



VAR models with fat tails and asymmetry

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Motivation

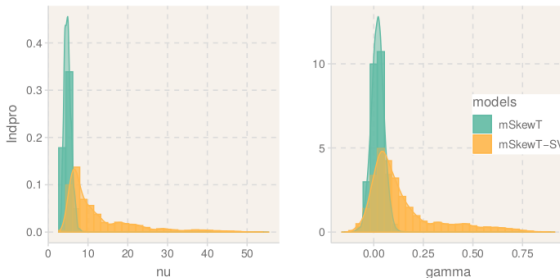
- The Gaussian vector autoregression (VAR) model has become one of the key macroeconomic models for policy makers and forecasters (Karlsson, 2013).
 - However, macroeconomic downturns during recessions and crisis can hardly be explained by a Gaussian structural shock (Mishkin, 2011; Acemoglu et al., 2017).
- There are several proposals for extending the Gaussian VAR model.
 - VAR models with Stochastic volatility (Uhlig, 1997; Clark, 2011).
 - VAR models with Time varying parameter (Primiceri, 2005; Cogley and Sargent, 2005).
 - VAR models with Heavy tail distributions (Clark and Ravazzolo, 2015; Chiu et al., 2017; Liu, 2019).

Related literature & Research questions

- The non-normality and fat tail characteristics of macroeconomic variables have been documented in many studies.
 - Fagiolo et al. (2008) analyze the output growth rates of OECD countries.
 - Ascari et al. (2015) shows findings on U.S. consumption, investment, employment, inflation and real wage.
 - Christiano (2007) found an evidence against the Gaussian assumption of an estimated VAR by inspecting the skewness and kurtosis properties of residuals.
 - Ni and Sun (2005) propose the VAR models with multivariate Student- t distribution (also Cúrdia et al. (2014) and Chib and Ramamurthy (2014)).
 - Panagiotelis and Smith (2008) first come up with an application of multivariate skew- t VAR models.
 - Liu (2019) estimates different fat tail and asymmetry distributions for macroeconomic variables, even though, the symmetric Student- t distribution is preferred among proposals for monthly data.
 - Cross and Poon (2016), Chiu et al. (2017) and Liu (2019) show that ignoring the stochastic volatility of the shocks will overestimate the fatness of the tail distribution.
- There is a gap in the literature on the combination of fat tails, asymmetry and stochastic volatility for the VAR model.

Preliminary results

- We propose a general class of Generalized Hyperbolic Skew Student's- t distribution with stochastic volatility (Skew- t .SV) VAR.
- In general, fat tail models improve in-sample goodness of fit and out-of-sample forecast.
- Slight evidence of asymmetry.
- Ignoring the stochastic volatility of the shocks will overestimate the fatness of the tail distribution **and underestimate the asymmetry**.



- 1 VAR Models
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 - Orthogonal Skew- t VAR Models
 - Multi-Skew- t VAR Models
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 - Model comparison and Forecast metrics
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- 5 Extension

Gaussian VAR Models

The Gaussian VAR model with stochastic volatility (Gaussian.SV) is discussed as following,

$$y_t = c + B_1 y_{t-1} + \dots + B_p y_{t-p} + A^{-1} H_t^{1/2} \epsilon_t, \quad (1)$$

where

- y_t is a k -dimensional vector of endogenous variables that $y_t = [y_{1t}, \dots, y_{kt}]'$;
- c is a k -dimensional vector of constant;
- B_j a $k \times k$ matrix of regression coefficients for $j = 1, \dots, p$;
- A is a $k \times k$ lower triangular matrix that describes the contemporaneous interaction of the endogenous variables;
- H_t is a $k \times k$ diagonal matrix that captures the heteroskedastic volatility; the vector of Gaussian shock $\epsilon_t \sim \text{Normal}(0, I_k)$. And

$$\log h_{it} = \log h_{it-1} + \sigma_i \eta_{it} \quad (2)$$

for $i = 1, \dots, k$ and $\eta_{it} \sim \text{Normal}(0, 1)$.

