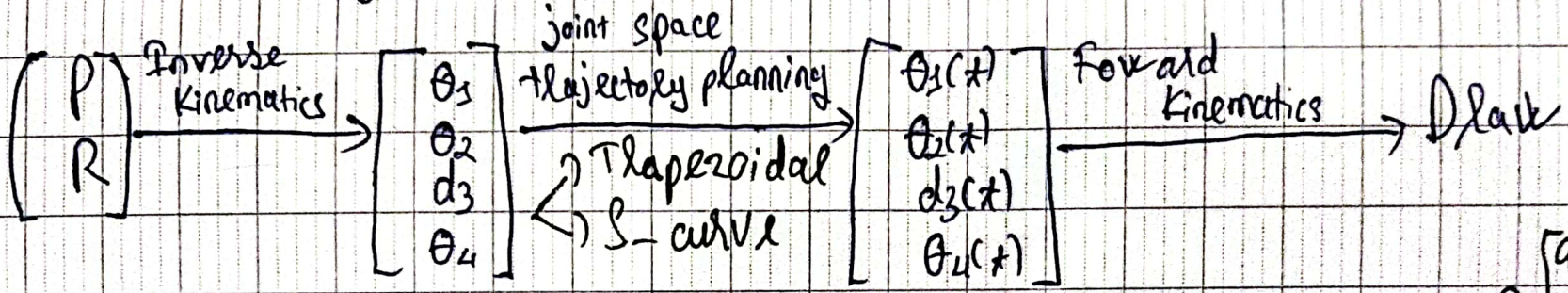
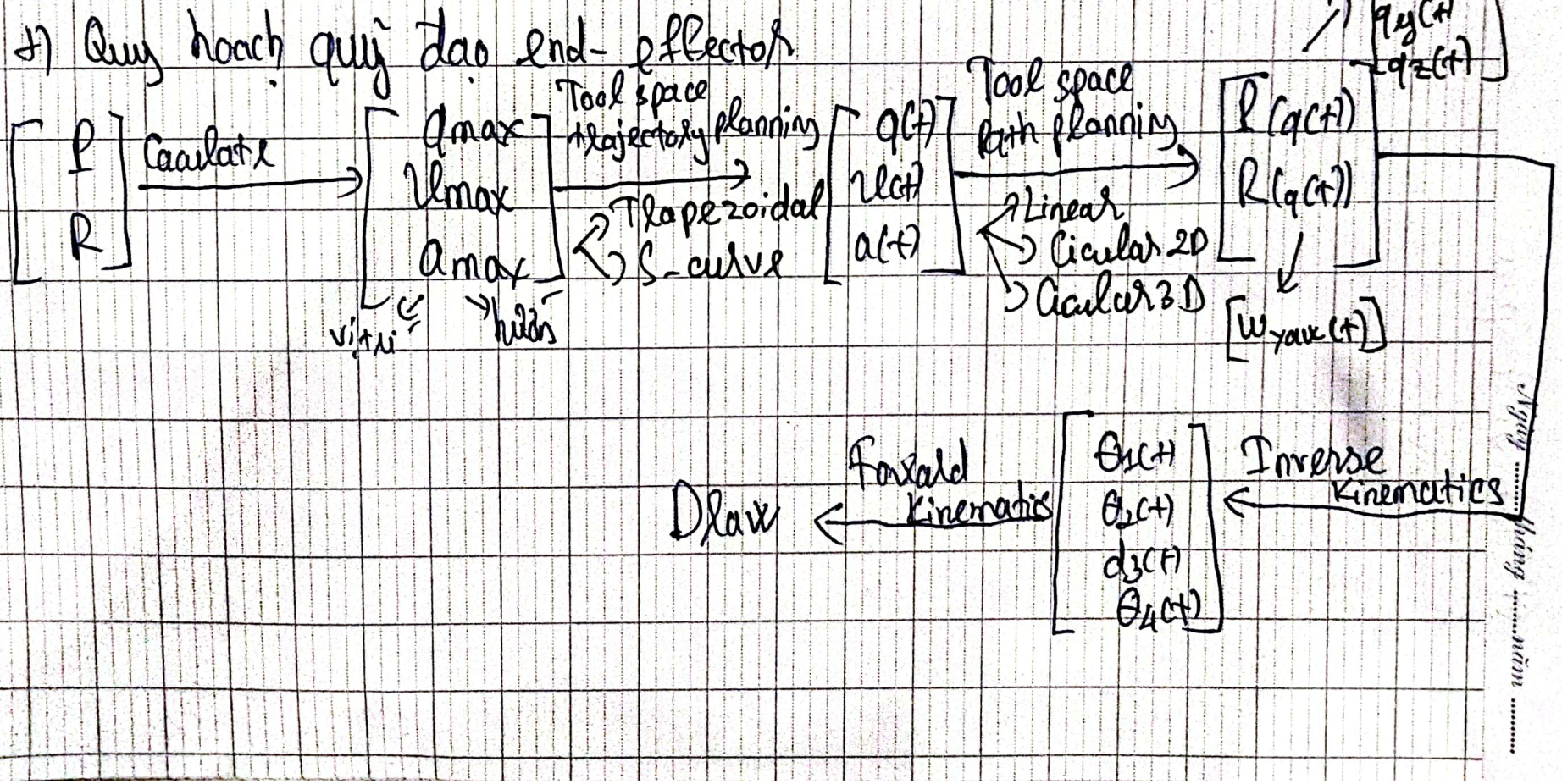
⑦ Trajectory and Path planning

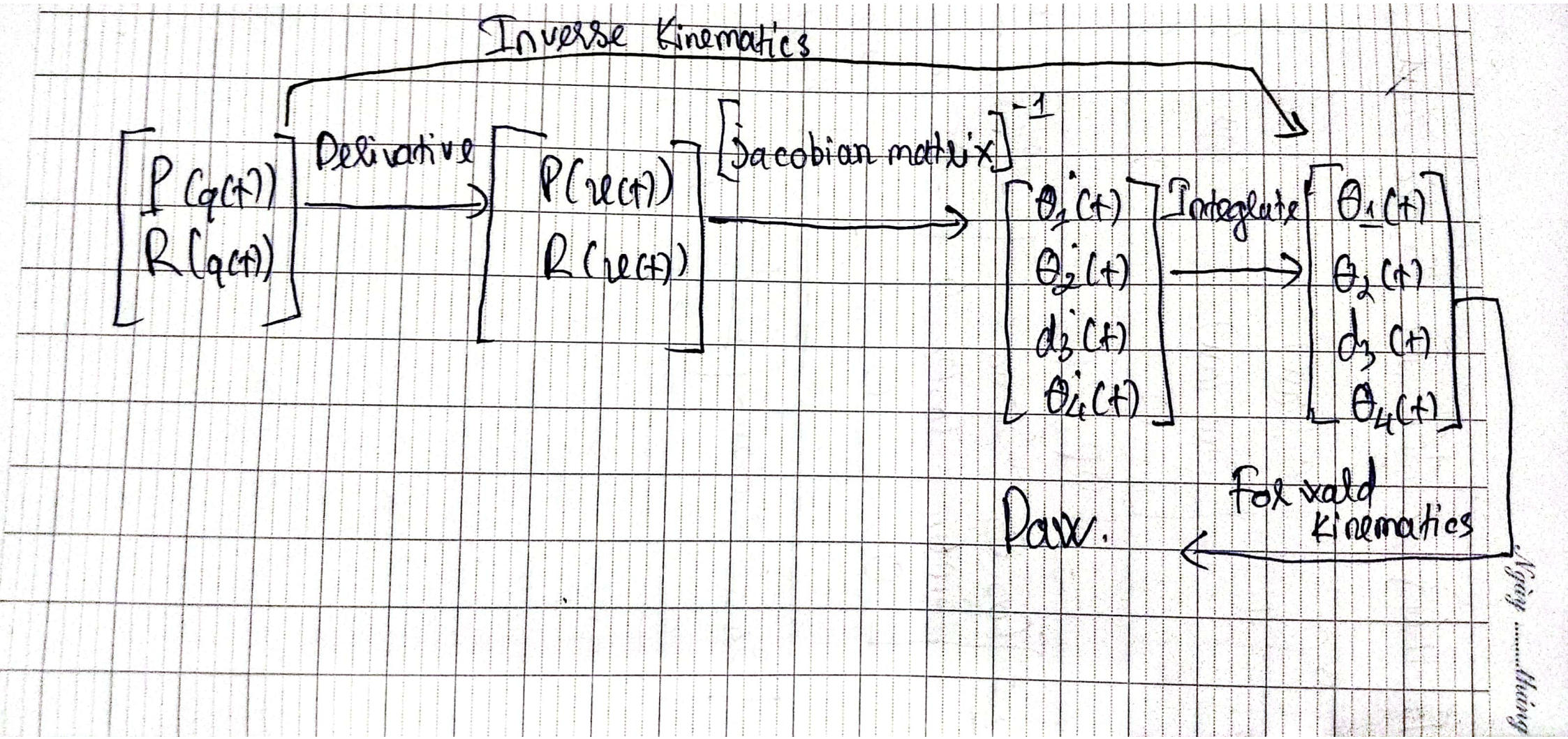
1) Quy hoạch quỹ đạo từng joint



2) Quy hoạch quỹ đạo end-effector

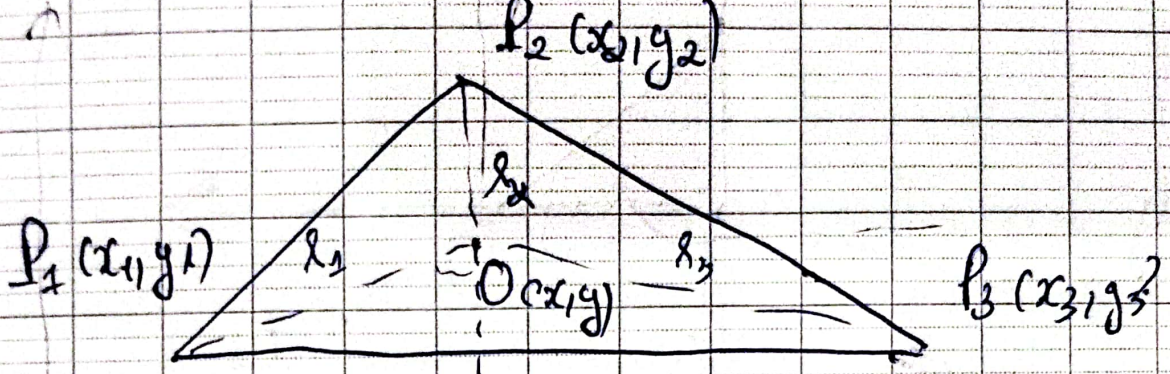


HONGHA


Inverse Kinematics
P(q(t))
R(q(t))
Derivative
P(ve(t))
R(ve(t))
[Jacobian matrix]^-1
θ1(t)
θ2(t)
d3(t)
θ4(t)
Integrate
θ1(t)
θ2(t)
d3(t)
θ4(t)
Forward Kinematics
Pdes.

Circular Interpolation 2D

Cho 3 đỉnh của 1 tam giác



$$r_1^2 = (x - x_1)^2 + (y - y_1)^2$$
$$r_2^2 = (x - x_2)^2 + (y - y_2)^2$$
$$r_3^2 = (x - x_3)^2 + (y - y_3)^2$$

$r_1^2 = r_2^2$ (= 0) $\Rightarrow y_1^2 + y_2^2 + x_2^2 - x_1^2 = 2x(x_2 - x_1) + 2y(y_2 - y_1)$

$r_2^2 = r_3^2$ (= 0) $\Rightarrow y_2^2 + y_3^2 + x_3^2 - x_2^2 = 2x(x_3 - x_2) + 2y(y_3 - y_2)$

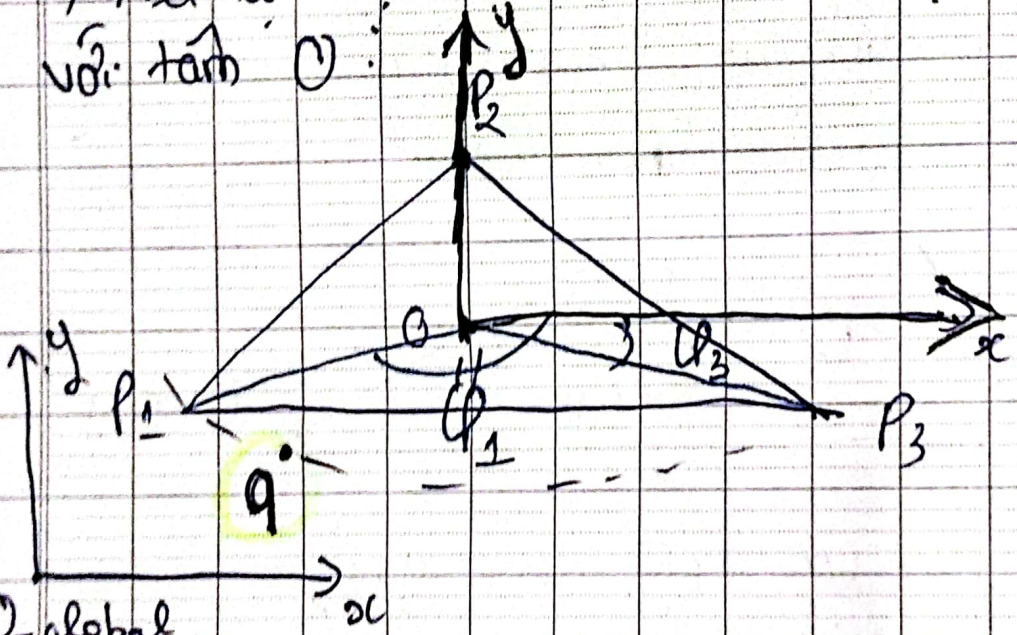
$$\Rightarrow 2\begin{bmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_2 & y_3 - y_2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_2^2 - x_1^2 \\ x_3^2 - x_2^2 \end{bmatrix} + \begin{bmatrix} y_2^2 - y_1^2 \\ y_3^2 - y_2^2 \end{bmatrix}$$

Ngày .....tháng .....năm .....

$$\Leftrightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} O_x \\ O_y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_2 \end{bmatrix}^{-1} \left(\begin{bmatrix} x_2^2 - x_1^2 \\ x_3^2 - x_2^2 \end{bmatrix} + \begin{bmatrix} y_2^2 - y_1^2 \\ y_3^2 - y_2^2 \end{bmatrix} \right)$$

