}{1-\sin 58^\circ} = 12.16 \qquad \Rightarrow \alpha T = 6.9\times10^{-6}$$

$$\Rightarrow T = \frac{1}{w\sqrt{\alpha}} = \frac{1}{6\times10^5\times\sqrt{17.3}} = 4.77\times10^{-7}$$

$$\Rightarrow \left| K_{lead}\left(\frac{1+\alpha T s}{\alpha(1+Ts)}\right)\left(\frac{5}{L_c s+R_s+R_c}\right)\right| = 1$$

4 =>

$$\left| K_{lead} \frac{1 + 5.7\times10^{-6} s}{12.16 + 5.7\times10^{-6} s} \times \left(\frac{5}{2\times10^{-4}s + 4.2} \right) \right| = 1$$

$$\Rightarrow \quad K_{lead} = 2.432.$$

We all the parameters and plot the transfer function, we can see that, It did not satisfy the requirement, so we changed K_{lead} by increasing it and some trial experiments, we got:

$$K_{lead} = 5{,}000\,000 \Rightarrow \frac{1}{R_2 C_1} = 5000000$$

$$\Rightarrow C_1 = 4\times10^{-11}\ F.$$

$$\left\{ \begin{array}{l} \dfrac{1}{R_3 C_2} = 2.6\times10^{-6} \\ \\ \dfrac{C_1 + C_2}{R_3 C_2 C_1} = 2.86\times10^{-6} \end{array} \right. \Rightarrow \boxed{\begin{array}{l} R_3 = 125\ K \\ C_1 = 4\times10^{-11}\ F \\ C_2 = 3\times10^{-12}\ F \end{array}}$$

5 FTS Plant System Identification

a) observe that the curve was shifted by -80 dB. so we shifted it back 80 dB, we have the resonance frequency at 10^3 with 20 dB

we have that

$$20\log_{10} M_{pw} = 20$$

$$\Rightarrow M_{pw} = 10$$

For second order system we have that

$$M_{pw} = \left(2\zeta\sqrt{1-\zeta^2}\right)^{-1}$$

$$\Rightarrow 10 = \frac{1}{2\zeta\sqrt{1-\zeta^2}} \Rightarrow \zeta = 0.05$$

5a)

$$\Rightarrow w_n = \frac{w_r}{\sqrt{1-2\zeta^2}} = \frac{10^3}{\sqrt{1-2\zeta^2}} \approx 1000$$

$$\Rightarrow T(s) = \frac{w_n^2}{s^2 + 2\zeta w_n s + w_n^2}$$

now we shifted it back -80 dB.

$$\Rightarrow T(s) = \frac{10^{-4}\, w_n^2}{s^2 + 2\zeta w_n s + w_n^2} = \frac{100}{s^2 + 100s + (1000)^2}$$

we know that:

$$\frac{X(s)}{F(s)} = \frac{1}{m_1 s^2 + b_1 s + k_1} = \frac{\frac{1}{m_1}}{s^2 + \frac{b_1}{m_1} s + \frac{k_1}{m_1}} = \frac{100}{s^2 + 100s + (1000)^2}$$

$$\Rightarrow \boxed{\begin{aligned} m_1 &= 0.01 \\ b_1 &= 100 \\ k_1 &= 10^4 \end{aligned}}$$

5(b,c) Transferfunction: with delay part at high frequency

$$\frac{X(s)}{F(s)} = \frac{1}{m_1 s^2 + b_1 s + k_1}\, e^{(-1.2\times 10^{-5})} = \frac{e^{(-1.2\times 10^{-5})}}{0.01 s^2 + 100s + 10^4}$$

$$\boxed{\frac{X(s)}{F(s)} = \frac{e^{(-1.2\times 10^{-5})}}{0.01 s^2 + 100s + 10^4}}$$

(6a) Plot the bode plot when the system is uncompensator, we saw that the phase was 200^0 which is unstable. now we designed a stable system with desiredable phase margin by choosesing $PM = 35^0$ at $w_g = 10^4$

$$\Rightarrow \phi_M = 35 - (180 - 200)$$

$$\Rightarrow \phi_M = 55^0 \Rightarrow \alpha = \frac{1 + \sin\phi_M}{1 - \sin\phi_M} = 10.1$$

$$\Rightarrow T = \frac{1}{w_g\sqrt{\alpha}} = 3,15 \times 10^{-5}$$

So we was gonna use lead compensator to add more possitive phase

$$\Rightarrow C_{lead} = \frac{1 + \alpha T S}{\alpha + \alpha T S} = \frac{1 + 3,15\times10^{-4} S}{10,1 + 3,15\times10^{-4} S}$$

now we use a controller to reduce the error of step input with K_i so we can increase or decrease phase margin $\Rightarrow$ It lead to increase or decrease overshoot and sensitivity.