Gaussian VAR Models

The Gaussian VAR model with stochastic volatility (Gaussian.SV) is discussed as following,

$$y_t = Bx_t + u_t, \quad (3)$$

where

- $B = [c, B_1, \dots, B_p]$ is a $k \times (1+kp)$ dimensional matrix,
- $x_t = [1, y'_{t-1}, \dots, y'_{t-p}]'$ is $(1+kp)$ dimensional vector
- $u_t = A^{-1}H_t^{1/2}\epsilon_t$ is a k -dimensional vector of heteroskedastic shocks associated with the VAR equations.
- $Au_t = H_t^{1/2}\epsilon_t$ is a k -dimensional vector of orthogonal shocks.

Orthogonal Skew- t (oSkew- t) VAR Models

Following Cúrdia et al. (2014), Clark and Ravazzolo (2015), and Chiu et al. (2017), we account for the asymmetric heteroskedastic shocks in the orthogonal residuals $Au_t = A(y_t - Bx_t)$ of the VAR models by assuming that

$$Au_t = W_t\gamma + \sqrt{W_t}H_t^{1/2}\epsilon_t, \quad (4)$$

where

- the mixing variable $W_t = \text{diag}(\xi_{1t}, \dots, \xi_{kt})$ is a diagonal matrix that $\xi_{it} \sim \text{InvGamma}(\frac{\nu_i}{2}, \frac{\nu_i}{2})$ and ν_i is the degree of freedom for $i = 1, \dots, k$ and $t = 1, \dots, T$.
- $\gamma = [\gamma_1, \dots, \gamma_k]'$ is a k -dimensional vector of the skewness parameters.
- Equation (4) represents the marginal distribution of the orthogonal shock Au_t as a vector of independent univariate generalized hyperbolic skew Student- t distributions (oSkew- t), see Aas and Haff (2006).
- Orthogonal Student- t VAR model and Gaussian VAR model as special cases.

Multi-Skew- t (mSkew- t) VAR Models

We propose a class of the multi-Skew- t (mSkew- t) VAR model by assuming the residuals $u_t = (y_t - Bx_t)$ as,

$$u_t = W_t\gamma + \sqrt{W_tA^{-1}H_t^{1/2}}\epsilon_t. \quad (5)$$

- Comparing to the oSkew- t VAR model, the mSkew- t VAR model imposes the fat tail and asymmetry of the structure shock directly in each VAR equation rather than in the idiosyncratic shock.
- The tail co-movement can be defined by restricting that $\xi_{1t} = \dots = \xi_{kt}$ so the marginal distribution of u_t is a multivariate generalized hyperbolic skew Student- t (Skew- t) distribution as a special case, see McNeil et al. (2015).
- It becomes a multivariate Student- t (Student- t) distribution when $\xi_{1t} = \dots = \xi_{kt}$ and $\gamma_1 = \dots = \gamma_k = 0$.

Bayesian Inference

- The set of the SkewVARSV model parameters

$$\theta = \{B, a, \gamma_{1:k}, \nu_{1:k}, \sigma_{1:k}^2, h_{1:k,0}, W_{1:T}, H_{1:T}\}.$$

- We use the Minnesota-style priors for the coefficients B , see Koop and Korobilis (2010), and vague prior distributions for other parameters.

