Modeling stock-oil co-dependence with Dynamic Stochastic MIDAS Copula models

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Abstract

Stock and oil relationship is usually time-varying and depends on the current economic conditions. In this study, we propose a new Dynamic Stochastic Mixed data sampling (DSM) copula model, that decomposes the stock-oil relationship into a short-run dynamic stochastic component and a long-run component, governed by related macro-finance variables. Inference and prediction is carried out using a novel Bayesian estimation strategy, that can efficiently estimate the latent states and delivers an estimate of the log marginal likelihood used for model comparison. We find that inflation/interest rate, uncertainty and liquidity factors are the main drivers of the long-run co-dependence. We show that the multi-step-ahead variance covariance forecasts constructed using the proposed approach are closer to the true values as compared to the benchmark model. Finally, investment portfolios, based on the proposed DSM copula model, are more accurate and produce better economic outcomes as compared to other alternatives.

Keywords: Copula, Hedging, MIDAS, Portfolio, SMC, Stock-Oil.

JEL Classification: C32, C52, C58, G11, G12

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1 Introduction

Modeling and forecasting the co-movements of financial returns has been one of major interests among the researchers and practitioners alike, especially in the presence of recent global crises (the 2008 financial crisis and the Covid pandemic). Understanding the equity return co-dependence is especially important in modern finance, in particular in derivative pricing, portfolio allocation, risk management and hedging. Commodities tend to have low or negative correlations with the conventional equity stocks, therefore, they have a high potential of providing diversification benefits (Sadorsky, 2014), especially in hedging the downside risk (Dai et al., 2022b). With the financialization of the commodity markets, it is now straightforward to include certain commodities in one's investment portfolio.

The stock-oil relationship has received a great deal of attention not only because oil is often used to hedge the market risk, but also because oil shocks have a direct effect on the financial markets, and vice-versa (Jones and Kaul, 1996; Sadorsky, 1999; Chao Wei, 2003; Akram, 2004; Kilian and Park, 2009; Dai and Zhu, 2023). This co-dependence is not constant in two senses: first, the relationship fluctuates in the short-run and, secondly, it presents more substantial changes in the levels in the long-run (Batten et al., 2017). Such changes in the co-dependence are affected by the changes in external political decisions, macroeconomic factors and the conditions in the financial markets (Huang et al., 1996; Amihud and Wohl, 2004; Sukcharoen et al., 2014). For example, there has been a dramatic increase in the long-run co-dependence between stock-oil returns after the 2008-2009 crisis, documented by numerous studies (Aloui et al., 2012; Pan, 2014; Zhu et al., 2014).

With this work, we contribute to the existing literature investigating the co-movements of stock-oil returns in four major ways. First, we decompose the time-varying dependence into two parts: short- and long-run. We allow the short-run component to be parameter-driven, resulting into greater flexibility in the dynamics, as opposed to the observation-driven models. Secondly, we allow for the long-run component to be affected by some external macro-finance variables, sampled at lower frequencies. Following some of the most recent works (Asgharian et al., 2016; Nguyen and Javed, 2021), we group these macro-finance variables via principal components (PCs). Thirdly, we employ a novel Bayesian estimation strategy based on density-tempered sequential Monte Carlo sampler. The method is especially well suited for non-linear non-Gaussian state space models and produces log marginal likelihoods as a by-product, which allows for consistent model comparison even for non-nested models. And, finally, we present an extensive empirical illustration using stock-oil return data from more than 30 years (including the latest Covid period). We perform multi-step variance covariance matrix forecasts and portfolio allocation exercises to quantify the economic gains of using our proposed approach.

The co-movement of financial returns is usually characterized via time-varying volatilities and correlations by employing multivariate GARCH or multivariate SV-type models (Bauwens et al., 2006; Asai et al., 2006; Chib et al., 2009). The usual assumption is that the conditional joint distribution of the returns follows a multivariate Normal or a multivariate Student-t distribution, which implies that it is the same for all marginals, and that the dependence structure must be linear. In order to relax the assumptions above, we model the co-dependence between financial assets via copulas (McNeil et al., 2015; Nelsen, 2006; Joe, 2015; Patton, 2009, 2012, 2013). The major advantage of the copula approach is that one can separate the modeling of the individual marginals from their dependence structure. Moreover, the marginals of the individual assets do not necessarily have to be the same, as in the multivariate GARCH or SV setting; and the choice of the appropriate copula (symmetric or asymmetric) is completely independent from the marginals. Such approach is rather standard in financial econometrics literature, see, for example, Dias and Embrechts (2004); Jondeau and Rockinger (2006); Ausin and Lopes (2010); Creal and Tsay (2015); Nguyen et al. (2019); Virbickaite et al. (2022), among many others. Copulas have also been used in modeling the stock-oil co-dependence in Aloui et al. (2013); Sukcharoen et al. (2014); Pan (2014); Mensi et al. (2017); Yu et al. (2020).

Copula function is characterized by a set of parameters, that can be either static or time-varying. In their widely cited paper, Hafner and Manner (2012) have proposed to model this copula dependence parameter as a stochastic process via latent AR(1)-type equation. They show that such approach allows for greater flexibility as compared to the observation-driven models. Therefore, we build on the dynamic stochastic copula (DSC) model of Hafner and Manner (2012) and interpret the unconditional mean of the AR(1) process as the long-run co-dependence, that can be driven by the related macro-finance factors. Since most of the economic factors are observed at a lower frequency than the financial variables, in order to account for the frequency mismatch, we rely on a mixed data sampling method (MIDAS, Ghysels et al., 2004). Here the effects of the lagged macro-finance variables are regularized by a polynomial weighting scheme to ensure a parsimonious model specification. We refer to the resulting model as the dynamic stochastic MIDAS (DSM) copula. The proposed model not only allows for a greater flexibility in the short-run dynamic dependence of the financial returns but also takes advantage of the exogenous macro-finance variables to explain the long-run changes in the dependence structure. The co-dependence between financial assets in the DSM copula models fluctuates around a long-run component rather than a mean reverting process, which is the case in the DSC model of Hafner and Manner (2012). Finally, modeling the co-dependence as a function of macroeconomic variables not only helps investors to accurately forecast the joint behavior of the financial assets and build efficient investment strategies, but also might serve as a warning signal for the changes of macroeconomic conditions, and vice-versa (Conrad and Stürmer, 2017). The proposed model can be applied to model the joint co-dependence of any type of financial assets of interest.

There have been several studies that model the long-run component of the correlations between financial returns based on the MIDAS framework. Conrad et al. (2014) employ the dynamic conditional correlation (DCC) MIDAS model (Engle, 2002a; Colacito et al., 2011) and find that the contraction in macroeconomic activity, measured using multiple current and forward-looking economic indicators, leads to an increase of the long-run correlation between crude oil and stock price returns, as vice-versa. Gong et al. (2018) employ a copula-MIDAS model to describe the asymmetric long-run return-liquidity dependence using the realized correlations. Gong et al. (2020) model the relationship between stock-oil returns via an observation-driven MIDAS copula model. The authors find that aggregate demand and stock-specific negative news have a significant impact on stock-oil co-dependence, however their proposed copula allows only for positive dependence. Alternative to our proposal, Nguyen and Javed (2021) use a generalized autoregressive score mixed frequency data sampling (GAS MIDAS) copula model to analyze the time varying co-dependence between stock-bond returns. The authors find that the inflation and interest rate, the state of the economy and the illiquidity are significant in modeling the long-run co-dependence.

Most of the aforementioned studies are based on a class of observation-driven models and employ the maximum likelihood approach to estimate the parameters. The uncertainty of parameters is quantified using a numerical approximation of the information matrix that can lead to the divergence of the likelihood or cannot guarantee the positive semi-definiteness of the information matrix. Moreover, maximum likelihood method becomes infeasible for a highly non-linear and (possibly) non-Gaussian state space models. Instead, we rely on Bayesian approach for inference and prediction for our proposed parameter-driven DSM copula model. We construct a density-tempered sequential Monte Carlo (DTSMC) sampler, similar to Tran et al. (2014) and Duan and Fulop (2015). This sampler combines the Anneal important sampling (Neal, 2001) and the Sequential Monte Carlo method (Del Moral et al., 2006). The DTSMC sampler provides an efficient alternative to the standard MCMC methods for estimating complex non-linear non-Gaussian state-space models. Same as MCMC, the results hold even for finite samples and one can make probabilistic statements on non-linear functions of the model parameters. More important, the sampler also provides the estimate of the log marginal likelihood (LML) as a by-product, which is used for model comparison even for non-nested models.

In an empirical illustration using stock-oil return data from more than 30 years we find that the stock-oil relationship is dynamic and fat-tailed. The inclusion of the macro-finance variables to model the stock-oil co-dependence increases the in-sample log marginal likelihoods. In particular, inflation/interest rate, uncertainty and liquidity factors are the main drivers of the long-run co-dependence. Moreover, the variance-covariance matrix forecasts, obtained using our proposed DSM copula models are more accurate, as compared to the benchmark specification. Finally, the hedging and minimum variance portfolios, built using these forecasts produce better economic outcomes out-of-sample.

The rest of the paper is organized as follows. Section 2 introduces the DSM copula model. Section 3 describes the Bayesian inference algorithm. Section 4 models the marginal distributions of the individual assets and then analyses the fundamental factors that affect the co-dependence of the stock-oil returns. Section 5 presents the forecasting and portfolio allocation exercises and conclusions are drawn in Section 6.

2 Model Specification

In this section, we describe the proposed DSM copula model. We start by briefly introducing the model for the marginals for the returns. We then proceed to define copulas and the DSC model of Hafner and Manner (2012). Finally, we extend the DSC model by incorporating the MIDAS effects as a function of past macro-finance variables, sampled at lower frequencies.

