# Supplemental Appendix for: Modelling Stock Dependence using Factor Copulas

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## Online Appendix

#### A Posterior estimation for simulated data

	True value	Estimation	Bias	SE	2.5%	50%	97.5%	T/F	Group
$a_1$	0.078	0.073	0.004	0.009	0.057	0.073	0.092	0	1
$a_2$	0.081	0.098	0.017	0.014	0.071	0.097	0.127	0	2
$a_3$	0.065	0.063	0.003	0.017	0.035	0.061	0.100	0	3
$a_4$	0.090	0.080	0.010	0.011	0.059	0.080	0.102	0	4
$a_5$	0.052	0.046	0.006	0.010	0.028	0.046	0.066	0	5
$a_6$	0.051	0.057	0.006	0.013	0.033	0.056	0.085	0	6
$a_7$	0.071	0.059	0.013	0.009	0.041	0.058	0.078	0	7
$a_8$	0.056	0.061	0.005	0.011	0.040	0.060	0.084	0	8
$a_9$	0.068	0.082	0.014	0.012	0.061	0.081	0.104	0	9
$a_{10}$	0.087	0.072	0.015	0.023	0.035	0.069	0.128	0	10
$b_1$	0.981	0.983	0.002	0.004	0.974	0.984	0.991	0	1
$b_2$	0.955	0.956	0.001	0.011	0.933	0.957	0.972	0	2
$b_3$	0.979	0.976	0.003	0.015	0.940	0.980	0.994	0	3
$b_4$	0.972	0.972	0.000	0.008	0.955	0.973	0.985	0	4
$b_5$	0.979	0.966	0.012	0.012	0.938	0.968	0.983	0	5
$b_6$	0.960	0.957	0.003	0.017	0.914	0.960	0.981	0	6
$b_7$	0.979	0.981	0.002	0.006	0.967	0.982	0.991	0	7
$b_8$	0.984	0.982	0.002	0.007	0.965	0.983	0.993	0	8
$b_9$	0.981	0.964	0.017	0.010	0.941	0.965	0.980	1	9
$b_{10}$	0.965	0.949	0.016	0.048	0.835	0.961	0.990	0	10
$\nu_1$	13.000	12.360	0.640	0.952	10.768	12.239	14.510	0	1
$\nu_2$	17.000	15.166	1.834	1.228	13.132	15.056	17.800	0	2
$\nu_3$	8.000	7.804	0.196	0.748	6.544	7.734	9.505	0	3
$\nu_4$	10.000	10.812	0.812	0.689	9.601	10.758	12.292	0	4
$\nu_5$	17.000	19.036	2.036	1.720	15.944	18.881	22.953	0	5
$\nu_6$	10.000	9.617	0.383	0.593	8.484	9.566	10.880	0	6

$\nu_7$	8.000	7.663	0.337	0.342	7.055	7.648	8.377	0	7
$\nu_8$	10.000	10.306	0.306	0.665	9.135	10.254	11.713	0	8
$ u_9$	15.000	14.074	0.926	1.024	12.252	14.044	16.124	0	9
$ u_{10}$	7.000	7.313	0.313	0.509	6.401	7.290	8.423	0	10
$\gamma_1$	-0.561	-0.534	0.027	0.038	-0.613	-0.532	-0.467	0	1
$\gamma_2$	-0.498	-0.489	0.009	0.039	-0.568	-0.489	-0.412	0	2
$\gamma_3$	-0.128	-0.136	0.008	0.028	-0.195	-0.136	-0.083	0	3
$\gamma_4$	-0.162	-0.202	0.040	0.022	-0.248	-0.201	-0.160	0	4
$\gamma_5$	-0.463	-0.477	0.014	0.039	-0.558	-0.476	-0.403	0	5
$\gamma_6$	-0.255	-0.264	0.009	0.024	-0.315	-0.263	-0.216	0	6
$\gamma_7$	-0.495	-0.471	0.024	0.022	-0.514	-0.470	-0.431	0	7
$\gamma_8$	-0.909	-0.912	0.003	0.048	-1.009	-0.907	-0.823	0	8
$\gamma_9$	-0.036	-0.064	0.028	0.024	-0.116	-0.064	-0.020	0	9
$\gamma_{10}$	-0.576	-0.620	0.044	0.045	-0.711	-0.618	-0.531	0	10
$f_{1,0}$	1.184	1.161	0.023	0.112	0.947	1.153	1.388	0	7
$f_{2,0}$	2.878	2.753	0.124	0.084	2.598	2.750	2.922	0	5
$f_{3,0}$	1.567	1.544	0.023	0.117	1.320	1.542	1.782	0	7
$f_{4,0}$	1.355	1.327	0.028	0.110	1.110	1.325	1.535	0	7
$f_{5,0}$	1.523	1.414	0.109	0.123	1.165	1.416	1.655	0	3
$f_{6,0}$	0.752	0.663	0.089	0.095	0.472	0.666	0.836	0	10
$f_{7,0}$	0.584	0.794	0.210	0.100	0.589	0.794	0.989	1	9
$f_{8,0}$	1.635	1.467	0.168	0.101	1.268	1.465	1.660	0	9
$f_{9,0}$	1.452	1.306	0.146	0.116	1.080	1.307	1.539	0	3