- $\text{vec}(B) \sim \mathcal{N}(b_0, V_{b_0})$

- $a \sim \mathcal{N}(0_k, 10I_k)$

- $\gamma \sim \mathcal{N}(0_k, I_k)$

- $\nu \sim \text{Gamma}(2, 0.1)$

- $\sigma_i^2 \sim \text{Gamma}(1/2, 1/2V_\sigma)$

- $\log h_{i0} \sim N\left(\log \hat{\Sigma}_{i,OLS}, 4\right)$

Gibb sampler scheme $\theta = \{B, a, \gamma_{1:k}, \nu_{1:k}, \sigma_{1:k}^2, h_{1:k,0}, W_{1:T}, H_{1:T}\}$

mSkew- t VAR Model: $y_t = Bx_t + W_t\gamma + \sqrt{W_t}A^{-1}H_t^{1/2}\epsilon_t$. Let's Ψ be a set of conditional parameters except the one that we sample from.

- Sample $\pi(\text{vec}(B)|\Psi)$ based on $y_t - W_t\gamma = Bx_t + \Sigma_t^{1/2}\epsilon_t$ where

$$\Sigma_t^{1/2} = \sqrt{W_t}A^{-1}H_t^{1/2}.$$

$$\pi(\text{vec}(B)|\Psi) \sim \mathcal{N}(b^*, V_b^*),$$

$$V_b^{*-1} = V_{b_0}^{-1} + \sum_{t=1}^T x_t x_t' \otimes \Sigma_t^{-1},$$

$$b^* = V_b^* \left[V_{b_0}^{-1} b_0 + \sum_{t=1}^T (x_t \otimes \Sigma_t^{-1} (y_t - W_t \gamma)) \right].$$

- Sample $\pi(\gamma|\Psi)$ based on $y_t - Bx_t = W_t\gamma + \Sigma_t^{1/2}\epsilon_t$.
- Sample $\pi(a|\Psi)$ based on $A\tilde{u}_t = H_t^{1/2}\epsilon_t$ where $\tilde{u}_t = W_t^{-1/2}(y_t - Bx_t - W_t\gamma)$.

Gibb sampler scheme $\theta = \{B, a, \gamma_{1:k}, \nu_{1:k}, \sigma_{1:k}^2, h_{1:k,0}, W_{1:T}, H_{1:T}\}$

mSkew- t VAR Model: $y_t = Bx_t + W_t\gamma + \sqrt{W_t}A^{-1}H_t^{1/2}\epsilon_t$:

- Sample $\pi(h_0, H_{1:T}|\Psi)$ and $\pi(\sigma^2|\Psi) = \pi(\sigma^2|h_0, H_{1:T})$,

Let $\tilde{u}_t = A\tilde{u}_t$, for each series $i = 1, \dots, k$, we have that $\log \tilde{u}_{it}^2 = \log h_{it} + \log \epsilon_t^2$.

Kim et al. (1998) approximated the distribution χ^2 of ϵ_t^2 using a mixture of 7 normal components.

- Sample $\pi(\nu_i|\Psi) \propto \mathcal{G}(\nu_i; 2, 0.1) \prod_{t=1}^T \mathcal{IG}\left(\xi_{it}; \frac{\nu_i}{2}, \frac{\nu_i}{2}\right)$ using RW Metropolis Hastings.
- Sample $\pi(W_t|\Psi)$ for $t = 1, \dots, T$ using the independent Metropolis Hastings.

Empirical illustration

- Data: Industrial Production, Inflation rate, Unemployment rate, Chicago board options exchange's volatility index (VIX).
- The data is collected at monthly frequency from 01/1970 - 12/2019 from the Federal Reserve Bank of St. Louis, see McCracken and Ng (2016).
- The growth changes are calculated using the first difference of the logarithm of Industrial Production Index and CPI. The VIX is calculated in the log scale.
- 14 VAR models ($p=4$ lags) w/wo SV: Gaussian, Student- t , hyperbolic skew Student- t (Skew- t), Orthogonal Student- t (oStudent- t), Orthogonal Hyperbolic skew Student- t (oSkew- t), multi-Student- t (mStudent- t), multi-hyperbolic skew Student- t (mSkew- t).
- 146 recursive forecast during 2007-2019.

