CHAPTER 30

Sources of the Magnetic Field

Magnetic Fields

The origin of the magnetic field is moving charges.

The magnetic field due to various current distributions can be calculated.

Ampère’s law is useful in calculating the magnetic field of a highly symmetric configuration carrying a steady current.

Magnetic effects in matter can be explained on the basis of atomic magnetic moments.

Biot-Savart Law – Introduction

Biot and Savart conducted experiments on the magnetic force exerted by an electric current on a nearby magnet.

They arrived at a mathematical expression that gives the magnetic field at some point in space due to a current.

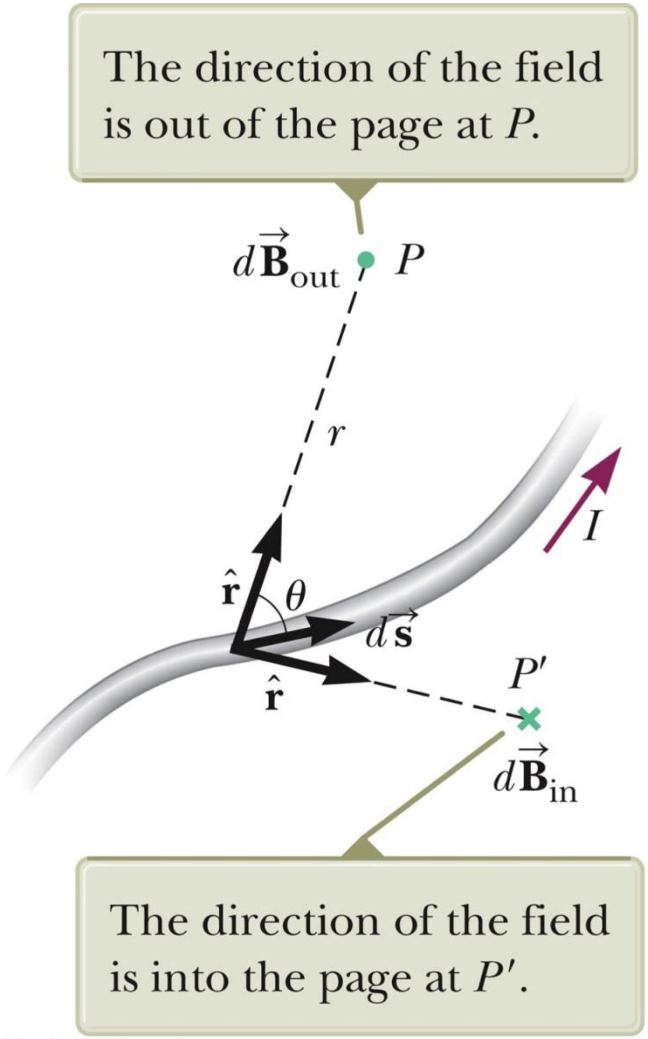
The magnetic field described by the Biot-Savart Law is the field *due to* a given <u>current carrying</u> conductor.

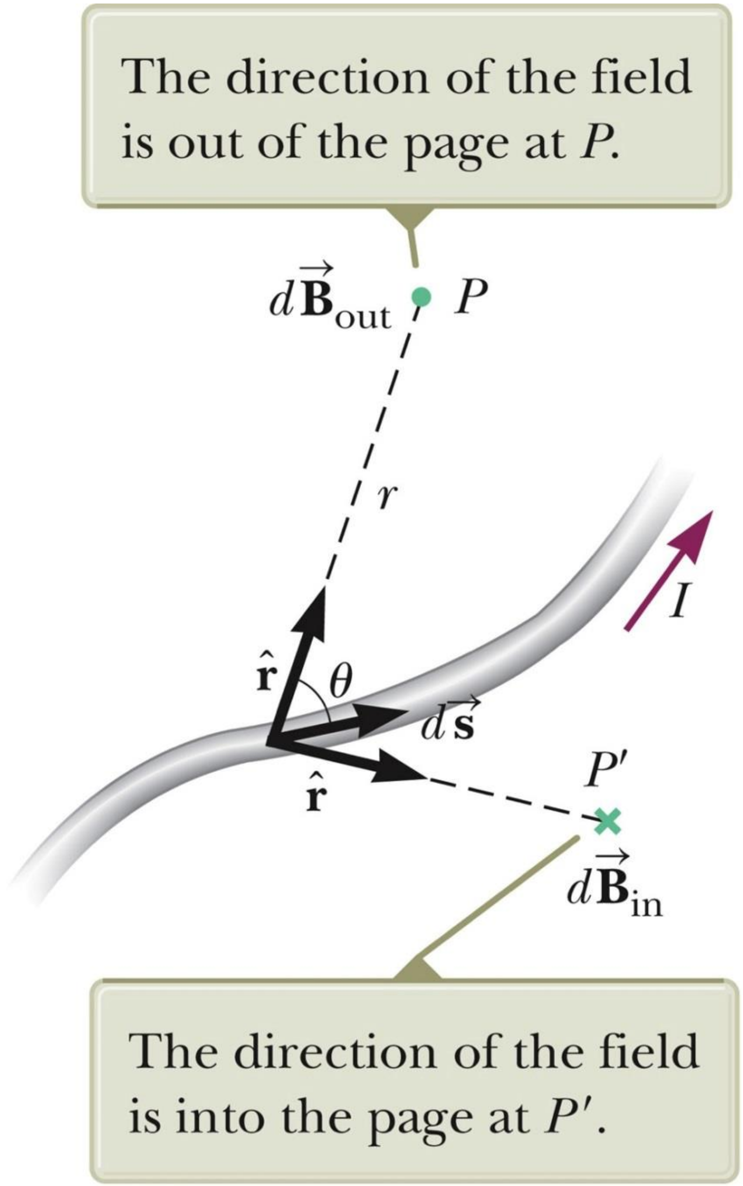
Do not confuse this field with any external field applied to the conductor from some other source.

Magnetic Field for a conducting wire:

The figure shown defines all of the essential parameters that we will require in order for us to use the Biot-Savart law.

From the right-hand rule, we can see the B-field directions on both of the labelled points that are shown (P and P')





The direction of the field is out of the page at P.
$d\vec{\mathbf{B}}_{\text{out}}$
P
r
I
$\hat{\mathbf{r}}$
θ
$d\vec{\mathbf{s}}$
$\hat{\mathbf{r}}$
P'
$d\vec{\mathbf{B}}_{\text{in}}$
The direction of the field is into the page at P'.

Biot-Savart Law – Observations

- The vector $d\vec{\mathbf{B}}$ is perpendicular both to $d\vec{\mathbf{s}}$ (which points in the direction of the current) and to the unit vector $\hat{\mathbf{r}}$ directed from $d\vec{\mathbf{s}}$ toward P.

- The magnitude of $d\vec{\mathbf{B}}$ is inversely proportional to r^2, where r is the distance from $d\vec{\mathbf{s}}$ to P.

- The magnitude of $d\vec{\mathbf{B}}$ is proportional to the current I and to the magnitude ds of the length element $d\vec{\mathbf{s}}$.

- The magnitude of $d\vec{\mathbf{B}}$ is proportional to $\sin\theta$, where θ is the angle between the vectors $d\vec{\mathbf{s}}$ and $\hat{\mathbf{r}}$.

Biot-Savart Law – Equation

All these observations are summarized in the mathematical equation called the **Biot-Savart law**:

$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

The constant μ_o is called the **permeability of free space.**

$$\mu_o = 4\pi \times 10^{-7} \approx 1.26 \times 10^{-6} \ \ T\, m / A$$

This is the magnetic equivalent of ε_o for the E-Field.

Total Magnetic Field

$d\vec{\mathbf{B}}$ is the field created by the current in the length segment ds. To find the total field, sum up the contributions from all the current elements $I\,d\vec{\mathbf{s}}$

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

The integral is over the entire current distribution.

The law is also valid for a current consisting of charges flowing through space. For example, this could apply to the beam in an accelerator.

Magnetic Field Compared to Electric Field

Distance

The magnitude of the magnetic field varies as the <u>inverse square</u> of the distance from the source.

The electric field due to a point charge also varies as the <u>inverse square</u> of the distance from the charge.

Direction

The electric field created by a point charge is <u>radial</u> in direction.

The magnetic field created by a current element is <u>perpendicular</u> to both the length element $d\vec{\mathbf{s}}$ and the unit vector $\hat{\mathbf{r}}$

Magnetic Field Compared to Electric Field, cont.

Source:

We note that an electric field is established by an isolated electric charge.

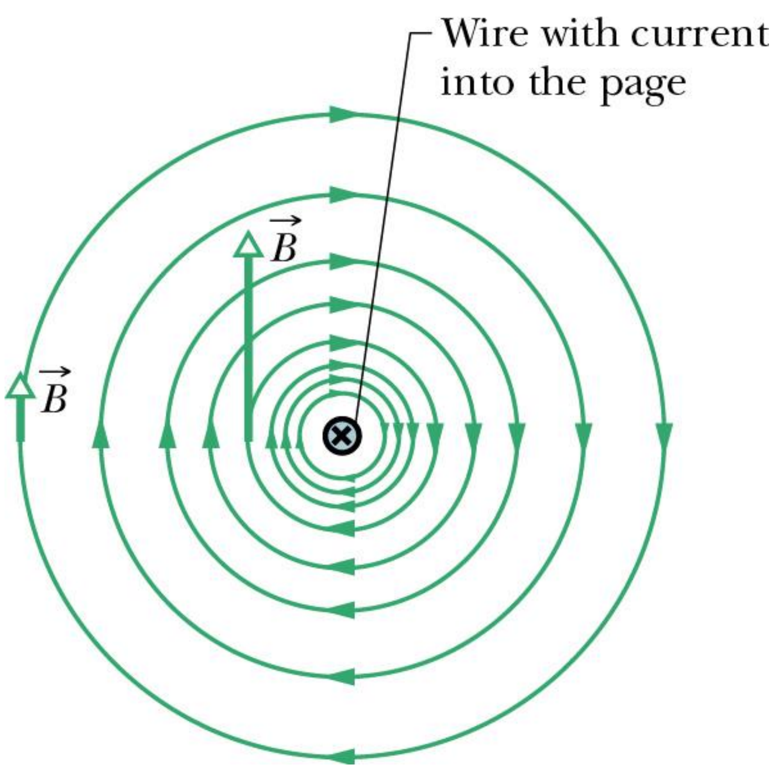
However, the current element that produces a magnetic field must be part of an extended current distribution.

Therefore it is necessary to integrate over the entire current distribution.

Magnetic Field – Right-Hand Rule

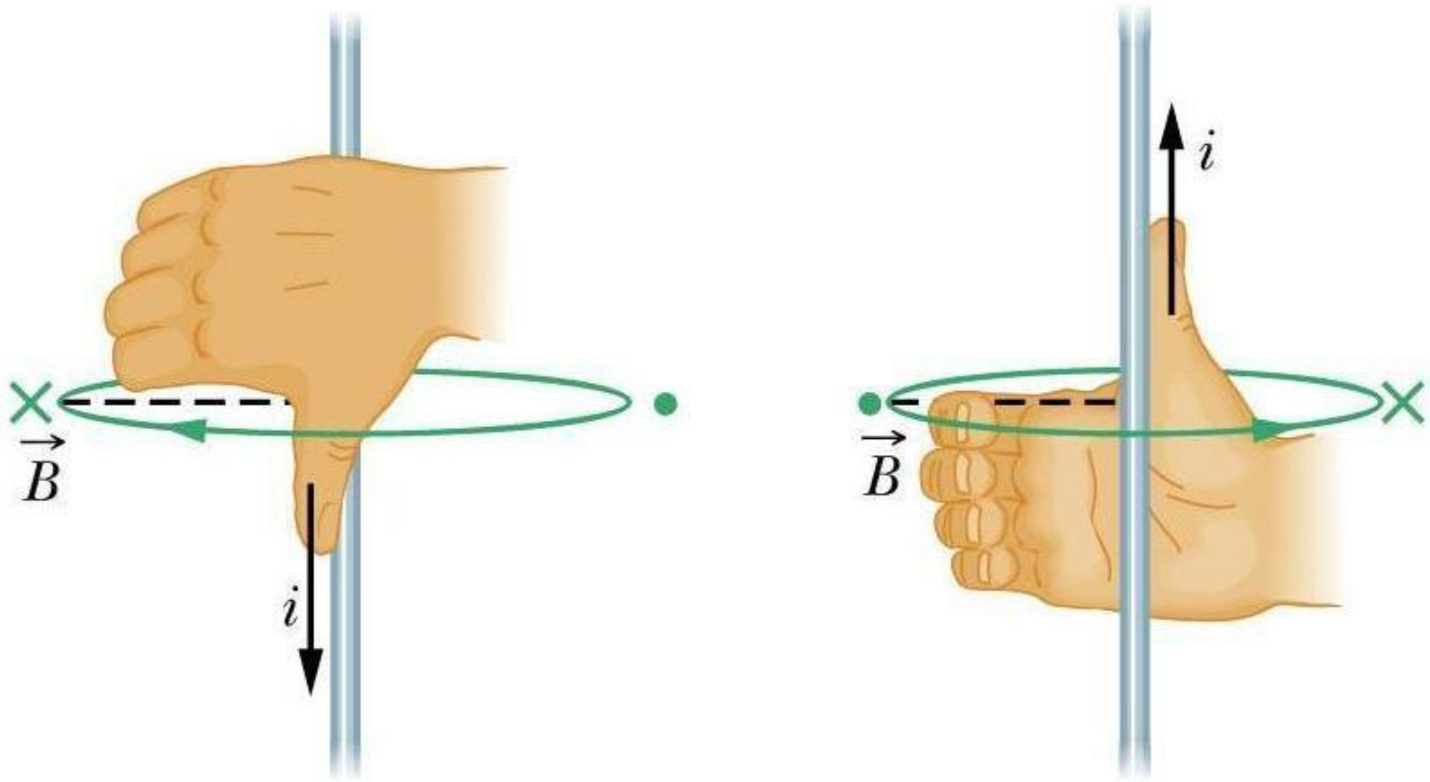
Direction of magnetic field lines:

Right-hand rule: Grasp the current-length element in your right hand with your extended thumb pointing in the direction of the current. Your fingers will then naturally curl around in the direction of the magnetic field lines due to that element. The field lines for a long straight wire are shown in the figure:



Magnetic Field – Right-Hand Rule

From the side for two different current directions:



Magnetic Field – Right-Hand Rule

By convention, there are two slightly different symbols that are frequently used to describe the direction of either a current or a magnetic (electric) field:

The symbol "× or ⊗" means "going directly away from you"

The symbol "• or ⊙" means "coming out towards you"

We shall now work through several helpful examples, using different shapes and lengths of current-carrying conductors.

