# Term 3 (Part 1) – Instructional Guide

# Grade 12/ASP-ES – Physics

# PHY70A-71

## Textbook Link:

| Learning Outcome | Number of Periods | Suggested Exercises /Assignments |
|---|---|---|
| **Topic 5: Electromagnetism**<br>Subtopic 5.2– Inductance<br>Subtopic 5.3– Maxwell's Equations<br>(KPIs 5.2.1 – 5.3.2) | 12 | Refer to the questions below |

- ## Practice Questions (Addition to Specified Example Questions)

**Multiple choice questions**

1. Which of the following are units for inductance?

I. Henry
II. Joules/(Ampere)$^2$
III. Volt . second/Ampere

A. I only
B. II only
C. III only
D. I and II only
√ E. I, II and III

2. Energy can be stored in many ways. What is the primary energy storage form of an inductor?

A. Kinetic
B. Electric field
√ C. Magnetic field
D. Gravitational field
E. Spring displacement

3. An inductor opposes the changes in___.

A. Voltage
√ B. Current
C. Resistance
D. Inductance
E. Capacitance

4. Which of the following statements is/are correct?

I. An inductor always resists having current flow through it when connected in a circuit
II. Inductors store energy by building up charge
III. An inductor always resists any change in the current through it

A. I only
B. II only
√ C. III only
D. I and II only
E. II and III only

5. How much energy is stored when 50 A flows through a 20 H inductor?

A. 500 J
B. 10000 J
C. 12500 J
√ D. 25000 J
E. 50000 J

6. How large a current must flow through a 1.2 H inductance so that the energy stored is 393 J?

A. 1.17 A
B. 2.31 A
√ C. 25. 6 A
D. 44.2 A
E. 107.1 A

7. What inductance should you put in series with a 200 Ω resistor to give a time constant of 4.6 ms?

A. 0.011 mH
B. 0.435 mH
C. 0.920mH
D. 43.5 mH
√ E. 920 mH

8. A 15 mH inductor is connected in series with a battery and a 12 Ω resistor, in a circuit with an open switch. 0.50 ms after the switch is closed, the current in the circuit is 1.2 A. What is the emf supplied by the battery?

A. 1.2 V
B. 1.8 V
C. 3.6 V
D. 21 V
√ E. 44 V

9. A 10 turn ideal solenoid has an inductance of 4.0 $mH$. To generate an emf of 2.0 $V$ the current should change at a rate of _____.

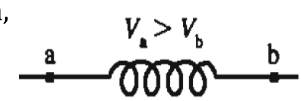
A. 0 A/s
B. 0/5 A/s
C. 50 A/s
D. 250 A/s
√ E. 500 A/s

10. A 5.6 $mH$ inductor is connected to a current source. What is the magnitude of the voltage in the inductor at time $t = 4.0\ s$ if the current is

$$I(t) = 10.0 + 9.0t - 3.0t^2?$$

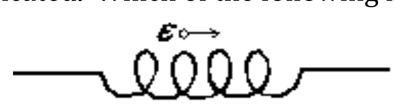
A. 11 mV
B. 22 mV
√ C. 84 mV
D. 58 mV
E. 185 mV

11. The potential at $a$ is higher than the potential at $b$. The inductor current $I$ is from,



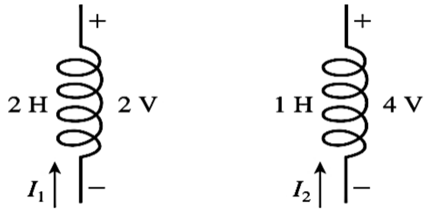

A. $a$ to $b$ and steady
B. $a$ to $b$ and decreasing
C. $b$ to $a$ and steady
D. $b$ to $a$ and increasing
√ E. $b$ to $a$ and decreasing

12. The diagram shows an inductor that is part of a circuit. The direction of the emf induced in the inductor is indicated. Which of the following is possible?




A. The current is constant and rightward
B. The current is constant and leftward
C. The current is increasing and rightward
√ D. The current is increasing and leftward
E. None of the above

13. In the diagram below, which current is changing more rapidly?




A. Current $I_1$

√ B. Current $I_2$

C. Both are constant

D. Both are changing at the same rate

E. Both are zero

14. When a switch is closed to complete a DC series *RL* circuit, which of the following is/are true?

I. the electric field in the wires increases to a maximum value

II. the magnetic field outside the wires increases to a maximum value

III. the rate of change of the electric and magnetic fields is greatest at the instant when the switch is closed

A. I only

B. II only

C. III only

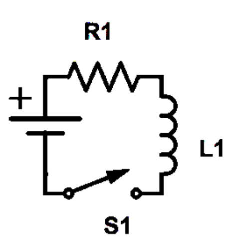
D. I and II only

√ E. I, II and III

**Questions 15 and 16.**

The figure below shows a *RL* circuit with the switch $S_1$ initially open. The battery voltage is 9 V, the resistor $R_1$ is 50 Ω and the inductance $L_1$ is 10 H.

15. What is the initial current, $i_0$ when $S_1$ is just closed?




√ A. 0 A

B. (9/50) A

C. (9/60) A

D. (50/9) A

E. (60/9) A

16. A long time passes, $t > 5\tau$, with the switch closed. What is the final steady state current $i_f$?

A. 0 A

√ B. (9/50) A

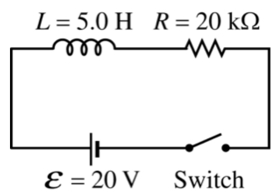
C. (9/60) A

D. (50/9) A

E. (60/9) A

**Questions 17 and 18**

After the switch is closed in the circuit below, the current in the circuit is given by $i = I(1 - e^{-t/\tau})$, where $I$ and $\tau$ are constants.




17. What is the value of $I$?

A. 0

√ B. 0.001 $A$

C. 0.80 $A$

D. 1.0 $A$

E. 4.0 $A$

18. What is the value of $\tau$?

A. $1.0 \times 10^{-5}$ $s$

√ B. $2.5 \times 10^{-4}$ $s$

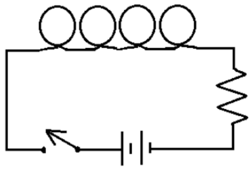
C. 1.0 $s$

D. $4.0 \times 10^{3}$ $s$

E. $1.0 \times 10^{5}$ $s$

**Questions 19 and 20**

A series circuit comprises of a battery with a voltage $V$, an open switch, a solenoid with inductance $L$ and a resistor with resistance $R$.



19. At time $t = 0$, the switch is closed. What is the current through the resistor immediately after the switch is closed?

√ A. 0

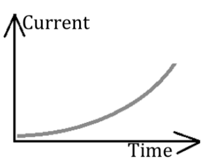
B. $\dfrac{V}{R}$

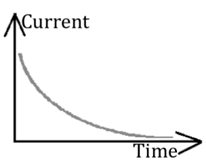
C. $\dfrac{V}{R}(1 - e^{-(R/L)t})$

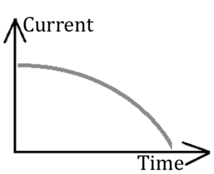
D. $\dfrac{V}{R}(1 - e^{-(RL)t})$

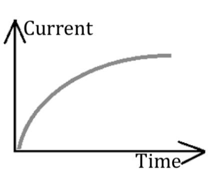
E. $\dfrac{V^2}{R}(1 - e^{-(RL)t})$

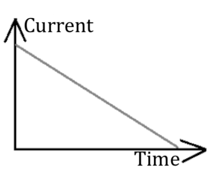
20. At time $t = 0$, the switch which had been closed is now opened. Which graph of current vs time through the resistor immediately after the switch is opened is the most accurate?

A.


