Xử lý tín hiệu số

Chương 4. Phân tích tín hiệu và hệ thống trên miền tần số

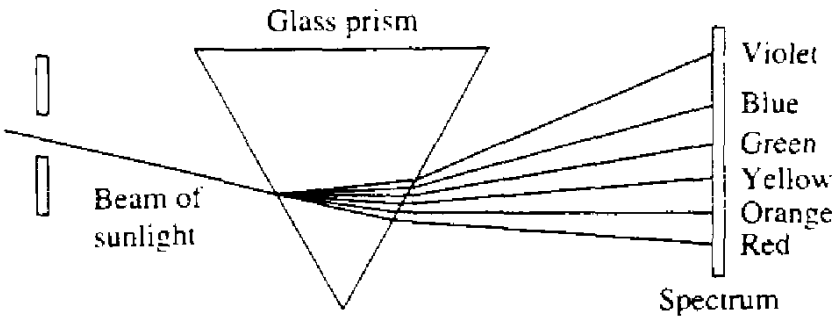
4.1 Biến đổi Fourier của tín hiệu liên tục

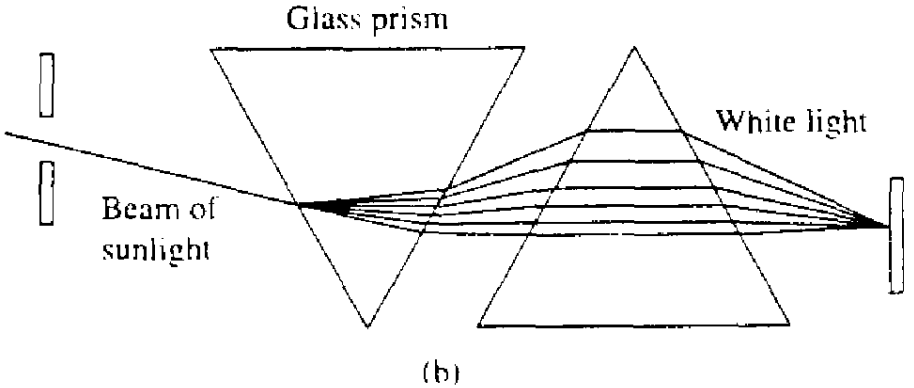
TS. Nguyễn Hồng Quang

Viện Công nghệ thông tin và Truyền thông
Trường Đại học Bách Khoa Hà Nội

4.1. Phân tích tín hiệu liên tục trên miền tần số.

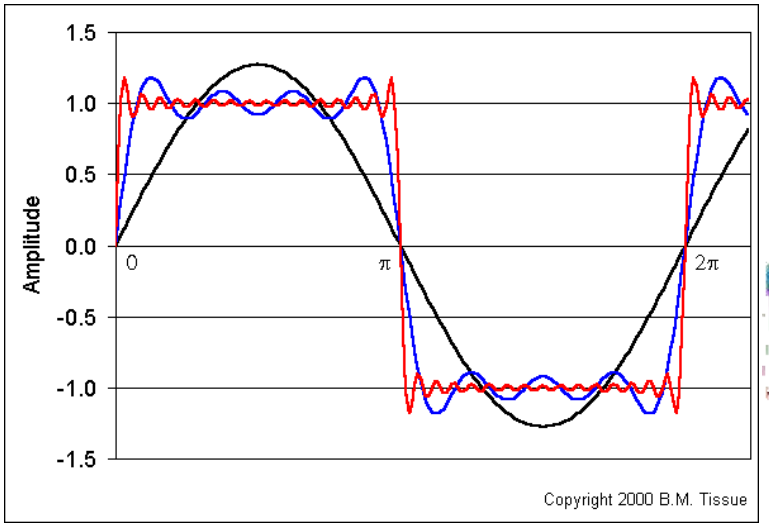
spectrum [Isaac Newton, 1672], Joseph Fourier (1768-1830)