+) $\text{Radius} = \sqrt{(x_1 - O_x)^2 + (y_1 - O_y)^2}$

+) Xét hệ toạ độ @ local - hệ toạ độ cục bộ với tâm O:



$$\varphi_3 = \arctan\left(\frac{y_3 - O_y}{x_3 - O_x}\right)$$

$$\varphi_1 = \arctan\left(\frac{y_1 - O_y}{x_1 - O_x}\right)$$

$$\Rightarrow q_{max} = \text{Radius} \cdot \| \varphi_3 - \varphi_1 \|$$

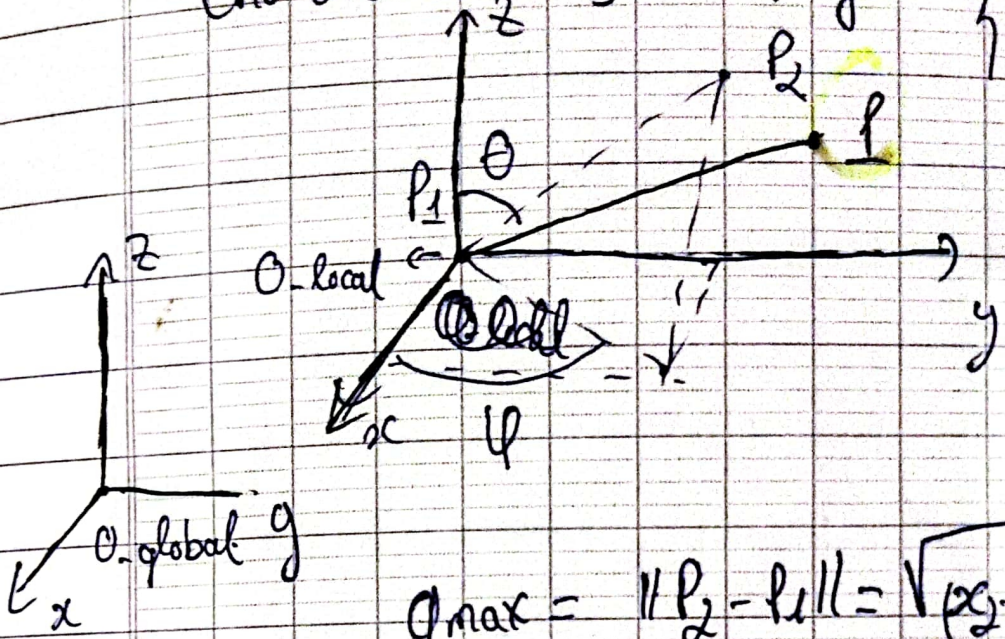
a) $\begin{cases} q_x = O_x + \lambda \cdot \cos\varphi \\ q_y = O_y + \lambda \sin\varphi \\ q_z = O_z \end{cases}$ $\left(\begin{matrix} {}^{local}q_x = \lambda\cos\varphi \\ {}^{local}q_y = \lambda\sin\varphi \\ {}^{local}q_z = 0 \end{matrix} \right)$

HONGHA

Ngày ..... tháng ..... năm .....

Linear Interpolation

Cho 2 điểm trong không gian $P_1(x_1, y_1, z_1)$, $P_2(x_2, y_2, z_2)$



$$q_{max} = \|P_2 - P_1\| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

Xét hệ toạ độ local là hệ toạ độ cầu (3D) gắn với P_1:

$$^{local}P_x = \|OP\| \cdot \sin\theta \cdot \cos\varphi$$
$$^{local}P_y = \|OP\| \sin\theta \sin\varphi$$
$$^{local}P_z = \|OP\| \cos\theta$$

Chuyển sang hệ global, ta được:

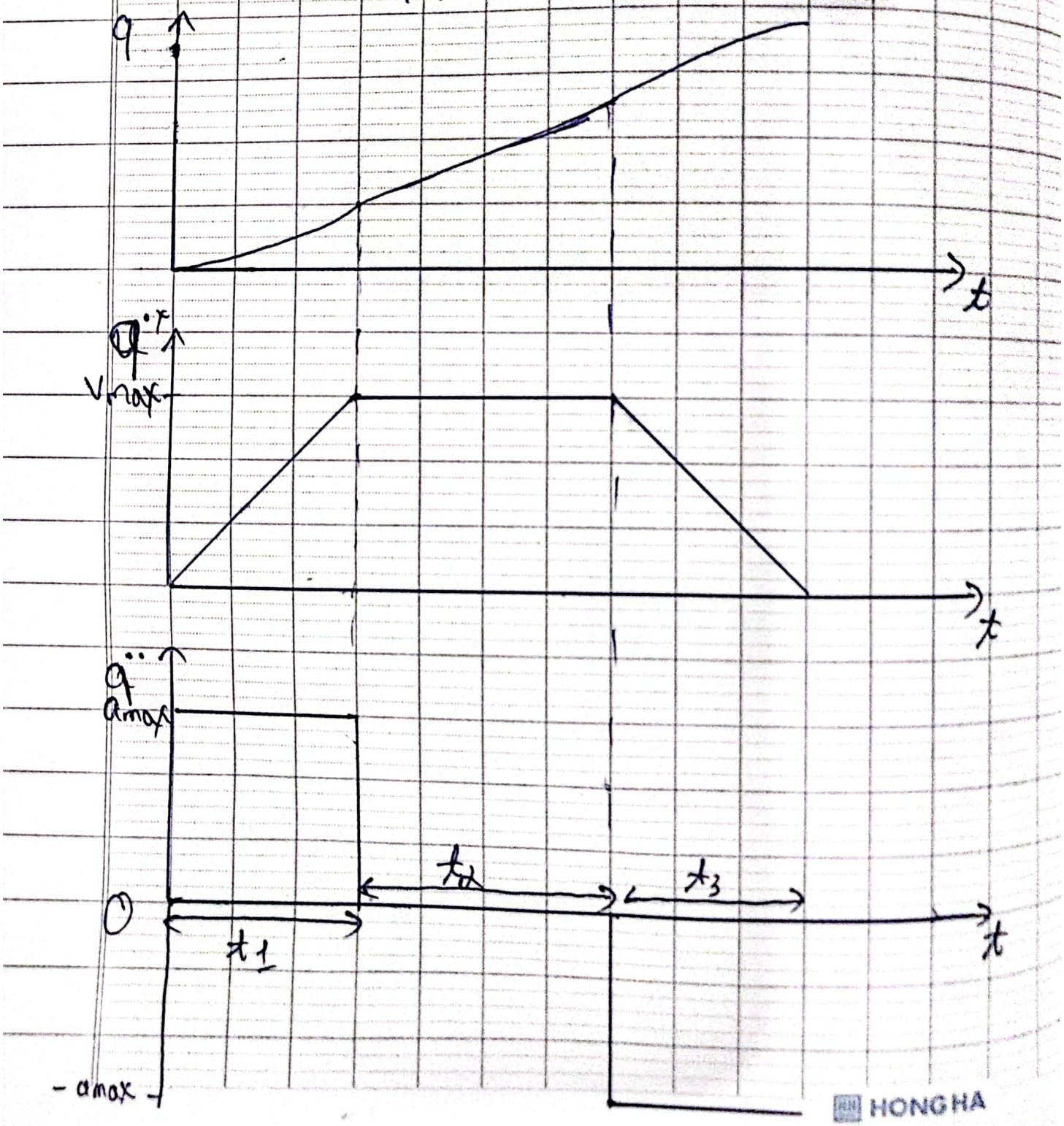
$$P_x = P_{1x} + {}^{local}P_x = P_{1x} + \|OP\| \sin\theta \cos\varphi$$
$$P_y = P_{1y} + {}^{local}P_y = P_{1y} + \|OP\| \sin\theta \sin\varphi$$
$$P_z = P_{1z} + {}^{local}P_z = P_{1z} + \|OP\| \cos\theta$$

Ngày ..... tháng ..... năm .....

Tlajectoly

⊛ Tlapezoidal tlajectoly

Input $\begin{bmatrix} q_{max} \\ v_{max} \\ a_{max} \end{bmatrix}$



HONGHA

Ngày .....tháng .....năm .....

Ta dễ thấy $v_{max} = t_1 . a_{max} \Rightarrow t_1 = \frac{v_{max}}{a_{max}}$

+) $0 < t \leq t_1$

$$\begin{cases} q''(t) = a_{max} \\ q'(t) = a_{max} . t \\ q(t) = \frac{1}{2} a_{max} t^2 \end{cases}$$

$. q(t_1) = \frac{1}{2} a_{max} . \frac{v_{max}^2}{a_{max}^2} = \frac{1}{2} \frac{v_{max}^2}{a_{max}}$

mà ta chọn $t_3 = t_1 \Rightarrow$ quãng đường đi trong khoảng t_1 và t_3 là bằng nhau và bằng $\frac{1}{2} \frac{v_{max}^2}{a_{max}}$

$\Rightarrow v_{max} . t_2 + 2 . \frac{1}{2} \frac{v_{max}^2}{a_{max}} = q_{max}$

$\Rightarrow t_2 = \frac{q_{max}}{v_{max}} - \frac{v_{max}}{a_{max}}$

+) $t_1 < t \leq t_1 + t_2$

$$\begin{cases} q''(t) = 0 \\ q'(t) = q'(t_1) = v_{max} \\ q(t) = q(t_1) + v_{max} (t - t_1) \end{cases}$$

HONG HA

CamScanner

Ngày .....tháng .....năm .....

+) $t_1+t_2 < t \le t_1+t_1+t_3 \ (= 2t_1+t_2)$

$$\begin{cases} \ddot{q}(t) = -a_{max} \\ \dot{q}(t) = \dot{q}(t_2) - a_{max}.(t-t_1-t_2) \\ \quad = v_{max} - a_{max}(t-t_1-t_2) \\ q(t) = q(t_2) + v_{max}(t-t_1-t_2) \\ \quad - \frac{1}{2} a_{max}(t-t_1-t_2)^2. \end{cases}$$

Ta có điều kiện để trajectory có thể thực hiện được là quãng đường đi khi $v = v_{max} \ (a=0) \ge 0$

$\Leftrightarrow q_{max} - 2q(t_1) \ge 0$

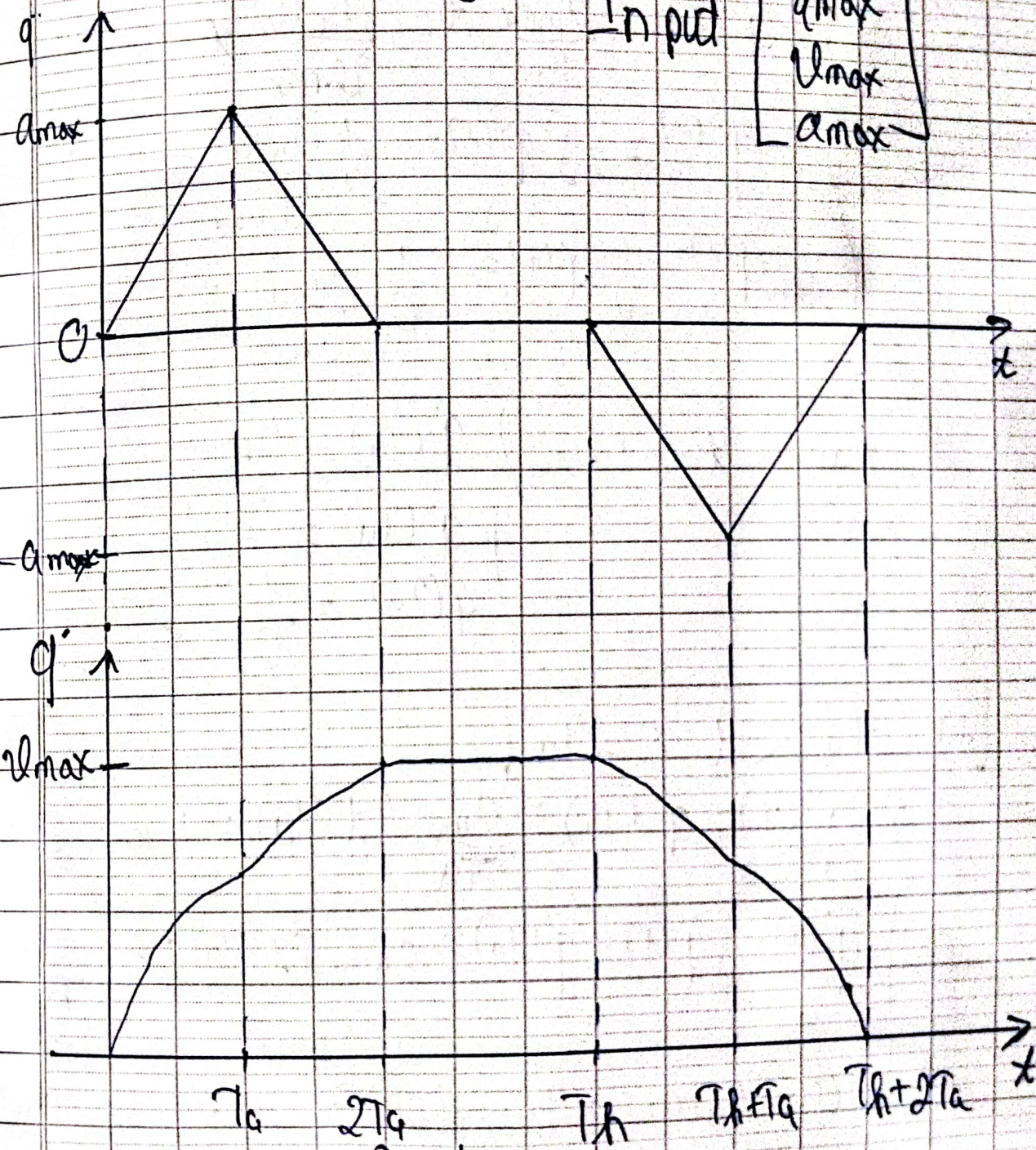
$\Leftrightarrow q_{max} - \frac{v_{max}^2}{a_{max}} \ge 0$

$\Leftrightarrow 0 \le v_{max} \le \sqrt{q_{max} \cdot a_{max}}$

Ngày ..... tháng ..... năm .....

✱ S_curve trajectory

Input $\begin{bmatrix} q_{max} \\ v_{max} \\ a_{max} \end{bmatrix}$



① . Ta có $v = \int a\,dt$

Ei $v_{max} = \frac{1}{2} \cdot a_{max} \cdot 2T_a$

$\Rightarrow T_a = \frac{v_{max}}{a_{max}}$

Ngày ..... tháng ..... năm .....

+) $0 < t \le T_a$

$$\begin{cases} \ddot{q}(t) = \dfrac{a_{max}}{T_a} t = \dfrac{a_{max}^2}{v_{max}} t \\ \dot{q}(t) = \int \ddot{q}(t) = \dfrac{1}{2} \dfrac{a_{max}^2}{v_{max}} t^2 + C \\ q(t) = \int \dot{q}(t) = \dfrac{1}{6} \dfrac{a_{max}^2}{v_{max}} t^3 + C \end{cases}$$

. tại $t = T_a$

$$\begin{cases} \ddot{q}(T_a) = a_{max} \\ \dot{q}(T_a) = \dfrac{v_{max}}{2} \\ q(T_a) = \dfrac{1}{6} \dfrac{v_{max}^2}{a_{max}} \end{cases}$$

+) $T_a < t \le 2T_a$

$$\begin{cases} \ddot{q}(t) = \ddot{q}(T_a) - \dfrac{a_{max}}{T_a}(t - T_a) = a_{max} - \dfrac{a_{max}}{T_a}(t - T_a) \\ \dot{q}(t) = \int \ddot{q}(t) = \dot{q}(T_a) + a_{max}(t - T_a) - \dfrac{1}{2} \dfrac{a_{max}}{T_a}(t - T_a)^2 \\ q(t) = \int \dot{q}(t) = q(T_a) + \dot{q}(T_a)(t - T_a) + \dfrac{1}{2} a_{max}(t - T_a)^2 - \dfrac{1}{6} \dfrac{a_{max}}{T_a}(t - T_a)^3 \end{cases}$$

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Ngày .....tháng .....năm .....