$$\Rightarrow G_{mech} = -K_b\left(1 + \frac{K_i}{S}\right)\left(\frac{1 + 3,15\times10^{-4} S}{10,1 + 3,15\times10^{-4} S}\right)$$

which $K_i = \frac{10^4}{5} = 0,2\times10^4$

$$K_b = 2.$$

$$\Rightarrow \boxed{G_{mech} = -2\left(1 + \frac{0,2\times10^4}{S}\right)\left(\frac{1 + 3,15\times10^{-4} S}{10,1 + 3,15\times10^{-4} S}\right)}$$

(6c) Yes, when we decrease K_b; the overshoot will be reduced but the sensitivile will be decreasing

Hoang Tran

505182304

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DC gain from Vset to Ic: -0.50
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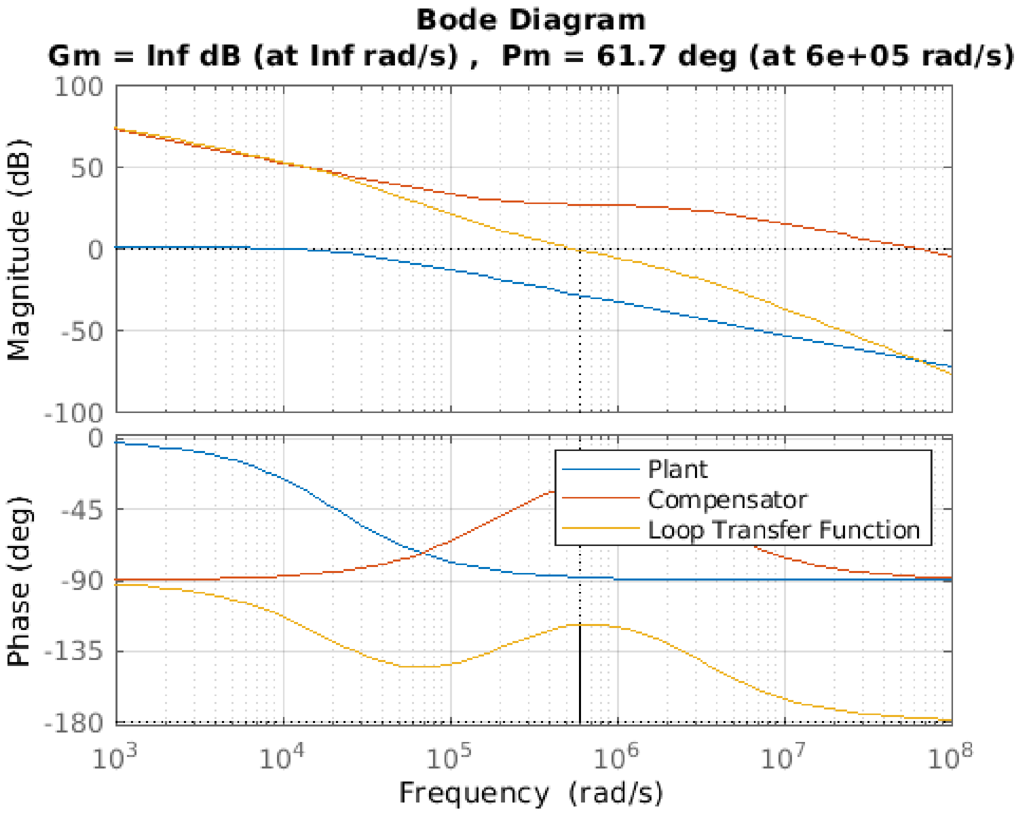
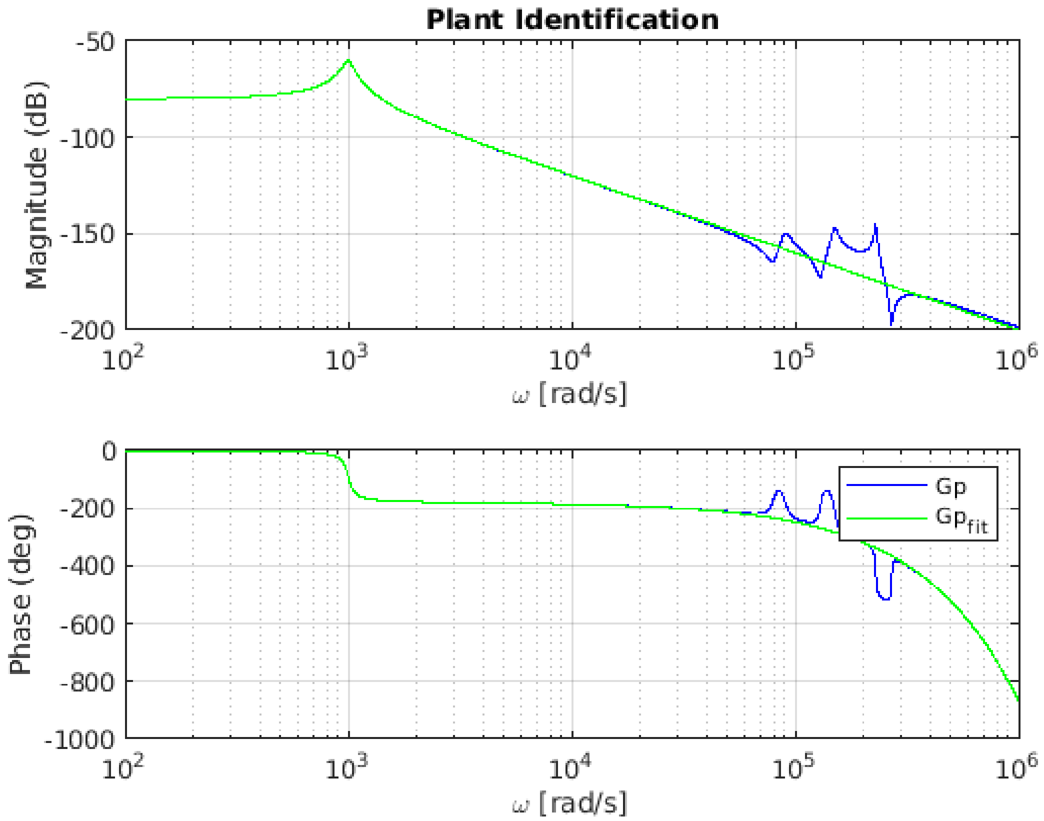
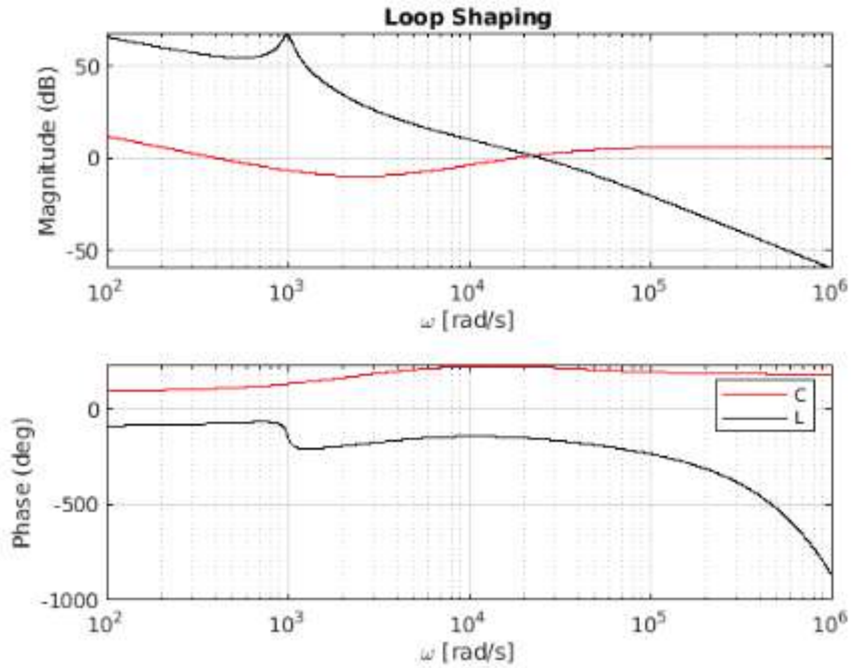
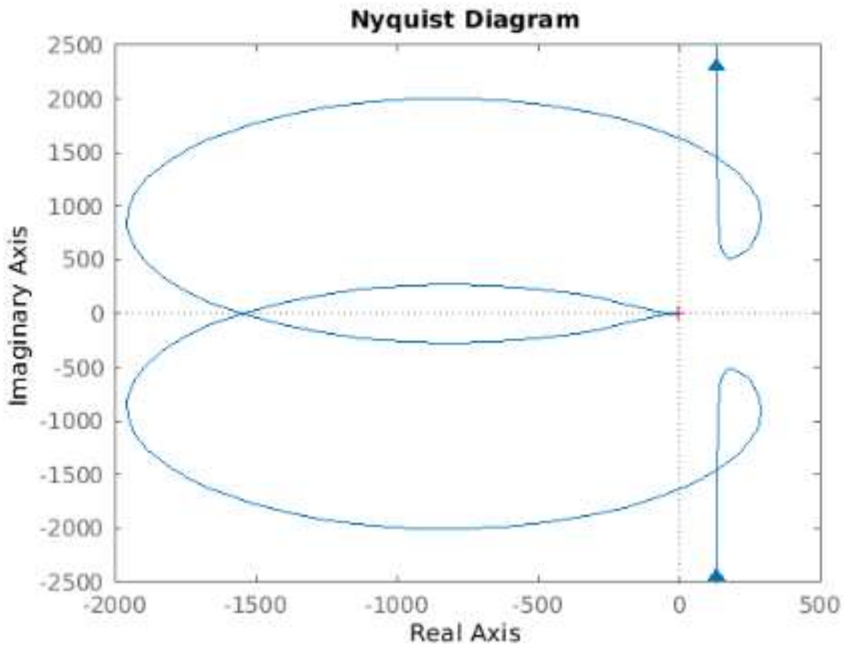