Data

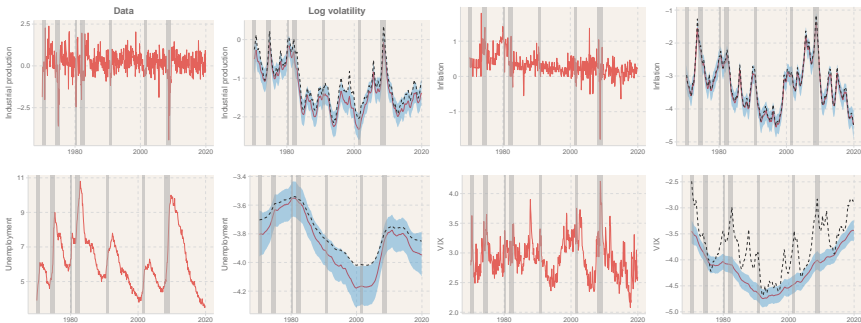


Figure: The plots show the growth of industrial production, inflation rate, unemployment rate, VIX in comparison to their estimated stochastic volatility from the oSkew- t .SV model.

The figures on the left-hand side show the growth changes of the variables while the figures on the right-hand side draw the estimated mean log volatility of the oSkew- t .SV model in red color with their 50% credible interval. The dash line shows the estimated mean log volatility from the Gaussian.SV model. The shaded areas highlight the recession periods based on the NBER indicators.

Posterior comparison of mSkew- t models w/wo SV

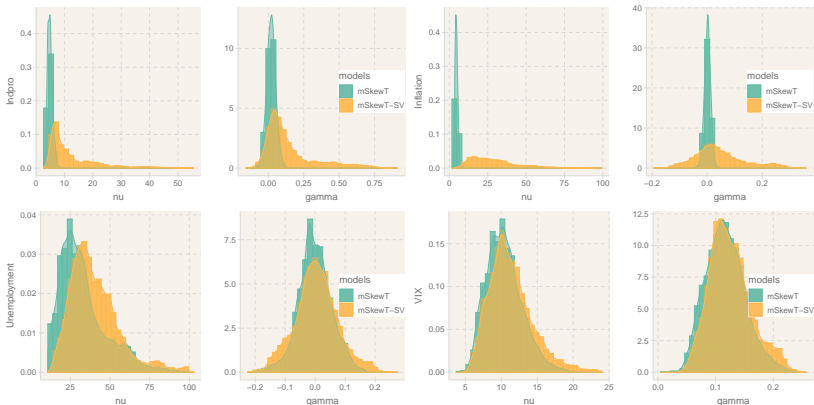


Figure: The plots show the posterior samples of the fat tail parameters and asymmetric parameters of the mSkew- t model with/without SV.

Posterior comparison of oSkew- t models w/wo SV

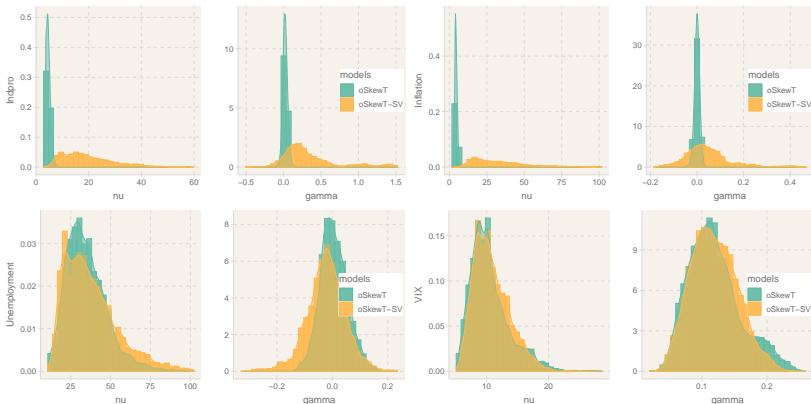


Figure: The plots show the posterior samples of the fat tail parameters and asymmetric parameters of the oSkew- t model with/without SV.