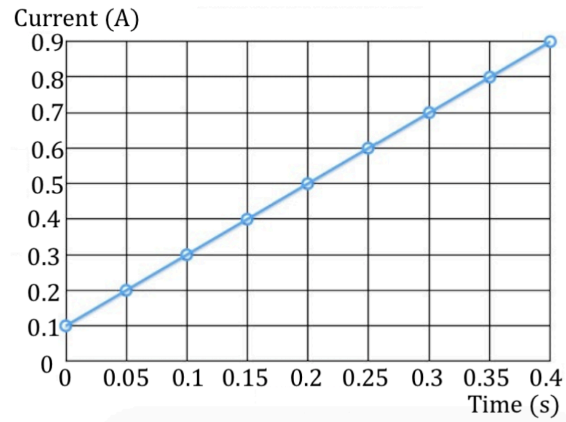

√ B.



C.



D.



E.



21. A 75 H inductor has s time-dependent current flowing through it over a short period of time as shown in the graph below.




What is the magnitude of the emf induced by the inductor at time $t = 0.3\ s$?

A. 7.5 V

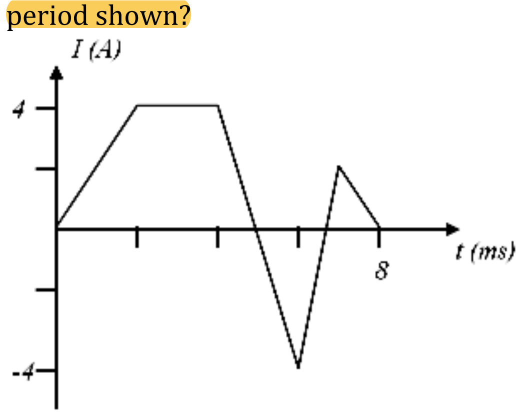
B. 22.5 V

C. 67.5 V

√ D. 150 V

E. 175 V

22. The figure shows the current through a 10 mH inductor. What is maximum magnitude of the potential difference $\Delta V_L$ across the inductor over the period shown?




A. 40 mV

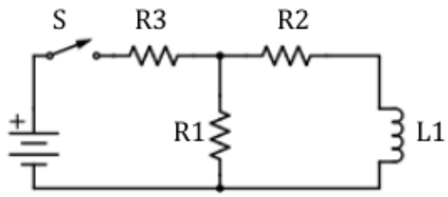
B. 60 mV

C. 20 V

D. 40 V

√ E. 60 V

23. A parallel circuit is composed of a switch, a battery, an inductor and 3 resistors. Initially, the switch is open, there is no current in the circuit and no energy in the inductor. Which statement is true about the initial current through this circuit immediately after the switch is closed?




A. There is no current anywhere

B. There is no current through $R_1$

√ C. There is no current through $R_2$

D. There is no current through $R_3$

E. There is current in all 3 resistors

24. If both the resistance and the inductance in an *LR* series circuit are halved, the new inductive time constant will be _____.

A. twice the old

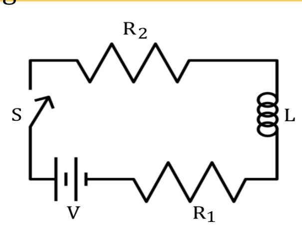
B. four times the old

C. half the old

D. one-fourth the old

√ E. unchanged

25. Which of the following equations is correct for the circuit below, a long time after the switch had been closed?




A. $V - S - IR_2 - L\dfrac{di}{dt} - IR_1 = 0$

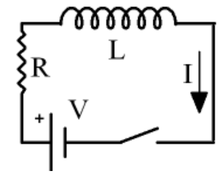
B. $V - IR_2 - L\dfrac{di}{dt} - IR_1 = V$

C. $V - IR_2 - L\dfrac{di}{dt} - IR_1 = 0$

D. $V - IR_2 - L - IR_1 = 0$

√ E. $V - IR_2 - IR_1 = 0$

26. When the switch in the circuit below is closed, what will happen?




A. No current will flow

B. The current will rise slowly to a final value of $I = RL$

√ C. The current will gradually rise to $I = V/R$ in the direction shown

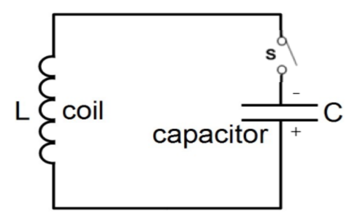
D. The current will start at $I = V/R$ in the direction shown and then drop to zero

E. The current will gradually rise to $I = V/R$ in the direction opposite to that shown

**Questions 27 and 28:**

In the circuit below, a fully charged capacitor with a capacitance C = 60 μF and charge Q = 6 μC is connected to an inductor L = 5 mH.

27. What is the current in the circuit at the instant when the switch is closed?




√ A. 0 A

B. 2 A

C. 6 A

D. 10 A

E. 30 A

28. Which of the following is/are true for the circuit above?

I. The capacitor stores energy in the electric field

II. The inductor stores energy in the magnetic field

III. There is a cyclic interchange of energy between capacitor and Inductor

A. I only

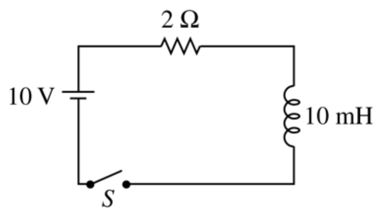
B. II only

C. III only

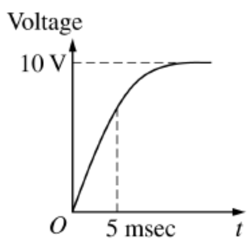
D. I and II only

√ E. I, II and III

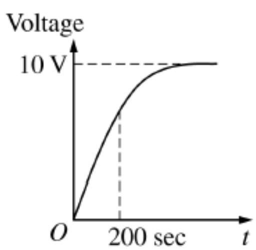
29. In the circuit shown below, the switch $S$ is closed at $t = 0$. Which of the following best represents the voltage across the inductor?



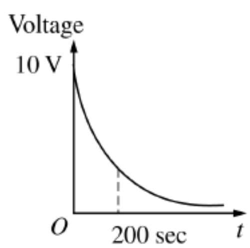

A.



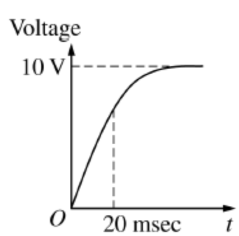

B.



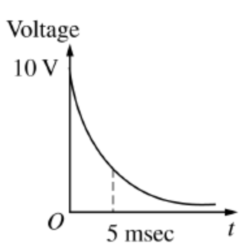

C.




D.




√ E.




**Questions 30 to 31**

A circuit contains a solenoid of inductance $L$ in series with a resistor of resistance $R$ and a battery with terminal voltage $\mathcal{E}$. At time $t = 0$, a switch is closed and the circuit is completed.

30. How long does it take for the current to reach 3/4 of its maximum (steady-state) value?

√ A. $(\ln 4)(L/R)$

B. $\left(\ln \frac{3}{4}\right)(L/R)$

C. $\left(\ln \frac{4}{3}\right)(L/R)$

D. $\left(\ln \frac{4}{3}\right)(R/L)$

E. $(\ln 4)(R/L)$

31. When the current reaches its maximum value, how much energy is stored in the magnetic field of the solenoid?

A. $L^2\mathcal{E}^2(4R^2)$

B. $L^2\mathcal{E}^2(2R^2)$

C. $L\mathcal{E}^2(4R^2)$

√ D. $L\mathcal{E}^2(2R^2)$

E. 0

32. When the current reaches its maximum value, what is the total magnetic flux through the solenoid?

A. $L\mathcal{E}$

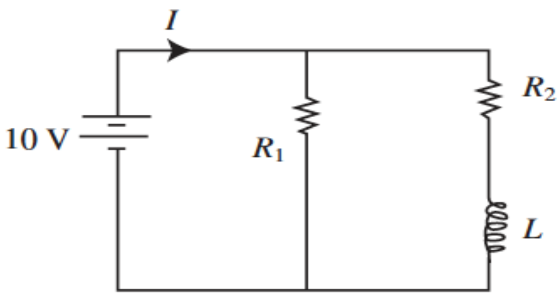
√ B. $L\mathcal{E}/R$

C. $\mathcal{E}/(RL)$

D. $RL/\mathcal{E}$

E. 0

33. If the two equal resistors $R1$ and $R2$ are connected in parallel to a 10-V battery with no other circuit components, the current provided by the battery is $I$. In the circuit shown above, an inductor of inductance $L$ is included in series with $R2$. What is the current through $R2$ after the circuit has been connected for a long time.