. Tại $t = 2T_a$

$q(2T_a) = \frac{v_{max}^2}{a_{max}}$

Ta có điều kiện để Hyperbol là quỹ đạo tàu vận tốc v_{max} đi được phải $\neq 0$:

$q_{max} - 2q(2T_a) \neq 0$

$\Leftrightarrow q_{max} - 2\frac{v_{max}^2}{a_{max}} \neq 0$

$\Leftrightarrow 0 < v_{max} \leq \sqrt{\frac{q_{max} \cdot a_{max}}{2}}$

$\Rightarrow$ Khi đó $T_h = 2T_a + \frac{q_{max} - 2q(2T_a)}{v_{max}}$

$= \frac{q_{max}}{v_{max}}$

+) $2T_a < t \leq T_h$

$\begin{cases} q''(t) = 0 \\ q'(t) = q'(2T_a) = v_{max} \\ q(t) = q(2T_a) + v_{max}(t - 2T_a) \end{cases}$

(2) $T_h < t \leq T_h + T_a$

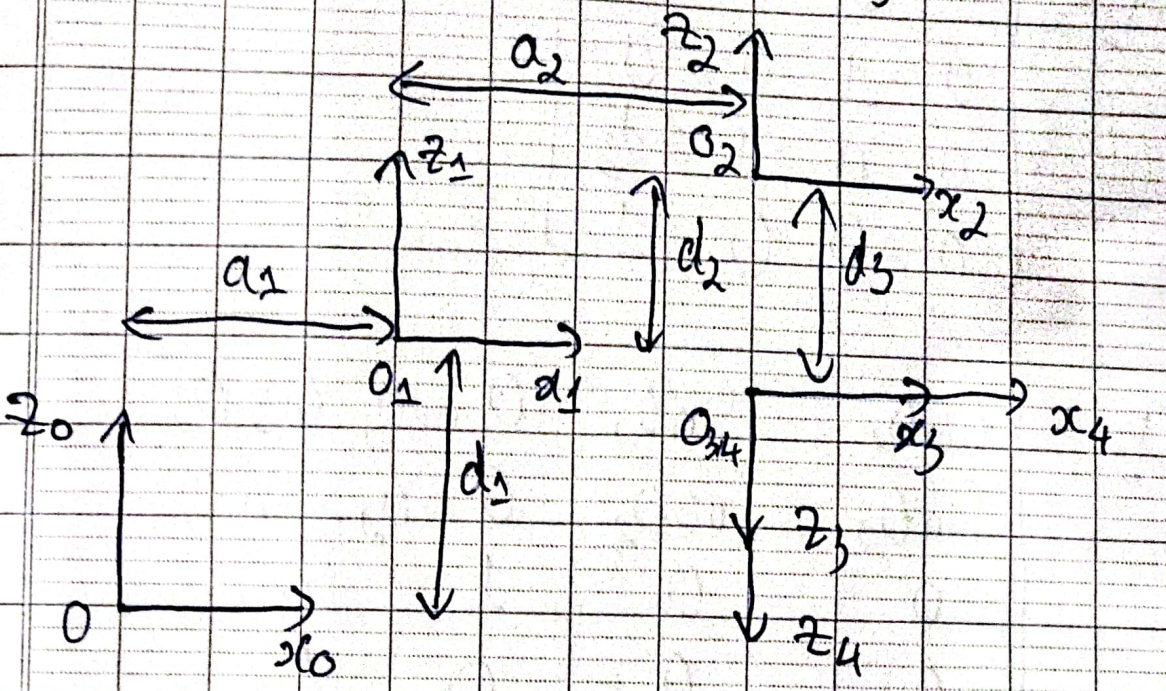
$$\begin{cases} \ddot{q}(t) = -\dfrac{a_{max}}{T_a}(t - T_h) \\ \dot{q}(t) = \int \ddot{q}(t) = \dot{q}(T_h) - \dfrac{1}{2}\dfrac{a_{max}}{T_a}(t-T_h)^2 \\ \qquad = v_{max} - \dfrac{1}{2}\dfrac{a_{max}}{T_a}(t-T_h)^2 \\ q(t) = \int \dot{q}(t) = q(T_h) + v_{max}(t-T_h) - \dfrac{1}{6}\dfrac{a_{max}}{T_a}(t-T_h)^3 \end{cases}$$

(3) $T_h + T_a \leq t \leq T_h + 2T_a$

$$\begin{cases} \ddot{q}(t) = -a_{max} \ddot{q}(T_h + T_a) + \dfrac{a_{max}}{T_a}(t - T_h - T_a) \\ \qquad = -a_{max} + \dfrac{a_{max}}{T_a}(t - T_h - T_a) \\ \dot{q}(t) = \int \ddot{q}(t) = \dot{q}(T_h + T_a) - a_{max}(t - T_h - T_a) \\ \qquad + \dfrac{1}{2}\dfrac{a_{max}}{T_a}(t - T_h - T_a)^2 \\ q(t) = \int \dot{q}(t) = q(T_h + T_a) + \dot{q}(T_h + T_a)(t - T_h - T_a) \\ \qquad - \dfrac{1}{2} a_{max}(t - T_h - T_a)^2 \\ \qquad + \dfrac{1}{6}\dfrac{a_{max}}{T_a}(t - T_h - T_a)^3 \end{cases}$$

Ngày .....tháng .....năm .....

④ Forward and Inverse Kinematics



DH table:

	d	θ	a	α	
z_0	d_1	θ_1	a_1	0	x_1
z_1	d_2	θ_2	a_2	0	x_2
z_2	$-d_3$	0	0	π	x_3
z_3	0	θ_4	0	0	x_4

$$T_3 = \begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & 0 \\ \sin\theta_4 & \cos\theta_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} {}^3R_4 & {}^3d_4 \\ 0 & 1 \end{bmatrix}$$

HONG HA

2 $T_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & -d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

1 $T_2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & a_2\sin\theta_2 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

0 $T_1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & a_1\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & a_1\sin\theta_1 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

4) Với thông số đầu vào là $\begin{bmatrix} x_d \\ y_d \\ z_d \end{bmatrix}$ ví tự, ta xét hệ ma trận ${}^0T_3$

${}^0T_3 = {}^0T_1 \cdot {}^1T_2 \cdot {}^2T_3$

Thực hiện nhân ma trận ta được:

Ngày ... tháng ... năm ...

$$^1T_2 \cdot {}^2T_3 = \begin{bmatrix} c_2 & s_2 & 0 & a_2c_2 \\ s_2 & -c_2 & 0 & a_2s_2 \\ 0 & 0 & -1 & -d_3+d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0T_3 = \begin{bmatrix} c_1c_2-s_1s_2 & c_1s_2+s_1c_2 & 0 & a_2c_1c_2-a_2s_1s_2+a_1c_1 \\ s_1c_2+c_1s_2 & s_1s_2-c_1c_2 & 0 & a_2s_1c_2+a_2c_1s_2+a_1s_1 \\ 0 & 0 & -1 & d_1+d_2-d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \left[\begin{array}{c|c} {}^0R_3 & {}^0d_3 \\ \hline 0 & 1 \end{array}\right]$$

+) đặt thêm $^0d_3 = \begin{bmatrix} x_d \\ y_d \\ z_d \end{bmatrix}$

$$\Leftrightarrow \begin{cases} a_2c_1c_2 - a_2s_1s_2 + a_1c_1 = x_d \\ a_2s_1c_2 + a_2c_1s_2 + a_1s_1 = y_d \\ d_1 + d_2 - d_3 = z_d \end{cases}$$

$$\Rightarrow \begin{cases} \boxed{d_3 = d_1 + d_2 - z_d} \\ a_1^2 + a_2^2 + 2a_1a_2c_2 = x_d^2 + y_d^2 \end{cases}$$

HONG HA

$$\Rightarrow C_2 = \frac{x_d^2 + y_d^2 - a_1^2 - a_2^2}{2 a_1 a_2} < 1$$

$$\Rightarrow \boxed{\theta_2 = \arctan2\left(\frac{\sqrt{1 - C_2^2}}{C_2}\right)}$$

$$\leftarrow) \; C_1 = \frac{x_d + a_2 S_1 S_2}{a_2 C_2 + a_1}$$

$$\Rightarrow a_2 S_1 C_2 + a_2 S_2 \frac{x_d + a_2 S_1 S_2}{a_2 C_2 + a_1} + a_1 S_1 = y_d$$

$$\leftarrow) \; S_1 \left(a_2 C_2 + \frac{a_2^2 S_2^2}{a_2 C_2 + a_1} + a_1 \right) = y_d - \frac{a_2 S_2 x_d}{a_2 C_2 + a_1}$$

$$\Rightarrow S_1 = \frac{y_d (a_2 C_2 + a_1) - a_2 S_2 x_d}{a_2^2 + a_1^2 + 2 a_1 a_2 C_2} < 1$$

$$\Rightarrow \boxed{\theta_1 = \arctan2\left(\frac{S_1}{\sqrt{1 - S_1^2}}\right)}$$

Ngày .....tháng .....năm .....

4) Với thông số đầu vào là góc Yaw

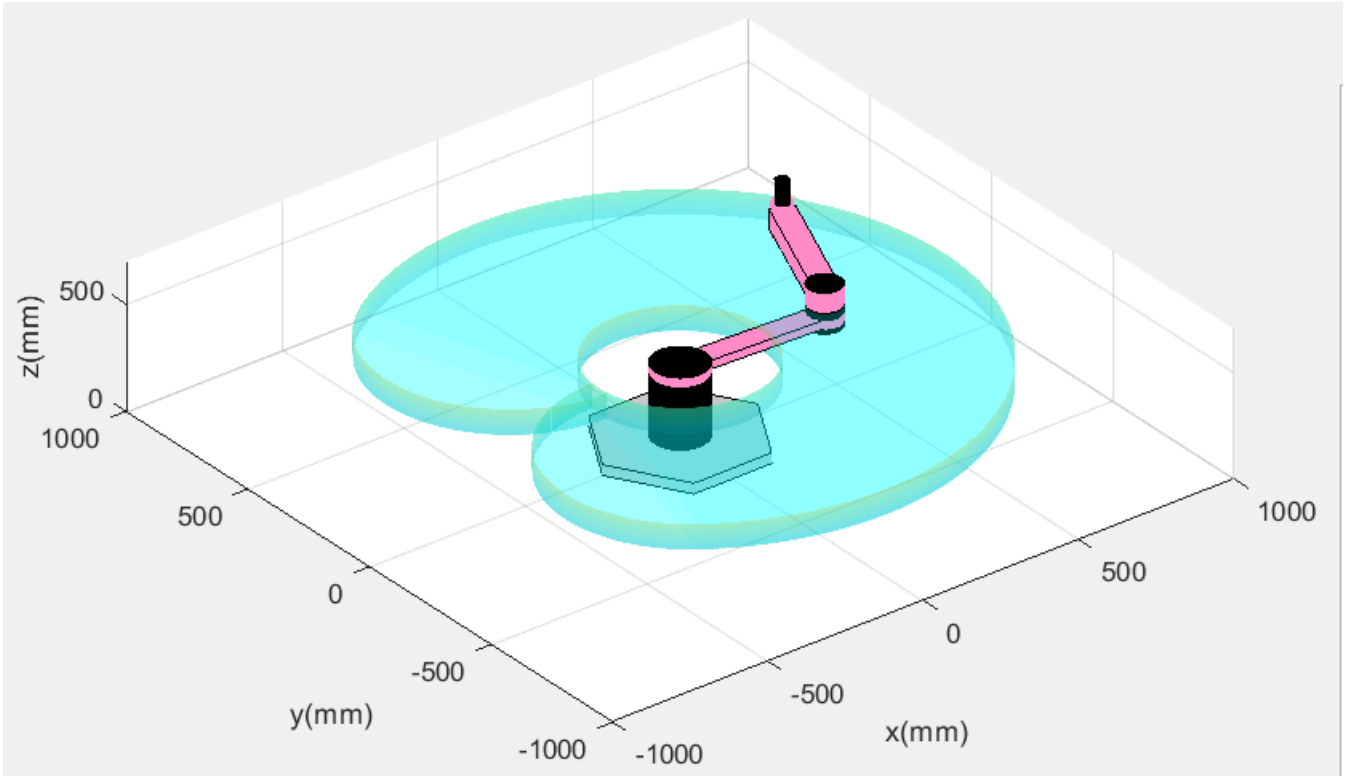
$$^0R_4 = \begin{bmatrix} \cos(Yaw) & -\sin(Yaw) & 0 \\ \sin(Yaw) & \cos(Yaw) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= {}^0R_3 \cdot {}^3R_4$$

$$\Rightarrow {}^3R_4 = \begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 \\ \sin\theta_4 & \cos\theta_4 & 0 \\ 0 & 0 & 1 \end{bmatrix} = ({}^0R_3)^{-1} \cdot {}^0R_4$$

$$= \begin{bmatrix} l_{11} & l_{12} & l_{13} \\ l_{21} & l_{22} & l_{23} \\ l_{31} & l_{32} & l_{33} \end{bmatrix}$$

$$\Rightarrow \boxed{\theta_4 = \operatorname{atan2}\left(\frac{\sqrt{1 - l_{11}^2}}{l_{11}}\right)}$$



Input_parameter

Theta1 [-125,125] degree		-2.521
Theta2 [-145,145] degree		70.0154
d3 [0,150] mm		0
Theta4 [-360,360] degree		0

End_effector_position (mm)

	Present	Desire
X (mm)	602.674	450
Y (mm)	349.743	-400
Z (mm)	443	350
Yaw (deg)	67.494	50