Model comparison and Forecast metrics

We compare 14 VAR models with different assumptions of tail distribution and stochastic volatility using the in-sample goodness of fit and out-of-sample forecast.

- The model marginal likelihood is calculated based on the cross entropy methods, see Chan and Eisenstat (2018).
- The out-of-sample forecast among models can also be measured by the mean square forecast error (MSFE), the log predictive density (LP), and the scale continuous rank probability scores (SCRPS) of the posterior predictive distribution.

Marginal likelihood

The marginal likelihood of the Skew- t -SV-VAR model requires high-dimensional integration

$$p(y_{1:T}) = \int p(y_{1:T}|\theta)\pi(\theta)d\theta. \quad (6)$$

- Approximate using the important sampling,

$$\hat{p}_{IS}(y_{1:T}) = \frac{1}{N} \sum_{n=1}^N \frac{\hat{p}(y_{1:T}|\theta^{(n)})p(\theta^{(n)})}{f(\theta^{(n)}|\lambda^*)}.$$

- Divide θ into two parameter groups $\theta_1 = \{\text{vec}(B)', a', \gamma', \nu', \sigma^{2'}, h_0'\}'$ and $\theta_2 = \{W'_{1:T}, H'_{1:T}\}'$. Then, calculate the integrate likelihood $\hat{p}(y_{1:T}|\theta_1^{(n)})$ first.

$$\hat{p}_{IS}(y_{1:T}) = \frac{1}{N} \sum_{n=1}^N \frac{\hat{p}(y_{1:T}|\theta_1^{(n)})p(\theta_1^{(n)})}{f(\theta_1^{(n)}|\lambda^*)}.$$

Marginal likelihood

The marginal likelihood of the Skew- t -SV-VAR model requires high-dimensional integration

$$p(y_{1:T}) = \int p(y_{1:T}|\theta)\pi(\theta)d\theta. \quad (7)$$

Algorithm 1. (Marginal likelihood estimation via the cross-entropy method)

- ① Two parameter groups $\theta_1 = \{\text{vec}(B)', a', \gamma', \nu', \sigma^{2'}, h_0'\}'$ and $\theta_2 = \{W'_{1:T}, H'_{1:T}\}'$.
Obtain the posterior samples $\theta_1^{(1)}, \dots, \theta_1^{(R)}$ from the posterior density $\pi(\theta|y_{1:T})$.
- ② Consider the parametric family $f(\theta_1; \lambda)$ parameterized by parameter λ such that

$$\lambda^* = \arg \max_{\lambda} \frac{1}{R} \sum_{r=1}^R \log f(\theta_1^{(r)}|\lambda)$$

- ③ Obtain new samples $\theta_1^{(1)}, \dots, \theta_1^{(N)}$ from $f(\theta_1; \lambda^*)$. Then the marginal likelihood is calculated via important sampling

$$\hat{p}_{IS}(y_{1:T}) = \frac{1}{N} \sum_{n=1}^N \frac{\hat{p}(y_{1:T}|\theta_1^{(n)})p(\theta_1^{(n)})}{f(\theta_1^{(n)}|\lambda^*)}.$$

Marginal likelihood

The integrated likelihood $p(y_{1:T}|\theta_1)$ require a high dimensional integral over the latent states $\theta_2 = \{W'_{1:T}, H'_{1:T}\}'$,

$$\begin{aligned} p(y_{1:T}|\theta_1) &= \int \int p(y_{1:T}|\theta_1, W_{1:T}, H_{1:T}) p(W_{1:T}, H_{1:T}|\theta_1) dW_{1:T} dH_{1:T} \\ p(y_{1:T}|\theta_1) &= \int p(y_{1:T}|\theta_1, H_{1:T}) p(H_{1:T}|\theta_1) dH_{1:T} \\ &\approx \sum_{l=1}^L \frac{1}{L} \frac{p(y_{1:T}|\theta_1, H_{1:T}^{(l)}) p(H_{1:T}^{(l)}|\theta_1)}{f(H_{1:T}^{(l)}|\lambda_H)} \end{aligned}$$