$f_{10,0}$	0.748	0.643	0.105	0.096	0.448	0.647	0.824	0	10
$f_{11,0}$	0.608	0.539	0.070	0.087	0.373	0.537	0.714	0	5
$f_{12,0}$	2.229	2.180	0.048	0.098	1.986	2.180	2.371	0	2
$f_{13,0}$	1.471	1.420	0.051	0.082	1.255	1.419	1.581	0	5
$f_{14,0}$	0.270	0.278	0.008	0.111	0.066	0.277	0.495	0	4
$f_{15,0}$	0.864	0.413	0.451	0.119	0.177	0.411	0.636	1	8
$f_{16,0}$	1.211	1.155	0.056	0.096	0.975	1.153	1.349	0	10
$f_{17,0}$	1.227	1.090	0.138	0.115	0.862	1.092	1.306	0	3
$f_{18,0}$	1.180	1.126	0.054	0.085	0.966	1.125	1.295	0	6
$f_{19,0}$	2.927	3.162	0.235	0.087	2.990	3.163	3.336	1	5
$f_{20,0}$	1.611	1.568	0.043	0.095	1.376	1.569	1.748	0	2
$f_{21,0}$	1.551	1.479	0.072	0.098	1.295	1.479	1.663	0	2
$f_{22,0}$	1.113	1.101	0.012	0.100	0.895	1.103	1.288	0	9
$f_{23,0}$	1.272	1.150	0.122	0.101	0.957	1.149	1.354	0	9
$f_{24,0}$	1.615	1.569	0.046	0.083	1.411	1.568	1.740	0	5
$f_{25,0}$	2.057	1.959	0.099	0.081	1.798	1.958	2.122	0	5
$f_{26,0}$	0.685	0.719	0.034	0.122	0.479	0.722	0.965	0	8
$f_{27,0}$	1.530	1.661	0.132	0.086	1.499	1.662	1.821	0	6
$f_{28,0}$	2.642	2.542	0.100	0.134	2.267	2.546	2.797	0	1
$f_{29,0}$	0.683	0.656	0.027	0.085	0.490	0.658	0.817	0	5
$f_{30,0}$	1.028	0.965	0.062	0.103	0.769	0.966	1.173	0	9
$f_{31,0}$	2.217	2.177	0.040	0.101	1.980	2.180	2.375	0	2
$f_{32,0}$	2.010	2.078	0.068	0.127	1.837	2.078	2.354	0	8
$f_{33,0}$	0.993	1.136	0.142	0.082	0.982	1.135	1.309	0	5
$f_{34,0}$	1.883	1.698	0.185	0.096	1.504	1.700	1.891	0	10
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f	1.753	1.660	0.093	0.113	1.437	1.661	1.893	0	4
$f_{35,0}$	0.316	0.296	0.093 $0.020$	0.113 $0.106$	0.083	0.298	0.503	0	9
$f_{36,0}$	0.510 $0.668$		0.020 $0.124$						6
$f_{37,0}$		0.544		0.083	0.373	0.541	0.710	0	9
$f_{38,0}$	1.787	1.672	0.115	0.100	1.470	1.670	1.875	0	
$f_{39,0}$	0.455	0.338	0.117	0.085	0.170	0.337	0.505	0	6
$f_{40,0}$	1.525	1.542	0.017	0.082	1.382	1.541	1.705	0	6
$f_{41,0}$	1.519	1.328	0.192	0.081	1.165	1.326	1.490	1	5
$f_{42,0}$	1.052	0.912	0.141	0.114	0.694	0.915	1.135	0	8
$f_{43,0}$	0.973	0.851	0.122	0.115	0.620	0.851	1.073	0	3
$f_{44,0}$	0.414	0.401	0.014	0.084	0.232	0.399	0.560	0	6
$f_{45,0}$	1.938	2.016	0.078	0.117	1.788	2.015	2.246	0	7
$f_{46,0}$	0.314	0.155	0.160	0.085	-0.022	0.158	0.315	0	6
$f_{47,0}$	0.225	0.100	0.125	0.130	-0.164	0.102	0.346	0	8
$f_{48,0}$	0.323	0.233	0.090	0.112	0.024	0.230	0.462	0	4
$f_{49,0}$	0.950	0.904	0.046	0.082	0.734	0.905	1.059	0	6
$f_{50,0}$	1.020	1.088	0.068	0.109	0.874	1.090	1.299	0	7
$f_{51,0}$	1.179	1.260	0.081	0.099	1.063	1.260	1.456	0	9
$f_{52,0}$	2.454	2.522	0.068	0.096	2.344	2.520	2.715	0	9
$f_{53,0}$	0.672	0.651	0.021	0.111	0.433	0.647	0.869	0	4
$f_{54,0}$	1.324	1.491	0.167	0.122	1.252	1.486	1.739	0	8
$f_{55,0}$	0.832	0.935	0.104	0.085	0.769	0.933	1.100	0	6
$f_{56,0}$	2.039	2.237	0.198	0.125	1.995	2.239	2.485	0	8
$f_{57,0}$	2.047	1.751	0.296	0.110	1.532	1.750	1.953	1	4
$f_{58,0}$	1.803	1.702	0.101	0.112	1.489	1.699	1.932	0	4
$f_{59,0}$	0.251	0.252	0.001	0.132	-0.012	0.253	0.511	0	1
$f_{60,0}$	1.202	1.338	0.136	0.081	1.181	1.338	1.500	0	5
$f_{61,0}$	1.432	1.365	0.067	0.125	1.124	1.362	1.615	0	1