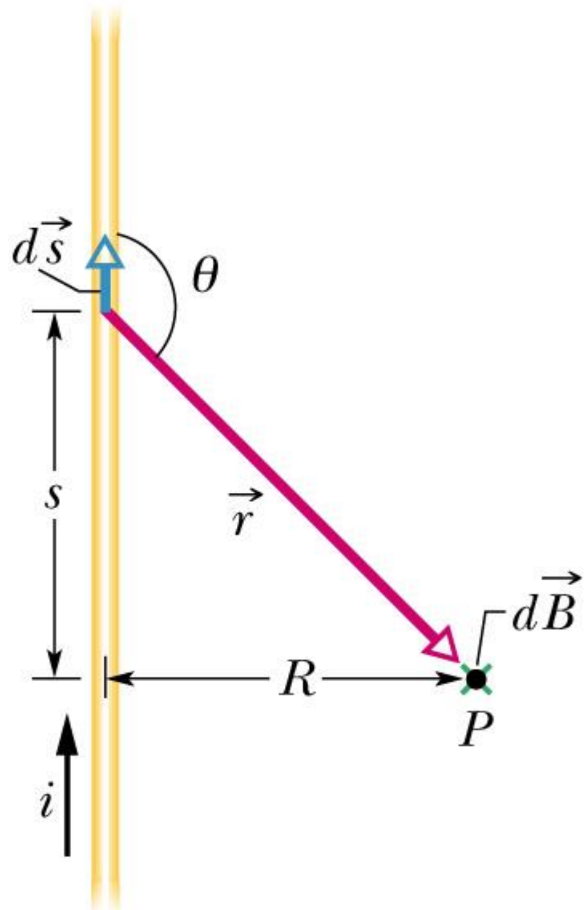
Magnetic Field – Long Straight Wire

Firstly, we consider an infinitely long straight vertical wire with the current going straight upwards:

$$r = \sqrt{s^2 + R^2}$$

$$\sin\theta = \sin(\pi - \theta) = \frac{R}{r}$$

$$\sin\theta = \frac{R}{\sqrt{s^2 + R^2}}$$



Magnetic Field – Long Straight Wire

The differential magnetic field associated with the current-length element, is directed <u>into</u> the plane of the page and has magnitude:

$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{i\,d\vec{s}\times\vec{r}}{r^3} \Rightarrow d\vec{B} = \frac{\mu_o}{4\pi}\frac{i\,d\vec{s}\times\hat{r}}{r^2} \Rightarrow dB = \frac{\mu_o}{4\pi}\frac{i\,ds\sin\theta}{r^2}$$

The direction of the magnetic field is the same for all of the current-length elements (centrally symmetric).

We need only find the magnitude of the magnetic field at point P from the centre of the wire to ∞ and then multiply by 2.

Magnetic Field – Long Straight Wire

$$B = 2\int dB = \frac{\mu_o i}{2\pi}\int_0^{\infty}\frac{\sin\theta\, ds}{r^2}$$ Substitution $\Rightarrow$

Giving:

$$B = \frac{\mu_o i}{2\pi}\int_0^{\infty}\frac{R\, ds}{(s^2+R^2)^{3/2}} = \frac{\mu_o i}{2\pi}\left[\frac{s}{R\sqrt{s^2+R^2}}\right]_0^{\infty}$$

We finally obtain:

$$B = \frac{\mu_o i}{2\pi R}$$

Magnetic Field – Long Straight Wire

Alternative integration limits $\pm\infty$ give the same result:

$$B = \int dB = \frac{\mu_o i}{4\pi} \int_{-\infty}^{\infty} \frac{\sin\theta \, ds}{r^2} = \frac{\mu_o i}{4\pi} \int_{-\infty}^{\infty} \frac{R \, ds}{(s^2 + R^2)^{3/2}}$$

Integration gives:

$$B = \frac{\mu_o i}{4\pi} \left[\frac{s}{R\sqrt{s^2 + R^2}} \right]_{-\infty}^{\infty} = \frac{\mu_o i}{2\pi R} \quad \text{(As above)}$$

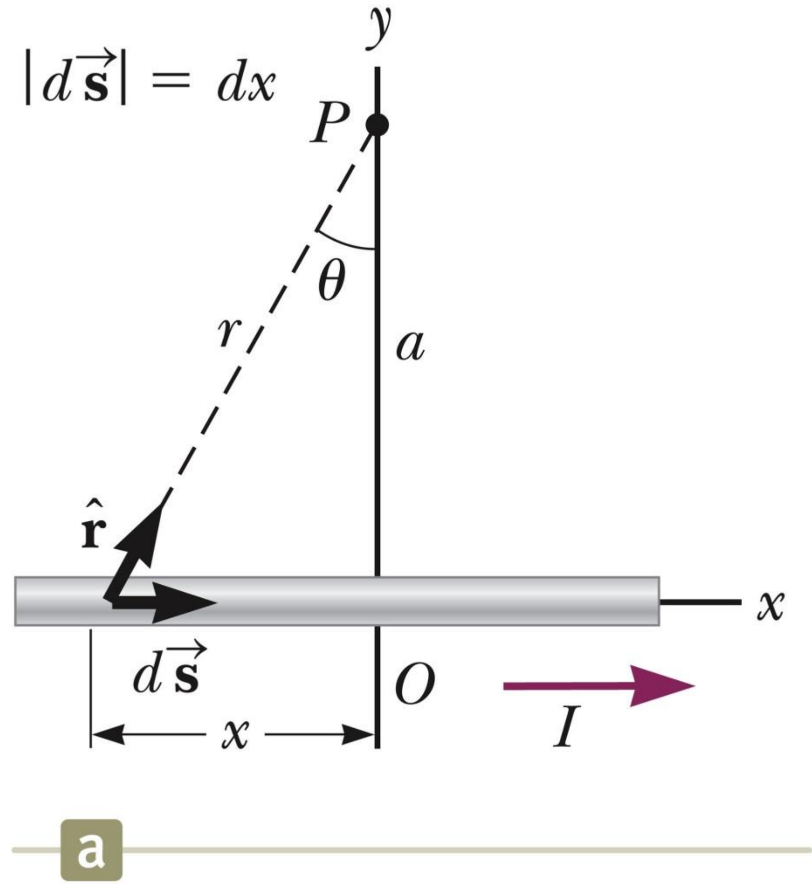
The magnitude of the magnetic field for either the upper or lower half (each is a semi-infinite wire) will be half of this.

Magnetic Field for a Long, Straight Conductor

To calculate the B-field from a long straight conductor:

Find the field contribution from a small element of current and then integrate over the current distribution.

The long thin, straight wire carries a <u>constant</u> current.

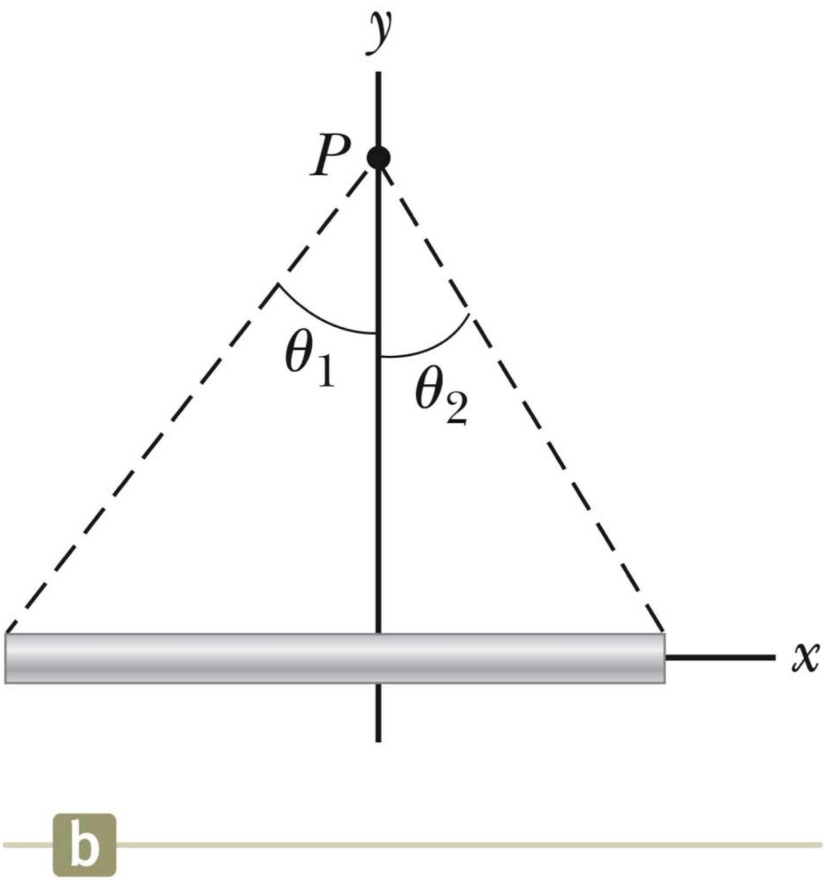


B-Field for a Long, Straight Conductor

We now consider a general solution where the angles at the apex are different.

This means that the lengths of current-carrying wire for each side of the coordinate origin must also be different.

This is shown in the figure:



B-Field for a Long, Straight Conductor

We start with the Biot-Savart law:

$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

The cross-product is:

$$d\vec{\mathbf{s}} \times \hat{\mathbf{r}} = \left|d\vec{\mathbf{s}} \times \hat{\mathbf{r}}\right|\hat{\mathbf{k}} = \left[dx \sin\left(\frac{\pi}{2} - \theta\right)\right]\hat{\mathbf{k}} = (dx\cos\theta)\hat{\mathbf{k}}$$

Giving:

$$d\vec{\mathbf{B}} = (dB)\hat{\mathbf{k}} = \frac{\mu_0 I}{4\pi}\frac{dx\cos\theta}{r^2}\hat{\mathbf{k}}$$

B-Field for a Long, Straight Conductor

Simple trigonometry gives:

$$r = \frac{a}{\cos\theta} \qquad \text{and} \qquad x = -a\tan\theta$$

The negative sign for x is because the vector $d\vec{\mathbf{s}}$ is at negative value of x. We now need the differential element dx calculated as shown:

$$dx = -a\sec^2\theta\ d\theta \quad = -\frac{a\,d\theta}{\cos^2\theta}$$

B-Field for a Long, Straight Conductor

Now substitute into the Biot-Savart equation:

$$dB = -\frac{\mu_0 I}{4\pi}\left(\frac{a\,d\theta}{\cos^2\theta}\right)\left(\frac{\cos^2\theta}{a^2}\right)\cos\theta$$

Giving us:

$$dB = -\frac{\mu_0 I}{4\pi a}\cos\theta\;d\theta$$

We now integrate over all length elements, using the fact that we have two values of θ to consider $\Rightarrow$

B-Field for a Long, Straight Conductor

The integral will now give us:

$$B = -\frac{\mu_0 I}{4\pi a}\int_{\theta_1}^{\theta_2} \cos\theta \; d\theta$$

$$B = \boxed{\frac{\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2)}$$

We can now use the final resulting equation to enable us to calculate the magnitude of the magnetic field for any straight current-carrying wire, if we know the angles.

B-Field for a Long, Straight Conductor

For the special case of an infinitely long wire (analyzed earlier in this section), we see that the two angles become:

$$\theta_1 = \pi/2 \qquad \text{and} \qquad \theta_2 = -\pi/2$$

For length elements ranging between x = - ∞ to x = + ∞, so that we obtain:

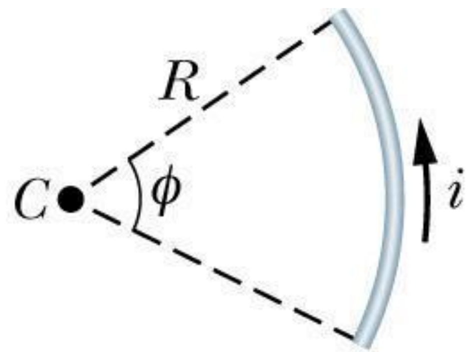
$$(\sin\theta_1 - \sin\theta_2) = [\sin\pi/2 - \sin(-\pi/2)] = 2$$

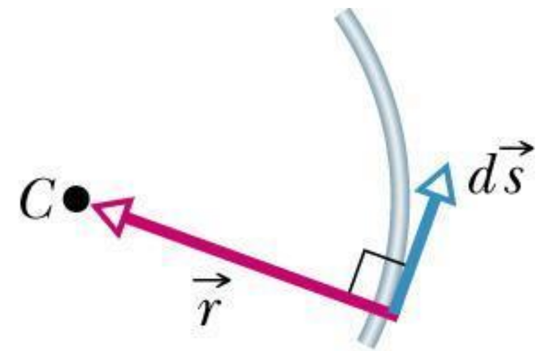
Finally:

$$B = \frac{\mu_0 I}{2\pi a}$$

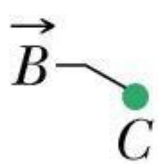
B-Field for a Circular Arc of Wire

The figures below show a circular arc of wire, with central angle ϕ centered at C and of radius R carrying a current i (directed upwards):





$$s = R\,\phi \quad \Rightarrow \quad d\,s = R\,d\,\phi$$



B-Field for a Circular Arc of Wire, cont

The right-hand rule shows that the magnetic field due to the wire is directed straight out of the plane of the page (dot) shown at point C. We note the different forms:

$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{i\,d\vec{s} \times \vec{r}}{r^3} \quad \Rightarrow \quad dB = \frac{\mu_o}{4\pi}\frac{i\,ds\sin\theta}{r^2}$$

Since $d\vec{s}$ is always $\perp$r to $\vec{r}$ and using $r = R$ gives $\Rightarrow$

Biot-savart Law Equation:

$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\,\frac{I\,d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

B-Field for a Circular Arc of Wire, cont

$$dB = \frac{\mu_o}{4\pi}\frac{i\,ds\sin 90^o}{R^2} = \frac{\mu_o\, i\, ds}{4\pi R^2} = \frac{\mu_o\, i\,(R\,d\phi)}{4\pi R^2}$$

Integration gives the full magnetic field for the arc of wire (for a central angle ϕ) as:

$$B = \int dB = \int_0^{\phi} \frac{\mu_o\, i\, R\, d\phi}{4\pi R^2} = \frac{\mu_o\, i}{4\pi R}\int_0^{\phi} d\phi = \frac{\mu_o\, i\,\phi}{4\pi R}$$

This applies <u>only</u> at the centre of curvature of the circular arc of current (and ϕ must be in radians).