A. *zero*

B. $(1/4)I$

√ C. $(1/2)I$

D. $I$

E. $\left(\frac{R_1+R_2}{LR_2}\right)I$

34. A long narrow solenoid has length $\ell$ and a total of $N$ turns, each of which has cross-sectional area $A$. Its inductance is:

A. $\mu_0 N^2 A\ell$

√ B. $\mu_0 N^2 A/\ell$

C. $\mu_0 NA/\ell$

D. $\mu_0 N^2\, \ell/A$

E. none of these

35. How much energy is stored in a long solenoid with 660 turns, length $L = 3.2\ m$, and cross sectional area $A = 121\ cm^2$, when the current in it is $I = 85\ A$?

A. 1.10 $mJ$

B. 1.10 $kJ$

C. 9.1 1$kJ$

D. 5.21 $kJ$

√ E. 7.48 $kJ$

36. A 10-turn ideal solenoid has an inductance of 3.5 mH. When the solenoid carries a current of 2.0 A the magnetic flux through each turn is:

A. 0 Wb

B. $3.5 \times 10^{-4}$ Wb

√ C. $7.0 \times 10^{-4}$ Wb

D. $7.0 \times 10^{-3}$ Wb

E. $7.0 \times 10^{-2}$ Wb

37. Which of the following has the largest inductance?

A. A solenoid of radius 1 cm, length 5 cm, and with 100 turns of wire per centimeter

B. A solenoid of radius 3 cm, length 1 cm, and with 100 turns of wire per centimeter

C. A solenoid of radius 2 cm, length 2 cm, and with 100 turns of wire per centimeter

D. A solenoid of radius 1 cm, length 1 cm, and with 200 turns of wire per centimeter

√ E. A solenoid of radius 2 cm, length 5 cm, and with 100 turns of wire per centimeter

38. An inductance $L$, resistance $R$, and ideal battery of emf $\mathcal{E}$ are wired in series. A switch in the circuit is closed at time $t = 0$, at which time the current is zero. At any later time $t$ the current $i$ is given by:

A. $(\mathcal{E}/R)(1 - e^{-Lt/R})$

B. $(\mathcal{E}/R)e^{-Lt/R}$

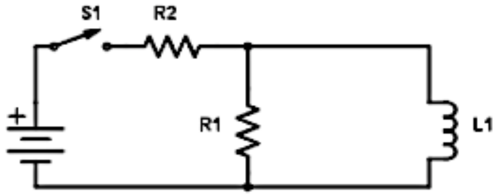
C. $(\mathcal{E}/R)(1 + e^{-Rt/L})$

D. $(\mathcal{E}/R)e^{-Rt/L}$

√ E. $(\mathcal{E}/R)(1 - e^{-Rt/L})$

**Questions 39 and 40**

A parallel circuit is composed of a switch, a battery, an indicator and 2 resistors. Initially the switch is open, there is no current in the circuit and no energy in the inductor.




39. Which statement is true about the initial current through this circuit immediately after the switch is closed?

A. There is no current anywhere

B. There is no current through $R_1$

C. There is no current through $R_2$

√ D. There is no current through $L_1$

E. There is current through $R_1, R_2, L_1$

40. After the switch is closed for a long time, what is true regarding the currents through $R_1, R_2$ and $L_1$?

A. $I_{R1} = I_{R2}$ and $I_{L1} = 0$

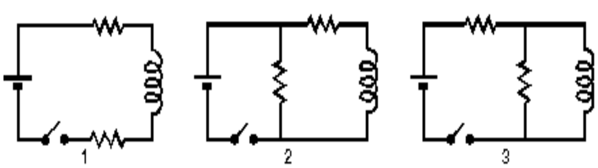
√ B. $I_{R2} = I_{L1}$ and $I_{R1} = 0$

C. $I_{L1} = I_{R1}$ and $I_{R2} = 0$

D. $I_{R1} = I_{R2} = I_{L1}$

E. $I_{R2} > I_{R1} = I_{L1}$

41. The diagrams show three circuits with identical batteries, identical inductors, and identical resistors. Rank them according to the current through the battery just after the switch is closed, from least to greatest.




A. $3 < 2 < 1$

B. $2 = 3 < 1$

√ C. $1 < 3 < 2$

D. $1 < 2 < 3$

E. $2 < 3 < 1$

**Questions 42 and 46**

The electric and magnetic fields are described by Maxwell's Equations below.

I. $\oint E \cdot dA = \frac{Q_{in}}{\varepsilon_0}$

II. $\oint B \cdot dA = 0$

III. $\oint E \cdot ds = -\frac{d\phi_m}{dt}$

IV. $\oint B \cdot ds = \mu_0 I_{through} + \varepsilon_0 \mu_0 \frac{d\phi_e}{dt}$

42. Which of the above states that a changing electric flux through a closed loop or an electric current through the loop creates a magnetic field around the loop?

A. I only

B. II only

C. III only

√ D. IV only

E. I and II only

43. Which of the equations must be modified if magnetic monopoles are discovered?

A. I only

√ B. II only

C. III only

D. IV only

E. I and II only

44. Which of the equations is associated with the Ampere-Maxwell law?

A. I only

B. II only

C. III only

√ D. IV only

E. I and III only

45. Which of the equations provide a method for calculating electric flux?

√ A. I only

B. II only

C. III only

D. I and II only

E. I, II and III

46. Which one of Maxwell's equations states that a changing electric field produces a magnetic field?

A. Gauss's law

B. Gauss's law for magnetism

C. Biot-Savart law

√ D. Ampere-Maxwell law

E. Faraday's law

47. A long, straight wire carrying a constant current creates a magnetic field around it. The magnetic field strength is measured with a magnetic field sensor at varying distances from the wire while the current is kept constant. Which of the following indicates a graph that will have a constant positive slope and the Maxwell equation most closely related to the graph?

A. A graph of the magnitude of the magnetic field as a function of the current. This is related to Faraday's law

B. A graph of the magnitude of the magnetic field as a function of distance from the center of the wire. This is related to Faraday's law

C. A graph of the magnitude of the magnetic field as a function of distance from the center of the wire. This is related to Ampere's law

D. A graph of the magnitude of the magnetic field as a function of the inverse of the distance from the center of the wire. This is related to Faraday's law

√ E. A graph of the magnitude of the magnetic field as a function of the inverse of the distance from the center of the wire. This is related to Ampere's law

48. One of the Maxwell's equations can be written as $\oint B \cdot dA = 0$. Which of the following statements follows directly from this statement?

I. Magnetic field lines must form closed loops
II. Moving charges create magnetic field
III. There exist no magnetic monopoles

A. I only

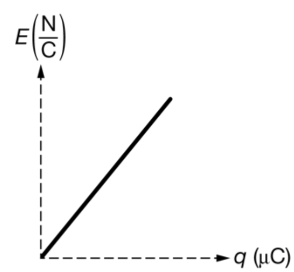
B. II only

C. III only

D. I and II only

√ E. I and III only

49. An experiment is run on a charged sphere. Students measure the charge $q$ on the sphere and the magnitude of the electric field $E$ a constant distance from the center of the sphere. The data are shown in the graph. Which of the following indicates a physical constant that can be determined from the graph and indicates the Maxwell equation associated with this graph?




| | | Constant | Maxwell's Equation |
|---|---|---|---|
| √ | A. | $\varepsilon_0$ | Gauss's Law |
| | B. | $\varepsilon_0$ | Ampere's Law |
| | C. | $\mu_0$ | Gauss's Law |
| | D. | $\mu_0$ | Ampere's Law |
| | E. | $\mu_0$ | Faraday's Law |

50. A student describes what happens when there is a constant magnetic field in the region of space. Then, the student describes what happens when there is a time-dependent magnetic field in the region of space. According to Maxwell's equations, which of the following is a correct description by the student about what occurs when the magnetic field changes from being constant to being time dependent?