$f_{62,0}$	0.351	0.184	0.167	0.100	-0.007	0.186	0.377	0	2
$f_{63,0}$	1.481	1.398	0.082	0.131	1.156	1.394	1.651	0	1
$f_{64,0}$	0.391	0.757	0.366	0.134	0.481	0.761	1.012	1	1
$f_{65,0}$	2.924	2.881	0.043	0.085	2.725	2.879	3.046	0	6
$f_{66,0}$	1.416	1.487	0.071	0.108	1.273	1.484	1.695	0	4
$f_{67,0}$	0.859	0.736	0.123	0.098	0.550	0.735	0.929	0	2
$f_{68,0}$	0.527	0.666	0.139	0.110	0.450	0.668	0.889	0	7
$f_{69,0}$	1.802	1.514	0.288	0.135	1.257	1.514	1.788	1	1
$f_{70,0}$	0.592	0.651	0.060	0.082	0.494	0.653	0.817	0	6
$f_{71,0}$	1.231	1.261	0.031	0.135	0.998	1.263	1.527	0	1
$f_{72,0}$	1.762	1.656	0.107	0.102	1.446	1.654	1.856	0	9
$f_{73,0}$	0.664	0.762	0.098	0.112	0.544	0.763	0.979	0	7
$f_{74,0}$	1.874	1.899	0.025	0.096	1.702	1.901	2.080	0	10
$f_{75,0}$	2.379	2.303	0.076	0.081	2.145	2.304	2.462	0	5
$f_{76,0}$	0.406	0.319	0.087	0.110	0.106	0.320	0.535	0	4
$f_{77,0}$	1.785	1.718	0.067	0.113	1.493	1.717	1.936	0	7
$f_{78,0}$	0.337	0.414	0.077	0.115	0.188	0.411	0.654	0	7
$f_{79,0}$	2.123	2.002	0.121	0.111	1.801	1.999	2.218	0	4
$f_{80,0}$	0.630	0.650	0.020	0.121	0.399	0.656	0.869	0	3
$f_{81,0}$	0.647	0.616	0.031	0.098	0.420	0.613	0.805	0	9
$f_{82,0}$	0.378	0.207	0.171	0.133	-0.075	0.210	0.455	0	1
J 04,0	0.010	J. <b>_</b> J.	V	0.100	0.0.0	U.=±U	0.100	•	_

$f_{83,0}$	1.208	1.143	0.065	0.137	0.870	1.149	1.404	0	1
$f_{84,0}$	1.591	1.628	0.037	0.105	1.437	1.626	1.826	0	2
$f_{85,0}$	1.056	1.138	0.082	0.083	0.985	1.137	1.312	0	5
$f_{86,0}$	1.198	1.222	0.025	0.135	0.958	1.218	1.491	0	1
$f_{87,0}$	0.233	0.344	0.112	0.115	0.128	0.344	0.568	0	4
$f_{88,0}$	0.501	0.480	0.021	0.102	0.302	0.476	0.688	0	10
$f_{89,0}$	1.787	1.894	0.107	0.116	1.658	1.897	2.116	0	7
$f_{90,0}$	1.047	1.162	0.114	0.098	0.964	1.161	1.345	0	2
$f_{91,0}$	0.305	0.233	0.071	0.110	0.017	0.232	0.443	0	7
$f_{92,0}$	0.247	0.275	0.028	0.098	0.076	0.275	0.470	0	2
$f_{93,0}$	1.532	1.488	0.044	0.139	1.219	1.486	1.767	0	1
$f_{94,0}$	0.285	0.325	0.040	0.125	0.058	0.331	0.562	0	8
$f_{95,0}$	2.927	2.897	0.030	0.104	2.686	2.901	3.097	0	9
$f_{96,0}$	0.211	0.273	0.062	0.103	0.072	0.274	0.488	0	2
$f_{97,0}$	0.966	1.111	0.146	0.115	0.891	1.113	1.339	0	4
$f_{98,0}$	1.518	1.499	0.019	0.101	1.304	1.500	1.699	0	9
$f_{99,0}$	0.317	0.255	0.062	0.120	0.036	0.255	0.501	0	8
$f_{100,0}$	2.628	2.464	0.164	0.121	2.239	2.461	2.703	0	7

## B The number of assets in each group sector

Table 2: Stock Groups

Group	SIC code	Industry	Number of Firms
1	1000 - 1999	Mining and Construction	5
2	2000 - 2099	Food and Beverage	6
3	2830 - 2839	Pharmacy	5
4	2800 - 2899	Plastic Material and Plant Chemical	8
5	2100 - 2999	Textile and Papers	12
6	3000 - 3599	Steels	13
7	3600 - 3799	Home Appliance and Automobile	14
8	3800 - 3999	Electronics	12
9	4000 - 4799	Transportation	12
10	4800 - 4899	Telecommunication	7
11	4900 - 5999	Retail and Distribution	8
12	6300 - 6399	Insurance Firms	14
13	6000 - 6999	Finance (not include Insurance)	17
14	7000 - 8999	Services and Others	17

## C Posterior estimation for multiple-group factor copula models

	Gaussian Cop.		Studer	nt Cop.	MGSt Cop.		