B-Field for a Circular Arc of Wire, cont

For example, at the centre of a full circle of current with a radius R, the central angle must be $\phi = 2\pi$ rad.

Therefore, the magnetic field will be:

$$B = \frac{\mu_o i \phi}{4\pi R} = \frac{\mu_o i (2\pi)}{4\pi R} = \frac{\mu_o i}{2R}$$

The smaller the radius, the stronger will be the magnetic field at the centre.

B-Field for a Circular Arc of Wire

Problem

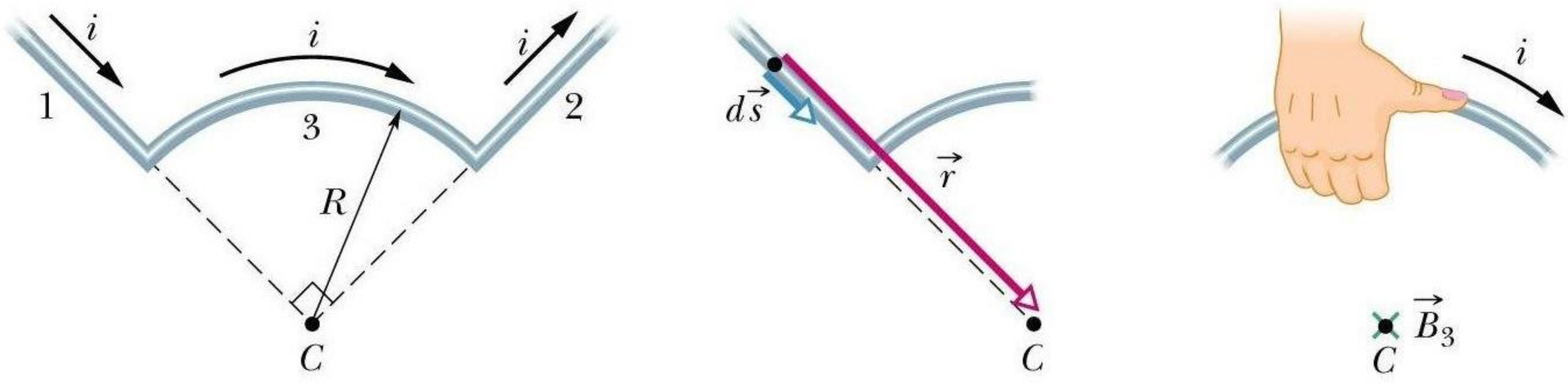
A wire consists of two straight sections (1 and 2) and a circular arc (3), and carries a constant current i.

The central angle for circular arc (3) is (π / 2) rad.

For a current-length element in section 1 (and also in section 2), the angle between and is zero.

B-Field for a Circular Arc of Wire

Problem: A wire consists of two straight sections (1 and 2) and a circular arc (3), and carries a constant current i. The central angle for circular arc (3) is (π / 2) rad. For a current-length element in section 1 (and also in section 2), the angle between $d\vec{s}$ and $\vec{r}$ is zero.



B-Field for a Circular Arc of Wire

Obtain a suitable equation for the magnitude of the magnetic field at C the centre point.

<u>Straight sections:</u> For any current-length element in section 1, the angle between $d\vec{s}$ and $\vec{r}$ is zero, so:

$$dB_1 = \frac{\mu_o}{4\pi}\frac{i\,ds\sin\theta}{r^2} \quad \Rightarrow \quad dB_1 = \frac{\mu_o}{4\pi}\frac{i\,ds\sin 0}{r^2} = 0 \quad \Rightarrow$$

$$B_1 = \int dB_1 = 0$$

B-Field for a Circular Arc of Wire

For straight section 2, the angle is now 180° so we have:

$$dB_2 = \frac{\mu_o}{4\pi}\frac{i\,ds\sin\theta}{r^2} \quad \Rightarrow$$

$$dB_2 = \frac{\mu_o}{4\pi}\frac{i\,ds\sin 180^o}{r^2} = 0 \quad \Rightarrow$$

$$B_2 = \int dB_2 = 0$$

So there is no magnetic field from sections 1 and 2.

B-Field for a Circular Arc of Wire

Circular arc: Direct application of the Biot-Savart law to any circular arc previously gave us:

$$B = \frac{\mu_o i \phi}{4\pi R}$$

So the magnitude of the magnetic field at C is:

$$B_3 = \frac{\mu_o i (\pi/2)}{4\pi R} = \frac{\mu_o i}{8R}$$

We can now calculate the net magnetic field as:

B-Field for a Circular Arc of Wire

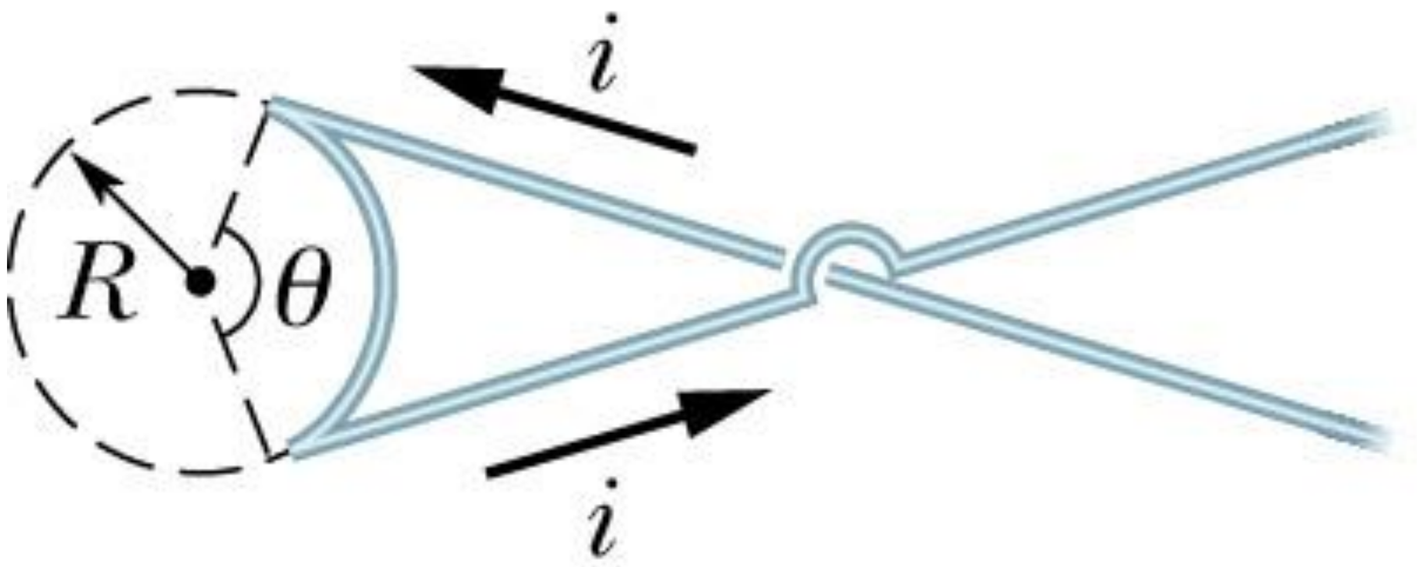
The magnitude of the net magnetic field will be:

$$B = B_1 + B_2 + B_3 = 0 + 0 + \frac{\mu_o i}{8R} = \frac{\mu_o i}{8R}$$

The right-hand rule gives the direction INTO the page.

B-Field for a Circular Arc of Wire

A wire carrying a current i is shaped as below, with two semi-infinite straight sections that are tangential to a circle, and a semicircular arc. Determine the angle θ for the magnitude of B to be zero at the centre of the circle.



B-Field for a Circular Arc of Wire

Solution:

The two semi-infinite wires give a magnetic field:

$$\underbrace{2B}_{\text{Semi-infinite wires}} = 2\left[\frac{\mu_o i}{4\pi R}\right]$$

This magnetic field points straight <u>out</u> of the page.

The current from the arc gives a magnetic field that points straight <u>into</u> the page, of magnitude: $\Rightarrow$

B-Field for a Circular Arc of Wire

Arcs:

$$\underbrace{B}_{\text{Circular arc}} = \frac{\mu_o i \theta}{4\pi R}$$

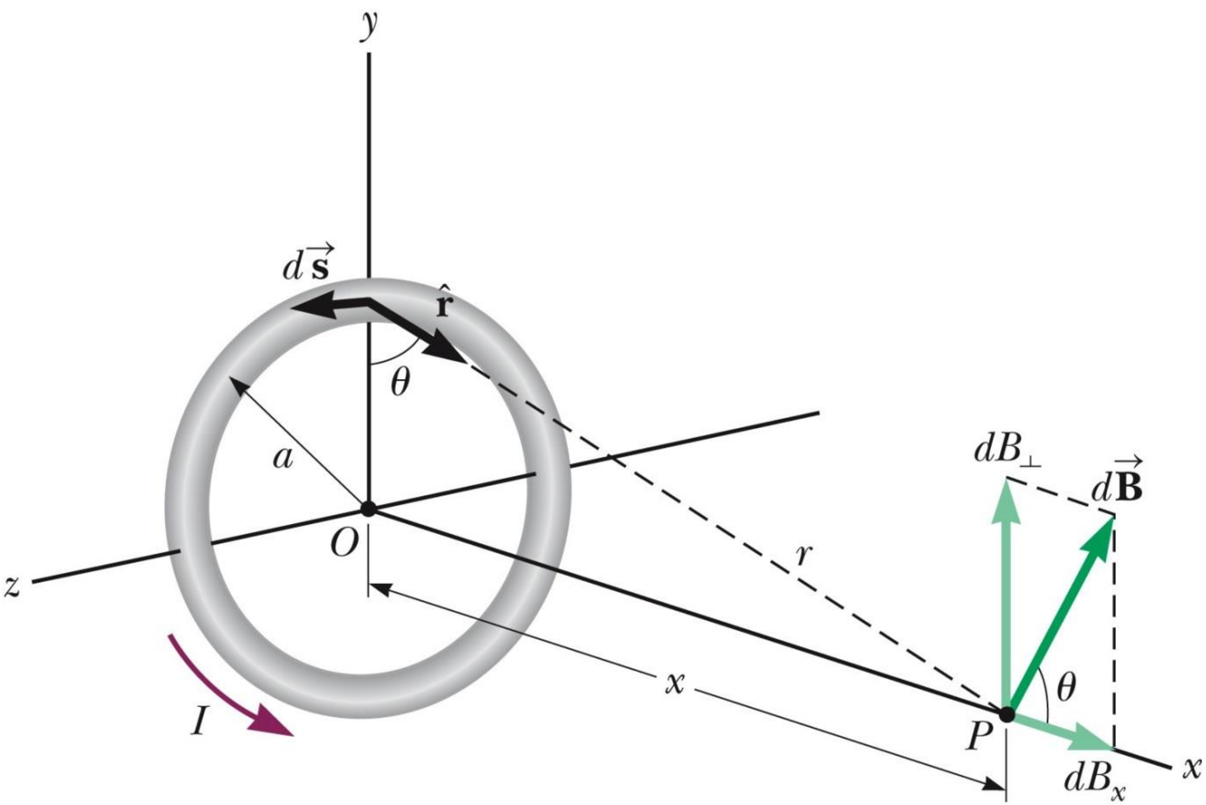
For zero net magnetic field:

$$\underbrace{2B}_{\text{Semi-infinite wires}} = \underbrace{B}_{\text{Circular arc}} \Rightarrow$$

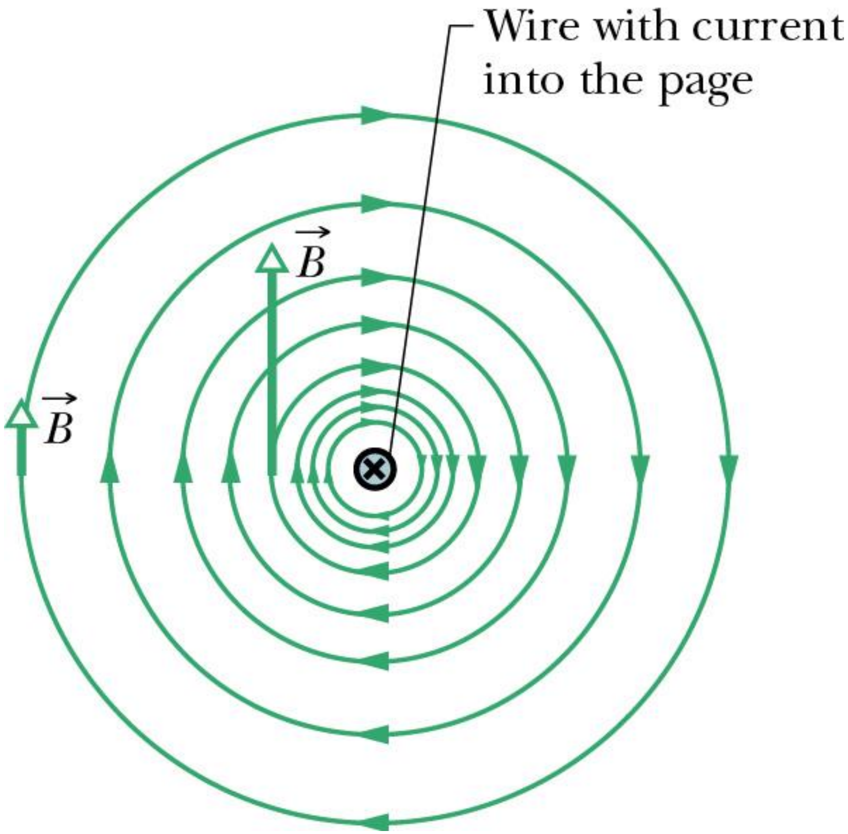
$$2\left[\frac{\mu_o i}{4\pi R}\right] = \frac{\mu_o i \theta}{4\pi R} \Rightarrow \theta = 2\, rad$$

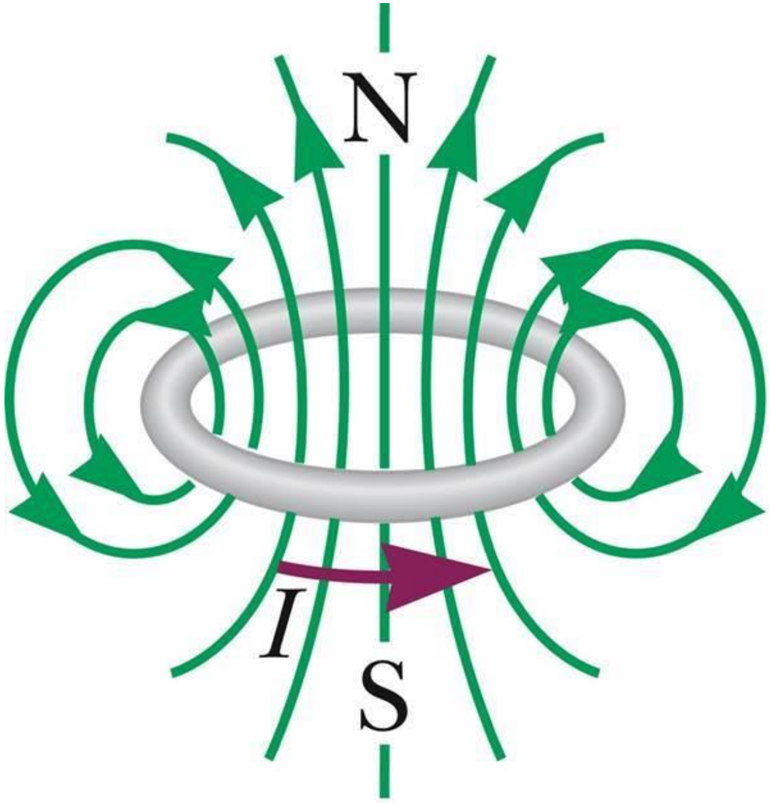
Magnetic Field on the Axis of a Circular Current Loop

Consider a circular wire loop of radius a located in the yz plane and carrying a steady current I as in Figure 30.5. Calculate the magnetic field at an axial point P a distance x from the center of the loop.