√ A. When the magnetic field is constant, there is only a magnetic field in the region of space. When the magnetic field is time dependent, there will be both a magnetic field and an electric field in the region of space

B. When the magnetic field is constant, there will be both a magnetic field and an electric field in the region of space. When the magnetic field is time dependent, there is only a magnetic field in the region of space

C. In both situations, there will be both a magnetic field and an electric field in the region of space

D. In both situations, there will only be a magnetic field in the region of space. If the magnetic field is not constant over the region, then there will be both a magnetic field and an electric field in the region of space

E. In both situations, there will only be a magnetic field in the region of space. If the magnetic field is not constant over the region, there will still be only a magnetic field in the region of space

51. An experiment is performed using a sensitive ammeter attached to a coil. In the first trial, a bar magnet is quickly and repeatedly pushed into and pulled out of the coil. The ammeter reading is positive as the bar magnet moves forward into the coil. For the second trial, the connections to the ammeter are reversed. The bar magnet is again quickly and repeatedly pushed into and out of the coil. The ammeter reading is again positive as the bar magnet moves forward into the coil. Which of the following is a correct claim about the second trial and indicates the Maxwell equation that provides the evidence?

A. According to the Biot-Savart law, the poles of the magnet are aligned relative to the loop, the same as in the first trial

B. According to Faraday's law, the poles of the magnet are aligned relative to the loop, the same as in the first trial

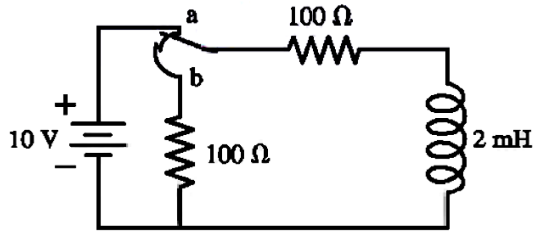
C. According to Ampere's law, the poles of the magnet are aligned relative to the loop, the same as in the first trial

√ D. According to Faraday's law, the poles of the magnet are reversed relative to the loop, as compared to the first trial

E. According to Ampere's law, the poles of the magnet are reversed relative to the loop, as compared to the first trial

**Answer the following questions:**

1. The switch in the figure below has been in position ***a*** for a long time. It is changed to position ***b*** at $t = 0s$.




a. What is the time constant?

$$\tau = \frac{L}{R} = \frac{2.0 \times 10^{-3} H}{200\ \Omega} = 1.0 \times 10^{-5} s = 10\ \mu s$$

b. What is the current in the circuit at $t = 5.0\ \mu s$?

$$I = I_0 e^{-\frac{t}{\tau}} = \left(\frac{10\ V}{100\ \Omega}\right) e^{-\frac{(5.0\mu s)}{10\mu s}} = 61\ mA$$

c. At what time has the current decayed to 1% of its initial value?

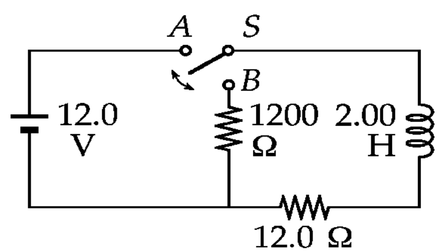
$$I = I_0 e^{-\frac{t}{\tau}}$$

$$0.01 \times 100\ mA = (100\ mA)\ e^{-\frac{t}{10\mu s}}$$

$$t = 10\mu s \ln\left(\frac{1\ mA}{100\ mA}\right)$$

$$t = 46\ \mu s$$

2. A 2.00 H inductor is connected in a circuit as shown below.




a. What is the current in the circuit a long time after the switch has been in position $a$?

After a long time, the current is steady: $i = \frac{\varepsilon}{R}$

$$i = \frac{12.0\ V}{12.0\ \Omega}$$

$$i = 1.00\ A$$

b. Now the switch is thrown quickly from $a$ to $b$. Compute the initial voltage across each resistor and across the inductor.

With the switch thrown to position $b$, the emf is no longer part of the circuit. The initial current is 1.00 A.

$$\Delta V_{12} = (1.00\ A)(12.00\ \Omega) = 12.0\ V$$

$$\Delta V_{1200} = (1.00\ A)(1200\ \Omega) = 1200\ V$$

$$\Delta V_L = \Delta V_R = (1200\ V + 12.0\ V) = 1212\ V$$

c. How much time elapses before the voltage across the inductor drops to 12.0 V?

With the switch thrown to position $b$, $\mathcal{E} = 0$.

$\Delta V_L = \Delta V_R = iR_{eff} = i(1200\,\Omega + 12\,\Omega) = i(1212\,\Omega)$

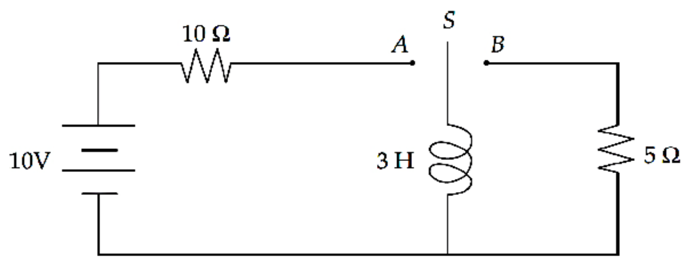
Then, when the voltage across the inductor has reached 12.0 V,

$$i = \frac{\Delta V_L}{R_{eff}} = \frac{12.0\ V}{1212\ \Omega} = 9.90 \times 10^{-3}$$

The current in the inductor decays as $i = I_i e^{-Rt/L}$. Solving for the time $t$,

$$t = \frac{L}{R} ln\left(\frac{I_i}{i}\right) = \left(\frac{2.00\ H}{1212\ \Omega}\right) ln\left(\frac{1.00\ A}{9.90 \times 10^{-3} A}\right) = 7.62 \times 10^{-3} s$$

3. A circuit is connected as shown below. The switch $S$ is initially open. Then it is moved to position $A$.




a. Determine the current in the circuit immediately after the switch is closed.
When the switch is initially moved, the inductor prevents an instantaneous change in current. So, the current in the inductor, and the rest of the circuit, is zero.

b. Determine the current in the circuit a long time after the switch is closed.
After the steady state condition is reached, the inductor has no emf (since $di/dt = 0$) so the current in the circuit can be found using Ohm's Law.

$$I = \frac{\mathcal{E}}{R}$$
$$I = \frac{10\ V}{10\ \Omega}$$
$$I = 1\ A$$

Sometime after the steady state situation has been reached, the switch is moved almost instantaneously from position $A$ to position $B$.

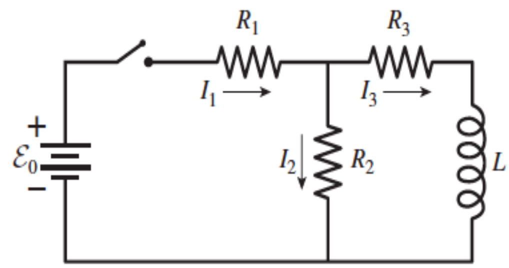
c. Determine the current through the 5 Ω resistor immediately after the switch has been moved.
The current in the inductor does not change immediately, so the current in the inductor is 1 $A$.

d. Determine the potential difference across the inductor immediately after the switch has been closed.
The current through the resistor is also 1 $A$ since it is in series with the inductor now. So the voltage drop across the resistor, $V_R$, can be found using Ohm's Law,

$$V_R = (1\ A)(5\ \Omega)$$
$$V_R = 5\ V$$

The voltage across the inductor is the same (since the voltage drops around the loop must add up to zero). Thus, $\mathcal{E}_L = 5\ V$

4. In the figure below, $\mathcal{E}_0 = 12\ V,\ \ R_1 = 4.0\ \Omega,\ \ R_2 = 8.0\ \Omega$, and $R_3 = 2.0\ \Omega$.




a. Find the current $I_2$:

i. Immediately after the switch is first closed.