We proposes an important sampling distribution $f(H_{1:T}|\lambda_H)$ and simulate $H_{1:T}^{(l)} \sim f(H_{1:T}|\lambda_H)$ for $l = 1, \dots, L$. Then, $p(y_{1:T}|\theta_1, H_{1:T}^{(l)})$ is the conditional likelihood which can be derived in a closed form multivariate distribution.

Model comparison

Table: Log marginal likelihood for models w/wo the SV

	Gaussian	Student- t	Skew- t	oStudent- t	mStudent- t	oSkew- t	mSkew- t
Non SV	-204.8	-110.6	-118.8	-110.5	-107.8	-110.1	-107.2
SV	-48.1	-31.1	-38.2	-37.5	-34.3	-27.2	-26.3

All models estimated using 20000 draws from the sampler and 5000 draws as burn-in.

Probability Integral Transform (PIT) histogram plots $u_t = F(y_t^{obs} | y_{1:t-1})$

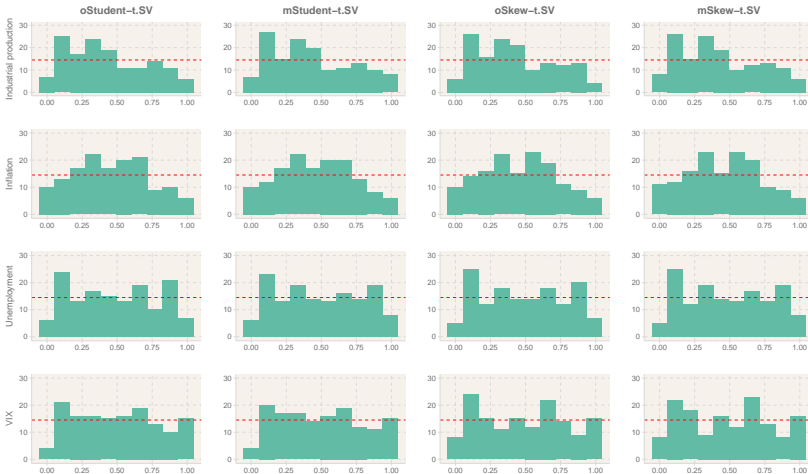


Figure: The plots show the PITs of one month forecast horizon of 146 recursive estimations (2007-2019).

Out-of-samples density forecast - LP

Table: Improvement in LP over the Gaussian VAR model (2007-2019)

	1M	3M	6M	12M	1M	3M	6M	12M
	(a) Industrial Production				(c) Unemployment rate			
Gaussian	-1.044	-1.122	-1.236	-1.266	-0.075	-0.600	-1.072	-1.712
Gaussian.SV	0.149	0.186	0.137	0.131	0.495	0.461	0.403	0.219
Student- <i>t</i> .SV	0.149	0.185	0.173	0.159	0.487	0.447	0.353	0.120
Skew- <i>t</i> .SV	0.142	0.189	0.179	0.175	0.489	0.442	0.326	0.171
oStudent- <i>t</i> .SV	0.153	0.206	0.198	0.179	0.493	0.450	0.350	0.195
mStudent- <i>t</i> .SV	0.156	0.202	0.200	0.182	0.493	0.447	0.353	0.145
oSkew- <i>t</i> .SV	0.157	0.205	0.187	0.168	0.494	0.465	0.369	0.148
mSkew- <i>t</i> .SV	0.150	0.199	0.190	0.183	0.492	0.441	0.348	0.132

The first line in each panel reports the LP of the benchmark Gaussian VAR model without stochastic volatility. The relative improvement over the benchmark is computed as the average of the LP obtained from 146 recursive estimations for different specifications of VAR models during 2007-2019. The entries greater than 0 indicate that the given model is better.