2.1 Marginals for the returns

Let $P_{i,t}$ be the daily price of a financial asset i (e.g. a stock index or a commodity, such as oil) for $t = 1, \ldots, T$ and define the daily log returns (in %) as $r_{i,t} = 100 \times (\ln P_{i,t} - \ln P_{i,t-1})$. Then, the dynamics of the individual asset log returns can be modeled as

$$r_{i,t} = \mu_i + \epsilon_{i,t}\sqrt{\sigma^2_{i,t}}. \tag{1}$$

Here μ_i is the unconditional mean, $\epsilon_{i,t}$ is an error term with a zero mean and unit variance, which can be assumed to be Gaussian, Student-t, Generalized Error Distribution or any other parametric or non-parametric distribution. $\sigma^2_{i,t}$ is the volatility, which again, can follow either GARCH, or SV dynamics, or, alternatively, can be estimated using the realized volatility measure (given the high-frequency data necessary for estimation is available). Hafner and Manner (2012) found that the estimated volatilities using either GARCH or SV models are very similar hence we model the dynamics of the returns in Eq. (1) via GJR-GARCH model with skew-t errors. Nonetheless, any other specification for the distribution term and the dynamics of the volatility is in order (e.g. Engle, 2002b; Ardia, 2009; Loaiza-Maya et al., 2018; Conrad and Kleen, 2020; Dai et al., 2022a; Virbickaite et al., 2023), as long as the probability integral transform of the standardized returns is uniformly distributed. In other words, denote $F_i(\cdot)$ as the cumulative distribution function (CDF) of the $\epsilon_{i,t}$, then if the distribution term and the volatility dynamics are correctly specified, the $u_{i,t} = F_i((r_{i,t} - \mu_i)/\sqrt{\sigma^2_{i,t}})$ should be *iid* uniformly distributed. Once the marginals have been specified, one can model the co-dependence between the resulting $u_{i,t}$ using copula.

2.2 Copula

Copulas allow to separate the marginal distributions from the dependence structure and permit for explicit control of tail dependence via specific parameters, unlike the classical multivariate distributions. The construction of flexible multivariate distributions using copulas has started with the seminal work of Sklar (1959). For a formal introduction to copulas, refer to Nelsen (2006) and Joe (2015). Consider a collection of random variables $Y_1, \ldots, Y_d$ with the corresponding marginal CDFs $F_i(y_i) = P[Y_i \leq y_i]$ for $i = 1 \ldots, d$ and a joint distribution function $H(y_1, \ldots, y_d) = P[Y_1 \leq y_1, \ldots, Y_d \leq y_d]$. According to Sklar (1959), there exists a copula C with parameter(s) θ such that $H(y_1, \ldots, y_d) = C_\theta(F_1(y_1), \ldots, F_d(y_d))$. Copulas are defined in the unit hypercube $[0, 1]^d$, where

d is the dimension of the data, and the univariate marginals $F_i(y_i) \forall i$ are uniformly distributed. Finally, let $h(y_1 \ldots, y_d)$ be the joint density of $Y_1, \ldots, Y_d$, and $f_i(y_i)$ the corresponding marginal probability density function (PDF) of Y_i. Then $h(\cdot)$ can be expressed as a product of the marginals and a copula density:

$$h(y_1, \ldots, y_d) = c_\theta(F_1(y_1), \ldots, F_d(y_d)) \cdot f_1(y_1) \cdots f_d(y_d).$$

For the sake of simplicity, in what follows we consider only bivariate case, i.e. $d = 2$. Joe (2015) summarizes several copula functions that allow for a flexible dependence in the bivariate context. Observing that elliptical copula and Archimedean copula families are most commonly used in finance literature due to the parsimonious specification and their ability to capture tail dependence, we consider Gaussian, Student-t (or simply t), Clayton, Gumbel, Frank, and Joe copulas to model the dynamic dependence (for details see the Online Appendix A). The Archimedean copula function has only one parameter that specifies the asymmetric tail dependence, thus the rotations of Archimedean copula are required. However, because of the uncertainty about the direction of the asymmetric tails in the dynamic context, we propose modified Archimedean copulas based on a mixture of their own rotations so that the Archimedean copula has a symmetric tail dependence,

$$c_{Arch}(u_1, u_2; \theta) = \begin{cases} c^+_{Arch}(u_1, u_2; \theta) = 0.5c(u_1, u_2; \theta) + 0.5c_{R180}(u_1, u_2; \theta), & \text{if } \theta > 0 \\ c^-_{Arch}(u_1, u_2; \theta) = 0.5c_{R90}(u_1, u_2; \theta) + 0.5c_{R270}(u_1, u_2; \theta), & \text{if } \theta < 0 \end{cases}$$

where c, c_{R90}, c_{R180}, c_{R270} are the Archimedean copula density and its 90-degrees, 180-degrees, 270-degrees rotations; (u_1, u_2) are uniformly distributed probability integral transforms $(F_1(y_1), F_2(y_2))$, and θ is a copula parameter. A key copula-related measure is Kendalls' tau $\tau_\kappa \in (-1, 1)$, also known as rank correlation coefficient. For most copulas there is a one-to-one relationship between the copula parameter θ and τ_κ. Kendall's tau provides a convenient way to compare the strength of the dependence across different copulas, because, unlike the copula parameter θ, τ_κ always lies in the same domain. As seen in the following section, we rely on Kendall's tau to map the copula-specific parameter θ to its corresponding domain. The copula CDFs, PDFs, and the $\theta \leftrightarrow \tau_\kappa$ transformations can be found in the Online Appendix A. Moreover, Online Appendix G contains a robustness check, where we also include an alternative specification for rotations based on a mixture of t copulas, as in Loaiza-Maya et al. (2018).

2.3 Dynamic stochastic copula

After we have specified the appropriate marginal model for financial time series, the next step is to model the joint dependence through a dynamic copula model. Hafner and Manner (2012) propose to model the dynamics of the copula parameter as a transformation of a mean reverting process, giving rise to the DSC model:

$$
\begin{aligned}
(u_{1,t}, u_{2,t}) &\sim c(u_{1,t}, u_{2,t}; \theta_t), \\
\theta_t &= \Lambda(\lambda_t), \\
\lambda_t &= \lambda_0(1-\beta) + \beta\lambda_{t-1} + \sigma_e e_t, \quad e_t \sim N(0,1).
\end{aligned} \tag{2}
$$

The function $\Lambda(\cdot)$ is a copula-specific transformation that ensures that a copula parameter θ_t is in the appropriate domain. In particular, we map λ to the Kendall's τ correlation by $\tau_\kappa = \frac{\exp(\lambda)-1}{\exp(\lambda)+1}$, then find the copula parameter θ by using a copula-specific $\theta \leftrightarrow \tau_\kappa$ transformation (see the Online Appendix A). The dynamics of the transformed copula parameter λ_t can be modeled via AR(1) process with unconditional mean λ_0, persistence β and variance σ_e^2. We assume that the persistence parameter $|\beta| < 1$ for stationary conditions and the variance $\sigma_e^2 > 0$. As a special case, Hafner and Manner (2012) show that the dynamic stochastic copula model becomes a bivariate Gaussian SV model with stochastic correlation when the copula and the marginals are all Gaussian. However, the use of other copula functions (potentially asymmetric and/or fat-tailed) allows for larger flexibility in modeling the tail dependence.

The DSC model belongs to a class of parameter-driven models where the dependence parameter follows a dynamic process with idiosyncratic innovations. It is more flexible than the alternative benchmark model such as the DCC (Engle, 2002a) in the sense that the dependence parameter in the DCC model is perfectly predictable one step ahead given the past information. Hence, the predictive distribution of the DSC model is a mixture of densities over random time-varying parameters, while the predictive density of the DCC model is simply the observation density which can produce neither over-dispersion nor heavier tails (Koopman et al., 2016).

2.4 Dynamic stochastic MIDAS copula

The DSC model, defined in Eq. (2), assumes a fixed level of long-run dependence, controlled by parameter λ_0, which might be restrictive and not in-line with the actually observed empirical evidence. Therefore, similar to Nguyen and Javed (2021), we allow the long-run dependence parameter to be time-varying and affected by some exogenous macro-finance variables. Even observed at a lower frequency, the macro-finance variables often provide new information on the current economic situation and business outlook. Hence, taking advantage of these sources of information can help us to better understand the dynamic co-dependence among the financial returns and improve the forecasting accuracy. In particular, we allow the unconditional mean of the latent AR(1) process to depend on some low-frequency explanatory variables through a MIDAS regression, giving rise to a novel DSM copula model:

$$
\begin{aligned}
(u_{1,t}, u_{2,t}) &\sim c(u_{1,t}, u_{2,t}; \theta_t), \\
\theta_t &= \Lambda(\lambda_t), \\
\lambda_t &= \lambda_\tau(1-\beta) + \beta\lambda_{t-1} + \sigma_e e_t, \quad e_t \sim N(0,1), \\
\lambda_\tau &= \lambda_0 + \sum_{j=1}^{N} \delta_j \left[\sum_{k=1}^{K_j} \phi_k(\omega_j) X_{j,\tau-k} \right].
\end{aligned} \tag{3}
$$

Here τ is an indicator for monthly time points which is related to the daily time points though $t = \tau L, \tau L+1, \ldots, (\tau+1)L$, where L is the number of trading days in a month; $X_\tau = (X_{1,\tau}, \ldots, X_{N,\tau})$ is N-dimensional vector of the low frequency explanatory variables at month τ. The effect of the j^{th} variable can be seen through the size of a regression coefficient δ_j, and the DSM copula model reduces to the DSC model when $\delta_j = 0, \forall j$. Also, $\phi_k(\omega_j)$ is a polynomial weighting function parameterized by ω_j of the variable j on its k^{th} lag, for $k = 1, \ldots, K_j$. The weighting parameter ω_j regulates for how long the effect lasts. Higher value of ω_j means that the effect of the lagged values of the explanatory variable decays at a faster speed. We use the restricted beta function to smooth the effect of the explanatory variables:

$$
\phi_k(w_j) = \frac{[1 - k/(K_j+1)]^{w_j - 1}}{\sum_{l=1}^{K_j} [1 - l/(K_j+1)]^{w_j-1}}.
$$

The use of a polynomial weighting function with a single parameter ω_j allows for parsimonious model specification. Some other alternative weighting functions could be considered, however, previous studies suggest that different weighting functions produce virtually identical estimation results (Nguyen and Javed, 2021).