	Est.	Se.	Est.	Se.	Est.	Se.	
$\overline{a_1}$	0.031	(0.005)	0.035	(0.006)	0.034	(0.006)	
$a_2$	0.042	(0.009)	0.045	(0.009)	0.045	(0.009)	

```
0.096
                             0.079
                                      (0.031)
                                                   0.049
                                                            (0.019)
                 (0.022)
a_3
        0.043
                 (0.008)
                             0.037
                                       (0.007)
                                                   0.037
                                                            (0.007)
a_4
a_5
        0.057
                 (0.010)
                             0.043
                                       (0.006)
                                                   0.045
                                                            (0.006)
        0.031
                 (0.003)
                             0.032
                                       (0.003)
                                                   0.033
                                                            (0.003)
a_6
        0.083
                 (0.014)
                             0.041
                                       (0.009)
                                                   0.038
                                                            (0.012)
a_7
        0.140
                 (0.015)
                             0.135
                                       (0.017)
                                                   0.132
                                                            (0.017)
a_8
        0.050
                             0.042
                                      (0.007)
                                                   0.042
                                                            (0.007)
                 (0.007)
a_9
        0.103
                 (0.019)
                             0.061
                                       (0.014)
                                                   0.065
                                                            (0.017)
a_{10}
                             0.023
                                      (0.003)
                                                            (0.004)
        0.023
                 (0.004)
                                                   0.023
a_{11}
        0.066
                 (0.014)
                             0.052
                                       (0.007)
                                                   0.054
                                                            (0.007)
a_{12}
        0.140
                 (0.013)
                             0.051
                                       (0.008)
                                                   0.052
                                                            (0.009)
a_{13}
        0.030
                 (0.003)
                             0.034
                                       (0.004)
                                                   0.034
                                                            (0.003)
a_{14}
b_1
        0.996
                 (0.002)
                             0.996
                                       (0.002)
                                                   0.996
                                                            (0.002)
        0.989
                 (0.006)
                             0.989
                                       (0.006)
                                                   0.991
                                                            (0.005)
b_2
b_3
        0.897
                 (0.065)
                             0.902
                                       (0.146)
                                                   0.976
                                                            (0.023)
                                       (0.005)
        0.981
                 (0.008)
                             0.988
                                                   0.989
                                                            (0.005)
b_4
        0.972
                 (0.011)
                             0.986
                                       (0.004)
                                                   0.985
                                                            (0.004)
b_5
b_6
        0.993
                 (0.001)
                             0.995
                                       (0.001)
                                                   0.995
                                                            (0.001)
        0.851
                 (0.043)
                             0.960
                                       (0.017)
                                                   0.957
                                                            (0.036)
b_7
        0.611
b_8
                 (0.053)
                             0.585
                                       (0.064)
                                                   0.597
                                                            (0.060)
        0.968
                 (0.008)
                             0.978
                                       (0.007)
                                                   0.974
                                                            (0.008)
b_9
        0.910
                             0.973
                                       (0.014)
                                                   0.969
                                                            (0.019)
b_{10}
                 (0.029)
b_{11}
        0.993
                 (0.004)
                             0.997
                                       (0.002)
                                                   0.997
                                                            (0.002)
                             0.972
                                                            (0.007)
        0.936
                 (0.028)
                                       (0.008)
                                                   0.971
b_{12}
b_{13}
        0.857
                 (0.022)
                             0.984
                                       (0.006)
                                                   0.982
                                                            (0.007)
b_{14}
        0.990
                 (0.002)
                             0.991
                                       (0.002)
                                                   0.991
                                                            (0.002)
                            12.562
                                                  12.971
\nu_1
                                       (1.215)
                                                            (1.389)
                            13.103
                                      (1.207)
                                                  14.069
                                                            (1.554)
\nu_2
                             7.766
                                       (0.596)
                                                  35.857
                                                            (5.343)
\nu_3
                            14.271
                                       (1.041)
                                                  15.368
                                                            (1.504)
\nu_4
                            14.916
                                       (0.906)
                                                  15.248
                                                            (1.062)
\nu_5
                            17.733
                                       (1.005)
                                                  17.982
                                                            (1.104)
\nu_6
                            15.883
                                                  17.012
                                                            (1.036)
                                       (0.804)
\nu_7
                            17.344
                                       (1.106)
                                                  17.560
                                                            (1.133)
\nu_8
                             9.287
                                                  10.220
                                       (0.388)
                                                            (1.028)
\nu_9
                             8.134
                                       (0.470)
                                                   8.121
                                                            (0.515)
\nu_{10}
                            21.253
                                                  21.196
                                                            (1.885)
                                       (1.963)
\nu_{11}
                             9.792
                                                   9.845
                                       (0.375)
                                                            (0.402)
\nu_{12}
                             9.483
                                       (0.356)
                                                            (0.345)
                                                   9.405
\nu_{13}
\nu_{14}
                            20.681
                                       (1.013)
                                                  21.333
                                                            (1.151)
                                                  -0.109