Just after the switch is closed, the inductor current is zero
Current flows through $R_2$, which is in series with $R_1$.

$$I_2 = I_1 = \frac{\mathcal{E}_0}{R_1 + R_2}$$
$$I_2 = \frac{12\ V}{4.0\ \Omega + 8.0\ \Omega} = 1.0\ A$$

ii. A long time later.

After a long time, the currents reach steady values. $dI/dt = 0$
This implies that the voltage across the inductor is zero.
$R_1 and\ R_2$ are in parallel with each other and series with $R_1$.

$$I_1 = \frac{\mathcal{E}_0}{R_1 + \left(\frac{R_2 R_3}{R_2 + R_3}\right)}$$
$$I_1 = \frac{12\ V}{4.0\Omega + \left(\frac{8.0 \times 2.0}{10}\right)\Omega}$$
$$I_1 = 2.14\ A$$

By Kirchhoff's rules, $I_1 = I_2 + I_3$ and $I_2 R_2 = I_3 R_3$

$$I_2 = \frac{R_3}{R_2 + R_3} I_1$$
$$I_2 = \frac{2.0\Omega}{(8.0 + 2.0)\Omega}(2.14\ A)$$
$$I_2 = 0.43\ A$$

b. What is the steady current $I_3$ through the inductor?
$I_3 = I_1 - I_2 = 2.14 - 0.43 = 1.71\ A$

After a long time, the **switch is opened**. Immediately after the switch is opened find the following:

c. What is the magnitude and direction of $I_2$ after the switch is just opened?
$I_2 = -I_3 = -1.71\ A$

Justify your answer.

When the switch is reopened, no current flows through $R_1$ The induced current tries to keep the current flowing through the inductor $I_3$ as it was before the switch was opened, that is 1.71 A, which will be current through $R_2$ as well, but it will be flowing in the opposite direction (upward).

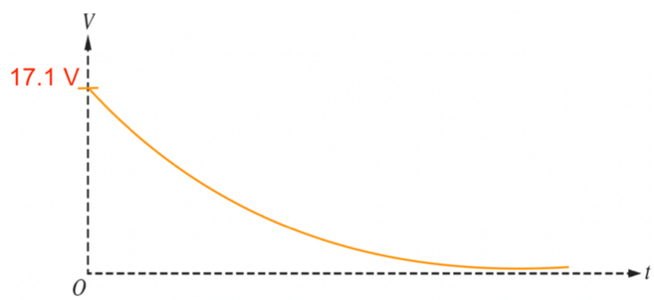
Conventional current would be going down through the inductor before the switch is opened. Inductors resists changes in current, so when the switch is opened, the current would continue to be down through the inductor and up through $R_2$

d. Which end of the inductor is at a lower potential?
The top end is at a lower potential and bottom is at a higher potential

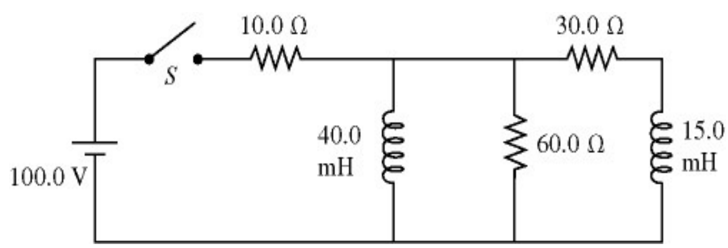
Justify your answer.
The inductor maintains a current going down through the inductor. Since conventional current comes out of the higher potential side, the bottom end of the inductor must be at the higher potential and the top end at lower potential

e. Compute the initial voltage across each resistor and across the inductor.
With the switch thrown to position $b$, the emf is no longer part of the circuit. The initial current is 1.00 A.
$\Delta V_1 = 0\ V$
$\Delta V_2 = (1.71\ A)(8.0\ \Omega) = 13.7\ V$
$\Delta V_3 = (1.71\ A)(1.0\ \Omega) = 3.42\ V$
$\Delta V_L = \Delta V_R = (13.7\ V + 3.42\ V) = 17.1\ V$

f. On the axes below, sketch a graph of the potential difference $V$ across the inductor as a function of time after the switch is opened. Explicitly label the vertical axis intercept with a numerical value.




5. For the circuit shown in the figure below, the inductors have no appreciable resistance and the switch has been open for a very long time.




a. The instant after closing the switch, what is the current through the 60.0-Ω resistor?
At $t = 0$, the inductor acts as an open switch. 10 Ω and 60 Ω are in series with each other.
Therefore, total resistance, $R_{total} = 10\ \Omega + 60\ \Omega = 70\ \Omega$

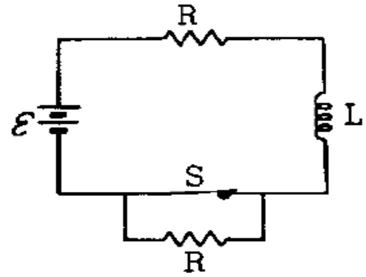
$$I_{60\ \Omega} = I_{total} = \frac{V}{R_{toatal}} = \frac{100\ V}{70\ \Omega} = 1.43\ A$$

b. The instant after closing the switch, what is the potential difference across the 15.0-mH inductor? Explain how you arrived at your answer.
At $t = 0$, the inductor acts as an open switch and so no current flows through the 30 Ω resistor.
Thus, the potential difference across the 15 $mH$ inductor is the same as the 60 Ω resistor as they are parallel, with no potential drop across the 30 Ω resistor.
$V_{15\ mH} = V_{60\ \Omega} = I_{60} \times 60$

$$V_{15\ mH} = V_{60\ \Omega} = \left(\frac{100}{10+60}\right) 60 = 85.7\ V$$

c. After the switch has been closed and left closed for a very long time, what is the potential drop across the 60.0-Ω resistor? Explain how you arrived at your answer.
After a long time, the currents reach steady values. $dI/dt = 0$
This implies that the voltage across the inductor is zero and the 40 mH inductor shorts the parallel circuit.
Therefore, the potential drop across the 60 Ω resistor is zero .

6. When the switch S in the circuit shown below is closed, an inductance L is in series with a resistance R and battery of Emf $\mathcal{E}$.



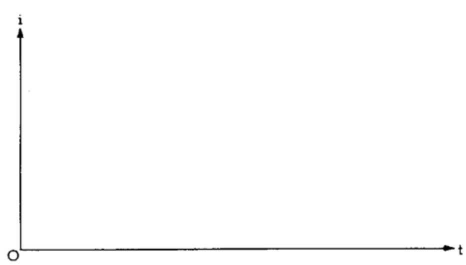

a. Determine the current $i_A$ in the circuit after the switch S has been closed for a very long time.

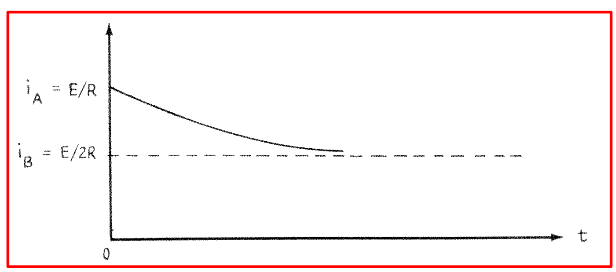
After the switch has been closed for a long time, the current will have ceased changing, so the inductor voltage $V_L = L\frac{di}{dt} = 0$. Therefore, the inductor can be ignored in this part of the problem.
The current can be calculated by using Ohm's Law, $V = IR$
Since the total resistance is just $R$,
$i_A = \mathcal{E}/R$

After being closed for a long time, the switch S is opened at time t = 0.

b. Determine the current $i_B$ in the circuit after the switch has been open for a very long time.
After the switch is opened, the two resistors are in series, and their combined resistance is 2R.
The current is $i_B = \mathcal{E}/2R$

c. On the axes below, sketch the graph of the current as a function of time t for $t \geq 0$ and indicate the values of the currents $i_A$ and $i_B$ on the vertical axis.