DH_para(mm)

a1	450
a2	400
d1	363
d2	80

Update

Velocity and acceleration

q_max	v max	a_max
	55	35
mm	mm/s	mm/s^2

PLOT

Trajectory Path: ○ Trapezoidal ○ S_curve

Linear_Interpolation

Kinematics

Forward_Kinematics

Inverse_Kinematics

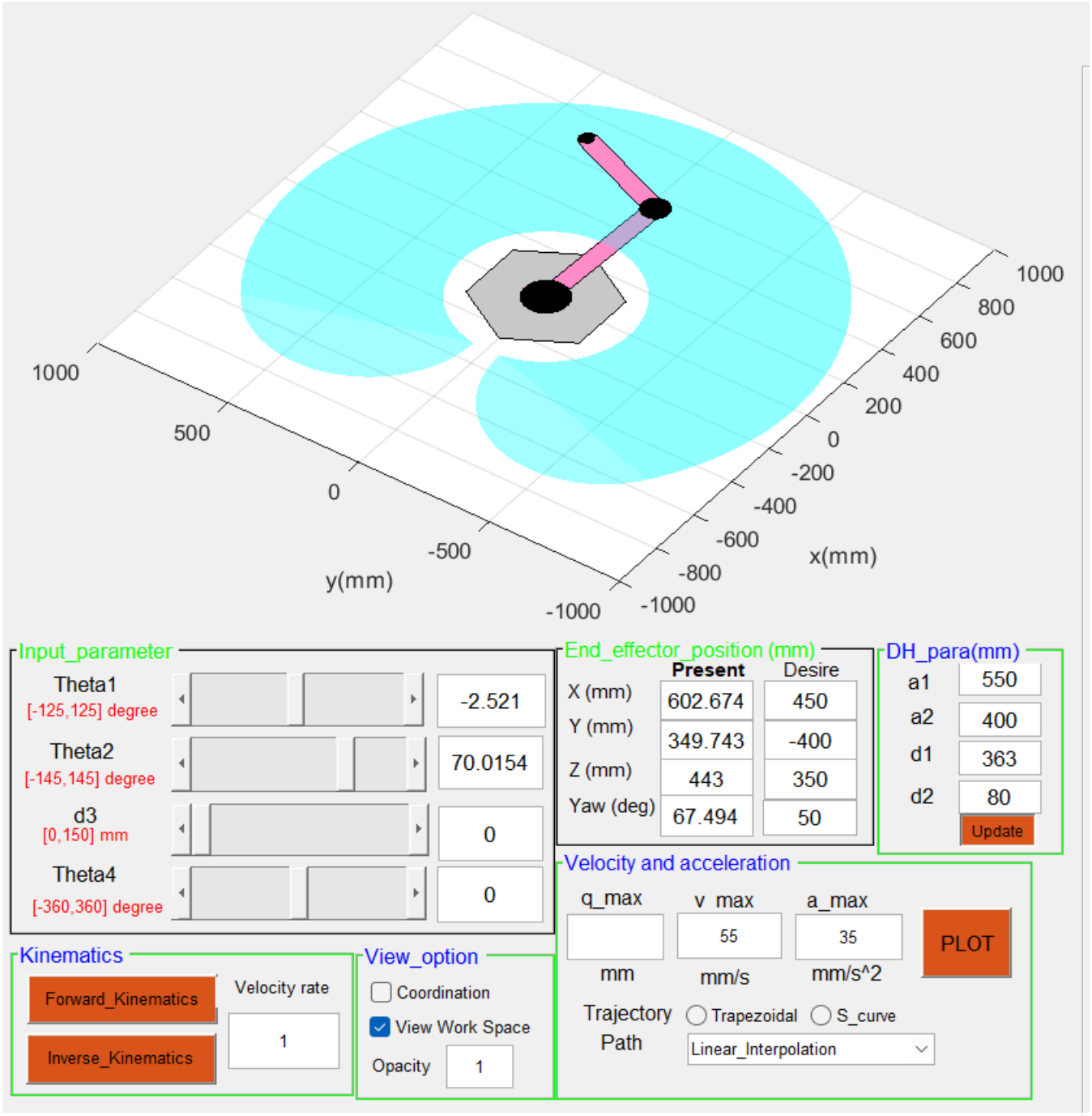
Velocity rate: 1

View_option

☐ Coordination

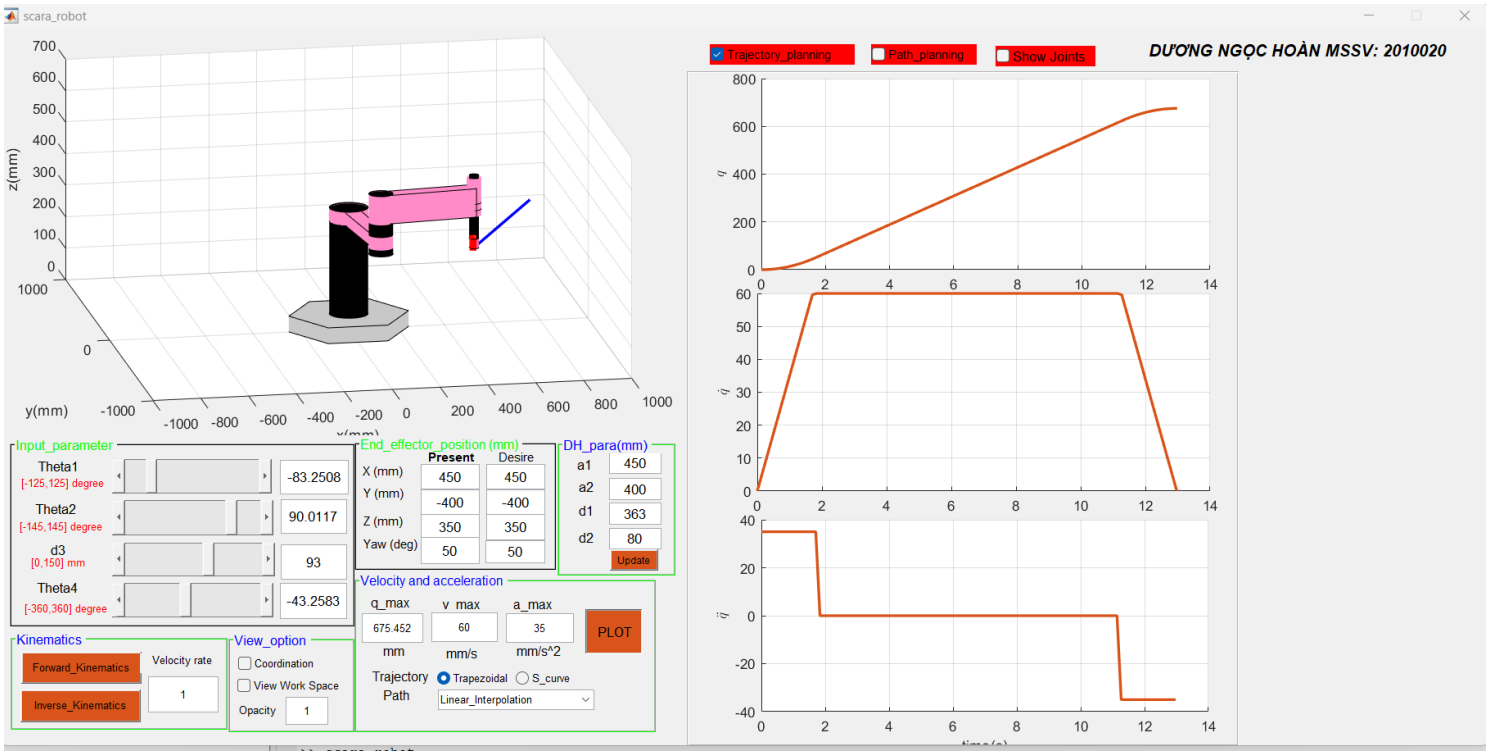
☑ View Work Space

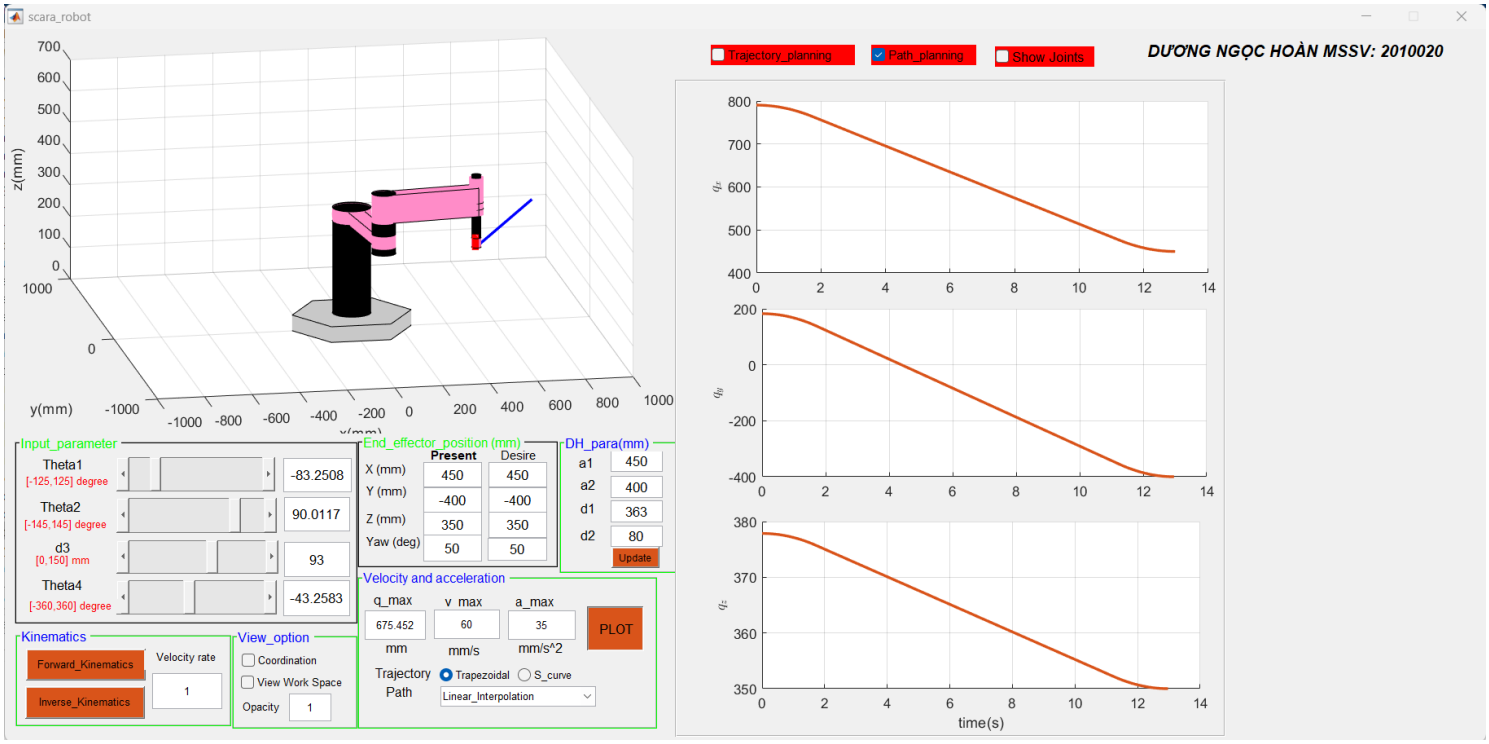
Opacity: 1

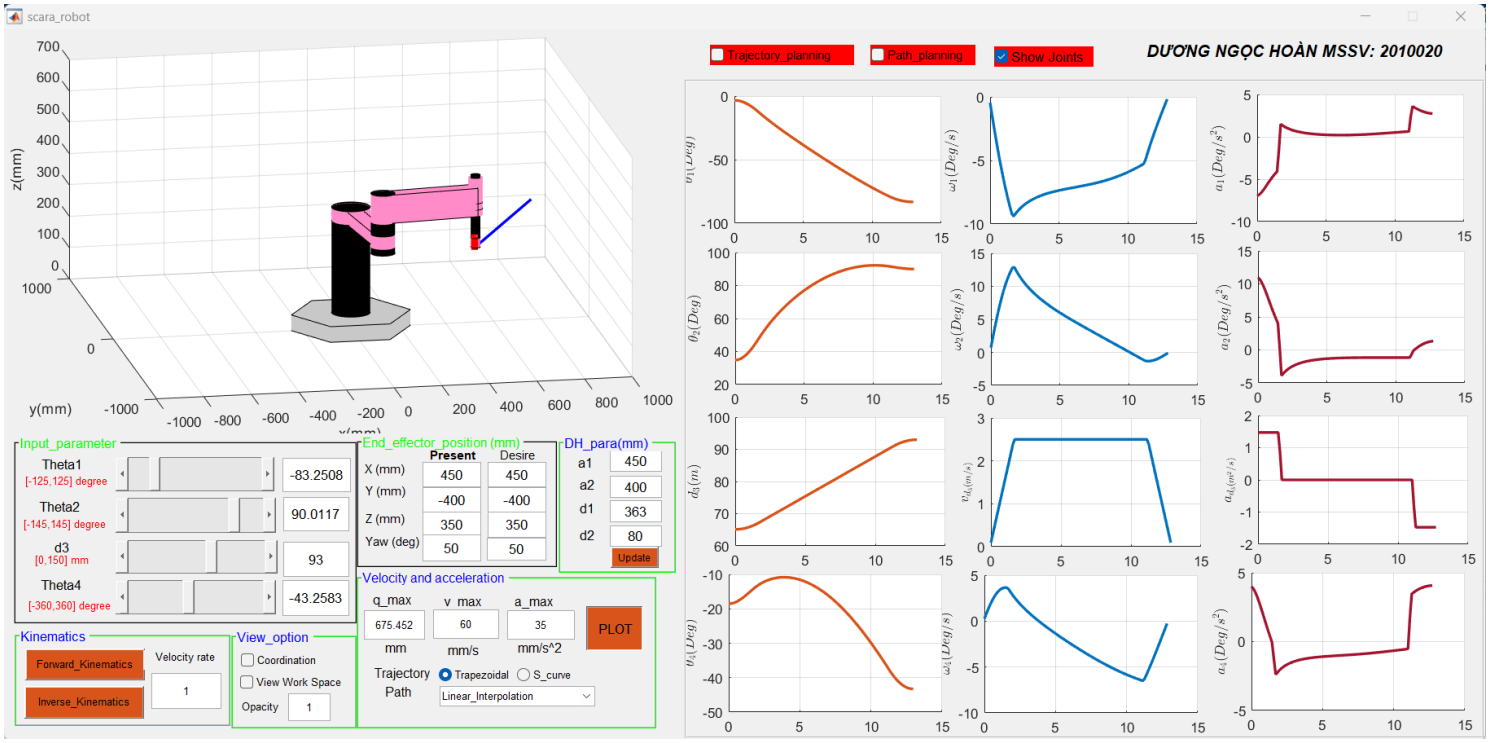

1000
500
0
-500
-1000
y(mm)
1000
800
600
400
200
0
-200
-400
-600
-800
-1000
x(mm)
Input_parameter
Theta1
[-125,125] degree
-2.521
Theta2
[-145,145] degree
70.0154
d3
[0,150] mm
0
Theta4
[-360,360] degree
0
End_effector_position (mm)
Present
Desire
X (mm)
602.674
450
Y (mm)
349.743
-400
Z (mm)
443
350
Yaw (deg)
67.494
50
DH_para(mm)
a1
550
a2
400
d1
363
d2
80
Update
Velocity and acceleration
q_max
v_max
a_max
55
35
mm
mm/s
mm/s^2
PLOT
Trajectory
Path
Trapezoidal
S_curve
Linear_Interpolation
Kinematics
Forward_Kinematics
Inverse_Kinematics
Velocity rate
1
View_option
Coordination
View Work Space
Opacity
1