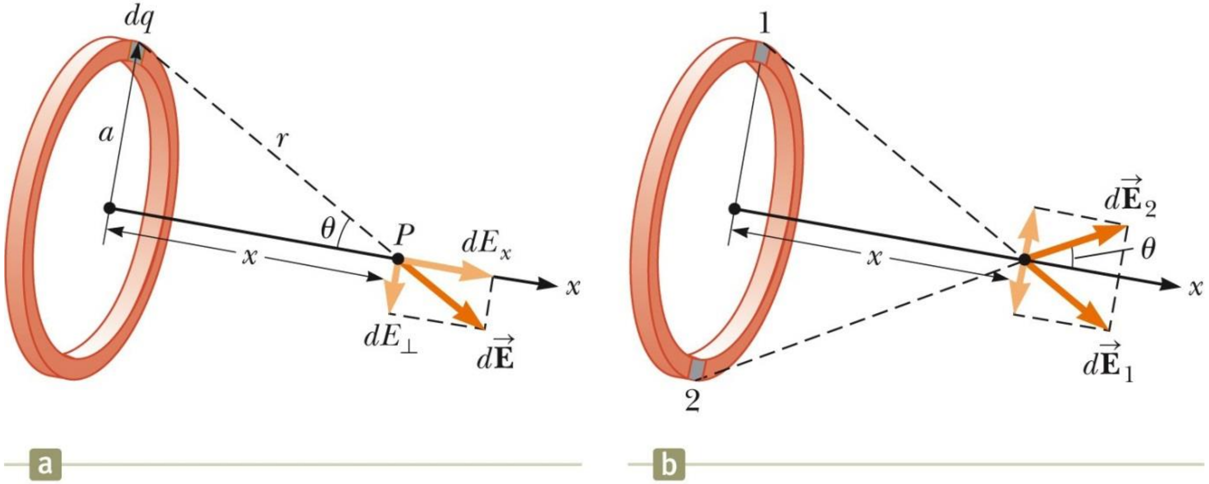
Magnetic Field – B-Field Lines





B-Field for a Circular Loop of Wire

It will help us to be reminded about the calculation of the electric field on the axis of a ring of charge:



Analysis gave:

$$E = \frac{k_e x}{(a^2 + x^2)^{3/2}} Q$$

B-Field for a Circular Loop of Wire

We continue with the self-explanatory analysis below:

$$dB = \frac{\mu_0 I}{4\pi} \frac{\left| d\vec{\mathbf{s}} \times \hat{\mathbf{r}} \right|}{r^2} = \frac{\mu_0 I}{4\pi} \frac{ds}{(a^2 + x^2)}$$

From the figure, we only need the axial *x*-component:

$$dB_x = \frac{\mu_0 I}{4\pi} \frac{ds}{(a^2 + x^2)} \cos\theta$$

Integrate round the loop:

$$B_x = \oint dB_x = \frac{\mu_0 I}{4\pi} \oint \frac{ds \cos\theta}{a^2 + x^2}$$

B-Field for a Circular Loop of Wire

The geometry gives us:

$$\cos\theta = \frac{a}{(a^2 + x^2)^{1/2}}$$

Substitute, and note that *x* and *a* are both constant:

$$B_x = \frac{\mu_0 I}{4\pi} \oint \frac{ds}{a^2 + x^2} \left[\frac{a}{(a^2 + x^2)^{1/2}} \right]$$

Simplify:

$$B_x = \frac{\mu_0 I}{4\pi} \frac{a}{(a^2 + x^2)^{3/2}} \oint ds$$

B-Field for a Circular Loop of Wire

The integral is now just the circumference of the current loop of radius a, which is simply equal to $(2\pi a)$ as shown:

$$B_x = \frac{\mu_0 I}{4\pi} \frac{a}{(a^2 + x^2)^{3/2}} (2\pi a)$$

Finally giving:

$$B_x = \boxed{\frac{\mu_0 I a^2}{2(a^2 + x^2)^{3/2}}}$$

Which is very similar to the E-field for a charged ring.

B-Field for a Circular Loop of Wire

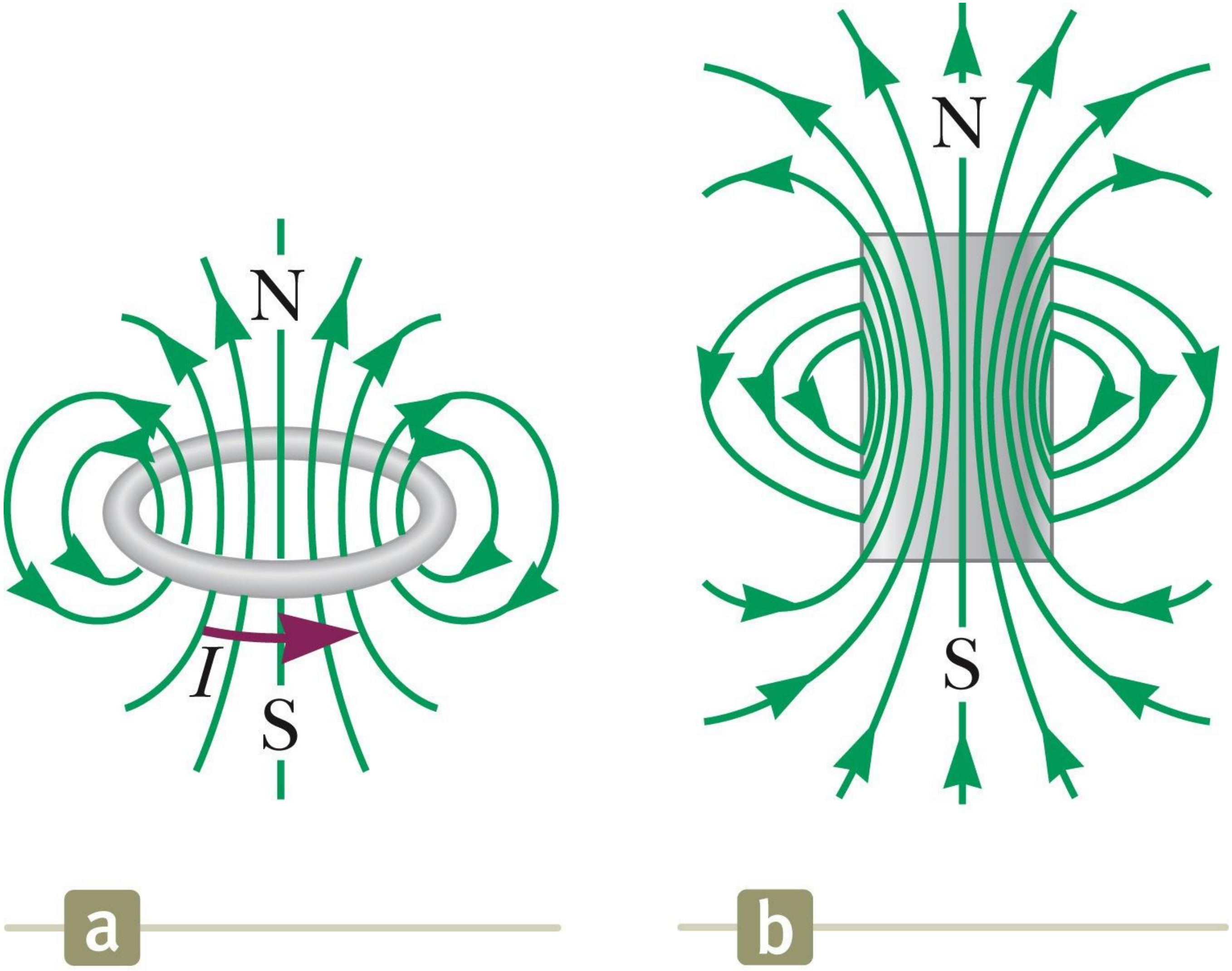
At the exact centre of the current-carrying circular loop, we set $x = 0$ in our equation to give:

$$B_x = \boxed{\frac{\mu_0 I a^2}{2(a^2 + x^2)^{3/2}}} \quad \Rightarrow \quad B = \frac{\mu_0 I}{2a} \quad (\text{at } x = 0)$$

Which gives exactly the same resulting equation, as our earlier analysis.

We can visualize the magnetic field lines from a circular loop and a permanent bar magnet as shown $\Rightarrow$

B-Field lines: Loop & Bar Magnet



B-Field for a Circular Loop of Wire

At distant points along the axis of the current-carrying circular loop, we set $x >> a$ in our equation to give:

$$B_x = \boxed{\frac{\mu_0 I a^2}{2(a^2 + x^2)^{3/2}}} \quad \Rightarrow \quad B \approx \frac{\mu_0 I a^2}{2x^3} \quad (\text{for } x >> a)$$

We now define the magnitude of the magnetic moment μ as the product of the current I and the loop area:

$$\mu = I(\pi a^2)$$

The final equation above can now be written as follows:

B-Field for a Circular Loop of Wire

The resulting equation becomes:

$$B \approx \frac{\mu_0}{2\pi} \frac{\mu}{x^3}$$

This is very similar to the form of the equation for the electric field due to an electric dipole:

$$E = k_e(p/y^3)$$

Where the electric dipole moment for two opposite charges separated by a distance 2a is: $p = 2aq$

Magnetic Force Between Two Parallel Conductors

We recall that the magnetic force can be written as:

$$\vec{F}_B = q\,\vec{v} \times \vec{B} \quad \Leftrightarrow \quad \vec{F}_B = I\,\vec{L} \times \vec{B}$$

Two current-carrying wires are positioned parallel to each other, so that each wire experiences the magnetic force from the other wire ($F_B = I\,l\,B$), that is due to the magnetic field from the current.

Each wire produces a magnetic field according to:

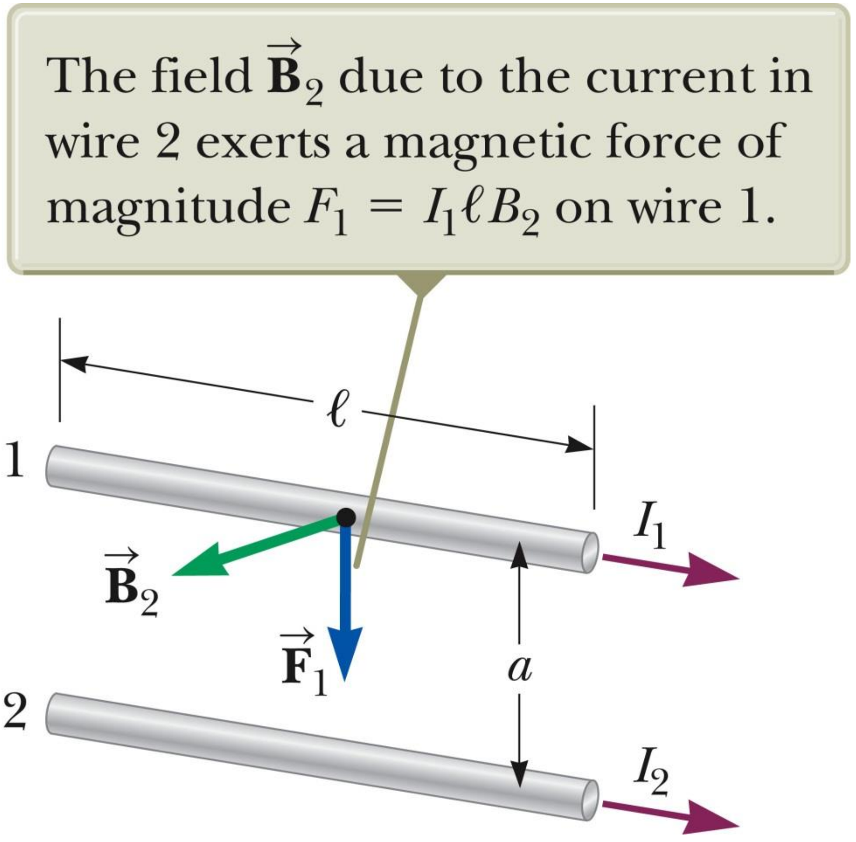
$$B = \frac{\mu_o I}{2\pi R}$$

Magnetic Force Between Two Parallel Conductors

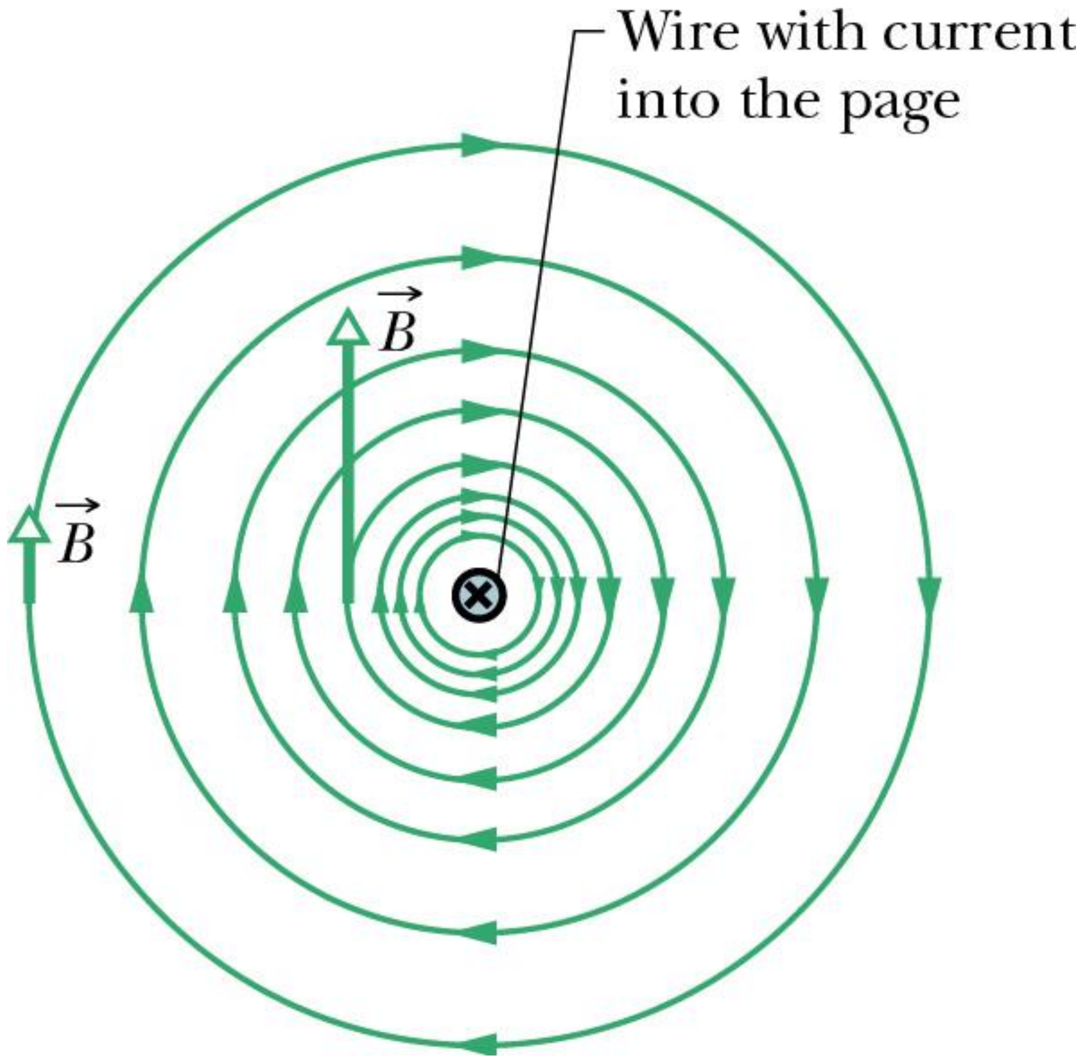
The two wires are assumed to be carrying two <u>different</u> currents I_1 and I_2 as shown:

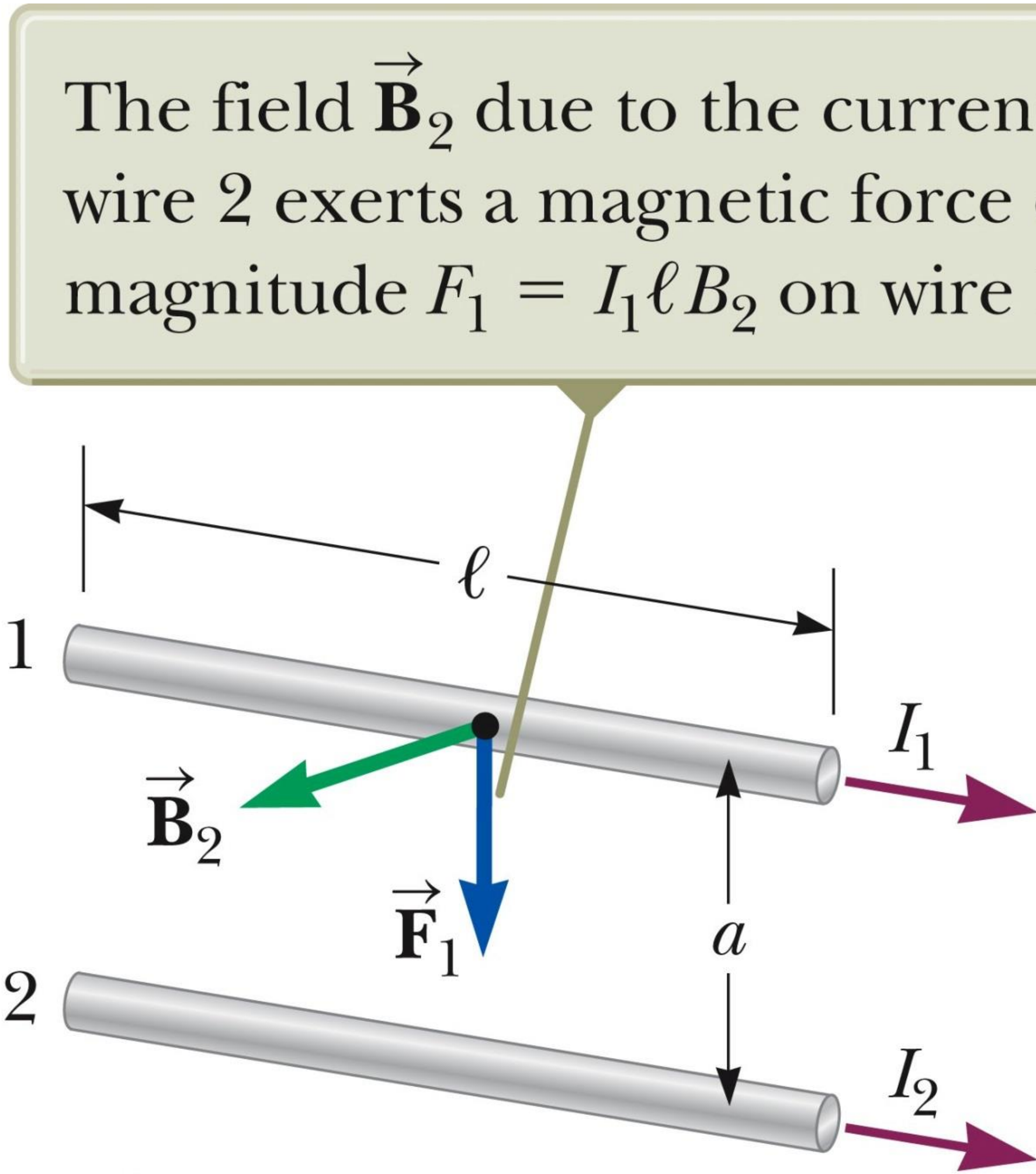
It is assumed for the present analysis that the currents are flowing <u>parallel</u> to each other.

Anti-parallel currents can be considered after the analysis has been performed.



Consider the magnetic field B_2 produced by the current I_2 in wire 2 (the lower wire), viewed from the left end:




The field $\vec{\mathbf{B}}_2$ due to the current in wire 2 exerts a magnetic force of magnitude $F_1 = I_1 \ell B_2$ on wire 1.
ℓ
1
2
I_1
I_2
$\vec{\mathbf{B}}_2$
$\vec{\mathbf{F}}_1$
a

Magnetic Force Between Two Parallel Conductors, cont.

Substituting the equation for the magnetic field (B_2) gives:

$$B_2 = \frac{\mu_0 I}{2\pi a}$$

So that:

$$F_1 = I_1 \ell B_2 = I_1 \ell \left(\frac{\mu_0 I_2}{2\pi a} \right) = \frac{\mu_0 I_1 I_2}{2\pi a} \ell$$

This is an <u>attractive</u> force towards wire 2, therefore, we note that parallel currents attract, so <u>anti-parallel</u> currents must <u>repel</u> each other.

Magnetic Force Between Two Parallel Conductors, final

The result can often be expressed as the magnetic force between the two wires, F_B. This can also be given as the force per unit length:

$$\frac{F_B}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi a}$$

The derivation assumes both wires are long compared with their separation distance:

Only one wire needs to be long.

The equations accurately describe the forces exerted on each other by a long wire and a straight, parallel wire of finite length ℓ.

Definition of the Ampere

The force between two parallel wires can be used to define the ampere:

When the magnitude of the force per unit length between two long, parallel wires that carry identical currents and are separated by 1 m is 2 x 10^{-7} N/m, the current in each wire is defined to be 1 A.

The value of 2 x 10^{-7} N/m comes about because the currents are: $I_1 = I_2 = 1$ A and the wires are separated by 1 m horizontally.

Definition of the Coulomb

A subsequent definition naturally follows:

The *SI* unit of charge, the coulomb, which is now defined in terms of the ampere.

When a conductor carries a steady current of 1 A, the quantity of charge that flows through a cross section of the conductor in 1 s is 1 C.

Andre-Marie Ampère

1775 – 1836

French physicist

Credited with the discovery of electromagnetism

The observed relationship between electric current and magnetic fields

Also worked in mathematics

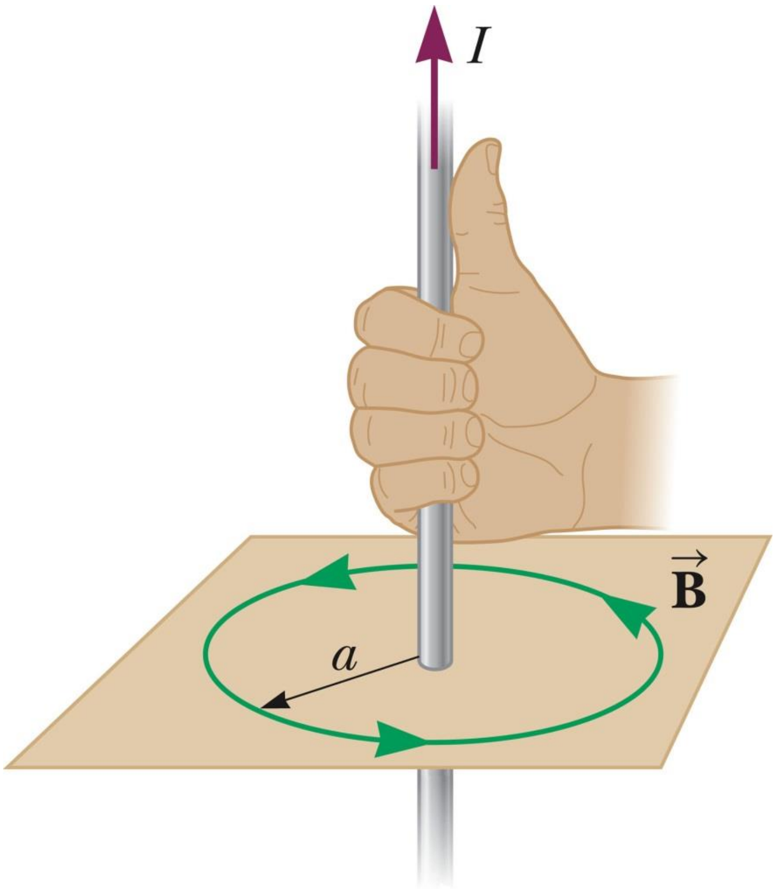
Magnetic Field for a Long, Straight Conductor: Direction

The magnetic field lines are circles concentric with the wire.

The field lines lie in planes perpendicular to the wire.

The magnitude of the field is constant on any circle of radius a.

The appropriate right-hand rule for determining the direction of the field is shown.

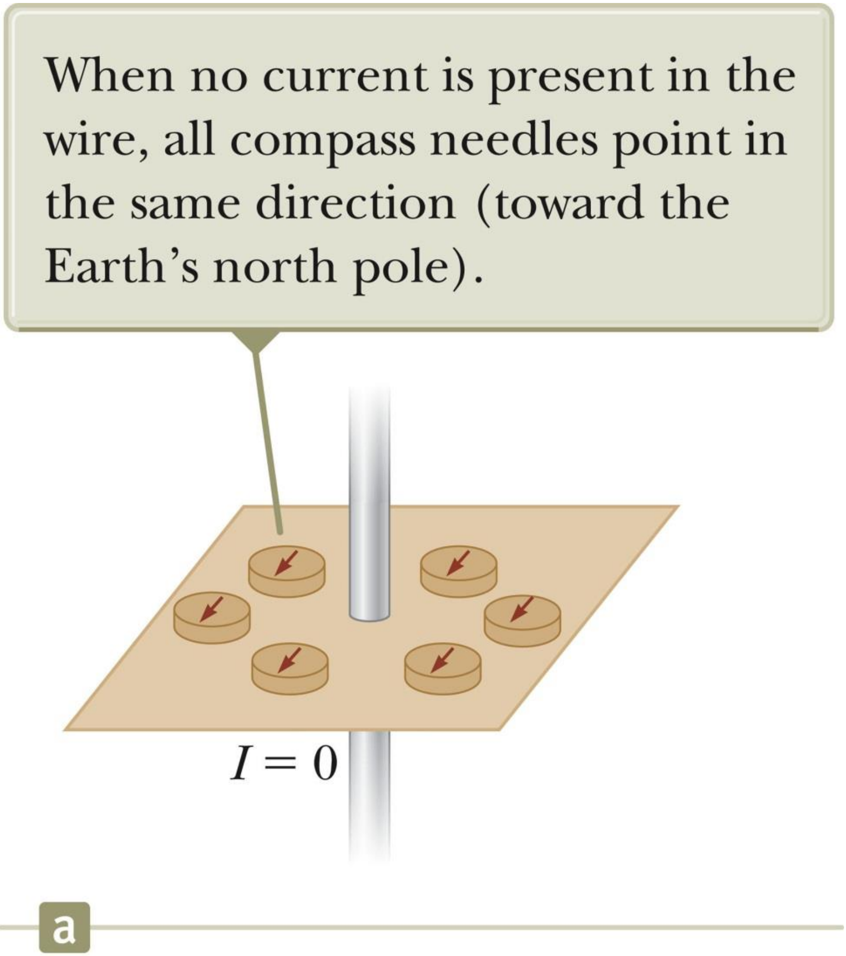


Magnetic Field of a Wire

A compass can be used to detect the magnetic field.

When there is no current in the wire, there is no field due to the current.