In the next section, we describe the Bayesian approach used for the parameter estimation.

3 Bayesian inference and model comparison

A standard way to estimate copula based models is via a two-step approach: first, estimate the univariate marginals and obtain the probability integral transforms $(u_{1,t}, u_{2,t})$, and secondly, estimate the copula-related parameters, see Junker and May (2005); Almeida and Czado (2012) for example. It is also possible to estimate the marginals and copula parameters in a one-step procedure, see Ausin and Lopes (2010), however, due to the latent nature of the evolution equation we opt for the two-step approach. The estimation of the univariate marginals can be performed using any standard estimation method, depending on the chosen specification for the dynamics of the individual volatilities and the distribution of the error term. Once the $(u_{1,t}, u_{2,t})$ data has been obtained, we then perform estimation of the copula related parameters.

The model in Eq. (3) is a non-linear and potentially non-Gaussian state space model. Estimation of such models entails estimating the set of fixed model parameters as well as the sequence of latent state variables λ_t. We rely on a variant of a novel Bayesian estimation approach, known as annealed importance sampling with an estimated likelihood in Tran et al. (2014), and density-tempered marginalized sequential Monte Carlo sampler in Duan and Fulop (2015). We call this procedure the Density-Tempered Sequential Monte Carlo (DTSMC). Note that even though the name has a word "sequential" in it, the algorithm, same as the standard MCMC methods, performs batch inference, as opposed to the "truly" sequential expanding-data approaches. However, differently than the standard MCMC methods, our approach is better suited for non-linear non-Gaussian state space models, especially when the exact likelihood is not available. Finally, the estimate of the marginal likelihood, allowing for consistent model comparison via Bayes Factors, in our case is a by-product, meanwhile it is notoriously difficult to obtain in the classical MCMC setting.

3.1 Prior Distributions

Denote $\Theta = \{\beta, \sigma_e^2, \lambda_0, \delta_{1:N}, \omega_{1:N}\}$ as the set of fixed parameters of the DSM copula model. Following Kastner and Frühwirth-Schnatter (2014), we use vague proper prior distributions: $\beta \sim \text{Beta}(20, 1.5)$ and $\sigma_e^2 \sim \text{Gamma}(\frac{1}{2}, \frac{1}{2B_\sigma})$ which is equivalent to $\pm\sqrt{\sigma_e^2} \sim \mathcal{N}(0, B_\sigma)$. The choice of the prior for σ_e is less influential when the true value is small, therefore, we set $B_\sigma = 1$. In the empirical illustration, we use the PCs as explanatory variables, therefore, weakly informative priors of the parameters in the MIDAS regression are considered in order to allow for a wide range of effects and the decay behavior of the effects: $\delta_j \sim N(0, 1)$ and $\omega_j \sim U(1, 30) \forall j$. The degree of freedom parameter in the Student copula follows a Gamma distribution, $\nu \sim \mathcal{G}(1, 0.1)$.

3.2 Density-tempered sequential Monte Carlo algorithm

Let $u_{1:T}$ be a collection of $(u_{1,t}, u_{2,t})$ for $t = 1, \dots, T$. We are interested in sampling from the posterior distribution:

$$\begin{aligned}
p(\Theta|u_{1:T}) &\propto p(u_{1:T}|\Theta)p(\Theta), \text{ where} \\
p(u_{1:T}|\Theta) &= \int p(u_{1:T}|\lambda_{1:T}, \Theta)p(\lambda_{1:T}|\Theta)d\lambda_{1:T} \\
&= \int \prod_{t=1}^{T} c(u_{1,t}, u_{2,t}|\lambda_t, \Theta, u_{1:t-1})p(\lambda_t|\lambda_{t-1}, \Theta, u_{1:t-1})d\lambda_{1:T}.
\end{aligned}$$

The likelihood $p(u_{1:T}|\Theta)$ is intractable due to the high dimensional integration over the latent process. Hafner and Manner (2012) use a particle filter to approximate the intractable likelihood and then use maximum likelihood to estimate Θ. Following this idea in the Bayesian context, we obtain an unbiased estimator $\widehat{p}(u_{1:T}|\Theta, e)$ of the likelihood $p(u_{1:T}|\Theta)$, where e is the set of pseudo random numbers used in the particle filter, and use the DTSMC algorithm to sample from the posterior distribution.

The DTSMC algorithm samples sequentially from the prior distribution $p(\Theta)$ to the posterior distribution $p(\Theta|u_{1:T})$ through a sequence of distributions $\pi_i(\Theta)$ that smoothly interpolate between the prior and the posterior distribution:

$$\pi_i(\Theta) := p_i(\Theta|u_{1:T}) \propto \widehat{p}(u_{1:T}|\Theta, e)^{\gamma_i} p(\Theta).$$

Here γ_i is a level temperature for $i = 0, \ldots, S$ such that $0 = \gamma_0 < \gamma_1 < \ldots < \gamma_S = 1$. We choose the tempering sequence $\gamma_i = (i/S)^3$ for $i = 0, \ldots, S$ so that the effective number of particles in each interpolation does not change dramatically.

In order to perform inference using the DTSMC algorithm, we first simulate a set of M weighted particles $\{W_0^j, \Theta_0^j\}_{j=1}^M$ from the prior distribution $p(\Theta)$ and a set of pseudo random e_0^j for the particle filter. For each level of temperature γ_i, the DTSMC algorithm performs reweighting, resampling and Markov moves. The reweighting step starts with a set of M weighted particles $\{W_{i-1}^j, \Theta_{i-1}^j\}_{j=1}^M$ obtained from the previous interpolation distribution $\pi_{i-1}(\Theta)$. The importance sampling is employed to re-weight the particles for approximating the target $\pi_i(\Theta)$,

$$\begin{aligned}
w_i^j &= W_{i-1}^j \widehat{p}(u_{1:T}|\Theta_{i-1}^j, e_{i-1}^j)^{\gamma_i - \gamma_{i-1}}, \;\; j = 1, ..., M, \\
W_i^j &= \frac{w_i^j}{\sum_{s=1}^{M} w_i^s}, \;\; j = 1, ..., M.
\end{aligned}$$

If the efficiency of these weighted particles, measured by the effective sample size (ESS), is below some threshold, the particles are resampled. Then, in order to avoid particle degeneracy, the particles are refreshed by a Markov kernel whose invariant distribution is $\pi_i(\Theta)$. We also incorporate the Correlated Pseudo Marginal (CPM) approach by Deligiannidis et al. (2018) into the Markov move step to deviate from the unstable integrated likelihood estimated by the particle filter, and use a random walk proposal for the new value of Θ. The DTSMC algorithm can be run in parallel for the particle moves in the Markov move step, so we can take advantage of the parallel computation in a high-performance computing server. The DTSMC algorithm is described in detail in the Online Appendix B. Finally, the log marginal likelihood is computed as

$$\log \widehat{p}(u_{1:T}) = \sum_{i=1}^{S} \log \left(\sum_{j=1}^{M} w_i^j \right).$$

3.3 Simulation

The Online Appendix D contains a simulation study that compares the proposed DSM copula model with the alternative models such as the DCC and the DSC. We consider different stress scenarios for the correlation dynamics and compare the estimation accuracy. In general, the DSM copula is preferred in terms of MSE and MAE. The time of inference is quite fast when using a multi-core

Ryzen 3950x CPU (16 cores at 3.5Hz). For a Gaussian bivariate copula with $T = 2000$ data points, it took on average 25 minutes and 28 minutes for DSC and DSM copula models respectively.

4 Empirical Illustration

This section contains the description of the data and the estimation results for the marginals, the DSC and DSM copula models for the stock-oil return data. For stock data we use daily S&P 500 observations and for oil data we use the daily spot prices of West Texas Intermediate (WTI) from 01/01/1990 to 31/12/2021. The S&P 500 data is obtained from Bloomberg and WTI daily spot prices are obtained from the US Energy Information Administration website. The negative WTI spot price that happened on April 20th 2020 was removed. The descriptive statistics tables, results for the Hong et al. (2007) test of asymmetry and the corresponding plots can be found in Online Appendix E. We first fit the GJR-GARCH model of Glosten et al. (1993) with skew-t errors on the daily log returns, see Online Appendix F for estimation results. Then we estimate the DSM copula to model the time-varying co-dependence between the stock-oil returns.

4.1 Dynamic co-dependence via DSM copula

In the DSM copula model the short-run co-dependence is modeled via latent AR-type process, where the long-run co-dependence is driven by several macro-finance factors. There is a large number of such possible macro-finance factors to be selected from. For example, Conrad et al. (2014) employs a number of indicators of current and future economic activity to model the stock-oil relationship. Such indicators include industrial production, unemployment, National Activity Index, among others. Gong et al. (2020), on the other hand, utilize the supply and demand shocks to oil as the economic factors, however the effect sizes of the supply and demand shocks are really small, therefore, the jumps in the level of the long term dependence can be hardly explained by a zero sum of uncorrelated shocks. Additionally, using shocks as explanatory variables is prone to model identification assumptions.