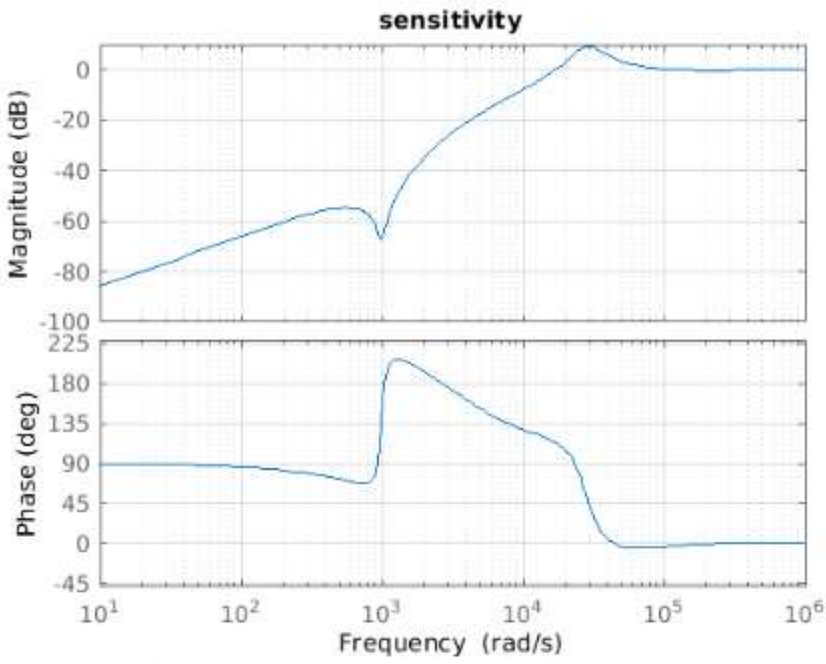


Plant Identification
Magnitude (dB)
Phase (deg)
ω [rad/s]
Gp
Gp_fit



Loop Shaping
Magnitude (dB)
Phase (deg)
ω [rad/s]
C
L



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Maximum Sensitivity (dB):   9.47