                                                            (0.075)
\gamma_1
                                                  -0.237
\gamma_2
                                                            (0.075)
                                                  -1.689
                                                            (0.163)
\gamma_3
                                                  -0.202
                                                            (0.073)
\gamma_4
                                                  -0.146
                                                            (0.044)
\gamma_5
                                                  -0.173
                                                            (0.048)
\gamma_6
                                                  -0.242
                                                            (0.043)
\gamma_7
                                                  -0.195
                                                            (0.043)
\gamma_8
```

```
-0.214
                                                               (0.089)
 \gamma_9
                                                     -0.075
                                                               (0.035)
\gamma_{10}
\gamma_{11}
                                                     -0.208
                                                               (0.055)
                                                     -0.109
                                                               (0.037)
\gamma_{12}
                                                     -0.036
                                                               (0.029)
\gamma_{13}
                                                     -0.318
                                                               (0.040)
\gamma_{14}
                  (0.089)
                               1.849
                                         (0.094)
                                                      1.831
                                                               (0.102)
         1.809
f_{1,0}
        2.075
                  (0.060)
                               2.197
                                         (0.074)
                                                      2.205
                                                               (0.074)
f_{2,0}
         1.922
                  (0.045)
                               2.016
                                         (0.051)
                                                      1.991
                                                               (0.050)
f_{3,0}
         1.641
                  (0.109)
                               1.673
                                         (0.132)
                                                      1.665
                                                               (0.135)
f_{4,0}
         1.188
                  (0.062)
                               1.261
                                         (0.072)
                                                      1.266
                                                               (0.110)
f_{5,0}
f_{6,0}
         1.239
                  (0.076)
                               1.302
                                         (0.088)
                                                      1.286
                                                               (0.092)
f_{7,0}
        0.959
                  (0.069)
                               1.000
                                         (0.075)
                                                     0.987
                                                               (0.070)
         1.804
                               1.849
                                         (0.076)
                                                      1.844
                                                               (0.075)
f_{8,0}
                  (0.060)
f_{9,0}
        2.436
                  (0.055)
                               2.520
                                         (0.101)
                                                      2.518
                                                               (0.095)
         2.144
                  (0.050)
                               2.229
                                         (0.059)
                                                      2.199
                                                               (0.063)
f_{10,0}
         1.298
                               1.375
                                         (0.069)
                                                      1.432
                                                               (0.093)
f_{11,0}
                  (0.062)
         1.884
                  (0.050)
                               2.012
                                         (0.061)
                                                      2.013
                                                               (0.060)
f_{12,0}
         1.235
                               1.302
                                         (0.075)
                                                      1.294
                                                               (0.072)
f_{13,0}
                  (0.063)
         1.350
                                                      1.442
f_{14,0}
                  (0.058)
                               1.452
                                         (0.107)
                                                               (0.097)
         1.249
                  (0.108)
                               1.333
                                         (0.133)
                                                      1.326
                                                               (0.141)
f_{15,0}
         1.476
                               1.493
                                         (0.125)
                                                      1.479
                                                               (0.131)
                  (0.113)
f_{16,0}
f_{17,0}
         1.718
                  (0.061)
                               1.797
                                         (0.073)
                                                      1.793
                                                               (0.075)
                               1.472
                                                      1.454
         1.416
                  (0.061)
                                         (0.088)
                                                               (0.082)
f_{18,0}
        2.152
                  (0.085)
                               2.244
                                         (0.102)
                                                      2.230
                                                               (0.099)
f_{19,0}
f_{20,0}
         1.349
                  (0.055)
                               1.399
                                         (0.061)
                                                      1.396
                                                               (0.062)
                                         (0.134)
                                                      1.469
f_{21,0}
         1.407
                  (0.108)
                               1.464
                                                               (0.129)
f_{22,0}
         1.656
                  (0.108)
                               1.730
                                         (0.131)
                                                      1.718
                                                               (0.131)
         1.458
                  (0.050)
                               1.508
                                         (0.051)
                                                      1.497
                                                               (0.051)
f_{23,0}
                                                      1.304
         1.298
                  (0.087)
                               1.311
                                         (0.095)
                                                               (0.100)
f_{24,0}
f_{25,0}
         1.769
                  (0.051)
                               1.826
                                         (0.059)
                                                      1.810
                                                               (0.060)
         1.843
                  (0.050)
                               1.883
                                         (0.057)
                                                      1.878
                                                               (0.062)
f_{26,0}
         1.905
                                                      2.025
                  (0.084)
                               2.014
                                         (0.098)
                                                               (0.097)
f_{27,0}
         1.632
                               1.732
                                         (0.076)
                                                      1.709
                                                               (0.067)
f_{28,0}
                  (0.068)
f_{29,0}
         1.843
                  (0.105)
                               1.953
                                         (0.129)
                                                      1.933
                                                               (0.130)
f_{30,0}
         1.789
                  (0.056)
                               1.798
                                         (0.095)
                                                      1.771