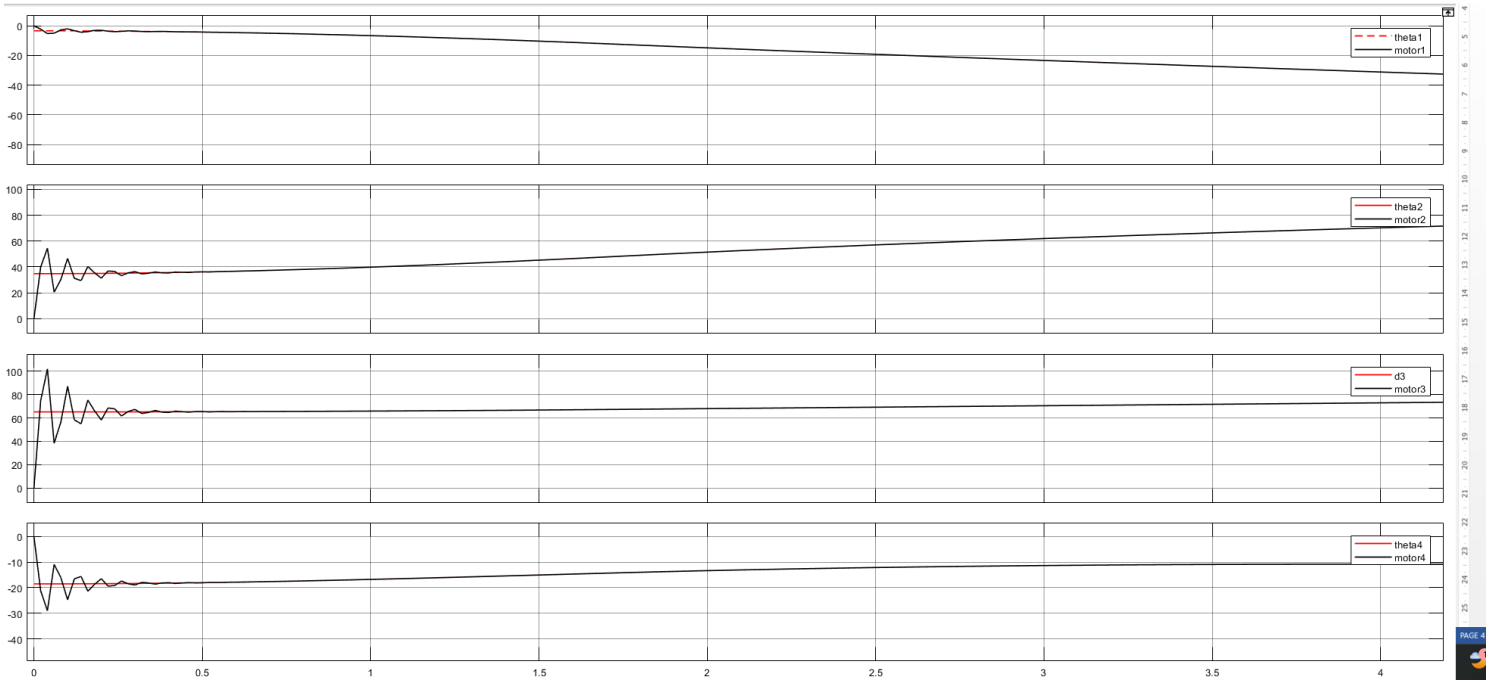
TRAPEZOIDAL TRAJECTORY

LINEAR INTERPOLATION



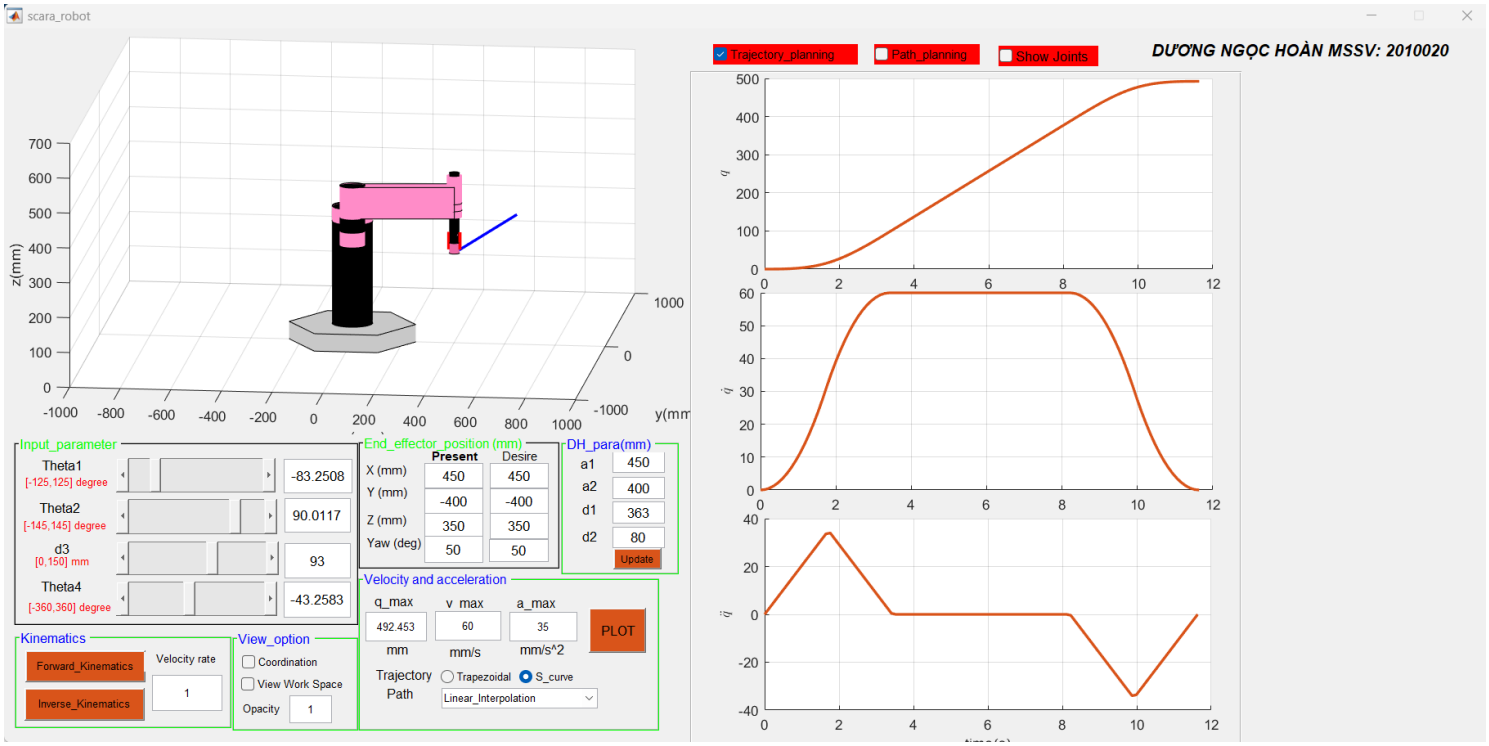


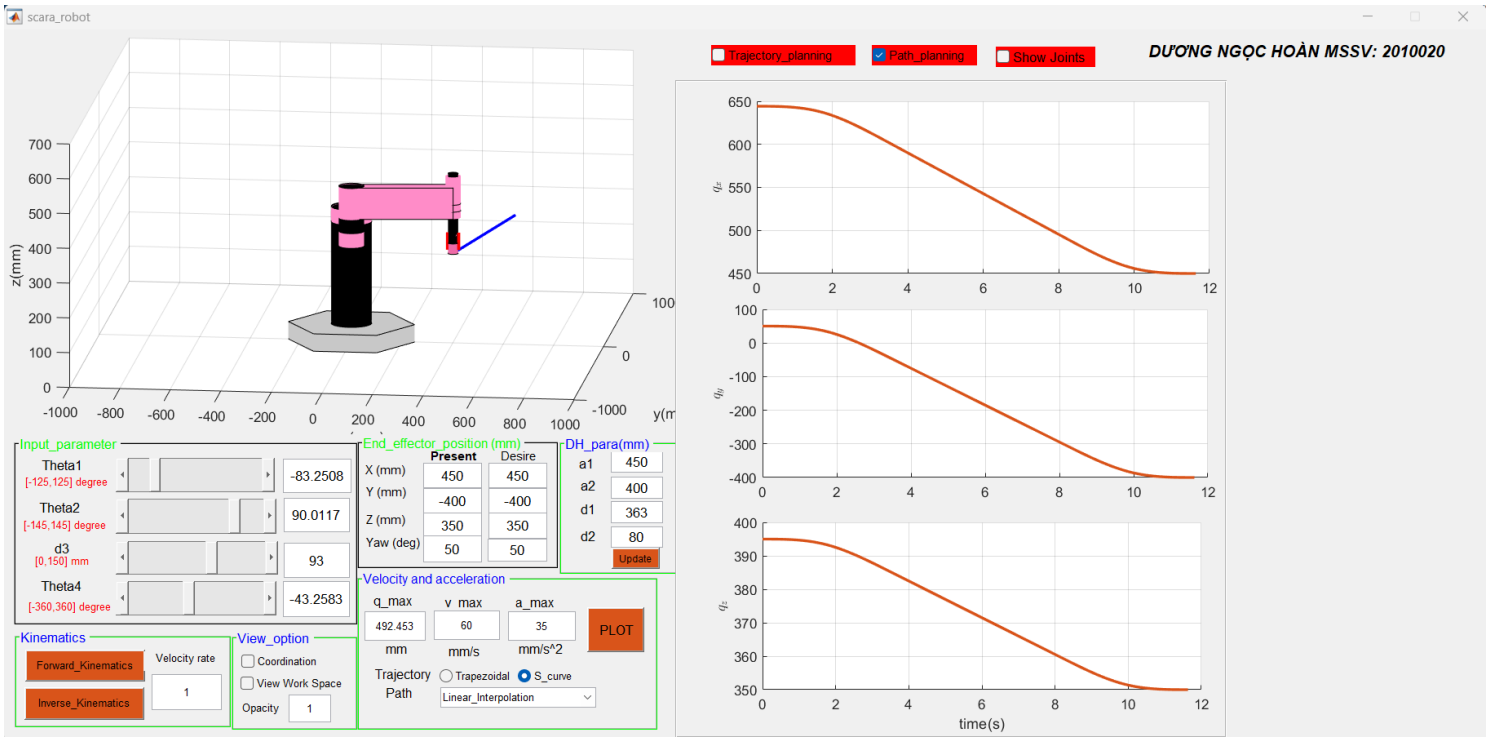

scara_robot
Trajectory_planning
Path_planning
Show Joints
DƯƠNG NGỌC HOÀN MSSV: 2010020
Input_parameter
Theta1
[-125,125] degree
-83.2508
Theta2
[-145,145] degree
90.0117
d3
[0,150] mm
93
Theta4
[-360,360] degree
-43.2583
End_effector_position (mm)
Present
Desire
X (mm) 450 450
Y (mm) -400 -400
Z (mm) 350 350
Yaw (deg) 50 50
DH_para(mm)
a1 450
a2 400
d1 363
d2 80
Update
Velocity and acceleration
q_max
v_max
a_max
675.452
60
35
mm
mm/s
mm/s^2
PLOT
Trajectory
Path
Trapezoidal
S_curve
Linear_Interpolation
Kinematics
Forward_Kinematics
Inverse_Kinematics
Velocity rate
1
View_option
Coordination
View Work Space
Opacity
1