When $t = 0$, the current is $i_A = \mathcal{E}/R$. It decreases after the switch has been opened. Eventually, it approaches the limiting value $i_B = \varepsilon/2R$. The decrease is exponential, as shown.

d. By relating potential differences and emf's around the circuit, write the differential equation that can be used to determine the current as a function of time.

Applying Kirchhoff's law;
$\varepsilon = V_R + V_L$
The sum of the potential differences across the resistors is $V_R = 2Ri$
The potential difference across the inductance is $V_L = L\frac{di}{dt}$

The differential equation for the current is therefore,

$\varepsilon = 2Ri + L\frac{di}{dt}$

e. Write the equation for the current as a function of time for all time t ≧ 0.

The expression for $i(t)$ must involve an exponential function of the form $e^{-t/T}$

The time constant $T = L/2R$

The solution must be the sum of a constant and a term involving $e^{-2Rt/L}$. It must satisfy the boundary conditions $i(0) = \varepsilon/R, and\ i(t) \to \varepsilon/2R\ as\ t \ \to \infty$

By inspection, $i(t) = \frac{\varepsilon}{2R}(1 + e^{-2Rt/L})$

7. A 1.0 A current passes through a 10 $mH$ inductor coil. What potential difference is induced across the coil if the current drops to zero in 5.0 μs?

The rate of current decrease is:

$$\frac{dI}{dt} = \frac{0 - 1.0\ A}{5.0 \times 10^{-6} s} = -2.0 \times 10^{5} A/s$$

$$\Delta V = -L\frac{dI}{dt}$$

$$\Delta V = -(0.010\ H)(-2.0 \times 10^{5} A/s)$$

$$\Delta V = 2000\ V$$

8. A 1.0 A current passes through an inductor coil connected across a 2000 V supply. What is the inductance of the coil if the current drops to zero in 5.0 μs?

The rate of current decrease is:

$$\frac{dI}{dt} = \frac{0 - 1.0\ A}{5.0 \times 10^{-6} s} = -2.0 \times 10^{5} A/s$$

$$\Delta V = -L\frac{dI}{dt}$$

$$L = -\frac{\Delta V}{dl/dt}$$

$$L = \frac{-(2000\ V)}{(-2.0 \times 10^{5} A/s)} = 0.01\ H = 10\ mH$$

9. An inductor with 190 turns is made by tightly wrapping a wire around a 4.0 mm diameter cylinder. What length cylinder has an inductance of 10 μH?

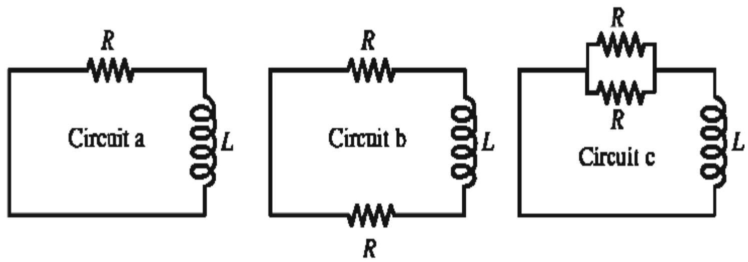
$$L = \frac{\mu_0 N^2 A}{l}$$

$$l = \frac{\mu_0 N^2 \pi r^2}{L}$$

$$l = \frac{(4\pi \times 10^{-7} Tm/A)(190)^2 \pi (2 \times 10^{-3}\ m)^2}{(10 \times 10^{-6} H)}$$

$$l = 0.057m$$

10. Rank in order, from largest to smallest, the three time-constants $\tau_a, \tau_b$ $and$ $\tau_c$ for the three circuits below. Explain how you arrived at your answer.




$$\tau_a = \frac{L}{R}$$

$$\tau_b = \frac{L}{2R} \quad (\textit{since the resistors are in series})$$

$$\tau_c = \frac{L}{(R/2)} = \frac{2L}{R} (\textit{since the resistors are in parallal})$$

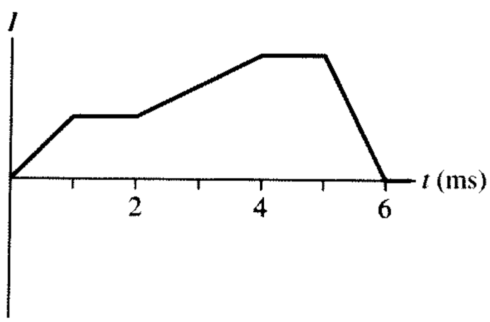
$$\therefore \tau_c > \tau_a > \tau_b$$

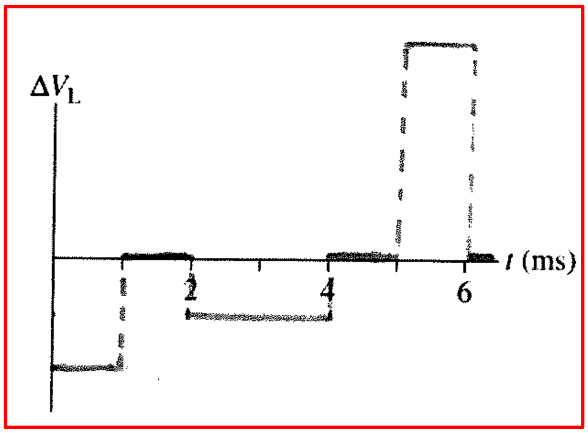
11. A 1 H inductor carries 10 A, and a 10 H inductor carries 1 A. Which inductor contains more stored energy? Explain your answer.

The energy stored in an inductor, $U \propto (LI^2)$

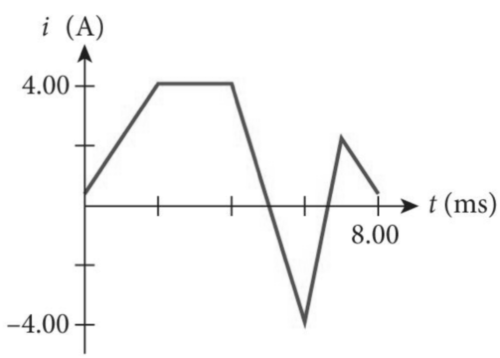
Therefore, the 1 H inductor carrying 10 A current stores more energy.

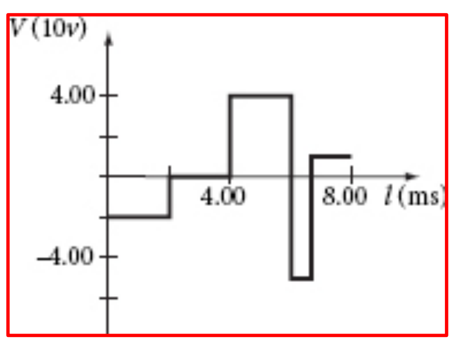
12. The figure shows the current through an inductor, changing with time. Sketch a corresponding graph of potential difference $\Delta V_L$ across the inductor.




13. The figure shows the current through a 10.0 mH inductor over a time interval of 8.00 ms. Draw a graph showing the self-induced potential difference, $\Delta V_L$ for the inductor over the same interval.

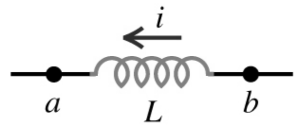


14. The inductor in the figure below has inductance 0.290 H and carries a current in the direction shown that is decreasing at a uniform rate, $di/dt = -1.90 \times 10^{-2} A/s$.

a. Find the self-induced emf.

$$\mathcal{E} = -L\frac{di}{dt} = -0.290\ H(-1.90 \times 10^{-2} A/s)$$
$$\mathcal{E} = 5.51 \times 10^{-3}\ V$$




b. Which end of the inductor, **a** or **b**, is at a higher potential?
End $a$ is at a higher potential as the current is decreasing.