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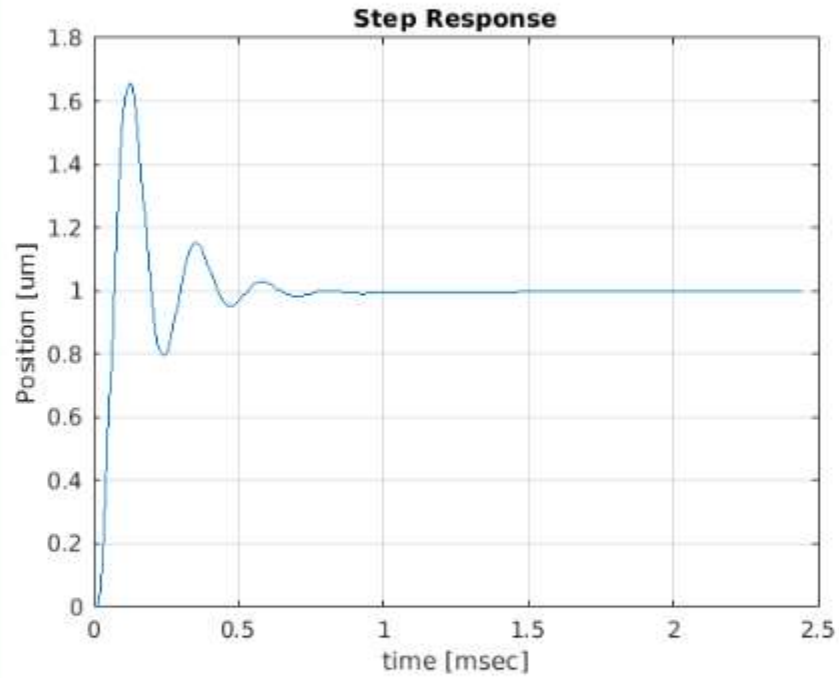
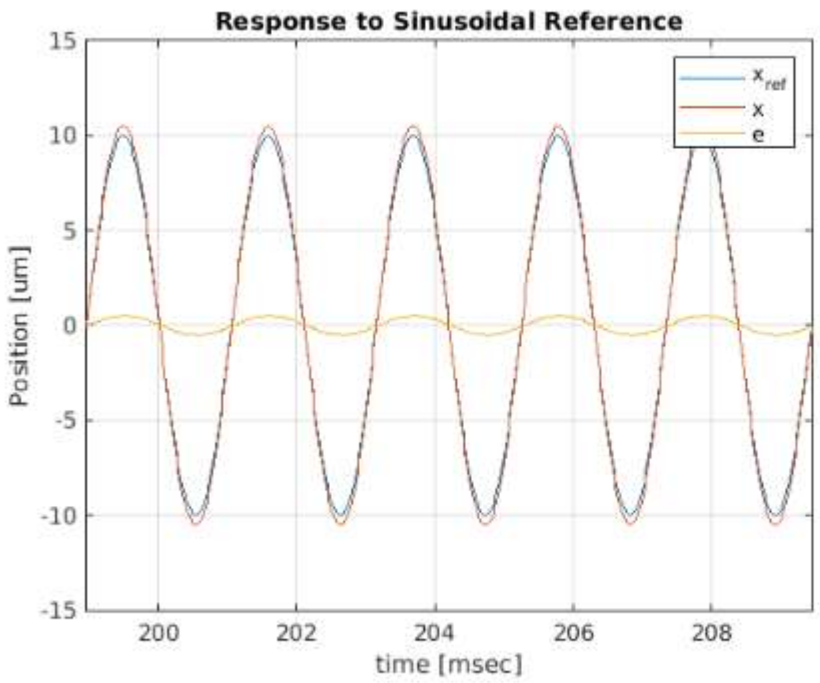
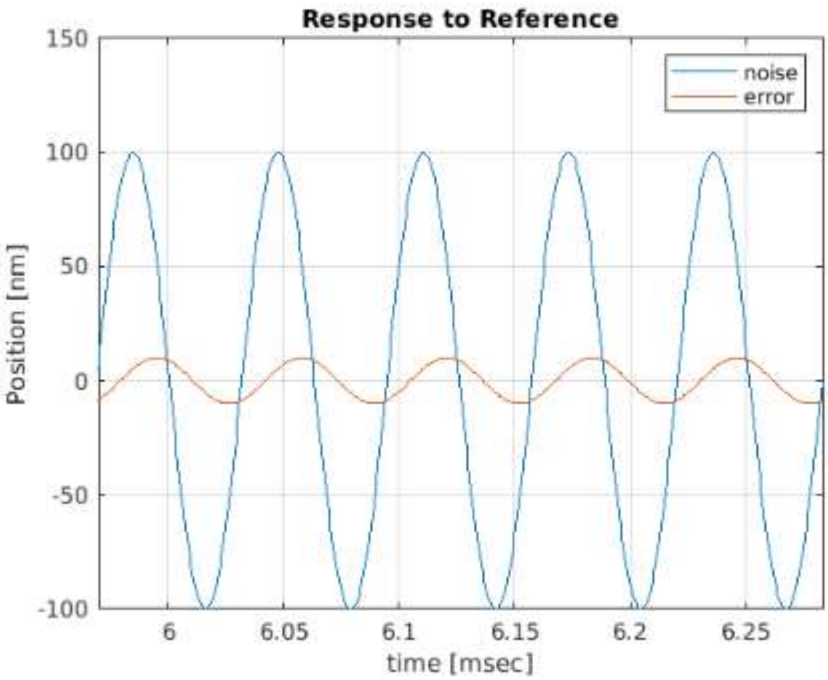


RMS SS step tracking error (nm): 0.1580



RMS SS sinuosoidal tracking error (um): 0.3606



RMS SS noise error (nm): 7.1191