In this work, we rely on several criteria for variable selection. First, we select such factors that have been used extensively in other related studies. Secondly, we exclude such variables that do not have long enough historical data. Finally, we pre-select such macro-finance factors that have reasonable economic interpretation and/or the correlation with the rolling window stock-oil correlation is high.

In the spirit of Asgharian et al. (2016); Nguyen and Javed (2021), we group the macro-finance variables into four major categories and use the first PC as a representative factor for each group. The use of principal components instead of "crude" variables mitigates the effect of the possibly noisy behavior of each of the individual factors, at the same time providing a reasonable representation of the underlying trend. Principal component approach was also used in Zhang et al. (2022)

to forecast the crude oil volatility. Besides, macro-finance variables are highly correlated to each other (inflation and interest rates, for example), and using MIDAS regression with such multiple factors can cause multicollinearity. Also, several PCs will better capture the overall macroeconomic conditions than using a specific explanatory variable alone. Below is the list of the macro-finance variables, used in this paper. Monthly data is obtained from FRED[1] and Bloomberg.

1. Inflation/interest rate variables (INR): Inflation (INF), calculated as changes (in %) of the consumer price index; Effective Federal Funds Rate (IR EFFR); The spread between 10-Year Treasury Constant Maturity and 3-Month Treasury Constant Maturity (T10Y3M).

2. Uncertainty variables (UNC): Economic policy uncertainty (EPU); Equity Market Volatility Tracker: Energy And Environmental Regulation (EMV); National Financial Conditions Index (NFCI).

3. (Il)liquidity variables (LIQ): liquidity LIX and Amivest (Amihud et al., 1997; Marshall et al., 2012; Danyliv et al., 2014) and Amihud illiquidity (Amihud, 2002) measures are calculated daily and transformed to monthly by taking the average over the corresponding month.

4. State of the economy variables (SOE) [2]: changes (in %) of Industrial Production (IP) index; changes (in %) of Coincident Economic Activity Index (CEAI), as a proxy for nonfarm payrolls, the unemployment, average hours worked and wages and salaries; National Activity Index (NAI); changes (in %) of US Consumer Confidence Index (CCI); changes (in %) of US Business Confidence Index (BCI); changes (in %) of US Composite Leading Indicator (CLI).

Table 1 reports the cross-correlation matrix among the explanatory variables, where the monthly correlation (RCor) is calculated as a sample correlation between daily stock and oil returns during a month.

The INR factor is positively correlated with inflation and with the effective interest rate and negatively with the term spread. High interest rates encourage more people to save due to the flight-to-quality phenomenon. As investors receive more on their savings rate than in the market, they would be getting rid of risky stocks. Also, while the spread is positive, the long term rates are higher than the short term rates: an indication of healthy economy, however, when the spread decreases (or becomes negative), it might be seen as a lead indicator of a recession, which, again encourages the selling-off of the stocks. On the other hand, some studies have suggested that low interest rates lead to high commodity prices (Frankel, 2008; Barsky and Kilian, 2001). However, Baumeister et al. (2015) found no empirical evidence of such relationship using several MIDAS configurations and concludes that "while there is no doubt about the theoretical link in question,

[1] Federal Reserve Bank of St. Louis Economic Data

[2] In the state of the economy group IP and CEAI variables are yearly differences instead of a monthly difference as during the pandemic, these variables dropped (increased) dramatically, and only a few months later they suffered an almost equal increase (decrease), returning to the (almost) pre-Covid level. Such noisy data might inadvertently affect the estimation results, therefore, for those variables we used lagged yearly data instead.

Table 1: Cross-correlation matrix for the macro-finance variables.

	RCor	PC INR	PC UNC	PC LIQ	PC SOE
INF	-0.192	0.556			
IR EFFR	-0.484	0.864			
T10Y3M	0.089	-0.768			
EPU	0.405		0.750		
EMV	0.217		0.675		
NFCI	0.179		0.724		
LIX wti	0.473			0.822	
Amih wti	-0.122			-0.572	
Amiv wti	0.265			0.625	
LIX sp500	0.110			0.817	
Amih sp500	-0.279			-0.833	
Amiv sp500	0.023			0.306	
IP	-0.329				0.865
CEAI	-0.261				0.782
NAI	-0.361				0.855
CCI	0.060				-0.291
BCI	0.068				-0.564
CLI	0.092				-0.458
PC INR	-0.363	1.000			
PC UNC	0.376	-0.295	1.000		
PC LIQ	0.335	-0.418	0.037	1.000	
PC SOE	-0.332	0.485	-0.218	-0.167	1.000

Cross-correlation matrix of individual macro-finance variables, the first principal component in each group and the monthly correlation (RCor) between oil-stock returns for 01/01/1990 till 31/12/2021 ($n = 8059$). Monthly RCor is calculated as a sample correlation using 22 days at the end of each period.

its quantitative importance has yet to be established." Therefore, increase in the INR factor signals the sell-off of the stock but will not necessary affect oil, hence, it is negatively correlated with the realized stock-oil correlation. We expect that the sign of INR variable in the DSM copula model will be negative.

The UNC factor is positively correlated with the three variables: EPU, EMV and the NFCI. EPU measures policy-related economic uncertainty, so higher values mean more uncertainty. EMV moves with the VIX (CBOE Volatility Index), so increase in EMV means increase in market uncertainty. Finally, positive values of the NFCI indicate financial conditions that are tighter than average, therefore, more uncertainty. To sum up, increase in the UNC factor signals the increase in the overall uncertainty, which, in turn, makes both stock and oil risky investments. Investors would be getting rid of both risky assets and the stock-oil co-dependence would increase. Therefore, the uncertainty factor and the realized monthly correlation is positively correlated and we expect that the sign of UNC variable in the DSM copula model will be positive.

The LIQ factor presents the overall liquidity: it is positively correlated with LIX and Amivest liquidity measures, and negatively with Amihud illiquidity. Therefore, higher LIQ values mean higher liquidity overall. When the supply of oil and stock is abundant they can be easily bought and sold, therefore, the co-dependence increases and we expect positive sign of the LIQ variable in the DSM copula model.

Finally, the SOE factor is positively correlated with positive economic indicators, signaling favorable economic conditions. In general, when the economy is not in distress, the co-dependence between financial assets tends to become weaker (Conrad et al., 2014). The opposite is also true: when the market is in turmoil the relationship becomes stronger. Therefore, the SOE factor is negatively correlated with the realized correlation and we expect the SOE variable to have a negative effect on the stock-oil co-dependence in the DSM copula model.

The variables are assigned to certain groups based on economic reasoning purely, even though a fully automatic factor construction, based solely on statistical criteria, is also possible. As a robustness check, Online Appendix Section G includes the PC analysis for all variables at once where the relevant factor groups are identified using the varimax/promax rotations. We find three important results. First, the resulting groups are strikingly similar to the ones formed using economic reasoning, however, in some cases, the economic interpretability is lost. Secondly, for those factors that maintain a clear economic meaning the sign and size of the effect are comparable to the main empirical application. Finally, in terms of log marginal likelihood, the models that use economic factors generally present better in-sample fit.

4.2 Results

First, in order to choose the most appropriate copula family based on the log marginal likelihood, we estimate the DSC and DSM copula models, with monthly lagged Rcor only as an explanatory variable. Then, for the selected copula family, we identify the set of macro-finance factors that affect the dynamic co-dependence by estimating the DSM copula with other explanatory variables. The number of lags in each DSM copula model is fixed at 24 months so that the marginal likelihood is stable (increasing the lag beyond 24 months does not change the value of the log marginal likelihood). We also include the DCC Copula model of Engle (2002a) for comparison purposes (Online Appendix C contains the detailed description of the model).

Table 2 reports the estimation results for the DSC models (without MIDAS effects) for a number of different copulas. Overall, the elliptical copula family is preferred over the Archimedean copula family according to the LML. The DSC Gaussian is slightly worse than the DCC, however, the flexibility of copula functions help to increase the in-sample fit. The persistence parameter β is close to unity in all cases which allows the dependence parameter to follow an almost random walk process and quickly move into different regimes rather than into a mean reverting process. It has been shown that if the structural shifts (jumps, regimes) are unaccounted in the AR(1)-type process, the persistence parameter tends to be overestimated, and, usually, is close to unity (Lamoureux and Lastrapes, 1990; So et al., 1998; Vo, 2009; Virbickaite and Lopes, 2019). This is in-line with the empirically observed co-dependence: before the global financial crisis of 2008, the realized stock-oil correlations fluctuated around zero, and during/after the crisis turned positive. The best log marginal likelihood is obtained with the t copula with an estimated 17 degrees of freedom. This indicates that the stock-oil co-dependence is moderately fat-tailed and tail-symmetric, in accordance with the results in Avdulaj and Barunik (2015). An interesting extension to the proposed model would be to consider a time-varying degrees of freedom parameter, as in Hansen (1994); Brooks et al. (2005); Creal et al. (2008), for example. We illustrate the possibility of including time-varying degrees of freedom parameter in the Online Appendix G. Next, Table 3 reports the estimation results for the DSM copula models with RCor as an explanatory variable. In all cases, the log marginal likelihoods have improved over the corresponding DSC alternatives. The DSM Gaussian is better than the DCC MIDAS due to the dynamic update mechanism adapted to the abrupt changes of the long-term dependence. The DSM with t copula is the best, and the persistence parameter is smaller than in the previous case. The decrease of β is not surprising: now some of the persistence is partially captured by the exogenous RCor variable. Hence, the RCor is effectively guiding the long-run stock-oil co-dependence, and the short-run component becomes a mean reverting process. The effect of lagged monthly RCor measured by δ is positive and statistically significant for all copulas. In the DSM t copula, the weighting parameter of the restricted beta function implies that the effect of RCor decays after approximately 12 months. Such specification could potentially lead to an increase in the log marginal likelihood, especially if the estimated static degrees of freedom parameter is very small.