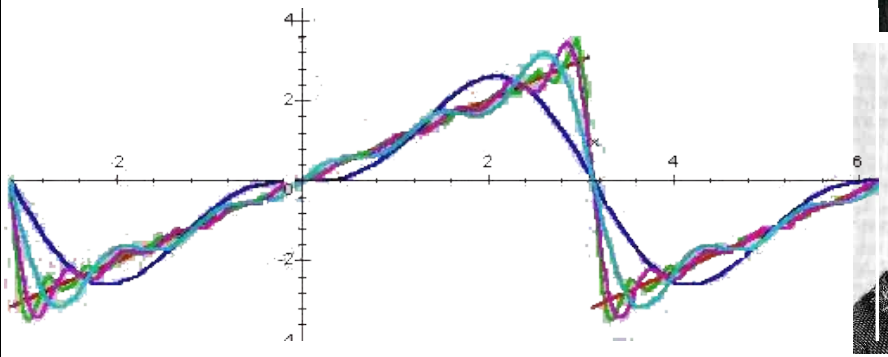


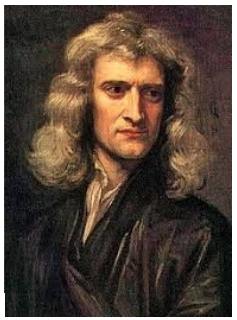


$$x(t) = a_1.x_1(t) + a_2.x_2(t) + \ldots$$

$$x(t) = A_1.\cos(\omega_1.t + \varphi_1) + A_2.\cos(\omega_2.t+\varphi_2) + \ldots$$







Chuỗi Fourier (Fourier Series)
x(t) tuần hòan với chu kỳ Tp, tần số $F_0 = 1/Tp$

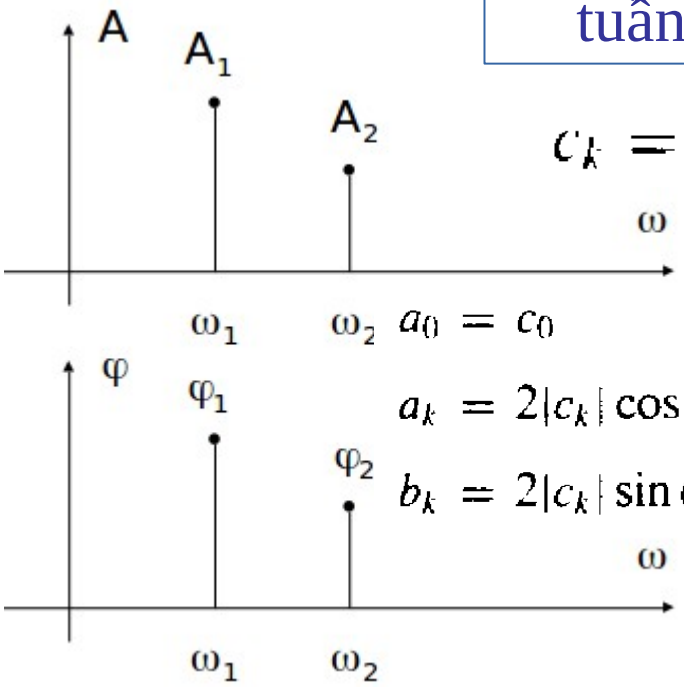
$$x(t) = A_1.\cos(\omega_1.t + \varphi_1) + A_2.\cos(\omega_2.t+\varphi_2) + \ldots$$

- A_1 , ω_1 , φ_1
- A_2 , ω_2 , φ_2
- ...

- $x(t) = A_1.e^{j(\omega 1.t + \varphi 1)} + A_2.e^{j(\omega 2.t+\varphi 2)} + \ldots$