Out-of-samples density forecast - LP

Table: Improvement in LP over the Gaussian VAR model (2007-2019)

	1M	3M	6M	12M	1M	3M	6M	12M
	(b) Inflation				(d) VIX			
Gaussian	-0.372	-0.591	-0.647	-0.665	-0.139	-0.523	-0.697	-0.860
Gaussian.SV	0.428	0.377	0.404	0.450	0.324	0.272	0.273	0.347
Student- <i>t</i> .SV	0.423	0.391	0.408	0.464	0.354	0.264	0.255	0.342
Skew- <i>t</i> .SV	0.420	0.374	0.408	0.439	0.397	0.291	0.285	0.352
oStudent- <i>t</i> .SV	0.425	0.366	0.404	0.448	0.361	0.264	0.266	0.340
mStudent- <i>t</i> .SV	0.424	0.364	0.402	0.446	0.360	0.265	0.269	0.341
oSkew- <i>t</i> .SV	0.427	0.371	0.405	0.457	0.403	0.317	0.311	0.357
mSkew- <i>t</i> .SV	0.424	0.370	0.406	0.457	0.402	0.310	0.311	0.357

The first line in each panel reports the LP of the benchmark Gaussian VAR model without stochastic volatility. The relative improvement over the benchmark is computed as the average of the LP obtained from 146 recursive estimations for different specifications of VAR models during 2007-2019. The entries greater than 0 indicate that the given model is better.

Improvement of LP for 3-month ahead forecast

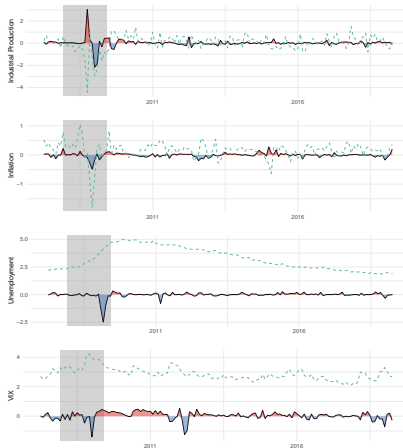


Figure: The difference of log predictive density for one quarter forecast horizon between the Gaussian.SV and mSkew- t .SV models. The red color means mSkew- t .SV predicts better and the blue color means vice versa. The dash lines illustrate the scale values of the original variables.

International results for OECD countries

Table: Fat tails and asymmetry evidence for OECD countries

OECD country	Industrial production		Inflation		Unemployment rate		Period	
	ν	γ	ν	γ	ν	γ	From	To
CAN	26.61 (18.02;33.72)	-0.09 (-0.35;0.17)	18.61 (12.23;24.04)	0.23 (0.12;0.32)	25.53 (16.31;24.04)	0.14 (0.09;0.19)	1961M02	2019M12
DEU	33.23 (20.82;44.05)	-0.35 (-0.86;0.16)	14.67 (8.79;19.49)	0.26 (0.12;0.39)	36.27 (22.94;19.49)	0.01 (-0.01;0.03)	1991M01	2019M12
ITA	28.71 (17.79;37.36)	-0.62 (-1.05;-0.18)	33.82 (21.28;44.00)	-0.03 (-0.12;0.07)	13.46 (8.29;44.00)	0.05 (0.02;0.07)	1983M01	2019M12
JPN	18.50 (12.28;23.07)	-0.74 (-1.16;-0.33)	16.16 (10.08;21.41)	0.31 (0.16;0.46)	45.89 (31.35;21.41)	0.02 (-0.02;0.06)	1970M02	2019M12
KOR	27.10 (15.72;36.92)	-0.37 (-1.03;0.24)	25.58 (16.98;33.38)	0.55 (0.29;0.79)	35.01 (20.13;33.38)	-0.01 (-0.07;0.05)	1990M01	2019M12
ESP	22.44 (11.28;32.19)	-0.15 (-0.52;0.20)	26.04 (16.17;34.43)	-0.35 (-0.54;-0.19)	30.89 (18.43;34.43)	-0.04 (-0.08;0.00)	1986M04	2019M12
SWE	24.92 (14.50;34.02)	-0.25 (-0.70;0.19)	5.70 (4.49;6.79)	0.08 (0.04;0.12)	22.85 (13.56;6.79)	0.11 (0.02;0.20)	1983M01	2019M12
UK	21.06 (10.03;31.28)	-0.35 (-0.60;-0.11)	6.00 (4.09;6.88)	-0.10 (-0.13;-0.04)	26.11 (16.49;6.88)	0.03 (0.00;0.06)	1983M01	2019M12