Next, we identify the set of macro-finance factors that affect the dynamic co-dependence by estimating the DSM copula with one of the macro-finance factors as explanatory variables. Note that we use the t copula only, because it provided the largest log marginal likelihood in previous estimations. Table 4 reports the estimation results for the DSM copula models with one macro-

Table 2: Estimation results for the DSC model.

	λ_0	β	σ_e	ν	LML
DCC Copula	0.170	0.980	0.017		244.138
	(0.076;0.255)	(0.975;0.985)	(0.014;0.021)		
DSC Gaussian	0.170	0.993	0.044		242.440
	(0.063;0.264)	(0.989;0.997)	(0.030;0.058)		
DSC Student	0.265	0.997	0.027	17.278	253.042
	(0.116;0.421)	(0.995;0.999)	(0.021;0.035)	(13.176;21.928)	
DSC Clayton	0.163	0.997	0.017		171.469
	(0.070;0.252)	(0.996;0.999)	(0.013;0.021)		
DSC Gumbel	0.088	0.997	0.024		210.163
	(-0.144;0.278)	(0.996;0.999)	(0.018;0.031)		
DSC Frank	0.057	0.997	0.029		228.706
	(-0.535;0.360)	(0.993;1.000)	(0.018;0.043)		
DSC Joe	0.202	0.999	0.010		142.158
	(0.071;0.352)	(0.998;1.000)	(0.007;0.013)		

The table reports the estimation results for the DCC Copula and the DSC model with different copulas. The numbers in the brackets show the [10% - 90%] credible intervals. The column σ_e reports the parameter σ_e in the DSC models or α in the DCC model.

Table 3: Estimation results for the DSM copula model using RCor

	λ_0	β	σ_e	δ	ω_2	ν	LML
DCC MIDAS	0.072	0.957	0.023	0.666	4.651		245.512
	(0.019;0.131)	(0.938;0.978)	(0.017;0.029)	(0.283;0.904)	(1.687;7.741)		
DSM Gaussian	0.062	0.910	0.137	1.346	4.625		253.499
	(0.023;0.100)	(0.867;0.948)	(0.099;0.177)	(1.183;1.514)	(2.849;6.754)		
DSM Student	0.084	0.963	0.075	1.248	4.423	22.514	258.025
	(0.034;0.135)	(0.935;0.984)	(0.049;0.106)	(1.056;1.444)	(2.359;6.792)	(16.205;30.167)	
DSM Clayton	0.150	0.995	0.017	0.187	12.343		173.164
	(0.057;0.257)	(0.990;0.999)	(0.011;0.024)	(-0.431;0.781)	(1.729;25.979)		
DSM Gumbel	0.162	0.976	0.045	0.924	6.901		213.739
	(0.028;0.313)	(0.949;0.999)	(0.021;0.071)	(0.396;1.229)	(2.342;16.192)		
DSM Frank	0.070	0.947	0.089	1.260	4.784		232.856
	(0.028;0.112)	(0.906;0.980)	(0.052;0.130)	(1.092;1.438)	(2.099;7.679)		
DSM Joe	0.051	0.989	0.015	0.469	12.642		146.091
	(-0.011;0.117)	(0.982;0.999)	(0.010;0.022)	(0.058;0.786)	(3.344;24.133)		

The table reports the estimation results of the DCC MIDAS Copula and the DSM copula model with different copulas. The long term dependence component is modelled using the RCor as an explanatory variable with the restricted beta weighting function. The selected lag length is such that the maximum likelihood becomes insensitive to the its choice ($K = 24$). The numbers in the brackets show the [10% - 90%] credible intervals. The column σ_e reports the parameter σ_e in the DSC models or α in the DCC MIDAS model.

finance explanatory variable. Only the model with UNC factor is comparable to the model with RCor in terms of log marginal likelihood. Among the four factors, only SOE gives an insignificant effect, while the signs of INR, UNC, and LIQ factors are in line with the economic reasoning and empirical findings. When the inflation and interest rates are high, the stock return is negative due to the flight-to-quality phenomenon while the oil prices do not necessary change. On the other hand, increase in the Uncertainty and Liquidity factors drives the stock-oil return co-dependence up. Uncertainty makes both stock and oil risky investments, thus, in uncertain times they both are sold-off in exchange for safer investments. When the supply of stock and oil is abundant (high liquidity), both assets are bought and sold at higher rates, therefore, the co-dependence moves higher. As a robustness check, we have also considered the five largest sectors in the S&P 500 index individually. We have found that the sign, size, and significance coincide with the results in Table 4, regardless of the sector. Hence we can conclude that inflation/interest rate, uncertainty and liquidity factors are the main drivers of the long-run co-dependence between stock and oil returns. All detailed results are in Section G of the Online Appendix.

Table 4: Estimation results for the DSM copula model with one macro-finance factor.

	λ_0	β	σ_e^2	δ	ω_2	ν	LML
DSM RCor	0.084	0.963	0.075	1.248	4.423	22.514	258.025
	(0.034;0.135)	(0.935;0.984)	(0.049;0.106)	(1.056;1.444)	(2.359;6.792)	(16.205;30.167)	
DSM INR	0.262	0.996	0.030	-0.112	17.522	18.313	251.311
	(0.143;0.402)	(0.994;0.999)	(0.022;0.039)	(-0.255;-0.017)	(4.894;28.881)	(13.780;23.730)	
DSM UNC	0.247	0.992	0.035	0.227	7.300	19.029	263.218
	(0.178;0.312)	(0.987;0.996)	(0.025;0.046)	(0.144;0.305)	(1.406;21.645)	(14.177;25.045)	
DSM LIQ	0.185	0.996	0.030	0.064	14.401	17.849	250.650
	(0.059;0.315)	(0.994;0.998)	(0.023;0.039)	(-0.020;0.140)	(3.546;25.920)	(13.236;23.173)	
DSM SOE	0.179	0.996	0.035	-0.061	14.732	18.637	253.186
	(0.073;0.285)	(0.994;0.998)	(0.027;0.042)	(-0.134;0.019)	(4.019;26.358)	(13.754;24.140)	

The table reports the estimation results of the DSM copula model (with t copula). The long term dependence component is modelled using one macro-finance factor as an explanatory variable with the restricted beta weighting function. The selected lag length is such that the maximum likelihood becomes insensitive to the its choice ($K = 24$). The numbers in the brackets show the [10% - 90%] credible intervals.

Figure 1 plots the equivalent long-run Pearson correlation between S&P 500 and WTI returns using DSM copula model with one macro-finance explanatory variable in comparison to the rolling annual correlation (RC1Y), calculated at the end of each month using previous 252 daily observations. The RCor model, as expected, tracks closely the long-run correlation. The models with INR and UNC (panels (b) and (c)) factors have a similar pattern with the RC1Y except during 2002-2008, which shows that one factor model might not be sufficient to capture the shifts in the co-dependence in a long run. The DSM copula models with LIQ and SOE factors estimate a flat long run co-dependence.

Finally, we estimate the DSM copula model with two explanatory variables, see Table 5. The top panel of the table contains results for RCor plus one of the macro-finance factors. In general, the log marginal likelihood in these two factor models is better than the model with one macro-finance factor, i.e., including RCor improves model fit. The signs of the coefficients of the macro-finance variables are consistent, although, the size of the effect is smaller. This is expected, as the RCor

Figure 1: Long-run co-dependence between S&P 500 and WTI returns: one explanatory variable. The figure shows the long-run co-dependence (the equivalent Person's correlation of λ_τ) between S&P 500 and WTI returns using DSM copula models with one explanatory variable in comparison to the annual rolling correlation (RC1Y), calculated at the end of each month using previous 252 daily observations. The PC factor is shown as a bar chart. The shaded areas highlight the recession periods based on the NBER indicators.

variable accounts for some long-run variation in the co-dependence, thus reducing the effect of the other explanatory variables.

Table 5: Estimation results for the DSM copula model with two macro-finance factors.

	λ_0	β	σ_e	δ_1	ω_1	δ_2	ω_2	ν	LML
DSM RCor-INR	0.079	0.943	0.092	1.127	5.534	-0.047	13.094	23.530	251.384
	(0.038;0.117)	(0.910;0.971)	(0.062;0.125)	(0.944;1.324)	(3.083;8.471)	(-0.082;-0.011)	(3.074;24.570)	(15.891;32.828)	
DSM RCor-UNC	0.105	0.953	0.081	0.979	6.050	0.103	14.145	22.666	260.343
	(0.059;0.156)	(0.928;0.973)	(0.059;0.105)	(0.694;1.216)	(2.561;11.164)	(0.054;0.160)	(1.691;27.648)	(16.092;31.412)	
DSM RCor-LIQ	0.087	0.960	0.076	1.090	5.641	0.024	11.842	21.573	253.338
	(0.029;0.123)	(0.935;0.982)	(0.053;0.103)	(0.848;1.388)	(2.391;9.635)	(-0.013;0.063)	(2.289;23.961)	(15.666;29.259)	
DSM RCor-SOE	0.096	0.973	0.059	1.191	3.906	-0.014	14.200	18.685	250.248
	(0.040;0.155)	(0.956;0.989)	(0.038;0.081)	(0.932;1.436)	(1.918;6.169)	(-0.047;0.020)	(4.470;24.640)	(14.330;23.972)	
DSM INR-UNC	0.268	0.992	0.032	-0.096	7.000	0.236	11.446	20.083	253.307
	(0.197;0.343)	(0.986;0.997)	(0.021;0.045)	(-0.174;-0.008)	(1.203;18.464)	(0.154;0.321)	(2.417;24.211)	(14.316;28.228)	
DSM INR-LIQ	0.573	0.996	0.028	0.080	12.691	0.036	15.212	16.956	247.633
	(0.121;1.786)	(0.990;1.000)	(0.016;0.044)	(-0.208;0.729)	(2.526;24.367)	(-0.157;0.161)	(2.550;26.225)	(12.736;22.159)	
DSM INR-SOE	0.269	0.998	0.026	-0.089	19.187	0.135	18.532	17.139	244.701
	(-0.032;0.641)	(0.996;1.000)	(0.020;0.033)	(-0.419;0.333)	(7.122;28.486)	(-0.102;0.486)	(6.432;27.968)	(12.972;22.110)	
DSM UNC-LIQ	0.236	0.986	0.045	0.246	6.635	0.090	14.844	20.624	256.699
	(0.189;0.284)	(0.979;0.992)	(0.032;0.060)	(0.194;0.299)	(1.990;17.102)	(0.049;0.131)	(4.834;25.641)	(14.849;27.799)	
DSM UNC-SOE	0.283	0.987	0.044	0.331	4.970	0.042	17.648	18.846	251.796
	(0.220;0.348)	(0.979;0.994)	(0.031;0.059)	(0.204;0.450)	(1.128;17.331)	(-0.045;0.112)	(6.300;27.357)	(14.038;24.434)	
DSM LIQ-SOE	0.213	0.997	0.029	0.123	16.749	0.041	16.505	16.961	250.412
	(0.083;0.357)	(0.994;0.999)	(0.022;0.037)	(0.030;0.215)	(6.534;27.167)	(-0.096;0.221)	(5.018;27.077)	(13.336;21.007)	