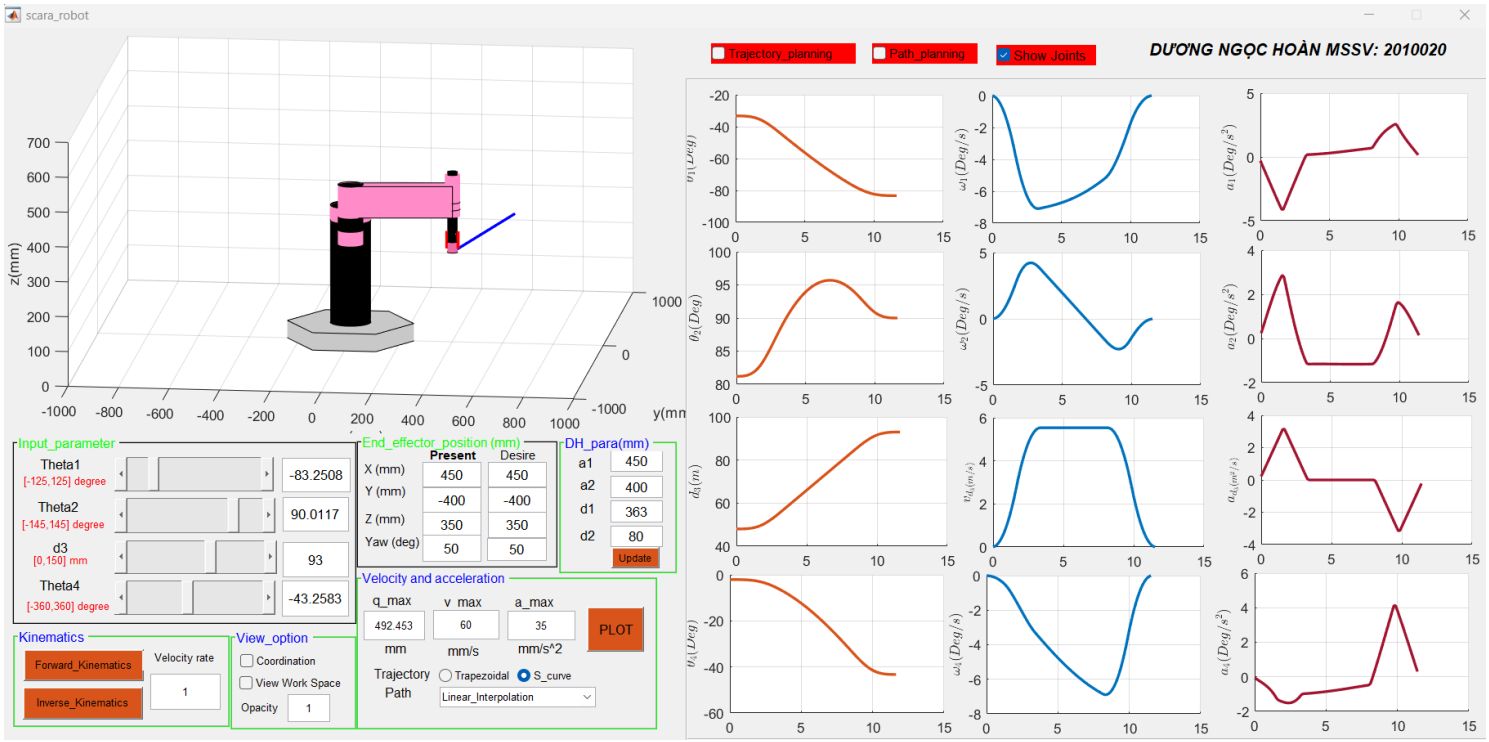

theta1
motor1
theta2
motor2
d3
motor3
theta4
motor4

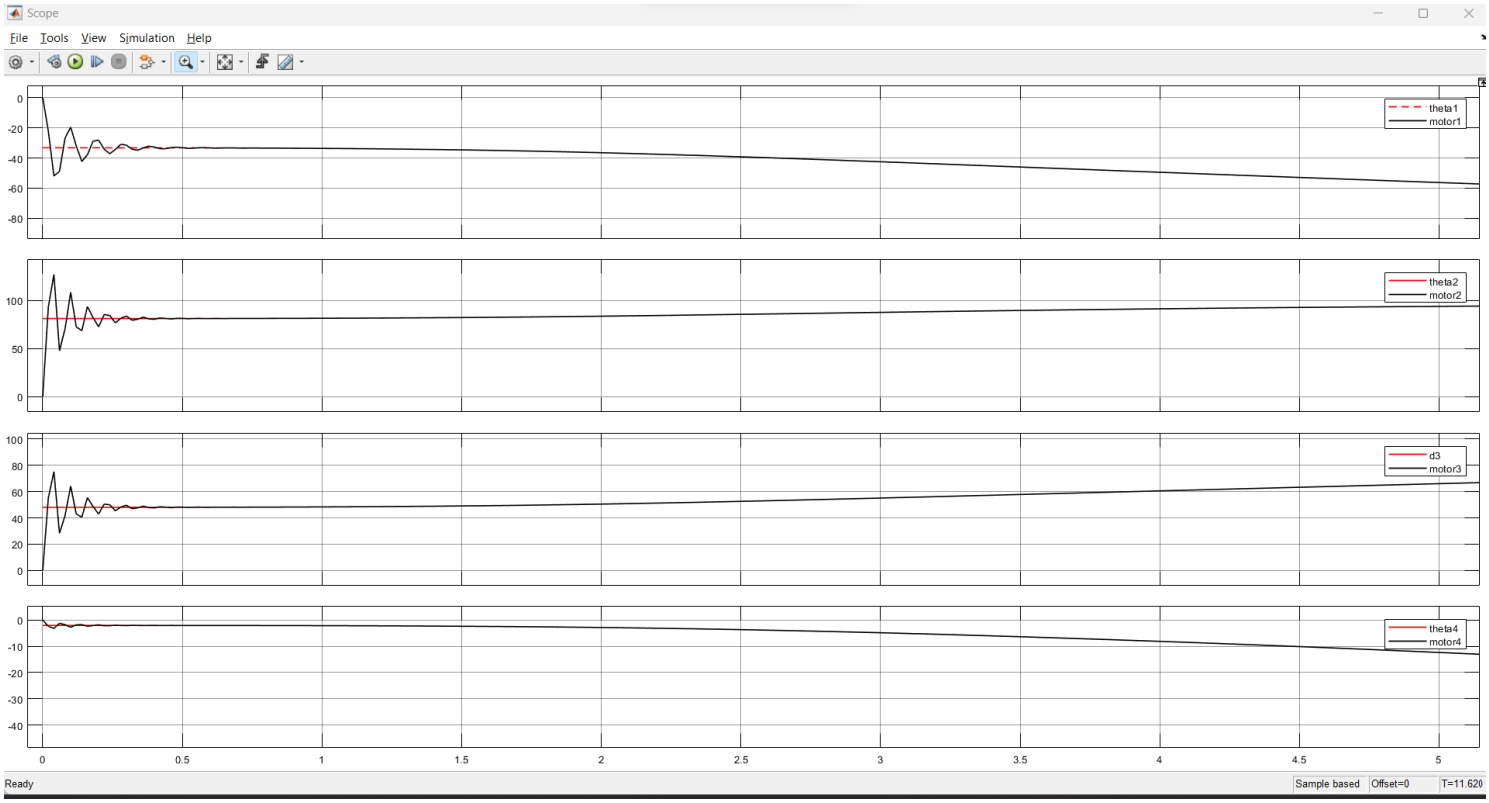
S-CURVE TRAJECTORY

LINEAR INTERPOLATION



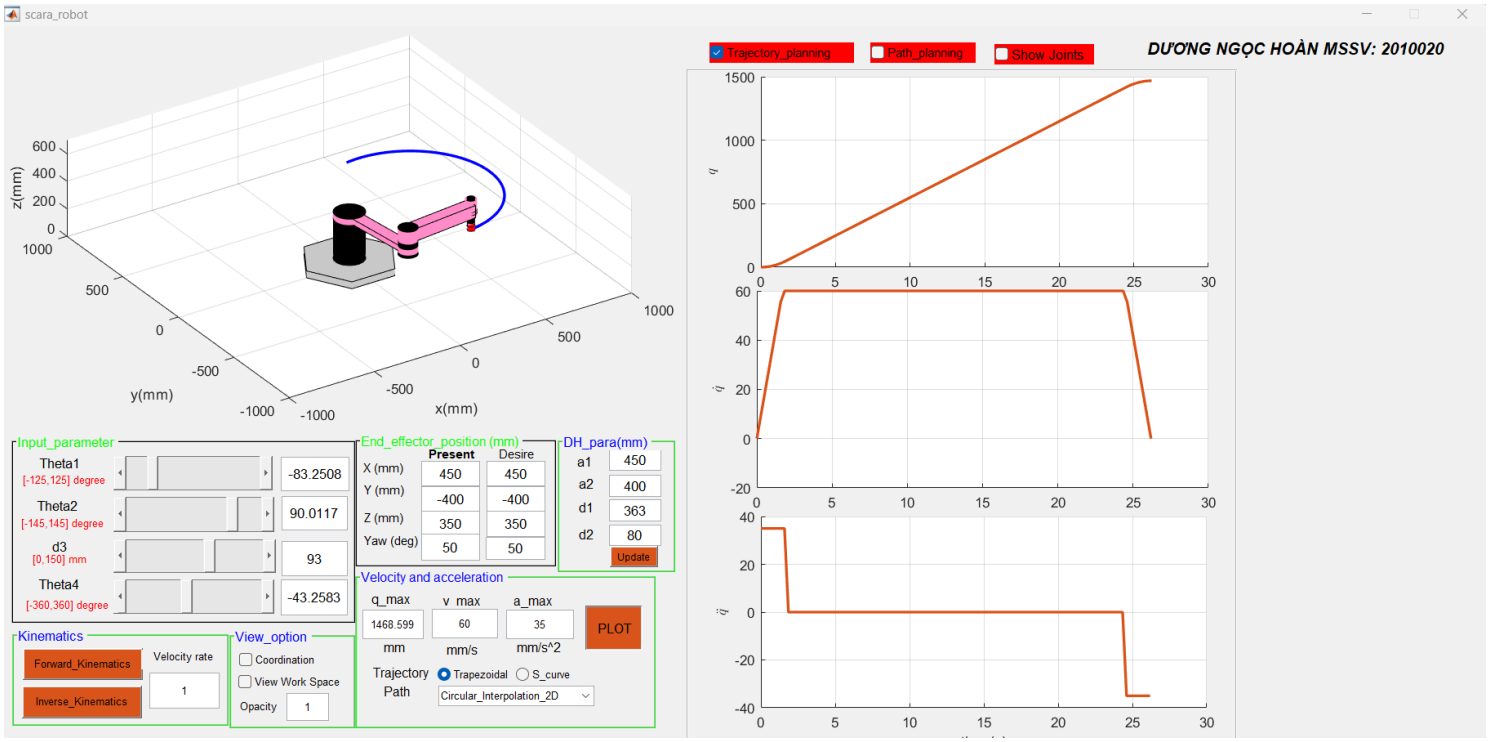


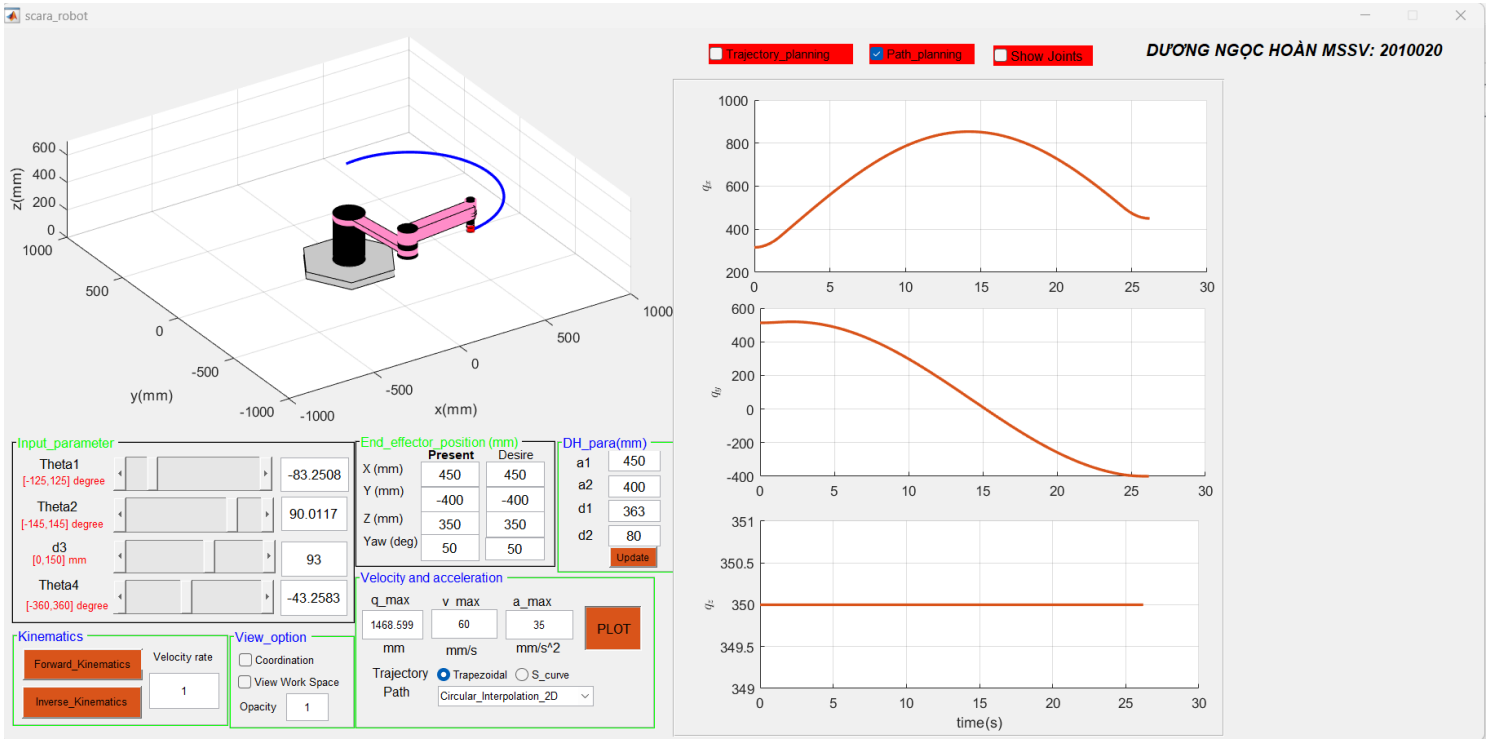

scara_robot
Trajectory_planning
Path_planning
Show Joints
DƯƠNG NGỌC HOÀN MSSV: 2010020
Input_parameter
Theta1
[-125,125] degree
-83.2508
Theta2
[-145,145] degree
90.0117
d3
[0,150] mm
93
Theta4
[-360,360] degree
-43.2583
End_effector_position (mm)
Present
Desire
X (mm) 450 450
Y (mm) -400 -400
Z (mm) 350 350
Yaw (deg) 50 50
DH_para(mm)
a1 450
a2 400
d1 363
d2 80
Update
Velocity and acceleration
q_max
v_max
a_max
492.453
60
35
mm
mm/s
mm/s^2
PLOT
Trajectory
Path
Trapezoidal
S_curve
Linear_Interpolation
Kinematics
Forward_Kinematics
Inverse_Kinematics
Velocity rate
1
View_option
Coordination
View Work Space
Opacity
1

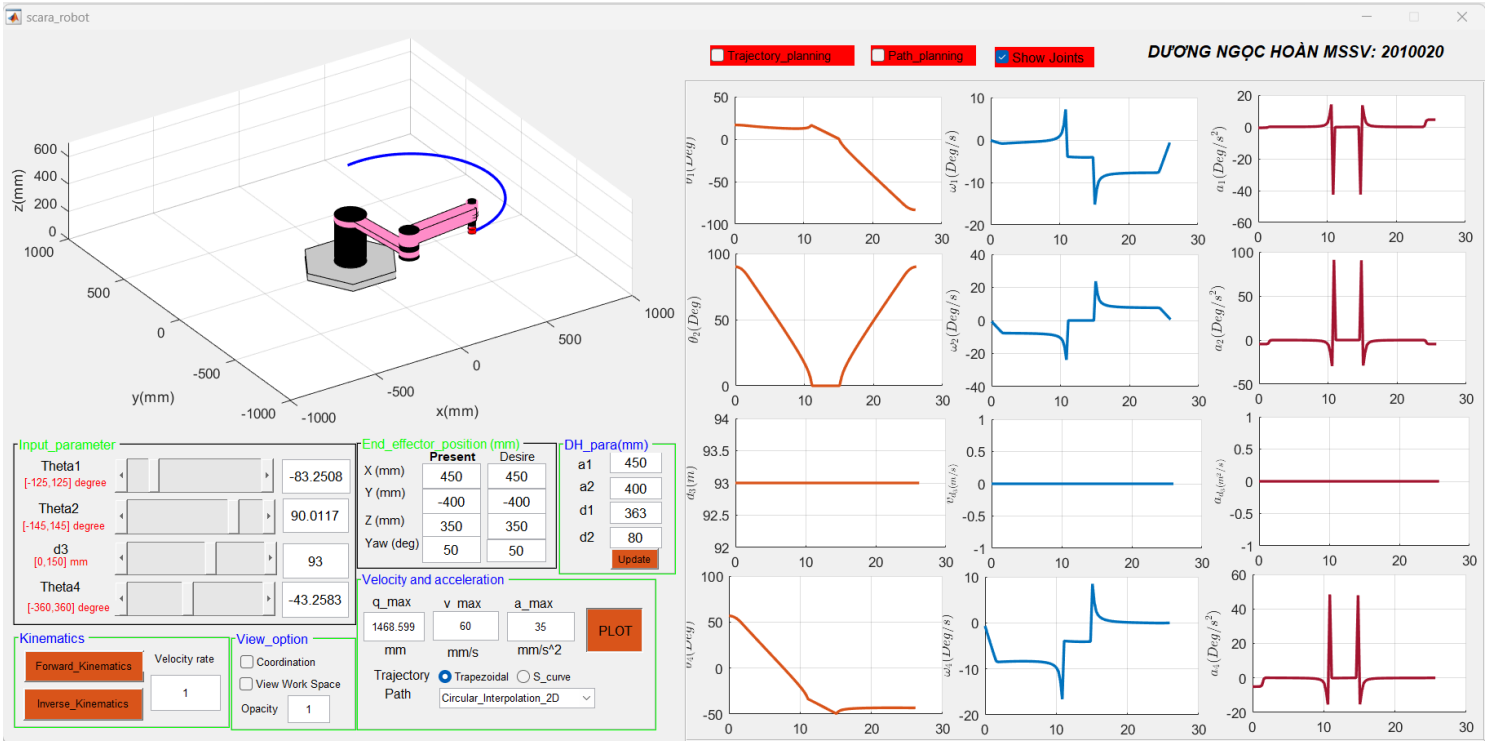

Scope
File Tools View Simulation Help
theta1
motor1
theta2
motor2
d3
motor3
theta4
motor4
Ready
Sample based
Offset=0
T=11.620