The compass needles all point toward the Earth's north pole, due to the Earth's magnetic field

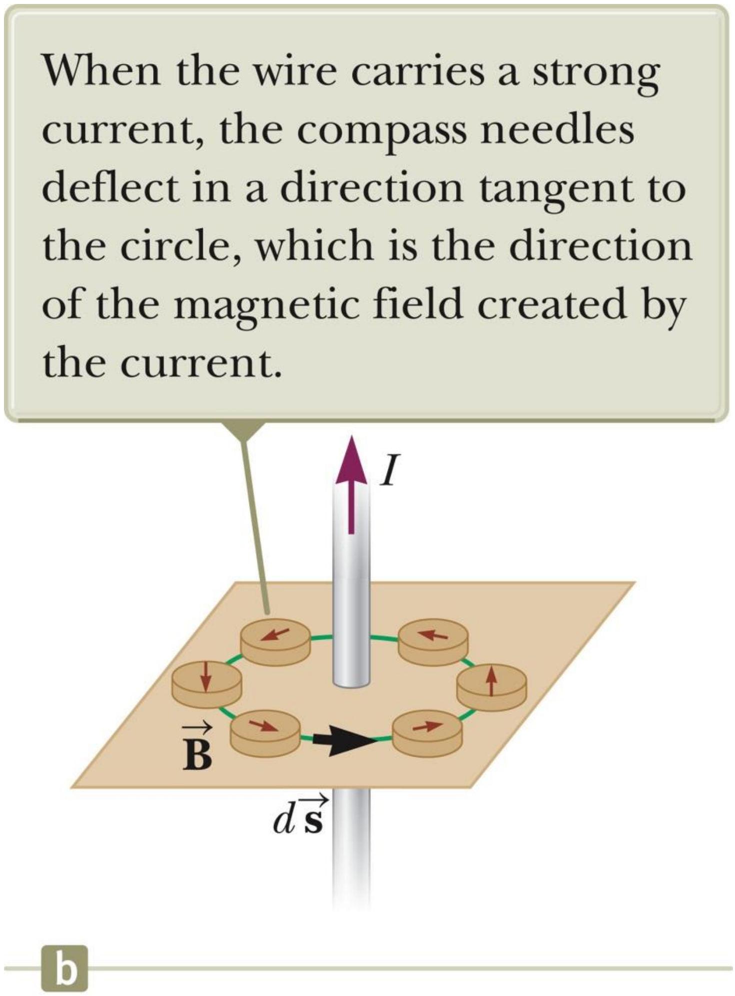


Magnetic Field of a Wire, cont.

Here the wire carries a strong current and the compass needles deflect in a direction tangent to the circle.

This shows the direction of the magnetic field produced by the wire.

If the current is reversed, the direction of the needles also reverse.



Ampere's Law

This is one of Maxwell's four equations, and is used to determine the net magnetic field from a series of currents. It is similar to Gauss' law:

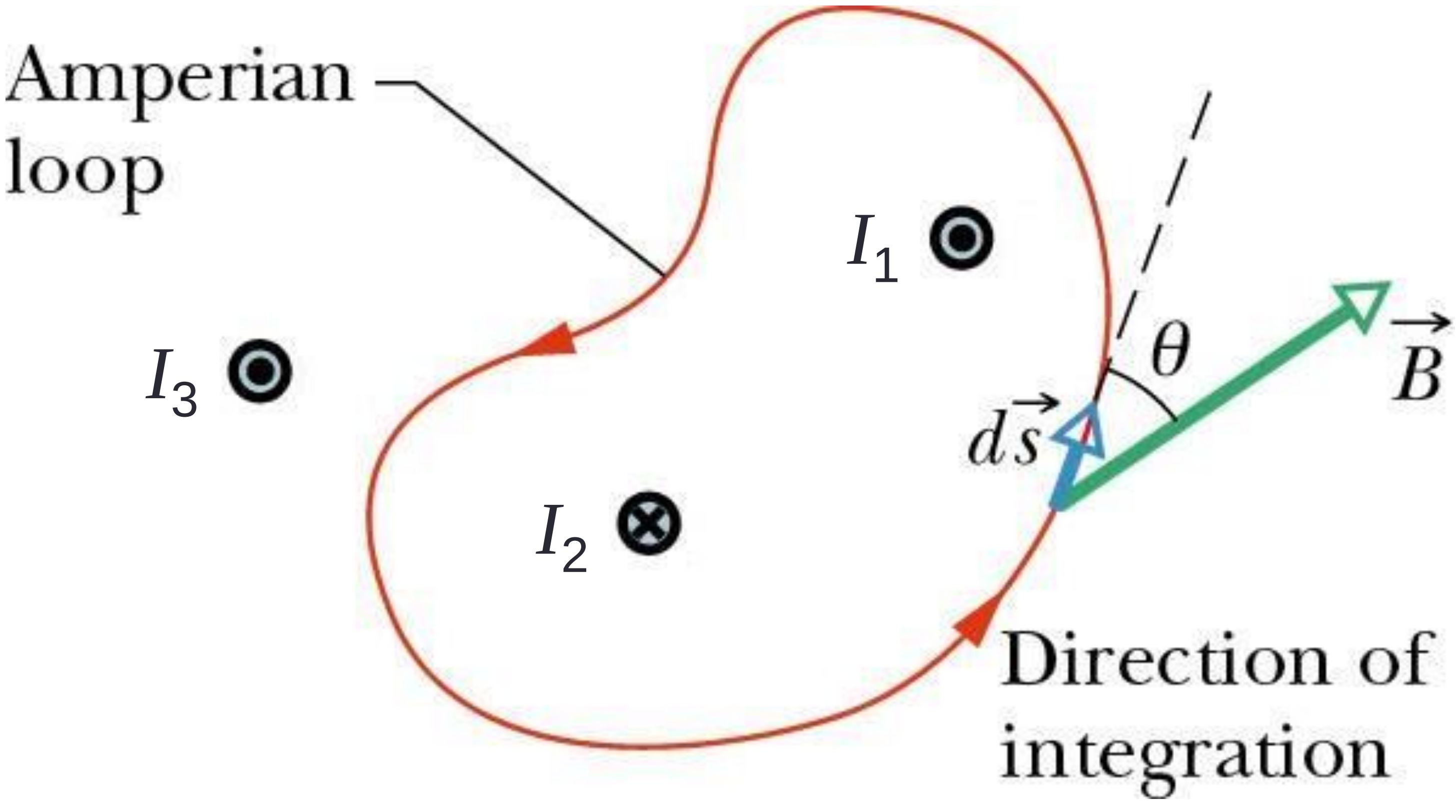
$$\text{Gauss' law (closed surface)}\ :\quad \varepsilon_o \oint \vec{E}\cdot d\vec{A} = Q_{enclosed}$$

$$\text{Ampere's law (closed loop)}\ :\quad \oint \vec{B}\cdot d\vec{s} = \mu_o I_{encircled}$$

The integration is performed around a closed loop, called an *Amperian loop*, shown in the figure.

Ampere's Law

The figure below shows an *Amperian loop*, with three current-carrying wires:



Ampere's Law

The counter-clockwise direction (marked with arrows) for the loop is chosen arbitrarily for the direction of integration.

In terms of the dot product, we can write:

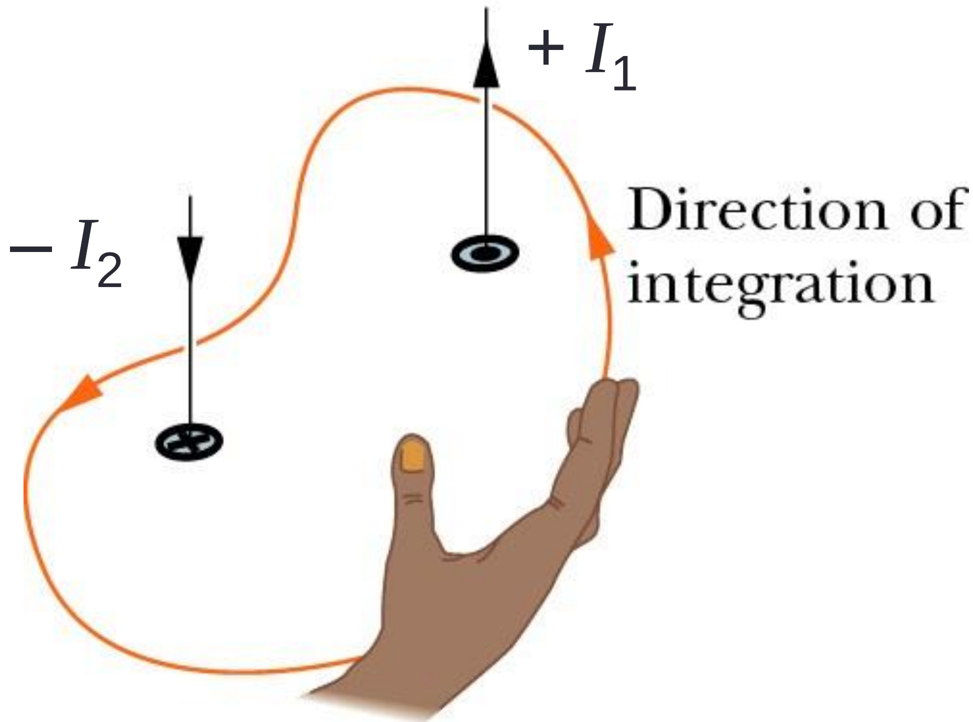
$$\oint \vec{B} \cdot d\vec{s} = \oint B \cos\theta \, d\,s = \mu_o \, I_{enc}$$

The sign of the directions of the currents is found by using another right-hand rule.

We assume that the magnetic field is in the direction of integration, as indicated in the next figure $\Rightarrow$

Ampere’s Law

To find the sign of the current: Curl your right hand round the *Amperian loop*, with your fingers pointing in the direction of integration:



Ampere's Law

A current through the loop in the general direction of your outstretched thumb is assigned a plus (+) sign, and a current generally in the opposite direction is assigned a negative (–) sign.

The third current is not encircled by the loop, so for this example:

$$I_{enc} = I_1 - I_2$$

Giving:

$$\oint B \cos\theta \, d s = \mu_o (I_1 - I_2)$$

Ampere’s Law

Ampere’s law describes the creation of magnetic fields by all continuous current configurations. Most useful for this course if the current configuration has a high degree of symmetry.

The encircled current in Ampere’s law, is the <u>total steady</u> current passing through any surface that is bounded by the closed path.

We note that the wire does <u>NOT</u> have to pass perpendicularly through the bounded surface and can be inclined.

Ampere's Law – Example 1

Magnetic Field outside a Long Current-Carrying Wire:

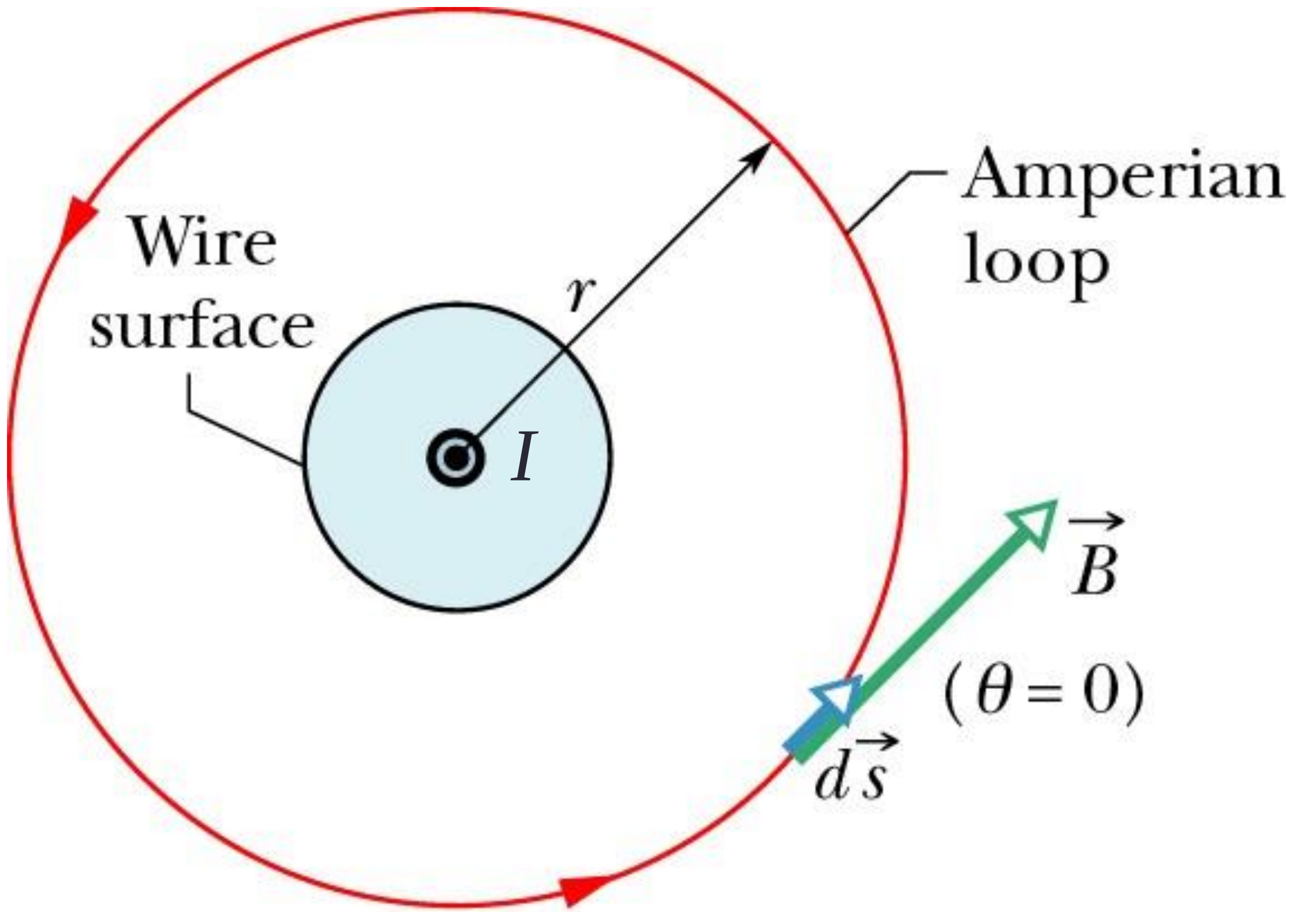
In the figure shown, a concentric Amperian loop surrounds a long current-carrying wire. The counter-clockwise direction of integration is selected arbitrarily for the application of Ampere's law.

$$\oint \vec{B} \cdot d\vec{s} = \mu_o I_{enc} \quad \Rightarrow$$

In scalar form:

$$\oint B \cos\theta \, ds = \mu_o I_{enc}$$

Ampere's Law – Example 1



Ampere's Law – Example 1

Note that the integration is taken counter-clockwise and at every point, the angle θ between $d\,\vec{s}$ and $\vec{B}$ is 0° because they are clearly parallel. So: cos θ = cos 0 = 1, and the integral is:

$$\oint \vec{B}\cdot d\vec{s} = \oint B\cos\theta\, d\,s = B\oint d\,s = B(2\pi r)$$