The table reports the estimation results of the DSM copula model (with t copula). The long term dependence component is modelled using two explanatory variables with the restricted beta weighting function. Top panel contains results for the RCor plus one other macro-finance factor, and bottom panel contains results for the two macro-finance factors as explanatory variables. The selected lag length in the weighting function is such that the maximum likelihood becomes insensitive to the its choice ($K = 24$). The numbers in the brackets show the [10% - 90%] credible intervals.

Figure 2 shows the equivalent long-run Pearson correlation between S&P 500 and WTI returns using DSM copula model with Rcor plus one of the macro-finance factors as an explanatory variable. The long-run components are quite similar among the two factor models but there are a few exceptions. The RCor-UNC (panel (b)) forecasts that the long-run co-dependence is higher during the two most recent recessions while the RCor-SOE (panel (d)) estimates lower long-run co-dependence overall.

The bottom part of Table 5 reports the DSM copula model with two macro-finance factors. The signs of the effects are consistent with the previous results, and the three factors, INR, UNC and LIQ, remain statistically significant. However, we do not observe a significant improvement over the one factor model in terms of log marginal likelihood. Some of the macro-finance factors are correlated, see Table 1, indicating possible multicollinearity of the explanatory variables, hence the choice of the two macro-finance factors in the DSM copula model should be performed with care. Figure 3 plots the equivalent long-run Pearson correlation of the two macro-finance factor models. The DSM copula models with INR-UNC and UNC-LIQ as explanatory variables track the long run correlation reasonably well (panels (a) and (d)), meanwhile the long-run trends produced by the combination of INR-LIQ and INR-SOE factors deviate from the true correlation (panels (b) and (c)). In these cases the persistence coefficient of the short-run component is very close to unity (0.990 and 0.999, see bottom part of Table 5) in order to compensate the lack of proper fit from the long-run component.

Figure 4 shows the equivalent total correlation (the equivalent Person's correlation of λ_t) between S&P 500 and WTI returns using DSC model (without MIDAS effects), DSM copula with

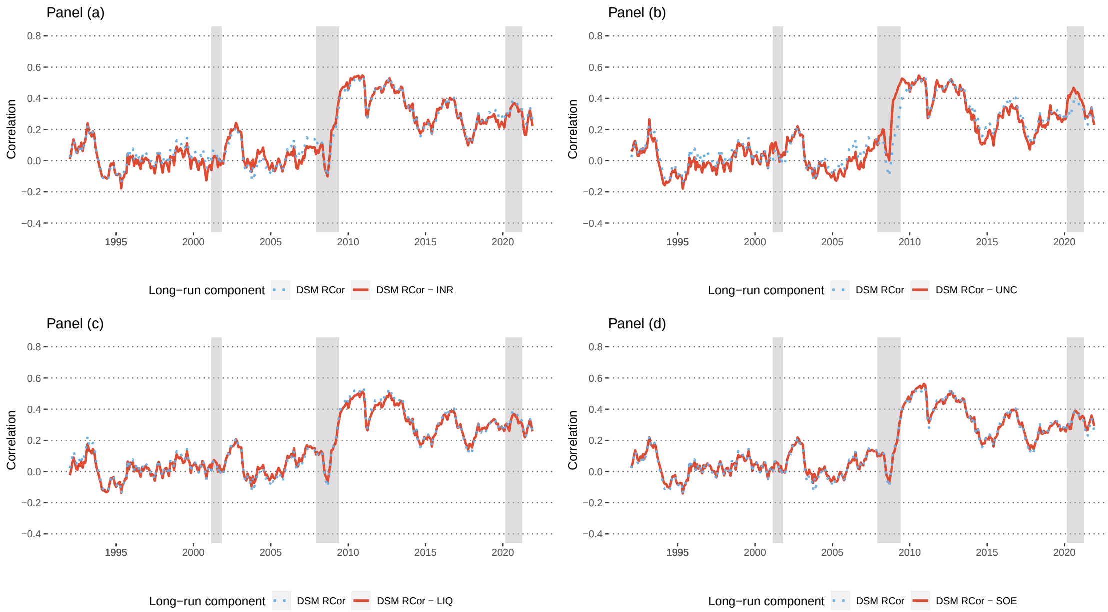

Figure 2: Long-run co-dependence between S&P 500 and WTI returns: RCor and one other macro-finance explanatory variable.

The figure shows the long-run co-dependence (the equivalent Person's correlation of λ_τ) between S&P 500 and WTI returns using DSM copula model with RCor plus one of the macro-finance factors as explanatory variable. The shaded areas highlight the recession periods based on the NBER indicators.

Figure 3: Long-run co-dependence between S&P 500 and WTI returns: two macro-finance explanatory variables.

The figure shows the long-run co-dependence (the equivalent Person's correlation of λ_τ) between S&P 500 and WTI returns using DSM copula models with two macro-finance explanatory variables in comparison to the rolling correlation (RC1Y), calculated at the end of each month using previous 252 daily observations. The shaded areas highlight the recession periods based on the NBER indicators.

RCor, and DSM copula with the RCor-UNC factor, all with t copula (highest LML overall). The total correlation is quite similar among all models since the total time varying co-dependence is mainly driven by the short-run component. However, the long-run component plays an important role in the long-run forecasts, which can have an effect on the optimal portfolio allocation. Hence in the next section we perform k-step ahead forecasts for the variance-covariance matrix and portfolio allocation exercises in order to quantify the economic gains from using the information from the macro-finance variables in the proposed DSM copula models.

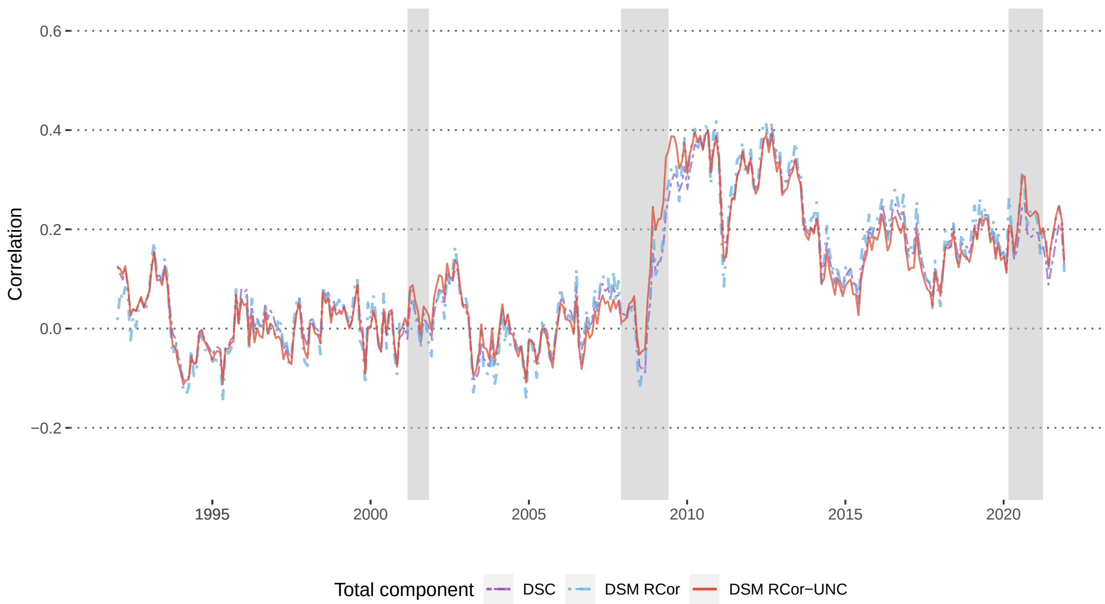

Figure 4: Total co-dependence between S&P 500 and WTI returns: DSC, DSM copula RCor and DSM copula with RCor-UNC factor models.

The figure shows the time varying co-dependence between S&P 500 and WTI returns using DSC, DSM copula RCor, and DSM copula RCor-UNC models. The shaded areas highlight the recession periods based on the NBER indicators.

5 Economic evaluation

In this section, we first obtain k-step-ahead variance-covariance matrix forecasts. We show that DSM copula models with two factors produce 22-days ahead cumulative covariance forecasts that are closer to the actually observed realized covariances than the simple DSC model. Next, we evaluate the potential economic gains of using the proposed DSM copula models for portfolio allocation problems. We split the sample into an in-sample period (from January 1990 to January 2018, 7059 observations) and an out-of-sample period (from January 2018 to December 2021 using the last 1000 observations). We re-estimate the DSM copula models using the in-sample data and use the out-of-sample observations to evaluate the performance of DSM copula model in portfolio allocation decisions.