$e^{j\omega_k t}$ $e^{j2\pi k F_0 t}$ $\omega_k = k.\Omega_0 = k.2\pi/Tp$

Tín hiệu thực tuần hòan



$$c_k = |c_k| e^{j\theta_k}$$

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k F_0 t}$$

$$c_l = \frac{1}{T_p} \int_{t_0}^{t_0+T_p} x(t) e^{-j2\pi l F_0 t} dt$$

$$a_0 = c_0$$

$$a_k = 2|c_k| \cos\theta_k$$

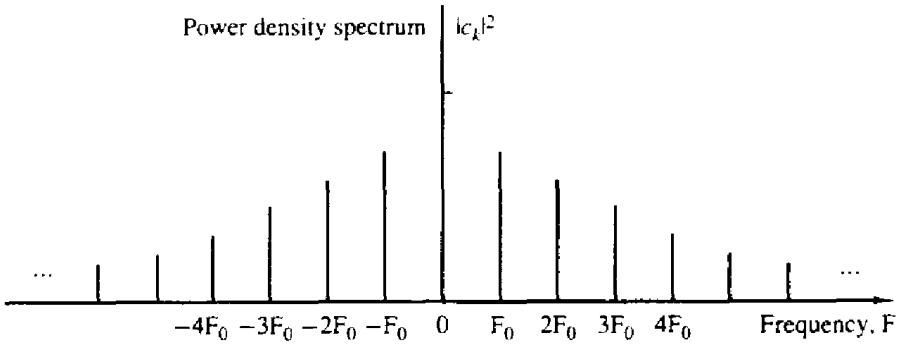
$$b_k = 2|c_k| \sin\theta_k$$

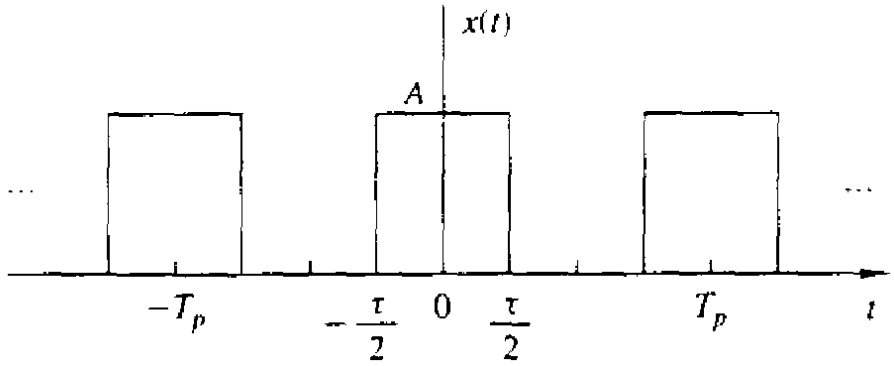
$$x(t) = c_0 + 2\sum_{k=1}^{\infty} |c_k| \cos(2\pi k F_0 t + \theta_k)$$

$$x(t) = a_0 + \sum_{k=1}^{\infty} (a_k \cos 2\pi k F_0 t - b_k \sin 2\pi k F_0 t)$$

Công suất và phổ mật độ của tín hiệu tuần hoàn

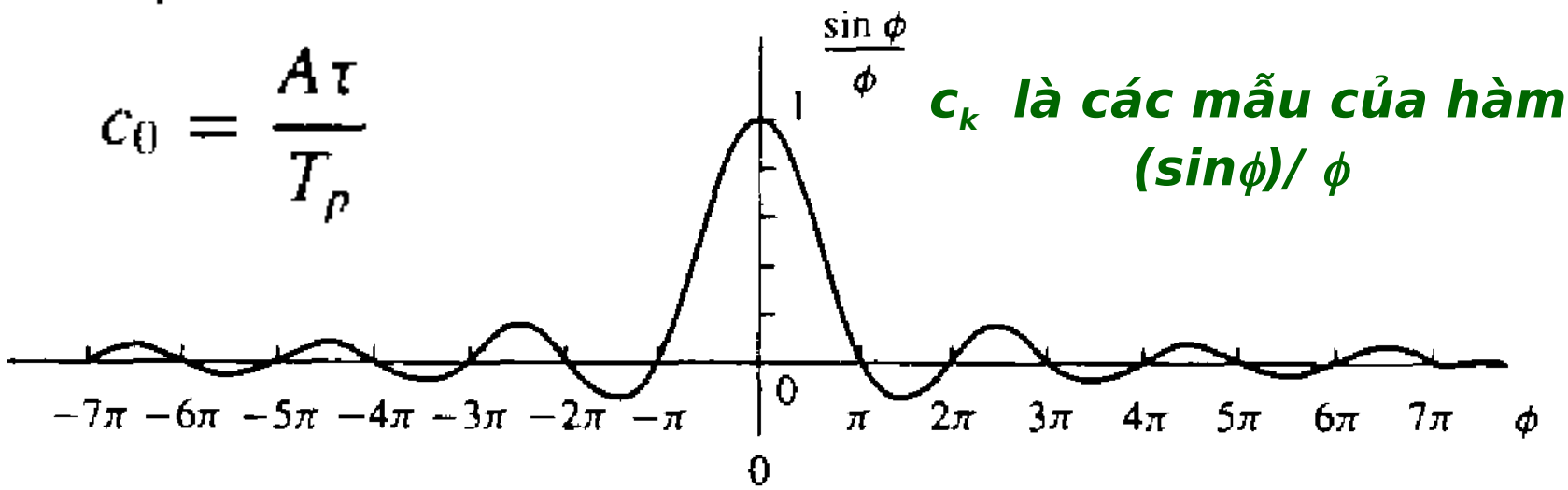
$$P_x = \frac{1}{T_p}\int_{T_p} |x(t)|^2 dt \qquad P_x = \sum_{k=-\infty}^{\infty} |c_k|^2$$