TRAPEZOIDAL TRAJECTORY

CIRCULAR 2D INTERPOLATION








```
>> scara_robot
Go through the singularity
Go through the singularity
Go through the singularity
Go through the singularity
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Go through the singularity
Go through the singularity
Go through the singularity
```
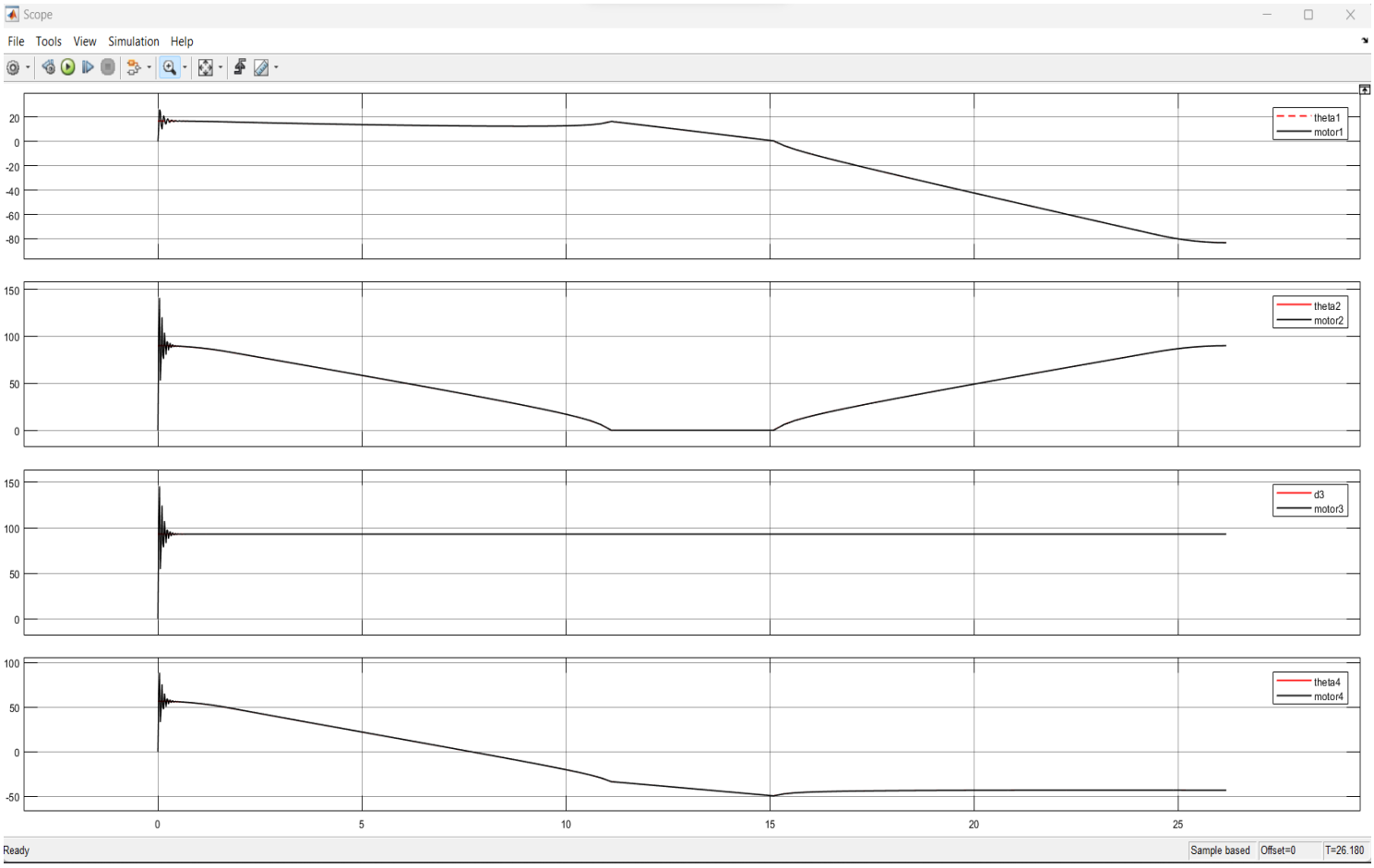
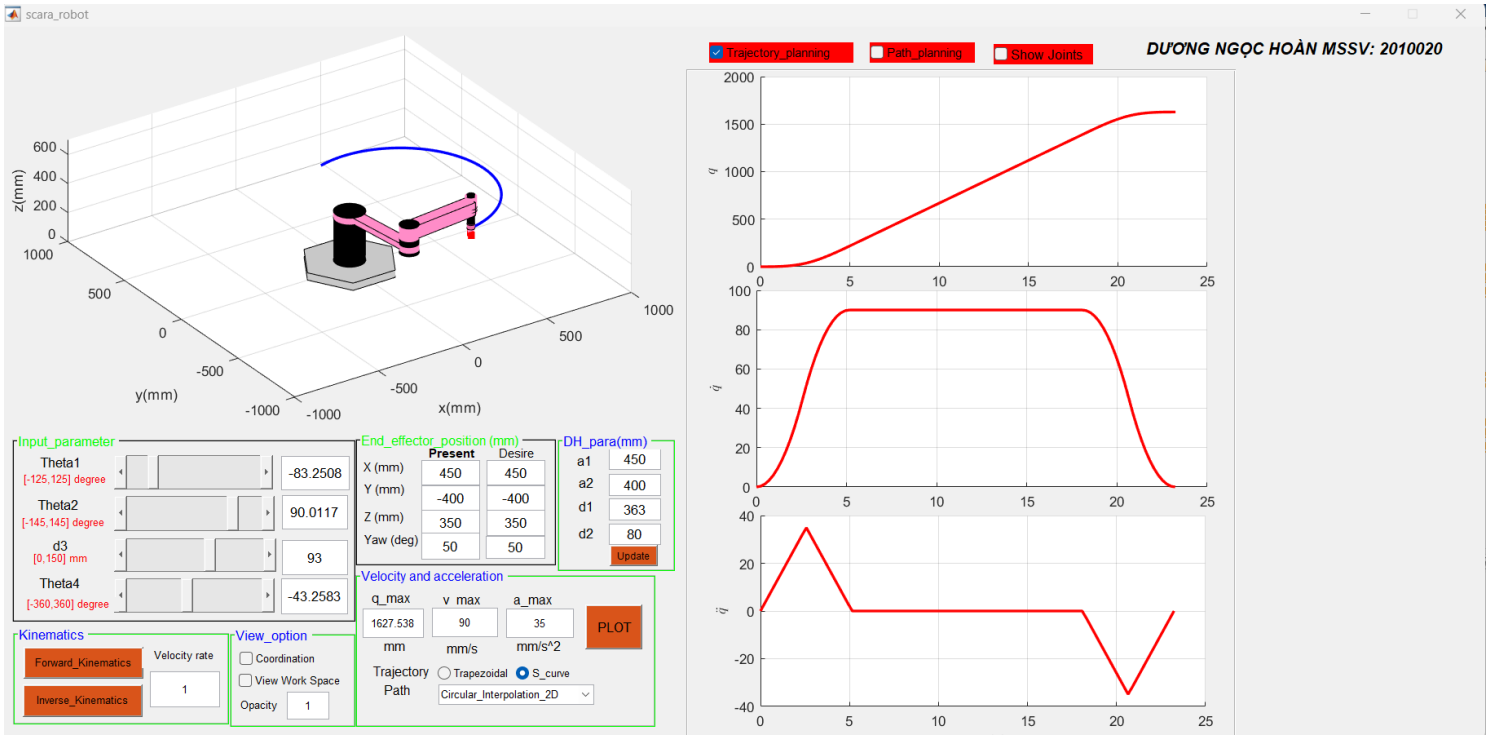
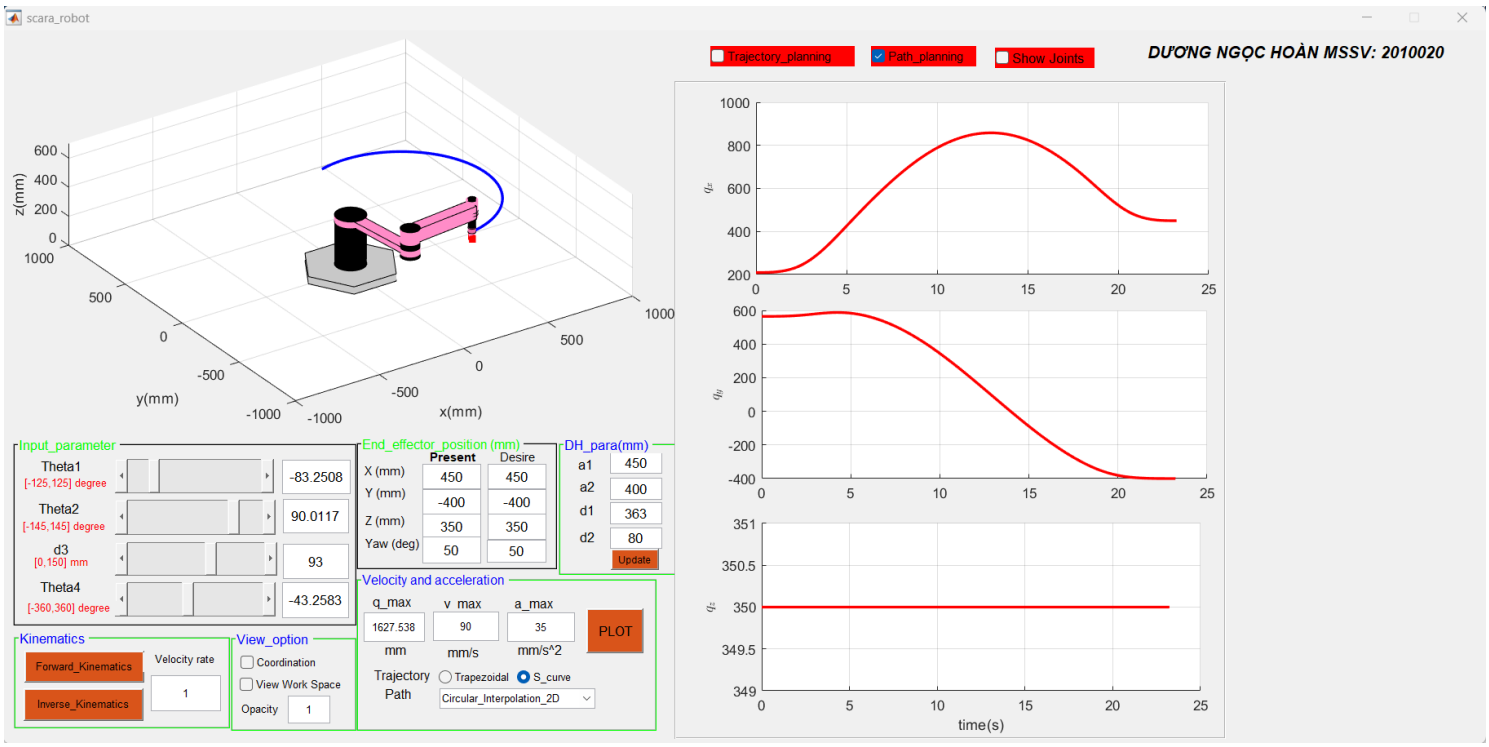
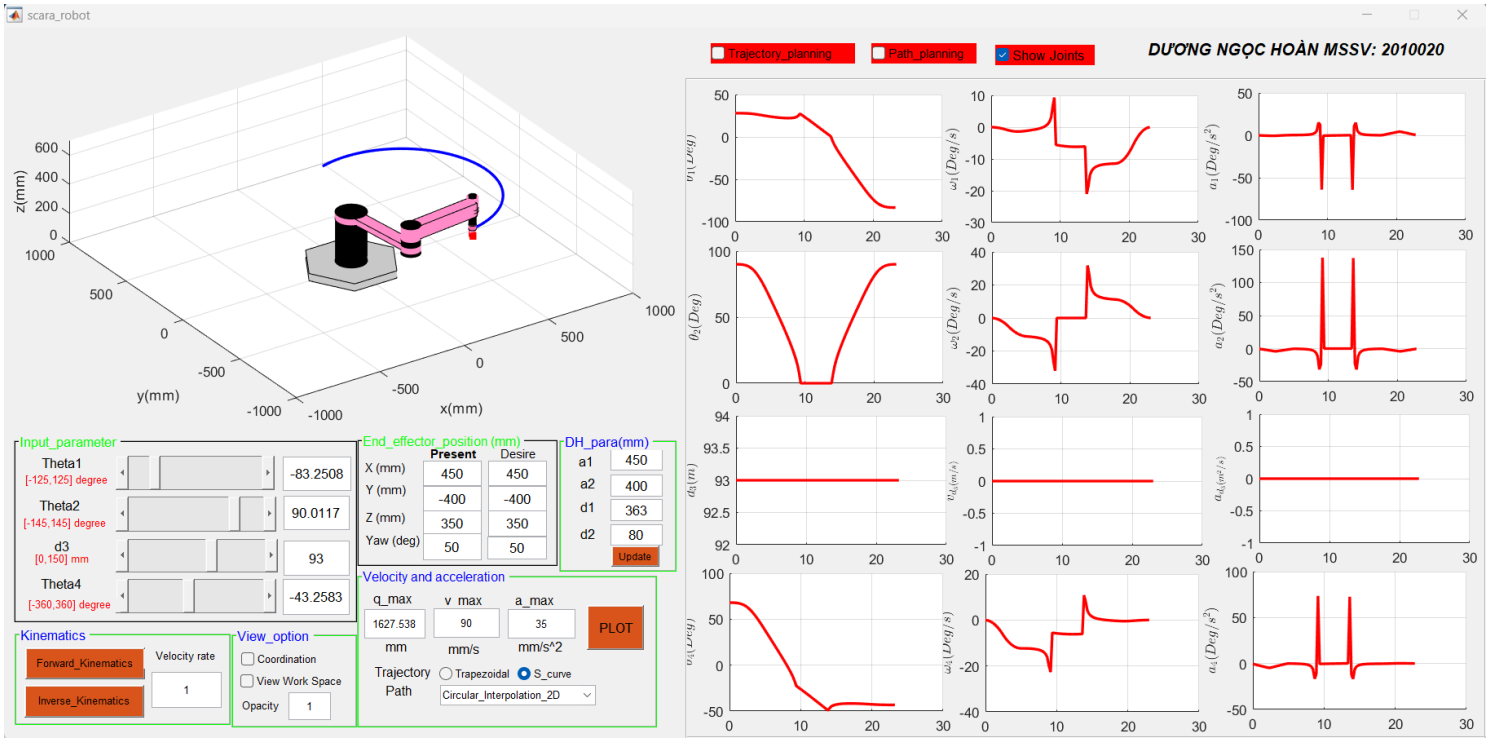