5.1 Portfolio allocation

In this section we quantify the economic gains by performing two portfolio allocation exercises: minimum variance portfolio and hedging portfolio. In particular, we compare portfolios formed using the proposed models with MIDAS effects against a benchmark model (BM) without MIDAS. The benchmark model is the DSC, and the models for comparison are DSM copula models with either one or two explanatory factors: monthly realized correlation RCor, Uncertainty factor UNC, Inflation/interest rate factor INR and liquidity factor LIQ. We select such DSM copula models that have statistically significant effects of the explanatory variables. Same as before, all models are with t copula.

Minimum variance portfolio

For the first portfolio allocation exercise we consider the classical dynamic minimum variance portfolio. Such portfolio minimizes the overall risk independently of the portfolio returns. The investor can obtain optimal weights by solving the following minimization problem:

$$\arg\min_{\mathbf{w}_t} \mathbf{w}_t^{'}\mathbf{H}_{t,k}\mathbf{w}_t, \qquad s.t. \mathbf{w}_t^{'}\mathbf{1} = 1,$$

where $\mathbf{w}_t = [w_{1,t}, w_{2,t}]'$ is the vector of optimal weights and the k-step-ahead forecast of the conditional variance-covariance is as follows:

$$\mathbf{H}_{t,k} = \begin{bmatrix} \mathrm{Var}(r_{1,t+1:t+k}) & \mathrm{Cov}(r_{1,t+1:t+k}, r_{2,t+1:t+k}) \\ \mathrm{Cov}(r_{1,t+1:t+k}, r_{2,t+1:t+k}) & \mathrm{Var}(r_{2,t,k}) \end{bmatrix},$$

where $r_{i,t+1:t+k}$ is the cumulative returns of stock (or oil) during the k periods at time t. If there is no short-selling constraint, then the the minimum variance portfolio weights can be obtained analytically: $\mathbf{w}_t = (\mathbf{H}_{t,k}^{-1}\mathbf{1})/(\mathbf{1}^{'}\mathbf{H}_{t,k}^{-1}\mathbf{1})$. We assume that the investor holds this portfolio for k-days and readjusts the portfolio weights after that.

Hedging portfolio

For the second portfolio allocation exercise, similarly as in Conrad and Stürmer (2017), we consider a dynamic hedging portfolio. In such portfolio the investor holds stock and then adds oil to reduce risk. The portfolio weights can be obtained by solving the following:

$$\arg\min_{\mathbf{w}_t} \mathbf{w}_t^{'}\mathbf{H}_{t,k}\mathbf{w}_t, \qquad s.t. \mathbf{w}_t^{'}\boldsymbol{\mu} = \mu_0,$$

where $\boldsymbol{\mu} = [\mu_0, 0]^{'}$ is the vector of the expected excess returns of stock and oil. Here μ_0 is the required excess return, which is the same as the expected excess returns of stock. Then the optimal hedging portfolio weights are given by:

$$\mathbf{w}_t = \left(1, -\operatorname{Cov}(r_{1,t+1:t+k}, r_{2,t+1:t+k}) / \operatorname{Var}(r_{2,t+1:t+k})\right)'.$$

Since $\mathbf{w}_t = [w_{1,t}, w_{2,t}]' = [1, w_{2,t}]'$, then, $1 - w_{1,t} - w_{2,t} = -w_{2,t}$ is held in cash. Apart from holding stocks, investor will either hold or short-sell oil. Because the sign of the weight of oil $w_{2,t}$ is opposite to the sign of the covariance, it means that investor will hold oil if covariance is negative and short-sell oil if covariance is positive.

Comparison in terms of portfolio variance

First, we are interested in whether the DSM copula models deliver smaller portfolio variance than the DSC model. Engle and Colacito (2006) show that the portfolio variance is minimized when using the true conditional covariance. The authors propose a test for the equality of the variances of two portfolios based on the predictive covariance $\mathbf{H}_{t,k}$. The difference between k-step-ahead portfolio variances at time t is defined as follows:

$$d_{t,k} = \left(\mathbf{w}_t^{'}(\mathbf{r}_{t,t+1:t+k} - k\bar{\mathbf{r}})\right)^2 - \left(\mathbf{w}_{BM,t}^{'}(\mathbf{r}_{t,t+1:t+k} - k\bar{\mathbf{r}})\right)^2,$$

where $\mathbf{w}_{BM,t}$ are the benchmark portfolio weights and $\bar{\mathbf{r}}$ is the sample mean of $\mathbf{r}_t$. The null hypothesis is that the mean of $d_{t,k}$ is 0. The testing can be done via Diebold-Mariano type test.

Similarly as in Conrad and Stürmer (2017), we estimate the gain/loss (G/L) of using the DSM copula model as compared to the DSC model (the benchmark):

$$G/L = 100\%(\sigma_{BM} - \sigma_P)/\sigma_P, \tag{1}$$

where σ_{BM}^2 and σ_P^2 are the overall portfolio variances that are formed based on the predicted variance-covariance matrices of the DSC model and some DSM copula model, respectively. The G/L ratio shows the percentage of the expected return gain if we use a DSM copula model over the benchmark. The preferred portfolio will have positive and higher G/L ratio.

Comparison in terms of other portfolio metrics

A commonly used metric for portfolio comparison is the total portfolio turnover from period $t-1$ to period t, defined as

$$TO_t = \sum_{i=1}^{2} \left| w_{i,t} - w_{i,t-1}\frac{1 + r_{i,t-1}}{1 + r_{P,t-1}} \right|,$$

where $r_P = \mathbf{w}_t'\mathbf{r}_t$. Larger turnover implies larger transaction costs, therefore, the portfolio return needs to be adjusted by such costs, resulting into portfolio excess return $r_{E,t} = r_{P,t} - cTO_t$. Here

c is some small constant, which we set to 1%. Finally, we also calculate the portfolio Sharpe ratio $SR = r_E/\sigma_P$. Note that for simplicity in all portfolio comparison metrics we assume that the risk-free interest rate is equal to zero.

5.2 Portfolio results

We consider dynamic portfolio allocation for the in-sample and out-of-sample periods, with latter also split in pre-Covid and post-Covid. The in-sample evaluation period is from January 1992 to January 2018 which contains 7059 daily and 337 monthly observations. The out-of-sample evaluation period is till December 2021 which contains 1000 daily and 48 monthly observations split at the end of January 2020 into pre- and post-Covid (518 and 482 daily observations, respectively). We consider the daily and monthly portfolios, where monthly portfolio is re-balanced every month. Monthly re-balancing means forming the portfolio, keeping it for 22 days and then observing the realized returns.

Tables 6 and 7 report the G/L criteria, for in- and out-of-sample periods. The G/L ratio shows the percentage of the expected return gain if we use a DSM copula model over the benchmark. The preferred portfolio will have positive and higher G/L ratio. The G/L ratio is calculated with respect to the benchmark portfolio, which is based on the DSC model (without MIDAS effects). As seen in Table 6, the in-sample G/L ratio is almost always positive, indicating that all MIDAS specifications produce portfolios with smaller variances, meaning that the covariance matrices are closer to the "true" conditional covariance matrix than the one produced by the benchmark model (Engle and Colacito, 2006). The asterisk next to the G/L ratio indicates that the difference in variances is statistically significant at 5% confidence level. Out-of-sample results are not as consistent, see Table 7. Top panel contains results for the minimum variance portfolio. Here the daily G/L is always positive, but the difference in portfolio variances is not statistically significant. Monthly portfolio results are not so overwhelmingly favorable to the DSM copula models, especially if we split the sample into pre- and post-Covid. However, overall, the proposed DSM copula specifications produce portfolios with either the same or statistically smaller variances than the benchmark model, except for one instance (pre-Covid period, the DSM-INR-UNR model). The bottom panel displays the results for the hedge portfolio. Here during the pre-Covid period daily and monthly portfolios are always better, albeit, statistically insignificant. For the post-Covid and the total in-sample period the results are mixed, but two models consistently provide positive G/L values over the benchmark: DSM copula with RCor and DSM copula with RCor and UNC factors. Incidentally, these two models are the ones with the highest overall log marginal likelihoods. However, the differences in variances is statistically insignificant. To sum up, results from Tables 6- 7 suggest that the portfolios formed using the DSM copula models have statistically equal or smaller variances than the benchmark model, both in-sample and out-of-sample, for daily and monthly portfolios, with only one exception.

Finally, we investigate the performance of the minimum variance and hedge portfolios in eco-

Table 6: In-sample portfolio evaluation: G/L.

Model	Minimum variance portfolio		Hedge portfolio	
	Daily	Monthly	Daily	Monthly
DSM-RCor	0.520*	0.338	1.543*	-0.232
DSM-UNC	0.299	0.603	1.099*	4.055
DSM-RCor-UNC	0.931*	0.798	3.051*	3.816
DSM-INR-UNC	0.255	0.406	0.763	3.145
DSM-UNC-LIQ	0.538*	0.969	1.654*	3.985

The table presents the G/L in percentage for the in-sample period (01/01/1992 - 10/01/2018) for daily and monthly minimum variance and hedge portfolios. The G/L ratio shows the percentage of the expected return gain if we use a DSM copula model over the benchmark. We test for the equality of the portfolio variances using the Diebold Mariano test. The asterisk next to the G/L ratio indicates that the test is significant at 5% level.

Table 7: Out-of-sample portfolio evaluation: G/L.