The right-hand rule gives a (+) sign for the direction of the current, so the right side of Ampere's law is positive:

$$\oint \vec{B}\cdot d\vec{s} = B(2\pi r) = +\mu_o I_{enc} = \mu_o I$$

$$B = \frac{\mu_o I}{2\pi r} \quad \text{(Outside a Long straight wire)}$$

Ampere's Law – Example 2

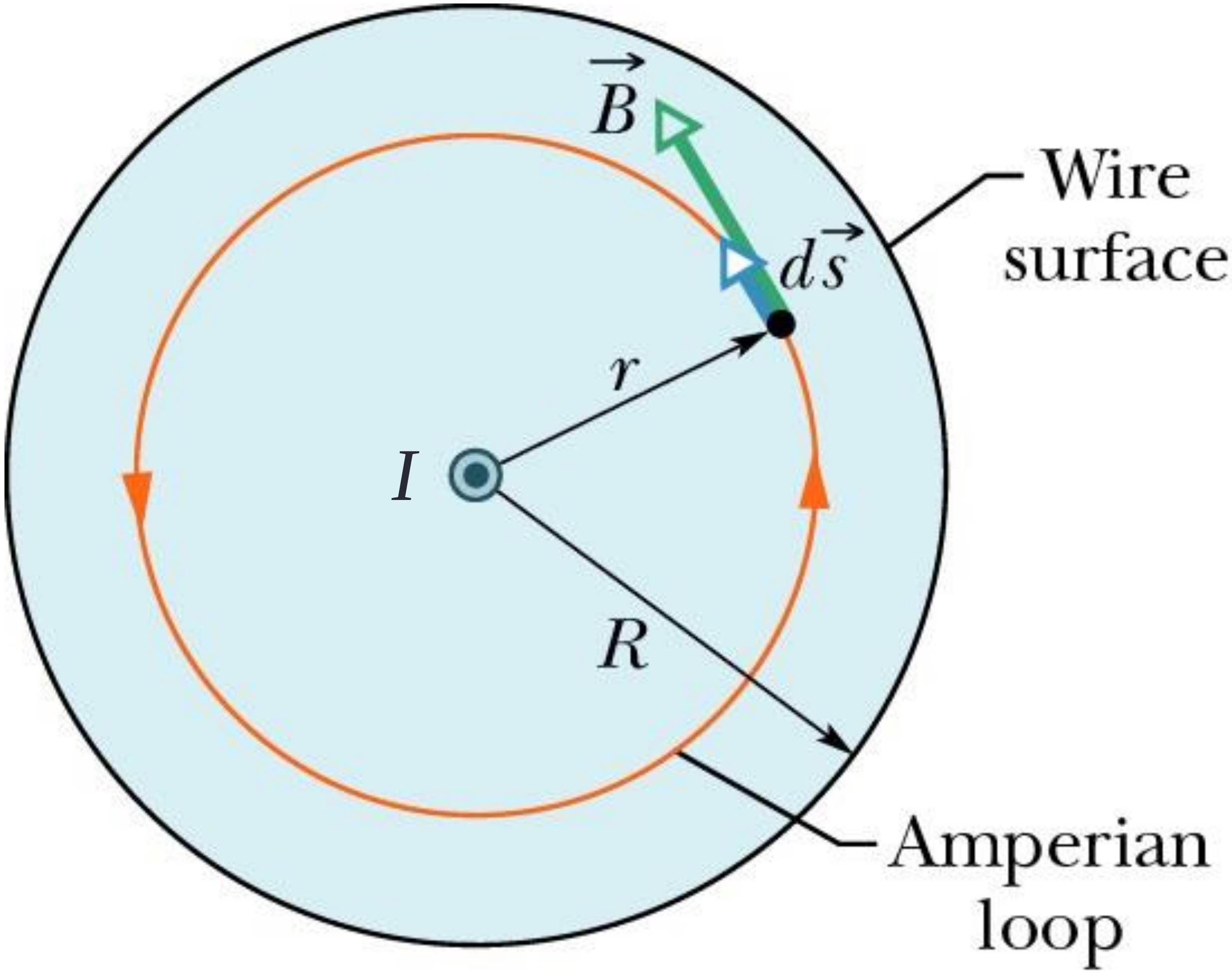
Magnetic Field <u>inside</u> a Long Current-Carrying Wire:

We apply Ampere's law to determine the field that a uniformly distributed current I (coming directly out of the page) produces, inside a long straight wire.

Now $r < R$ so that the uniformly distributed current encircled by the Amperian loop is proportional to the area encircled by the loop:

$$I = J\,A \quad \Rightarrow \quad I_{enc} = I\left(\frac{\pi r^2}{\pi R^2}\right)$$

Ampere’s Law – Example 2



Ampere's Law – Example 2

Note that the integration is taken counter-clockwise and at every point, the angle θ between $d\vec{s}$ and $\vec{B}$ is 0° because they are clearly parallel. So: cos θ = cos 0 = 1, and the right-hand rule gives a (+) sign:

$$\oint \vec{B}\cdot d\vec{s} = B(2\pi r) = +\mu_o I_{enc} = \mu_o I\left(\frac{\pi r^2}{\pi R^2}\right)$$

Finally:

$$B = \left(\frac{\mu_o I}{2\pi R^2}\right) r \quad \text{(Inside a long straight wire)}$$

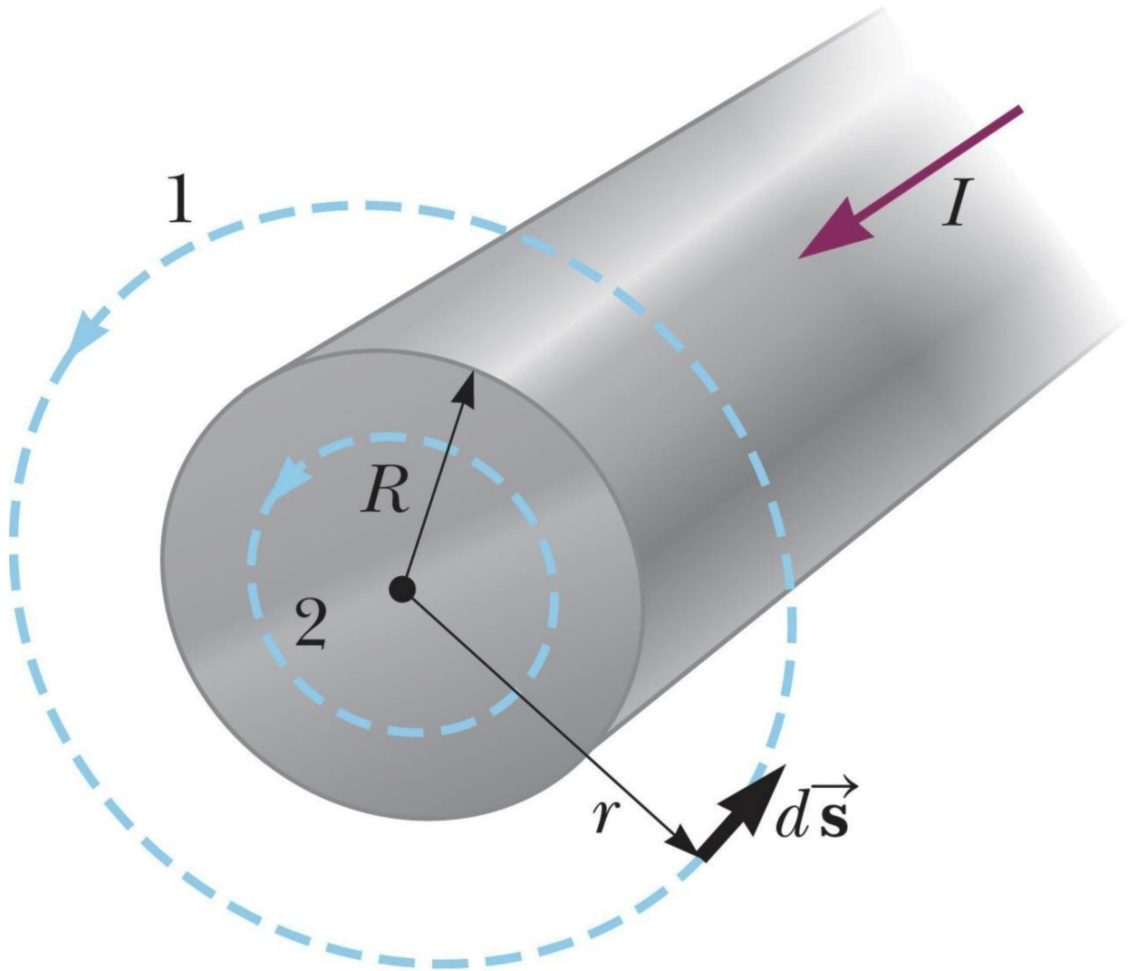
Ampere's Law – Example 2

Note that inside the wire, the magnitude B of the magnetic field is directly proportional to r (the radius of the Amperian loop).

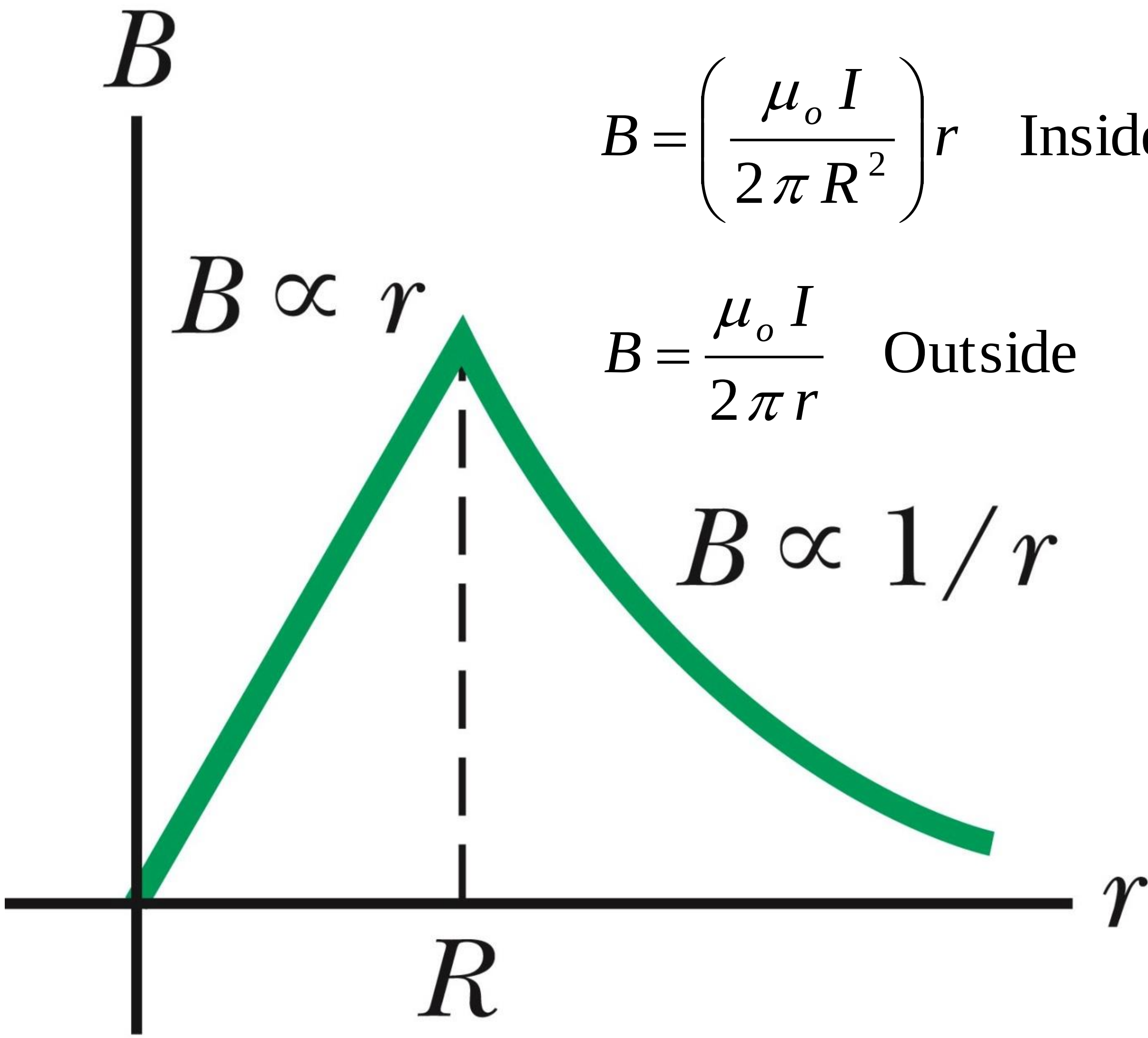
The magnitude of B is zero at the centre of the wire and is a maximum at the surface, where $r = R$. Both of the equations (inside and outside) give the same result when $r = R$.

An alternative schematic figure shows a long straight wire of radius R, that carries a steady (constant) current I along the wire and outwards to the left. $\Rightarrow$

Ampere's Law – Figure (a)



Ampere’s Law – Figure (b)



$$B = \left(\frac{\mu_o I}{2\pi R^2}\right) r \quad \text{Inside}$$

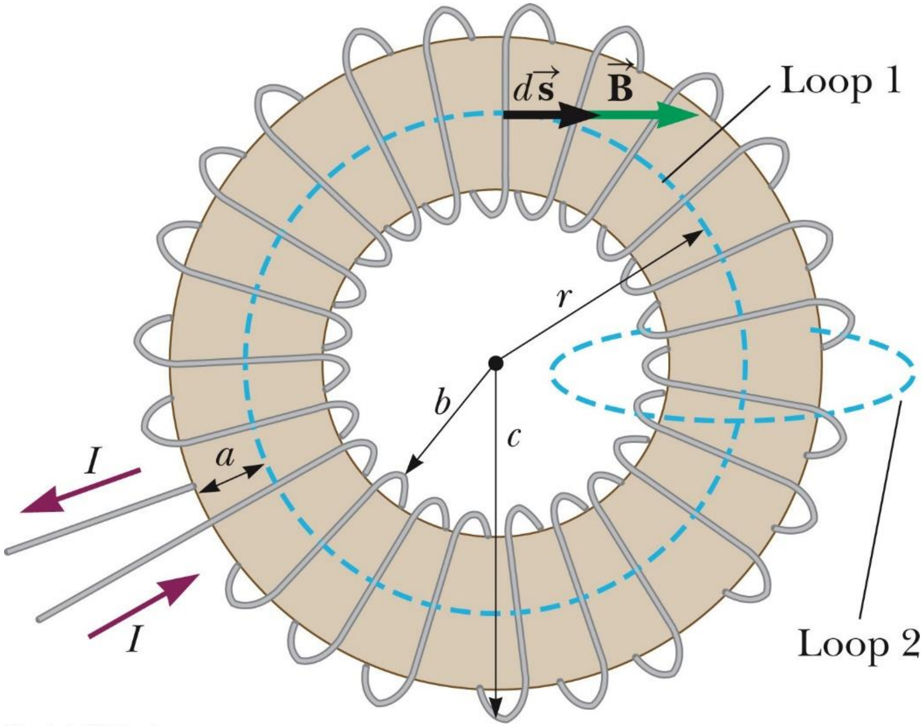
$$B = \frac{\mu_o I}{2\pi r} \quad \text{Outside}$$

Magnetic Field of a Toroid

- Find the field at a point at distance r from the center of the toroid.
- The toroid has N turns of wire.