Scope
File Tools View Simulation Help
theta1
motor1
theta2
motor2
d3
motor3
theta4
motor4
Ready
Sample based
Offset=0
T=26.180

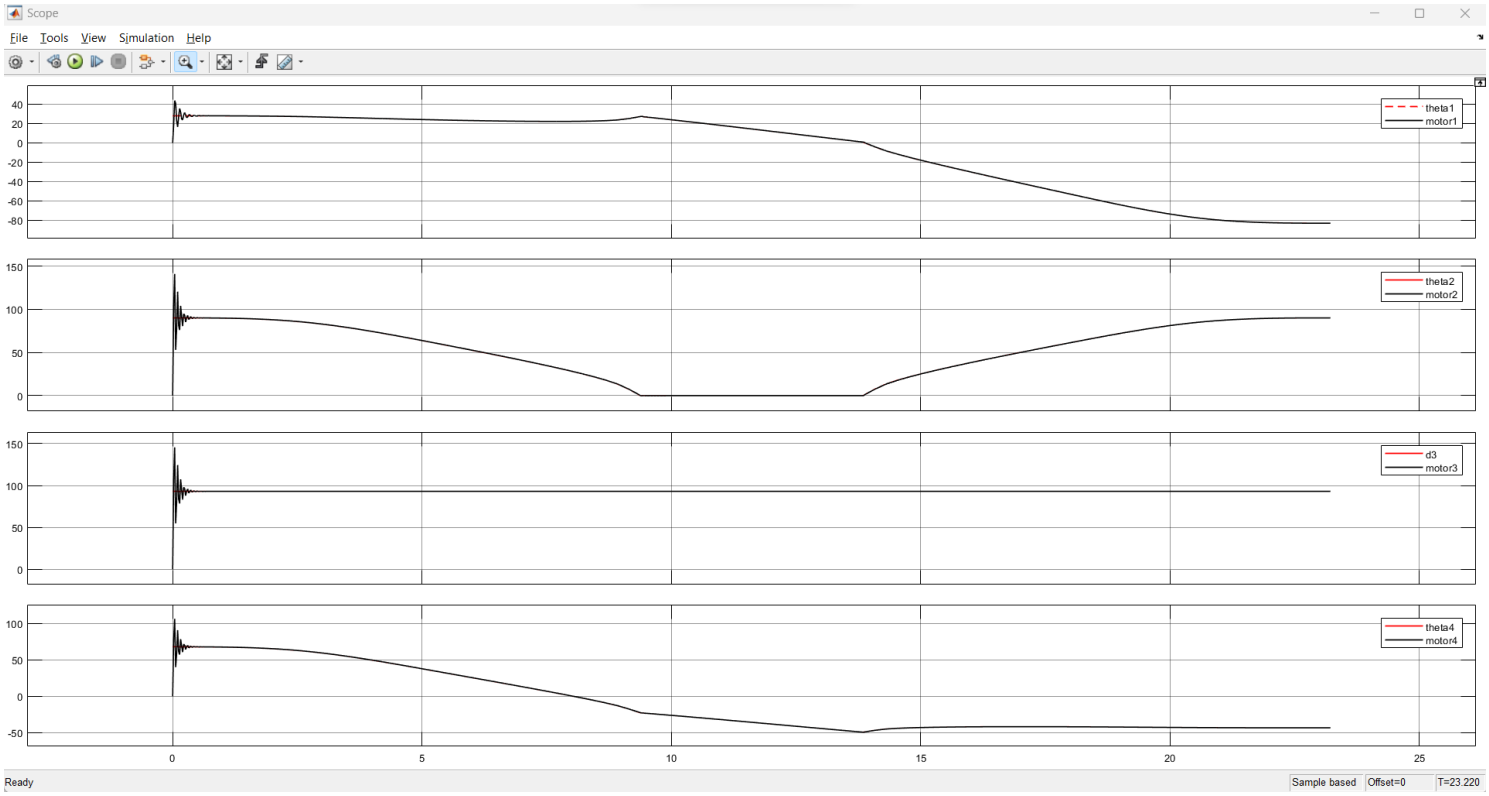
S-CURVE TRAJECTORY

CIRCULAR 2D INTERPOLATION






scara_robot
Trajectory_planning
Path_planning
Show Joints
DƯƠNG NGỌC HOÀN MSSV: 2010020
z(mm)
y(mm)
x(mm)
Input_parameter
Theta1
[-125,125] degree
-83.2508
Theta2
[-145,145] degree
90.0117
d3
[0,150] mm
93
Theta4
[-360,360] degree
-43.2583
End_effector_position (mm)
Present
Desire
X (mm) 450 450
Y (mm) -400 -400
Z (mm) 350 350
Yaw (deg) 50 50
DH_para(mm)
a1 450
a2 400
d1 363
d2 80
Update
Velocity and acceleration
q_max
v_max
a_max
1627.538
90
35
mm
mm/s
mm/s^2
PLOT
Trajectory
Path
Trapezoidal
S_curve
Circular_Interpolation_2D
Kinematics
Forward_Kinematics
Inverse_Kinematics
Velocity rate
1
View_option
Coordination
View Work Space
Opacity
1


Scope
File Tools View Simulation Help
theta1
motor1
theta2
motor2
d3
motor3
theta4
motor4
Ready
Sample based
Offset=0
T=23.220

```
Go through the singularity
Go through the singularity
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Go through the singularity
Go through the singularity
Go through the singularity
>>
```
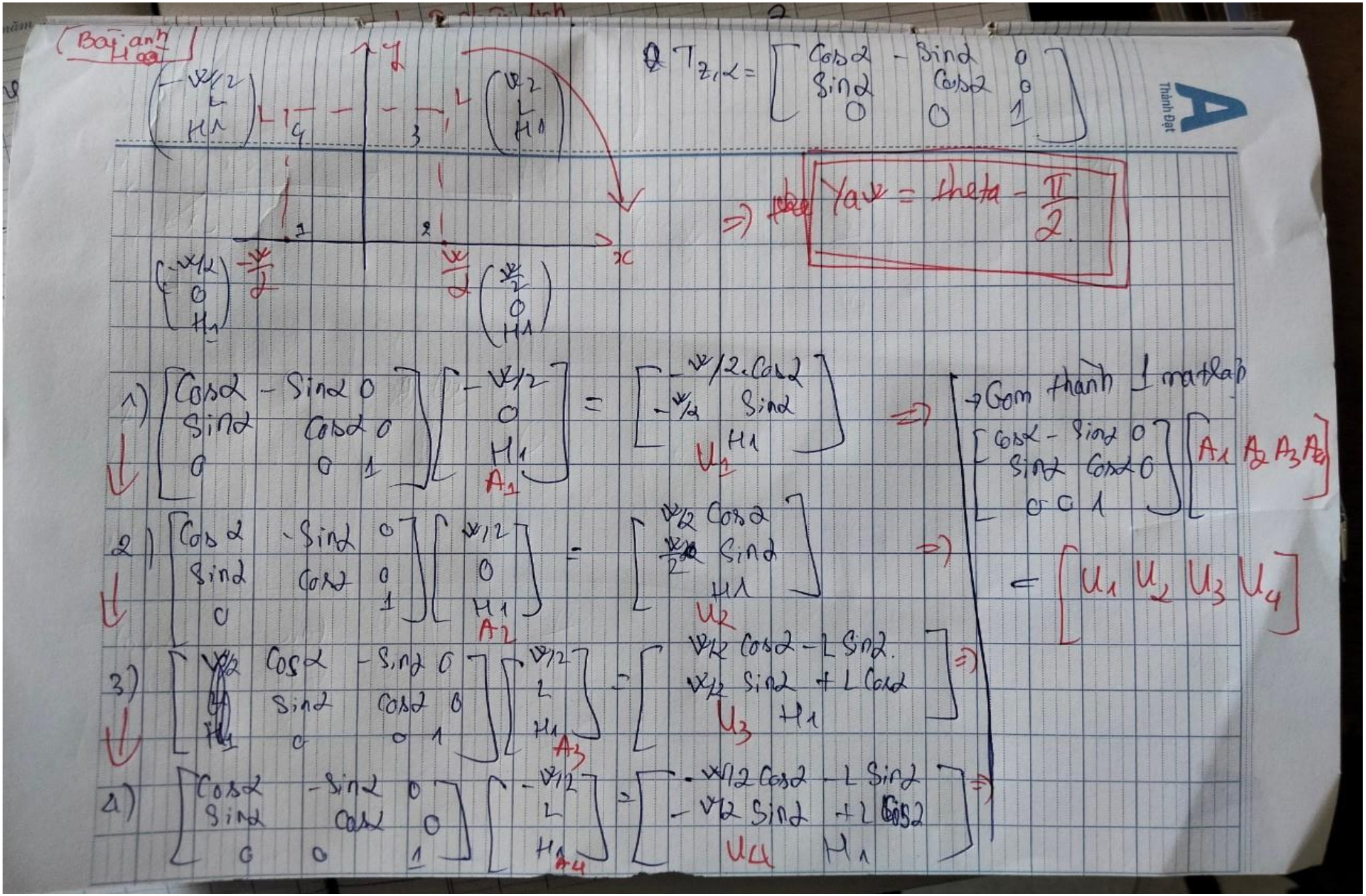
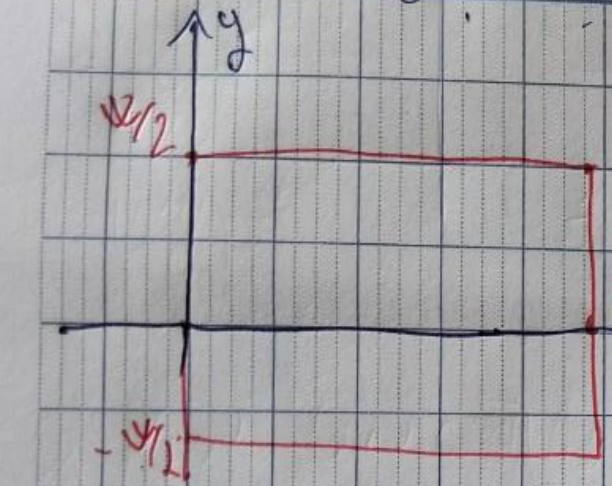

IDEAL TO DRAW LINKS

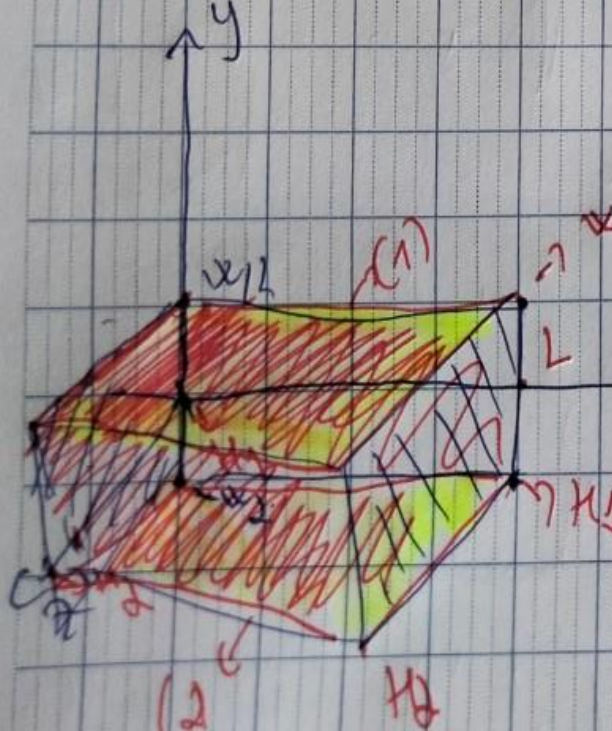


- phát triển cái gì trên 1 ta chỉ cần xài tính đc 4 điểm vào 1 Fill3 => sẽ vẽ được mặt đó. => chuyển Yaw = theta(1) cho dễ hiểu



2 mặt phẳng =>

$$\begin{bmatrix} 0 & 0 & L & L \\ w/2 & -w/2 & -w/2 & w/2 \\ H & H & H & H \end{bmatrix} \Rightarrow H \begin{cases} H_1 \\ H_2 \end{cases}$$



$$\Rightarrow \begin{cases} \begin{bmatrix} 0 & 0 & L & L \\ w/2+w/2 & +w/2 & +w/2 & +w/2 \\ H_1 & H_2 & H_2 & H_1 \end{bmatrix} & \text{(mặt bên 1) màu đỏ đậm} \\ \quad \downarrow \\ \quad y = w/2 \\ \begin{bmatrix} 0 & 0 & L & L \\ -w/2 & -w/2 & -w/2 & -w/2 \\ H_1 & H_2 & H_2 & H_1 \end{bmatrix} & \text{(mặt bên 2) màu đỏ đậm} \\ \quad \downarrow \\ \quad y = -\frac{w}{2} \end{cases}$$

Re_write source code:

```
  Yaw = theta(1);
% Ma trận quay quanh truc Z mot goc Yaw
    Z_yaw = [cos(Yaw) -sin(Yaw) 0;
             sin(Yaw) cos(Yaw)  0;
             0          0       1]
```

Khai báo ma trận khi quay quang Z một góc yaw

```
% Ma trận quay quanh truc Z mot goc Yaw
    Z_yaw = [cos(Yaw) -sin(Yaw) 0;
             sin(Yaw) cos(Yaw)  0;
             0          0       1]
 %Ve mat phang chieu cao z = H1
    A = [  0     0    L    L;
         W/2 -W/2 -W/2  W/2;
          H1    H1   H1   H1];
    U = Z yaw*A;
```

Khai báo ma trận A là 4 điểm trong không gian của mặt phẳng cần vẽ, để tính toán gọn ta gom 4 điểm vào 1 ma trận , khi đó ma trân U có được có các cột là các điểm tương ứng sau khi xoay quang Z một góc Yaw.

```
X = U(1,:);
Y = U(2,:);
Z =  U(3,:);
fill3(handles.axes1, X, Y, Z, Color, 'FaceAlpha', opacity)
```

Lấy các tọa độ X, Y, Z ở 3 cột trong ma trận U và vẽ mặt phẳng đó bằng lệnh fill3.