                                                               (0.089)
         1.316
                               1.389
                                                      1.433
                                                               (0.099)
f_{31,0}
                  (0.061)
                                         (0.067)
f_{32,0}
         1.472
                  (0.082)
                               1.566
                                         (0.098)
                                                      1.577
                                                               (0.096)
         1.831
                               1.972
                                                      1.958
                                                               (0.075)
f_{33,0}
                  (0.060)
                                         (0.075)
f_{34,0}
         1.292
                  (0.078)
                               1.359
                                         (0.097)
                                                      1.356
                                                               (0.100)
         1.341
                               1.430
                                         (0.073)
                                                      1.421
                                                               (0.071)
f_{35,0}
                  (0.063)
         2.230
                                                      2.379
f_{36,0}
                  (0.055)
                               2.368
                                         (0.073)
                                                               (0.075)
         1.905
                  (0.053)
                               2.022
                                         (0.097)
                                                      2.023
                                                               (0.085)
f_{37,0}
f_{38,0}
         1.355
                  (0.086)
                               1.422
                                         (0.096)
                                                      1.441
                                                               (0.094)
                               1.691
                                                               (0.081)
         1.619
                  (0.060)
                                         (0.084)
                                                      1.670
f_{39.0}
         1.755
                  (0.058)
                               1.863
                                         (0.103)
                                                      1.858
                                                               (0.092)
f_{40,0}
         1.726
                                                      1.799
                                                               (0.094)
                  (0.078)
                               1.795
                                         (0.095)
f_{41,0}
         1.843
                               1.995
                                         (0.075)
                                                      1.983
                                                               (0.073)
                  (0.069)
f_{42,0}
```

```
(0.160)
                                                     1.327
                                                              (0.162)
f_{43,0}
        1.350
                  (0.141)
                               1.330
        2.265
                  (0.046)
                               2.354
                                        (0.046)
                                                     2.330
                                                              (0.049)
f_{44,0}
f_{45,0}
        1.270
                  (0.086)
                               1.316
                                        (0.097)
                                                     1.312
                                                              (0.096)
        1.636
                  (0.106)
                               1.719
                                        (0.139)
                                                     1.710
                                                              (0.134)
f_{46,0}
        1.120
                  (0.068)
                               1.172
                                        (0.074)
                                                     1.151
                                                              (0.075)
f_{47,0}
        1.973
                  (0.048)
                               2.008
                                        (0.048)
                                                     1.998
                                                              (0.048)
f_{48,0}
        1.225
                                                     1.265
                                                              (0.178)
                  (0.157)
                               1.283
                                        (0.178)
f_{49,0}
        1.169
                  (0.101)
                               1.234
                                        (0.118)
                                                     1.222
                                                              (0.123)
f_{50,0}
                                                              (0.095)
        1.910
                  (0.078)
                               1.967
                                        (0.099)
                                                     1.969
f_{51,0}
        1.806
                  (0.086)
                               1.847
                                        (0.095)
                                                     1.837
                                                              (0.097)
f_{52,0}
        1.776
                  (0.078)
                               1.845
                                        (0.097)
                                                     1.832
                                                              (0.098)
f_{53,0}
        2.084
                  (0.049)
                               2.122
                                        (0.060)
                                                     2.139
                                                              (0.061)
f_{54,0}
f_{55,0}
        1.707
                  (0.065)
                               1.772
                                        (0.075)
                                                     1.753
                                                              (0.070)
        1.849
                               1.898
                                        (0.094)
                                                     1.892
                                                              (0.089)
f_{56,0}
                  (0.077)
        1.695
                  (0.094)
                               1.787
                                        (0.140)
                                                     1.792
                                                              (0.141)
f_{57,0}
        1.439
                  (0.058)
                               1.547
                                        (0.097)
                                                     1.525
                                                              (0.092)
f_{58,0}
        1.801
                               1.913
                                        (0.075)
                                                     1.902
                                                              (0.069)
f_{59,0}
                  (0.068)
        1.717
                  (0.088)
                               1.809
                                        (0.103)
                                                     1.785
                                                              (0.099)
f_{60,0}
        1.834
                               1.948
                                                     1.930
                                                              (0.136)
f_{61,0}
                  (0.111)
                                        (0.139)
f_{62,0}
        0.913
                  (0.063)
                               0.933
                                        (0.086)
                                                     0.907
                                                              (0.083)
        1.495
                  (0.105)
                               1.571
                                        (0.134)
                                                     1.566
                                                              (0.130)
f_{63.0}
        1.732
                               1.791
                                        (0.061)
                                                     1.777
                                                              (0.062)
                  (0.051)
f_{64,0}
f_{65,0}
        1.352
                  (0.051)
                               1.391
                                        (0.051)
                                                     1.378
                                                              (0.050)
                               1.700
        1.639
                  (0.076)
                                        (0.094)
                                                     1.703
                                                              (0.093)
f_{66,0}
        2.461
                  (0.049)
                               2.499
                                        (0.059)
                                                     2.475
                                                              (0.061)