Model	Daily	Monthly	Daily	Monthly	Daily	Monthly
	Pre-Covid		Post-Covid		Total	
Minimum variance portfolio						
DSM-RCor	0.309	-1.001	0.115	-0.042	0.157	-0.218
DSM-UNC	0.128	0.143	0.893	1.130*	0.725	0.971*
DSM-RCor-UNC	0.050	-2.010*	0.566	0.365	0.452	-0.064
DSM-INR-UNC	0.122	-0.048	0.778	1.221*	0.634	1.016*
DSM-UNC-LIQ	0.186	1.031*	0.685	1.259*	0.575	1.238*
Hedge portfolio						
DSM-RCor	0.243	0.769	0.626	0.017	0.527	0.265
DSM-UNC	0.023	0.723	-0.768	2.451	-0.566	1.418
DSM-RCor-UNC	0.093	0.091	0.527	2.679	0.415	1.227
DSM-INR-UNC	0.155	0.975	-0.863	4.670	-0.603	2.846
DSM-UNC-LIQ	0.348	1.406	-0.289	-4.005	-0.126	-1.877

The table presents the G/L in percentage for the out-of-sample period (11/01/2018 - 31/12/2021) for daily and monthly minimum variance and hedge portfolios. The out-of-sample is split on 31/01/2020 for pre- and post-Covid. The G/L ratio shows the percentage of the expected return gain if we use a DSM copula model over the benchmark. We test for the equality of the portfolio variances using the Diebold Mariano test. The asterisk next to the G/L ratio indicates that the test is significant at 5% level.

nomic terms. We are especially interested in the portfolio standard deviation (σ_P) for the global minimum variance portfolio and the portfolio Sharpe ratio for the hedge portfolio. The Sharpe ratio is calculated as the ratio of portfolio excess return over the standard standard deviation; where the excess return is obtained as the overall portfolio return minus 0.01$\times$ the turnover (also reported).

Left panel of Table 8 contains the results for the in-sample period for the minimum variance portfolio. The first three smallest overall variances are produced by the DSM copula specifications with RCor-UNC, UNC-LIQ and Rcor factors as explanatory variables, in that order, for daily; and UNC-LIQ, Rcor-UNC and UNC factors as explanatory variables, in that order, for monthly portfolios. The results remain virtually the same for the out-of-sample period as well, see left panel of Table 9. The first three smallest overall variances are produced by the DSM copula specifications with UNC, INR-UNC and UNC-LIQ factors as explanatory variables, in that order, for daily; and UNC-LIQ, Rcor-UNC and UNC factors as explanatory variables - identical to the in-sample ordering - for monthly portfolios. To sum up, incorporating information from the low frequency macro-finance variables into modeling the co-dependence of stock-oil returns results in more precise 1- and 22-step ahead variance covariance matrix forecasts. This translates into daily and monthly global minimum variance portfolios with the lowest variance overall, in- and out-of-sample.

As for hedging portfolio, we are primarily interested in the Sharpe ratios. In our case the calculated Sharpe ratio is a rather comprehensive metric for portfolio comparison because it also penalizes the large turnover. For the in-sample period (right panel of the Table 8), the highest Sharpe ratios are produced by the DSM copula models with essentially the same explanatory variables as in the best minimum variance portfolios: Uncertainty, Liquidity and RCor factors, either alone or in combinations. The results are the same for daily and monthly horizons, with a slight change in model ordering. Out-of-sample results change only very slightly (right panel of the Table 9), where the DSC specification (without MIDAS effects) is among the top three models, and the remaining best models are again the ones using RCor, UNC and LIQ as explanatory variables (either alone or in combinations), for daily and monthly hedging portfolios.

6 Conclusions

The paper has proposed a new approach to model the time varying stock-oil co-dependence via dynamic stochastic mixed data sampling (MIDAS) copula model, or DSM copula in short. Such model decomposes the stock-oil relationship into a short-run dynamic stochastic component and a long-run component, driven by a set macro-finance factors, assembled into four major groups via principal components: inflation/interest rate, uncertainty, liquidity and state of the economy factors.

The proposed approach leads to a better understanding of the economic drivers of the stock-oil co-dependence. In particular, by using data from more that 30 years, we found that inflation/interest rate factor has a negative and statistically significant effect on the co-dependence of

Table 8: In-sample portfolio evaluation: economic gains.

Model	σ_P	r_E	Sharpe	TO	σ_P	r_E	Sharpe	TO
Daily	Minimum variance portfolio				Hedge portfolio			
DSC	15.654	9.281	0.593	7.724	16.734	7.578	0.453	2.062
DSM-RCor	15.613	9.403	0.602	7.996	16.606	7.781	0.469	2.567
DSM-UNC	15.630	9.223	0.590	7.774	16.643	7.839	0.471	2.068
DSM-RCor-UNC	15.581	9.345	0.600	8.061	16.484	8.111	0.492	2.662
DSM-INR-UNC	15.634	9.228	0.590	7.765	16.670	7.799	0.468	2.056
DSM-UNC-LIQ	15.612	9.290	0.595	7.852	16.597	7.888	0.475	2.231
Monthly	Minimum variance portfolio				Hedge portfolio			
DSC	15.295	6.139	0.401	0.413	14.199	7.845	0.553	0.180
DSM-RCor	15.271	6.216	0.407	0.418	14.216	7.783	0.548	0.201
DSM-UNC	15.248	6.022	0.395	0.412	13.920	8.109	0.583	0.197
DSM-RCor-UNC	15.234	6.085	0.399	0.416	13.937	8.081	0.580	0.206
DSM-INR-UNC	15.264	6.043	0.396	0.412	13.982	8.075	0.578	0.200
DSM-UNC-LIQ	15.222	6.079	0.399	0.411	13.927	8.138	0.584	0.202

The table presents various portfolio metrics, all annualized, for the in-sample period (01/01/1992 - 10/01/2018) for daily and monthly minimum variance and hedge portfolios. σ_P is portfolio standard deviation, r_E is portfolio excess return calculated as the overall portfolio return minus $0.01 \times TO$, where TO is the turnover.

Table 9: Out-of-sample portfolio evaluation: economic gains.

Model	σ_P	r_E	Sharpe	TO	σ_P	r_E	Sharpe	TO
Daily	Minimum variance portfolio				Hedge portfolio			
DSC	22.881	2.464	0.108	8.107	20.290	18.869	0.930	2.573
DSM-RCor	22.863	2.127	0.093	8.347	20.237	18.836	0.931	2.880
DSM-UNC	22.799	2.365	0.104	7.991	20.349	18.587	0.913	2.636
DSM-RCor-UNC	22.830	1.874	0.082	8.401	20.249	18.430	0.910	2.970
DSM-INR-UNC	22.809	2.324	0.102	8.012	20.353	18.456	0.907	2.629
DSM-UNC-LIQ	22.816	2.235	0.098	8.243	20.303	18.864	0.929	2.826
Monthly	Minimum variance portfolio				Hedge portfolio			
DSC	27.365	5.958	0.218	0.461	13.812	18.369	1.330	0.359
DSM-RCor	27.393	5.709	0.208	0.455	13.811	18.096	1.310	0.344
DSM-UNC	27.243	5.870	0.215	0.448	13.758	17.741	1.289	0.387
DSM-RCor-UNC	27.376	5.561	0.203	0.448	13.774	17.654	1.282	0.340
DSM-INR-UNC	27.235	5.783	0.212	0.450	13.664	17.536	1.283	0.386
DSM-UNC-LIQ	27.205	5.882	0.216	0.450	13.969	18.215	1.304	0.401

The table presents various portfolio metrics, all annualized, for the out-of-sample period (11/01/2018 - 31/12/2021) for daily and monthly minimum variance and hedge portfolios. σ_P is portfolio standard deviation, r_E is portfolio excess return calculated as the overall portfolio return minus $0.01 \times TO$, where TO is the turnover.

the stock-oil returns. Moreover, uncertainty and liquidity factors have a positive and statistically significant effect on the co-dependence. We have shown that incorporating low frequency macro-finance factors leads to better in-sample fit in terms of log marginal likelihoods as compared to the benchmark specification of Hafner and Manner (2012). The results have important practical implications from the investor's and/or policy maker's perspectives. If the investor anticipates changes in the external macroeconomic/financial conditions, such as inflation, interest rates, uncertainty, and liquidity, she can revise her investment or hedging portfolios accordingly. On the other hand, a policy maker can foresee how her decisions regarding adjustments in the macroeconomic policy will affect the conditions in the financial and commodity markets in the long run.

We have performed several multi-step-ahead variance-covariance matrix forecasting and portfolio allocation exercises. There are three major findings. Firstly, the portfolios formed using the DSM copula models have statistically equal or smaller variances than the benchmark model, both in-sample and out-of-sample, for daily and monthly portfolios, with one exception only. According the Engle and Colacito (2006) this implies that the estimated variance-covariance matrix is closer to the true one. Secondly, DSM copula models produce daily and monthly global minimum variance portfolios with the lowest variance overall, in- and out-of-sample. Past realized correlation, liquidity and uncertainty are the most useful explanatory factors. Finally, the same realized correlation, liquidity and uncertainty factors remain essential for forming hedge portfolios with the highest Sharpe ratios: in-sample best daily and monthly portfolios are produced using the DSM copula model with either one or combination of two factors, and for out-of-sample the models using these factors are still in the top three best models, with a benchmark DSC model sometimes being better. The results of the portfolio allocation exercises reveal that the use of the exogenous macroeconomic/financial information to model the co-dependence between financial assets not only results in more precise variance-covariance matrix forecasts but also leads to tangible economic benefits from the investor's perspective.

It is straightforward to extend the DSM model in higher dimensions, as a bivariate copula is a building block for vine copulas and factor copulas (Aas et al., 2009; Kurowicka and Joe, 2011; Dißmann et al., 2013; Joe, 2015; Czado, 2019; Nguyen et al., 2020).

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