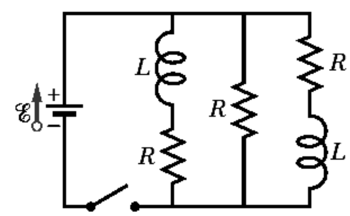
15. In a particular circuit, assume that $\mathcal{E}$ = 12V, R =40 Ω, and L = 5 mH. How much energy is stored in the inductor's magnetic field when the current reaches its maximum steady-state value?
When the current in the circuit reaches its maximum steady state value, I = $\mathcal{E}$ /R. At this point, the energy stored in the magnetic field of the inductor is

$$U_L = \frac{1}{2}LI^2 = \frac{1}{2}L\left(\frac{\mathcal{E}}{R}\right)^2 = \frac{(5 \times 10^{-3} H)(12\ V)^2}{2(40\ \Omega)^2} = 2.3 \times 10^{-4}\ J$$

16. Figure below shows a circuit that contains three identical resistors with resistance $R$ = 9.0 Ω, two identical inductors with inductance $L$ = 2.0 mH, and an ideal battery with emf $\mathcal{E}$ = 18 V.

a. What is the current *i* through the battery just after the switch is closed?
Just after the switch is closed, the inductor acts to oppose a change in the current through it.

$$i = \frac{\mathcal{E}}{R} = \frac{18\ V}{9.0\ \Omega} = 2.0\ A$$



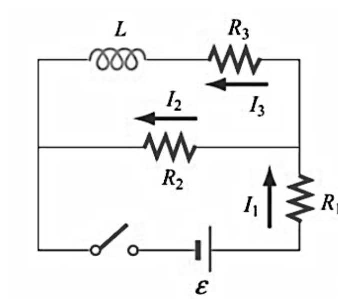

b. What is the current *i* through the battery log after the switch is closed?
We now have a circuit with three identical resistors in parallel; their equivalent resistance is

$$R_{eq} = \frac{R}{3} = \frac{9.0\ \Omega}{3} = 3.0\ \Omega$$
$$i = \frac{\mathcal{E}}{R_{eq}} = \frac{18\ V}{3.0\ \Omega} = 6.0\ A$$

17. Consider the circuit shown in Figure below.

Determine the current through each resistor

(a) immediately after the switch is closed.

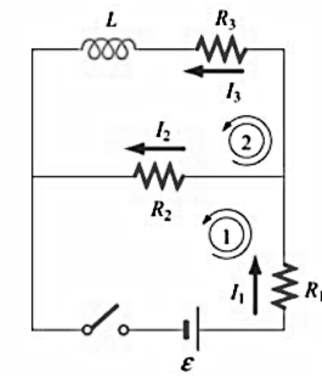



Immediately after the switch is closed, the current through the inductor is zero because the self-induced emf prevents the current from rising abruptly. Therefore,
$I_3 = 0$. Since $I_1 = I_2 + I_3$ we have $I_1 = I_2$
Applying Kirchhoff's rules to the first loop shown in figure yields

$$I_1 = I_2 = \frac{\mathcal{E}}{R_1 + R_2}$$

(b) a long time after the switch is closed.

After the switch has been closed for a long time, there is no induced emf in the inductor and the currents will be constant. Kirchhoff's loop rule gives
$\mathcal{E} = I_1R_1 - I_2R_2 = 0$ for the first loop, and
$I_1R_1 - I_2R_2 = 0$ for the second.
Combining the two equations with the junction rule:
$I_1 = I_2 + I_3$, we obtain:




$$I_1 = \frac{(R_2 + R_3)\mathcal{E}}{R_1R_2 + R_1R_3 + R_2R_3}$$
$$I_2 = \frac{(R_3)\mathcal{E}}{R_1R_2 + R_1R_3 + R_2R_3}$$
$$I_3 = \frac{(R_2)\mathcal{E}}{R_1R_2 + R_1R_3 + R_2R_3}$$

Suppose the switch is reopened a long time after it's been closed. What is each current

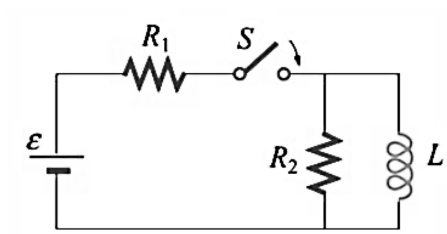
(c) immediately after it is opened?

Immediately after the switch is opened, the current through $R_1$ is zero, i.e.. $I_1 = 0$. This implies that $I_2 + I_3 = 0$ On the other hand, loop 2 now forms a decaying RL circuit and $I_3$ starts to decrease. Thus,
$$I_3 = -I_2 = \frac{(R_2)\mathcal{E}}{R_1R_2 + R_1R_3 + R_2R_3}$$

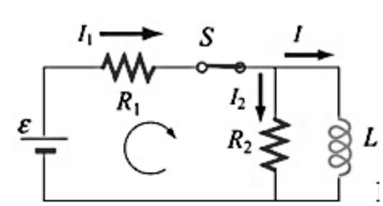
(d) after a long time?

A long time after the switch has been closed, all currents will be zero.
That is, $I_1 = I_2 = I_3$

18. In the circuit shown in Figure, suppose the circuit is initially open. At time t = 0 it is thrown closed. What is the current in the inductor at a later time t?




Let the currents through $R_1$, $R_2$ and L be $I_1$, $I_2$ and I, respectively, as shown in Figure.
From Kirchhoff's junction rule, we have $I_1 = I_2 + I_3$ Similarly, applying Kirchhoff's loop rule to the left loop yields
$\mathcal{E} - (I + I_2)R_1 - I_2R_2 = 0$ ***Equation 1***
Similarly, for the outer loop, the modified Kirchhoff's loop rule gives
$$\mathcal{E} - (I + I_2)R_1 = L\frac{dI}{dt} \qquad \textbf{\textit{Equation 2}}$$




The two equations can be combined to yield
$$I_2R_2 = L\frac{dI}{dt} \Rightarrow I_2 = \frac{L}{R_2}\frac{dI}{dt}$$
Substituting into equation 1 the expression obtained for $I_2$, we have
$$\mathcal{E} - \left(I + \frac{L}{R_2}\frac{dI}{dt}\right)R_1 - L\frac{dI}{dt} \Rightarrow \mathcal{E} - IR_1 - \left(\frac{R_1 + R_2}{R_2}\right)L\frac{dI}{dt} = 0$$
Dividing the above equation by $\left(\frac{R_1+R_2}{R_2}\right)$ leads to:

$$\mathcal{E}' - IR_1' - L\frac{dI}{dt} = 0$$

$$Where, \quad R_1' = \frac{R_1R_2}{R_1 + R_2}, \quad \mathcal{E}' = \frac{R_2\mathcal{E}}{R_1 + R_2}$$

The differential equation can be solved and the solution is given by:

$$I(t) = \frac{\mathcal{E}'}{R'}\left(1 - e^{-Rt/L}\right)$$

Since, $\dfrac{\mathcal{E}'}{R'} = \dfrac{R_2\mathcal{E}/R_1 + R_2}{R_1R_2/R_1 + R_2} = \dfrac{\mathcal{E}}{R_1}$

The current through the inductor can be written as

$$I(t) = \frac{\mathcal{E}'}{R'}\left(1 - e^{-Rt/L}\right) = \frac{\mathcal{E}}{R_1}\left(1 - e^{-Rt/L}\right)$$

Where $\tau = L/R'$ is the time constant

19. A coil has an inductance of 53 mH and a resistance of 0.35 Ω.

(a) If a 12 V emf is applied across the coil, how much energy is stored in the magnetic field after the current has built up to its equilibrium value?