f_{67,0}
f_{68,0}
        1.885
                  (0.047)
                               1.966
                                        (0.049)
                                                     1.961
                                                              (0.050)
                                        (0.062)
f_{69,0}
        1.513
                  (0.054)
                               1.592
                                                     1.598
                                                              (0.064)
        1.756
                  (0.158)
                               1.834
                                        (0.180)
                                                     1.837
                                                              (0.183)
f_{70,0}
        1.965
                               2.127
                                        (0.098)
                                                     2.111
                                                              (0.091)
f_{71,0}
                  (0.057)
                                                     1.856
f_{72,0}
        1.789
                  (0.067)
                               1.876
                                        (0.074)
                                                              (0.069)
f_{73,0}
        1.283
                  (0.076)
                               1.336
                                        (0.092)
                                                     1.311
                                                              (0.092)
        1.134
                  (0.096)
                               1.105
                                        (0.164)
                                                     1.094
                                                              (0.160)
f_{74,0}
        1.588
                                                     1.622
                  (0.107)
                               1.641
                                        (0.132)
                                                              (0.125)
f_{75,0}
        1.977
                               1.993
                                        (0.121)
                                                     1.949
                                                              (0.137)
f_{76,0}
                  (0.110)
                                                     1.220
f_{77,0}
        1.210
                  (0.062)
                               1.231
                                        (0.089)
                                                              (0.080)
f_{78,0}
        2.238
                  (0.059)
                               2.452
                                        (0.075)
                                                     2.439
                                                              (0.075)
        2.371
                               2.453
                                                     2.451
                                                              (0.075)
f_{79,0}
                  (0.057)
                                        (0.077)
f_{80,0}
        1.503
                  (0.096)
                               1.600
                                        (0.152)
                                                     1.601
                                                              (0.150)
                                                              (0.093)
f_{81,0}
        1.493
                  (0.057)
                               1.592
                                        (0.097)
                                                     1.581
        1.462
                  (0.155)
                               1.500
                                        (0.181)
                                                     1.492
                                                              (0.172)
f_{82,0}
        1.653
                               1.671
                                        (0.094)
                                                     1.659
                                                              (0.091)
f_{83,0}
                  (0.078)
f_{84,0}
        1.741
                  (0.089)
                               1.815
                                        (0.097)
                                                     1.793
                                                              (0.095)
        1.481
                  (0.049)
                               1.523
                                        (0.052)
                                                     1.519
                                                              (0.053)
f_{85,0}
f_{86,0}
        1.347
                  (0.101)
                               1.405
                                        (0.134)
                                                     1.393
                                                              (0.142)
        1.183
                               1.238
                  (0.107)
                                        (0.116)
                                                     1.225
                                                              (0.120)
f_{87,0}
        1.687
                  (0.047)
                               1.749
                                        (0.048)
                                                     1.736
                                                              (0.049)
f_{88,0}
        1.354
                                                              (0.094)
                  (0.063)
                               1.414
                                        (0.071)
                                                     1.406
f_{89,0}
        2.158
                               2.330
                                                     2.317
                                                              (0.072)
                  (0.060)
                                        (0.072)
f_{90,0}
```

```
1.844
                               1.956
                                        (0.059)
                                                     1.958
                                                              (0.062)
f_{91,0}
                  (0.053)
         1.658
                  (0.074)
                               1.685
                                        (0.096)
                                                     1.665
                                                               (0.089)
f_{92,0}
f_{93,0}
         1.765
                  (0.087)
                               1.809
                                        (0.095)
                                                     1.774
                                                               (0.099)
         1.883
                  (0.060)
                               2.004
                                        (0.102)
                                                     1.999
                                                               (0.096)
f_{94,0}
f_{95,0}
         1.369
                  (0.060)
                               1.413
                                        (0.073)
                                                     1.374
                                                               (0.102)
         1.600
                  (0.111)
                               1.710
                                        (0.132)
                                                     1.678
                                                               (0.128)
f_{96.0}
                               2.077
                                                     2.064
                                                               (0.069)
f_{97.0}
         1.941
                  (0.067)
                                        (0.076)
         1.968
                  (0.055)
                               2.019
                                        (0.108)
                                                     2.019
                                                               (0.094)
f_{98,0}
                               1.936
                                                     1.929
                                                               (0.138)
         1.870
                  (0.108)
                                        (0.131)
f_{99,0}
         1.907
                  (0.105)
                               2.033
                                        (0.138)
                                                     2.024
                                                               (0.135)
f_{100,0}
         1.576
                  (0.061)
                               1.601
                                        (0.104)
                                                     1.592
                                                               (0.092)
f_{101,0}
f_{102,0}
         1.362
                  (0.110)
                               1.446
                                        (0.120)
                                                     1.464
                                                               (0.126)
f_{103,0}
         1.921
                  (0.049)
                               1.995
                                        (0.049)
                                                     1.985
                                                               (0.047)
         1.344
                  (0.072)
                               1.407
                                        (0.091)
                                                     1.391