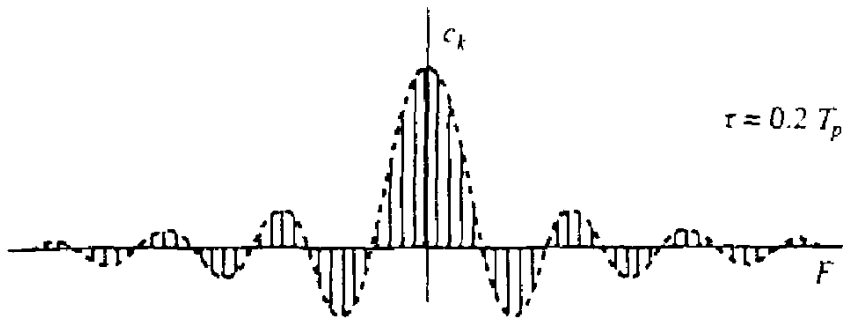
$$c_k = \frac{A\tau}{T_p}\frac{\sin \pi k F_0 \tau}{\pi k F_0 \tau} \qquad k = \pm 1, \pm 2, \ldots$$

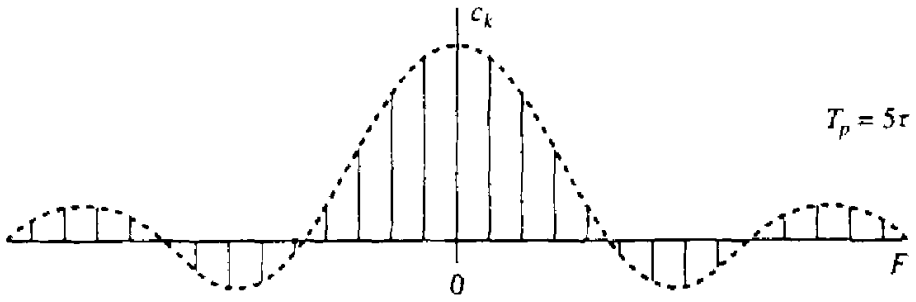
$$c_0 = \frac{A\tau}{T_p}$$

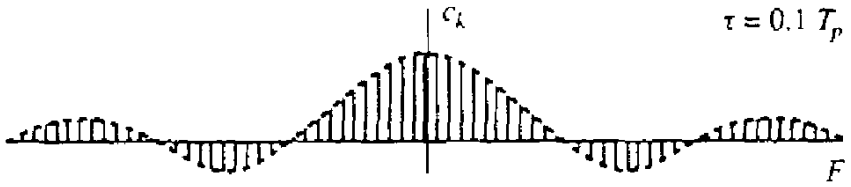


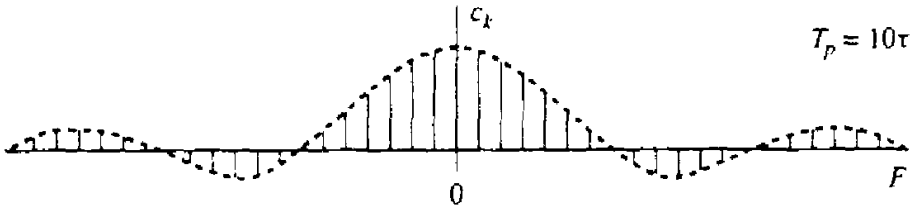
c_k là các mẫu của hàm $(\sin\phi)/\phi$

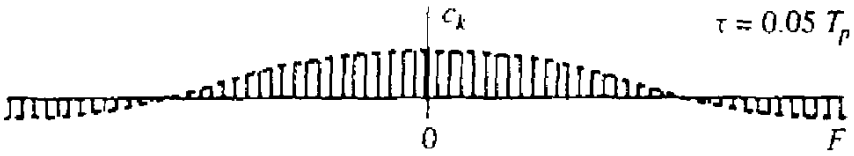
Cố định Tp, thay đổi τ
Cố định T và thay đổi chu kỳ tuần hoàn Tp

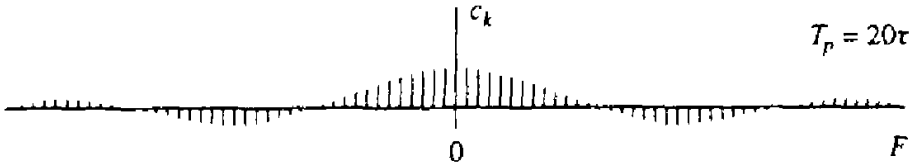












khi Tp → ∞

Phổ của tín hiệu không tuần hoàn chính là đường bao các vạch phổ của tín hiệu tuần hòan tương ứng

Bài tập 4.1

4.1 Consider the full-wave rectified sinusoid in Fig. P4.1.

(a) Determine its spectrum $X_a(F)$.