The table reports the means of the posterior samples of the mSkew VAR model for 8 OECD countries. The numbers in the brackets report the [20% - 80%] credible intervals.

Conclusion and Discussion

Contributions:

- VAR models with fat tail and asymmetry + an R package.

Findings:

- Stochastic volatility improves point and density forecast.
- Ignoring the stochastic volatility of the shocks will overestimate the fatness of the tail distribution **and underestimate the asymmetry**.
- In general, fat tail improves in-sample goodness of fit and out-of-sample forecast.
- Slight evidence of asymmetry.

Extension - Dynamic Skew VAR models

Dynamic Skew VAR models:

$$\begin{aligned}y_t &= Bx_t + u_t, \\u_t &= w_t\gamma_t + \sqrt{w_t}A^{-1}H_t^{1/2}\epsilon_t. \\ \log h_{it} &= \log h_{it-1} + \sigma_i\eta_{it} \\ \gamma_t &= \gamma_{t-1} + \sigma_\gamma\varsigma_t\end{aligned}\tag{8}$$

Extension - Stack Mixed-Frequency VAR models

Stack Mixed-Frequency VAR models:

- Let the vector of mixed-frequency quarterly data be

$$y_t = [y_{t-2/3}^{(m)}, y_{t-1/3}^{(m)}, y_t^{(m)}, y_t^{(q)}]'$$

$$y_t = Bx_t + u_t,$$

$$u_t = A^{-1} \left(W_t \gamma + \sqrt{W_t} H_t^{1/2} \epsilon_t \right), \quad (9)$$

$$\log h_{it} = \log h_{it-1} + \sigma_i \eta_{it}.$$

Thank you

Model comparison and Forecast metrics

Following Andersson and Karlsson (2008), the LP of the posterior predictive distribution is computed using the Rao-Blackwellization.

$$\begin{aligned} \text{LP}_{i,h} &= \log p(y_{i,t+h}^o | y_{1:t}) = \log \int_{\theta} p(y_{i,t+h}^o | \theta, y_{1:t}) p(\theta | y_{1:t}) d\theta \\ &= \log \sum_{r=1}^R \frac{1}{R} p(y_{i,t+h}^o | \theta^{(r)}, y_{1:t}) \end{aligned}$$

where $\theta^{(1)}, \dots, \theta^{(R)}$ are the posterior samples of the VAR models.

Bolin and Wallin (2019) proposed a scaled version of the CRPS

$$\text{SCRPS}_{i,h} = - \frac{E_f \left| y_{i,t+h|t} - y_{i,t+h}^o \right|}{E_f \left| y_{i,t+h|t} - y'_{i,t+h|t} \right|} - \frac{1}{2} \log E_f \left| y_{i,t+h|t} - y'_{i,t+h|t} \right|,$$

where f is the predictive density of the variable $y_{i,t+h|t}$, and $(y_{i,t+h|t}, y'_{i,t+h|t})$ are independent random draws from the predictive density f . SCRPS is locally scale invariant and robust to extreme observations.

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