                                                               (0.095)
f_{104,0}
         1.564
                  (0.159)
                               1.585
                                        (0.187)
                                                     1.572
                                                               (0.178)
f_{105,0}
         1.572
                  (0.105)
                               1.707
                                        (0.133)
                                                     1.690
                                                               (0.129)
f_{106,0}
         1.913
                  (0.081)
                               1.968
                                        (0.095)
                                                     1.948
                                                               (0.101)
f_{107,0}
f_{108,0}
         1.304
                  (0.078)
                               1.361
                                        (0.092)
                                                     1.344
                                                               (0.095)
                                        (0.077)
         1.758
                  (0.059)
                               1.803
                                                     1.791
                                                               (0.075)
f_{109,0}
         1.749
                                                     1.842
                  (0.058)
                               1.851
                                        (0.097)
                                                               (0.087)
f_{110,0}
         2.229
                  (0.058)
                               2.482
                                        (0.074)
                                                     2.465
                                                               (0.071)
f_{111.0}
         1.514
                               1.608
                                        (0.095)
                                                     1.605
                                                               (0.092)
f_{112,0}
                  (0.060)
f_{113,0}
         1.265
                  (0.078)
                               1.285
                                        (0.096)
                                                     1.283
                                                               (0.094)
                                                     1.589
                                                               (0.061)
         1.516
                  (0.054)
                               1.587
                                        (0.061)
f_{114,0}
f_{115,0}
         1.678
                  (0.050)
                               1.716
                                        (0.049)
                                                     1.696
                                                               (0.048)
f_{116,0}
         2.031
                  (0.046)
                               2.110
                                        (0.050)
                                                     2.115
                                                               (0.047)
                                                     1.808
f_{117,0}
         1.711
                  (0.065)
                               1.834
                                        (0.077)
                                                               (0.069)
         1.434
                  (0.092)
                               1.466
                                        (0.100)
                                                     1.463
                                                               (0.095)
f_{118,0}
         1.736
                  (0.078)
                               1.788
                                        (0.098)
                                                     1.760
                                                               (0.101)
f_{119,0}
         1.425
                  (0.093)
                               1.454
                                        (0.142)
                                                     1.461
                                                               (0.149)
f_{120,0}
f_{121,0}
         1.550
                  (0.058)
                               1.688
                                        (0.094)
                                                     1.679
                                                               (0.089)
f_{122,0}
         1.630
                  (0.057)
                               1.735
                                        (0.097)
                                                     1.717
                                                               (0.088)
                                                     1.426
         1.369
                  (0.066)
                               1.436
                                        (0.076)
                                                               (0.070)
f_{123,0}
                                                     1.797
         1.669
                               1.808
                                        (0.179)
                                                               (0.184)
f_{124,0}
                  (0.096)
f_{125,0}
         1.927
                  (0.087)
                               1.976
                                        (0.098)
                                                     1.941
                                                               (0.100)
f_{126,0}
         1.456
                  (0.088)
                               1.563
                                        (0.100)
                                                     1.574
                                                               (0.095)
                               2.514
         2.476
                                                     2.507
                                                               (0.096)
f_{127,0}
                  (0.054)
                                        (0.096)
f_{128,0}
         1.905
                  (0.098)
                               1.870
                                        (0.174)
                                                     1.849
                                                               (0.166)
                               1.128
                                                     1.120
                                                               (0.093)
f_{129,0}
         1.105
                  (0.077)
                                        (0.091)
f_{130,0}
         1.874
                  (0.050)
                               1.989
                                        (0.059)
                                                     1.995
                                                               (0.062)
         1.983
                  (0.051)
                               2.063
                                        (0.060)
                                                     2.047
                                                               (0.063)
f_{131,0}
                                                     1.215
                                                               (0.095)
f_{132,0}
         1.151
                  (0.079)
                               1.210
                                        (0.094)
         1.027
                  (0.107)
                               1.104
                                        (0.113)
                                                     1.091
                                                               (0.124)
f_{133,0}
f_{134,0}
         1.272
                  (0.079)
                               1.290
                                        (0.096)
                                                     1.274
                                                               (0.097)
         1.726
                               1.839
                                                     1.828
                                                               (0.072)
f_{135.0}
                  (0.057)
                                        (0.074)
         1.606
                  (0.093)
                               1.680
                                        (0.139)
                                                     1.682
                                                               (0.138)
f_{136,0}
                                                     1.850
                                                               (0.081)
         1.753
                  (0.061)
                               1.877
                                        (0.087)
f_{137,0}
                                        (0.085)
         1.750
                               1.999
                                                     1.973
                                                               (0.085)
                  (0.060)
f_{138,0}
```

$f_{139,0}$	1.849	(0.059)	2.102	(0.084)	2.078	(0.081)
$f_{140,0}$	1.904	(0.066)	2.069	(0.074)	2.051	(0.069)
$f_{141,0}$	1.306	(0.062)	1.376	(0.072)	1.370	(0.072)
$f_{142,0}$	2.007	(0.064)	2.122	(0.073)	2.102	(0.070)
$f_{143,0}$	1.623	(0.085)	1.652	(0.096)	1.639	(0.093)
$f_{144,0}$	1.550	(0.079)	1.647	(0.094)	1.643	(0.091)
$f_{145,0}$	1.323	(0.079)	1.346	(0.095)	1.347	(0.091)
$f_{146,0}$	1.478	(0.086)	1.527	(0.095)	1.519	(0.096)
$f_{147,0}$	1.597	(0.083)	1.697	(0.099)	1.672	(0.099)
$f_{148,0}$	1.607	(0.052)	1.635	(0.061)	1.618	(0.058)
$f_{149,0}$	1.661	(0.050)	1.746	(0.047)	1.743	(0.052)
$f_{150,0}$	1.682	(0.058)	1.772	(0.095)	1.752	(0.087)