The energy stored in the magnetic field of a coil at any time depends on the current through the coil at that time, according to the equation, at equilibrium is:

$$U_\infty = \frac{1}{2}Li_\infty{}^2$$

Thus, to find the energy stored at equilibrium, we must find the equilibrium current.

$$i_\infty = \frac{\mathcal{E}}{R} = \frac{12\ V}{0.35\ \Omega} = 34.3\ A$$

$$U_\infty = \frac{1}{2}Li_\infty{}^2 = \frac{1}{2}(23 \times 10^{-3} H)(34.3\ A)^2 = 31\ J$$

(b) After how many time constants will half this equilibrium energy be stored in the magnetic field?

$$U = \frac{1}{2}U_\infty$$

$$\frac{1}{2}Li^2 = \frac{1}{2}\left(\frac{1}{2}Li_\infty{}^2\right)$$

$$i = \frac{1}{\sqrt{2}}i_\infty$$

This equation tells us that, as the current increases from its initial value of 0 to its final value of $i_\infty$, the magnetic field will have half its final stored energy when the current has increased to this value.

$$Since, \quad i_\infty = \frac{\mathcal{E}}{R}$$

$$i = \frac{1}{\sqrt{2}}i_\infty \Rightarrow \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{t}{\tau}}\right) = \frac{\mathcal{E}}{\sqrt{2}R}$$

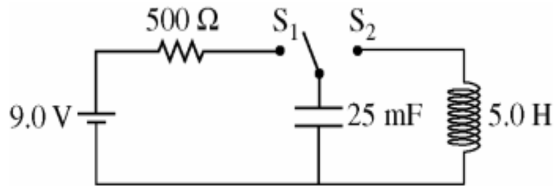
$$e^{-\frac{t}{\tau}} = 1 - \frac{1}{\sqrt{2}} = 0.293$$

$$\frac{t}{\tau} = -\ln(0.293) = 1.23$$

$$t \approx 1.2\ \tau$$

Thus, the energy stored in the magnetic field of the coil by the current will reach half its equilibrium value 1.2 time constants after the emf is applied.

20. The circuit represented above contains a 9.0 V battery, a 25 mF capacitor, a 500 Ω resistor, and a switch with two positions, $S_1$ and $S_2$. Initially the capacitor is uncharged and the switch is open.

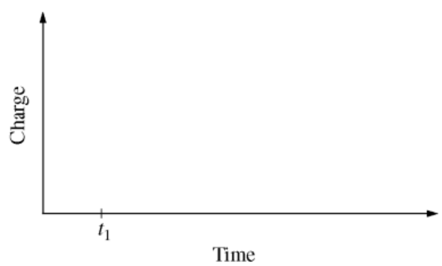


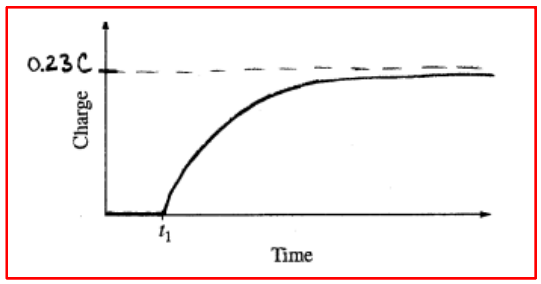

a. In experiment 1 the switch is closed to position $S_1$ at time $t_1$ and left there for a long time.

i. Calculate the value of the change on the bottom plate of the capacitor a long time after the switch is closed.

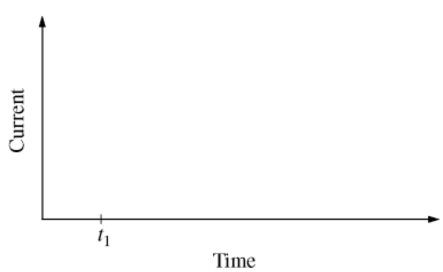
$Q = CV = (25 \times 10^{-3} F)(9.0\ V) = 0.23\ C$

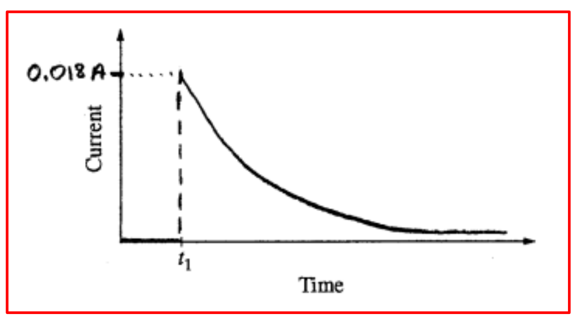
ii. On the axes below, sketch a graph of the magnitude of the charge on the bottom of plate of the capacitor as a function of time. On the axes, explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.




iii. On the axes below, sketch a graph of the current through the resistor as a function of time. On the axes, explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate




The maximum current occurs just after the switch is closed, when there is no charge on the capacitor.

V =IR

$I_{max}$ =VR = 9.0 V/ 500 Ω= 0.018 A

I = 0 for $t < t_1$

b. In experiment 2 the capacitor is again charged when the switch is closed to position $S_1$ at time $t_1$. The switch is then moved to position $S_2$ at time $t_2$ when the magnitude of the charge on the capacitor plate is 105 mC, allowing electromagnetic oscillations in the LC circuit.

i. Calculate the energy stored in the capacitor at time $t_2$.

$$U_c = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{(105\times10^{-3}C)^2}{25\times10^{-3}\,F} = 0.22\,J$$

ii. Calculate the maximum current that will be present during the oscillations.

$$U_L = \frac{1}{2}LI^2 = U_c$$

$$I = \sqrt{2U_c/L} = \sqrt{\frac{2(0.22\,J)}{5.0\,H}} = 0.30\,A$$

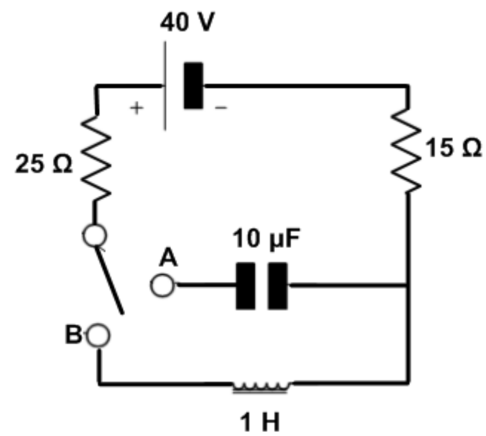
iii. Calculate the time rate of change of the current when the charge on the capacitor plate is 50 mC.

Using the loop rule:

$$L\frac{dI}{dt} + \frac{Q}{C} = 0$$

$$\frac{dI}{dt} = -\frac{Q}{LC} = \frac{50\times10^{-3}C}{(25\times10^{-3}F)(5.0\,H)} = -0.40\,A/s$$

21. In the circuit below, the switch is initially open, all currents are zero and the capacitor is uncharged.




a. At time $t = 0$, the switch is closed to position A. Determine the voltage across 15 Ω resistor.

$$I = \frac{\varepsilon}{R_1 + R_2} = \frac{40\,V}{40\,\Omega} = 1\,A$$

$$V = IR = (1\,A)(15\,\Omega) = 15\,V$$

b. After a long time when the switch is closed to position A, determine the voltage across 15 Ω resistor.

$V = 0$

c. After a long time when the switch is closed to position A, determine the charge on the capacitor.

$Q = CV = (10\times10^{-6}\,F)(40\,V) = 4\times10^{-4}\,C$

d. At new time $t = t_1$, the switch is closed to position B, determine the voltage across 15 Ω resistor at time $t_1$.

$V = 0$

e. Determine the current in 15 Ω resistor after long time from$t_1$.

$$I = \frac{\varepsilon}{R_{net}} = \frac{40V}{40\,\Omega} = 1\,A$$

f. Determine the energy stored in the inductor long time after $t_1$.

$U = ½(LI^2) = 0.5(1\,H)(1\,A)^2 = 0.5\,J$