(b) Compute the power of the signal.

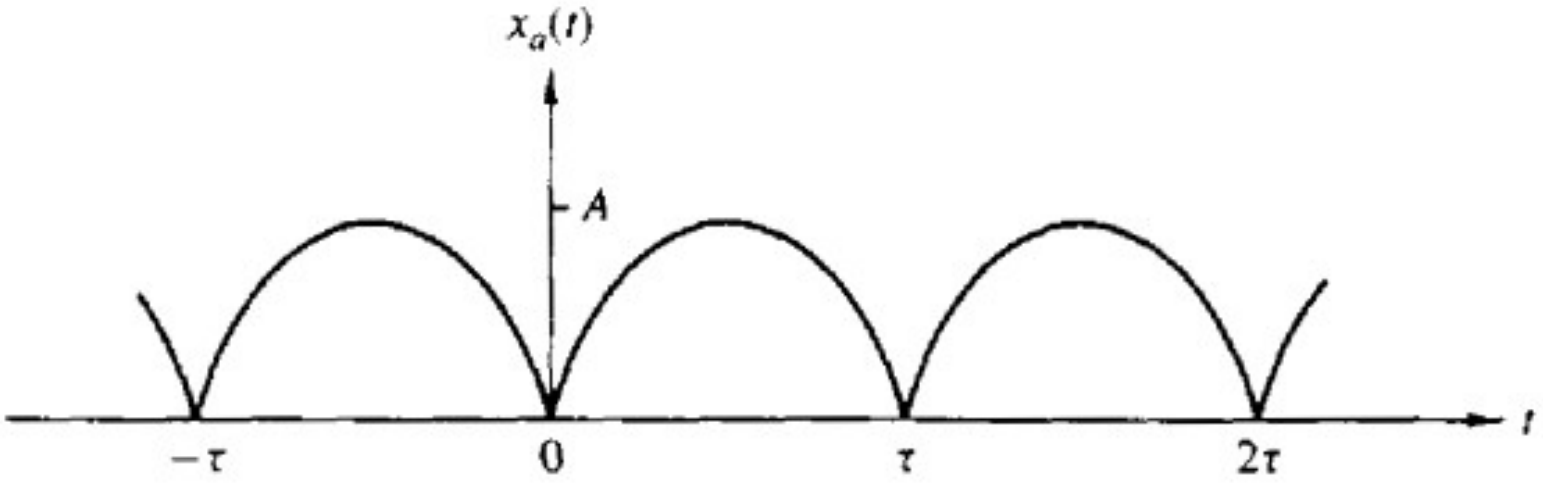
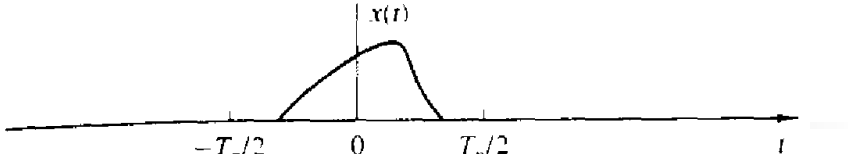


Figure P4.1

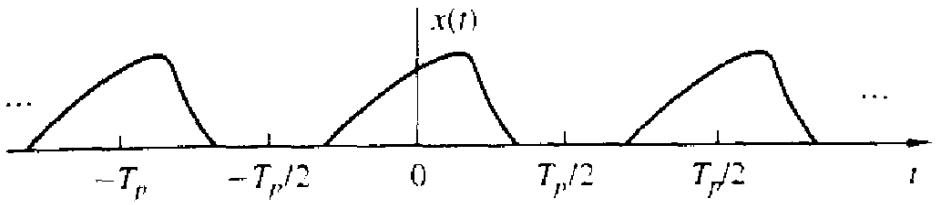
(c) Plot the power spectral density.

(d) Check the validity of Parseval's relation for this signal.

Biến đổi Fourier của tín hiệu liên tục không tuần hòan



$$x(t) = \lim_{T_p \to \infty} x_p(t)$$



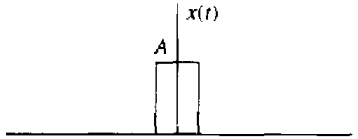
Xác định phổ của x(t) từ phổ của $x_p(t)$ bằng cách tính giới hạn $T_p \to \infty$.

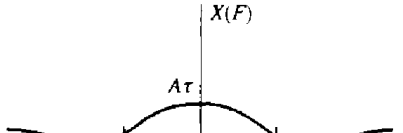
$$x(t) = \int_{-\infty}^{\infty} X(F) e^{j2\pi Ft} dF$$

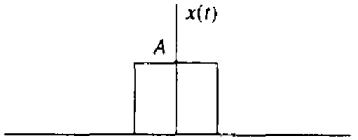
$$X(F) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi Ft} dt$$

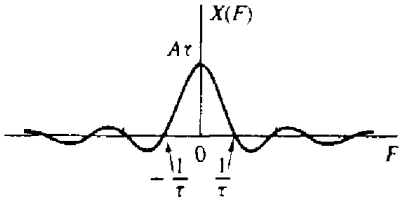
$$x(t) = \begin{cases} A, & |t| \le \tau/2 \\ 0, & |t| > \tau/2 \end{cases}$$

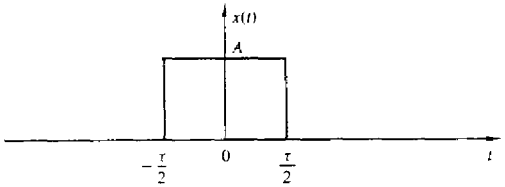
$$X(F) = \int_{-\tau/2}^{\tau/2} A e^{-j2\pi Ft} dt = A\tau \frac{\sin \pi F\tau}{\pi F\tau}$$

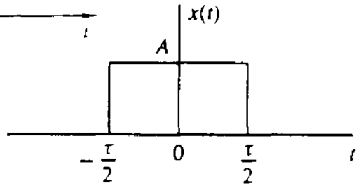


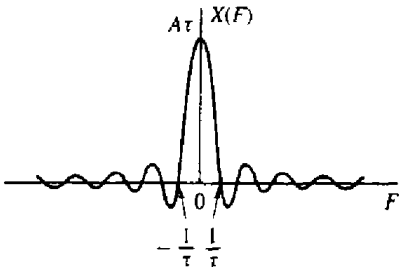


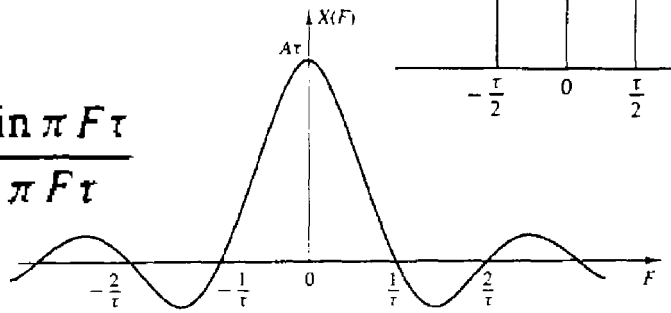












τ giảm

Bài tập

4.2 Compute and sketch the magnitude and phase spectra for the following signals ($a > 0$).

(a) $x_a(t) = \begin{cases} Ae^{-at}, & t \geq 0 \\ 0, & t < 0 \end{cases}$ A = 2, a = 4

(b) $x_a(t) = Ae^{-a|t|}$ A = 2, a = 6

4.3 Consider the signal

$$x(t) = \begin{cases} 1 - |t|/\tau, & |t| \leq \tau \\ 0, & \text{elsewhere} \end{cases}$$

(a) Determine and sketch its magnitude and phase spectra, $|X_a(F)|$ and $\measuredangle X_a(F)$, respectively.

(b) Create a periodic signal $x_p(t)$ with fundamental period $T_p \geq 2\tau$, so that $x(t) = x_p(t)$ for $|t| < T_p/2$. What are the Fourier coefficients c_k for the signal $x_p(t)$?

(c) Using the results in parts (a) and (b), show that $c_k = (1/T_p)X_a(k/T_p)$.