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B(2\pi r) = \mu_o N I$$

$$B = \frac{\mu_o N I}{2\pi r}$$

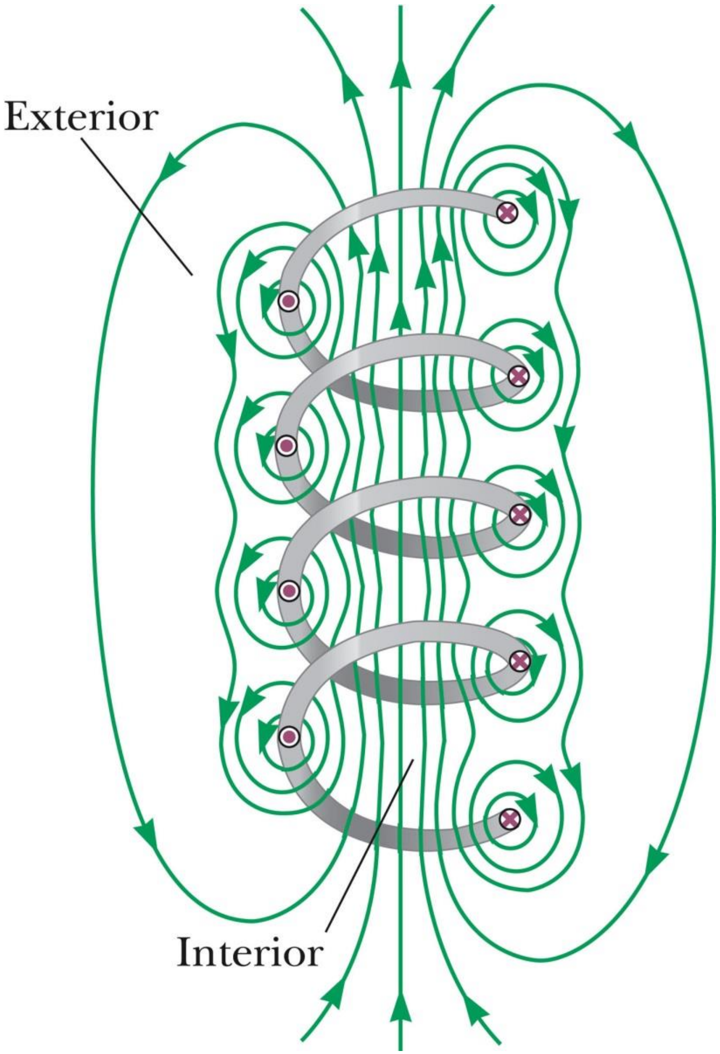


Magnetic Field of a Solenoid

A solenoid is a long wire wound in the form of a helix.

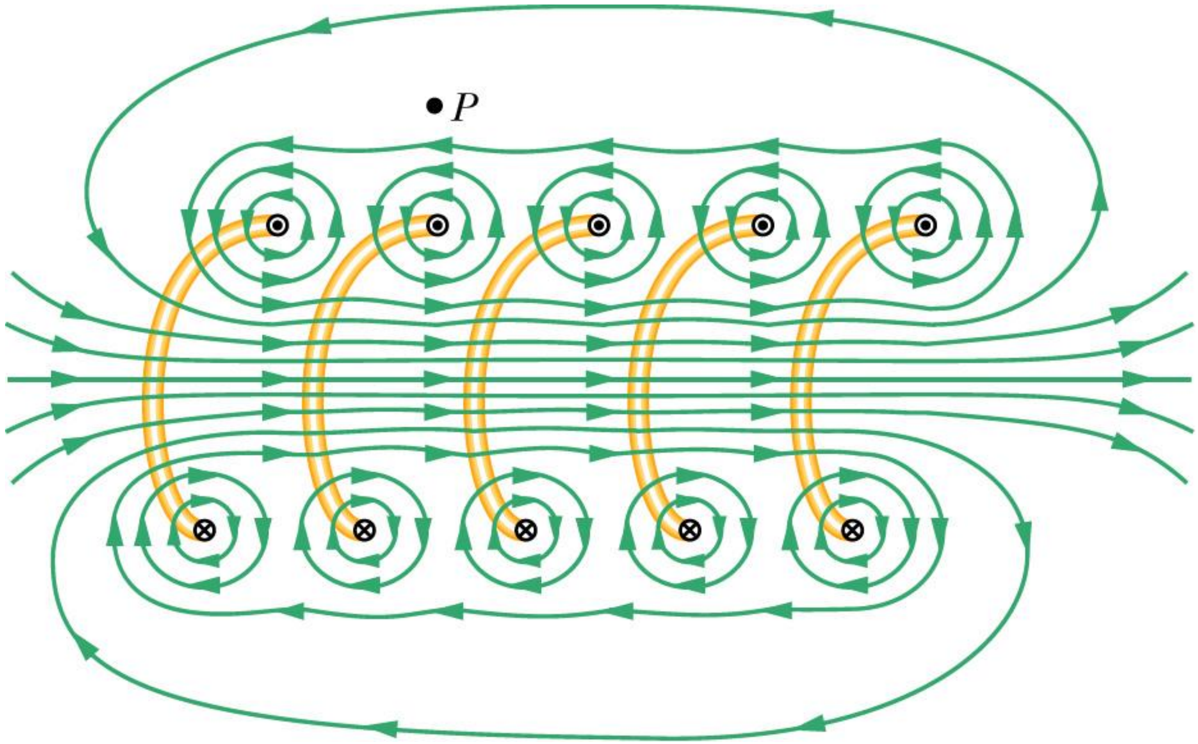
An approximately uniform magnetic field can be produced in the space surrounded by the turns of the wire.

Namely, the interior of the solenoid



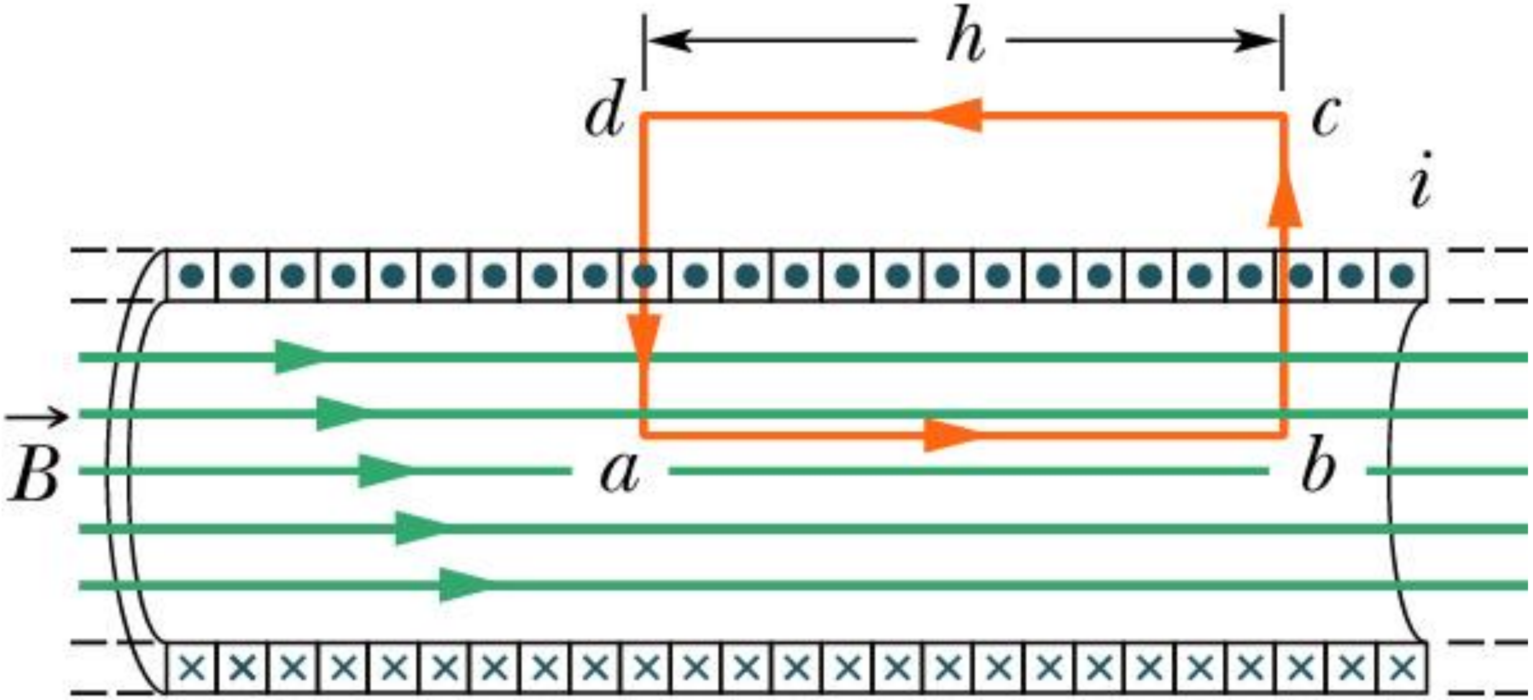
Magnetic Field of a Solenoid

A vertical cross-section through the central axis of a "stretched-out" solenoid (so the coils are not making contact with each other) is shown:



Magnetic Field of a Solenoid

We now apply Ampere's law to the ideal solenoid shown below, in which the path of the Amperian loop is shown to be ($a \rightarrow b \rightarrow c \rightarrow d \rightarrow a$):



Magnetic Field of a Solenoid

It is assumed that the magnitude of the magnetic field is uniform inside the solenoid and *h* is chosen (arbitrarily) as the length of segment ($a \rightarrow b$).

The integral can be written as the sum of:

$$\oint \vec{B}\cdot d\vec{s} = \int_a^b \vec{B}\cdot d\vec{s} + \int_b^c \vec{B}\cdot d\vec{s} + \int_c^d \vec{B}\cdot d\vec{s} + \int_d^a \vec{B}\cdot d\vec{s}$$

We now consider each term individually: $\Rightarrow$

Magnetic Field of a Solenoid

The first integral gives:

$$\int_a^b \vec{B} \cdot d\vec{s} = B\,h$$

For the second integral, for every element *ds* of the segment, $\vec{B}$ is either perpendicular to *ds* or zero:

$$\int_b^c \vec{B} \cdot d\vec{s} = 0$$

Magnetic Field of a Solenoid

The <u>third</u> integral lies outside the solenoid, therefore for every element ds of the segment, $B = 0$:

$$\int_c^d \vec{B} \cdot d\vec{s} = 0$$

In the <u>fourth</u> integral, for every element ds of the segment, $\vec{B}$ is either perpendicular to ds or zero:

$$\int_d^a \vec{B} \cdot d\vec{s} = 0$$

Magnetic Field of a Solenoid

So for the entire rectangular loop:

$$\oint \vec{B} \cdot d\vec{s} = B\,h$$

We note that the net current I_{enc} encircled by the rectangular Amperian loop is not the same as the current I in the solenoid coils (or windings). This is because the current-carrying windings pass more than once through the loop.

If there are n turns per unit length of the solenoid, then the loop encloses (nh) turns for a length h.

Magnetic Field of a Solenoid

Therefore, we can write:

$$I_{enc} = I\,(n\,h)$$

So Ampere's law gives:

$$\left.\begin{aligned} &\oint \vec{B}\cdot d\vec{s} = \mu_o I_{enc} \\ &\oint \vec{B}\cdot d\vec{s} = B\,h \\ &\mu_o I_{enc} = \mu_o I\,(n\,h) \end{aligned}\right\} \Rightarrow \quad B = \mu_o I\,n \quad \text{(Ideal solenoid)}$$

Magnetic Field of a Solenoid, Description

The field lines in the interior are:

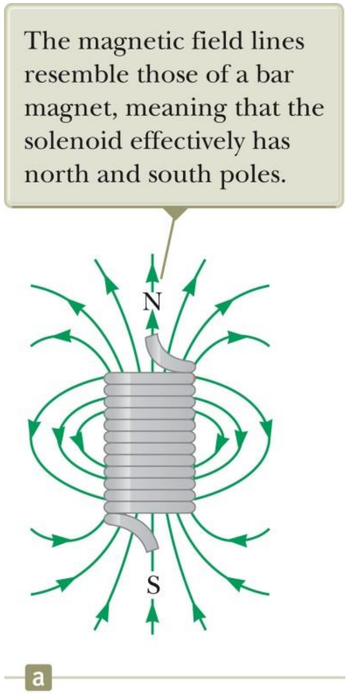
Nearly parallel to each other

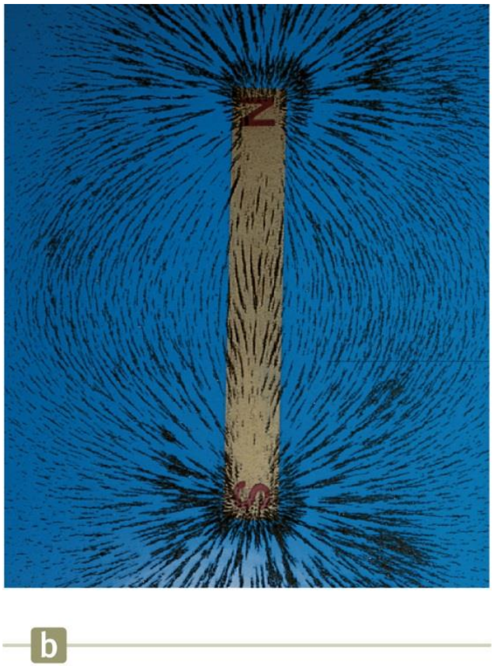
Uniformly distributed

Close together

This indicates the field is strong and almost uniform.

Magnetic Field of a Tightly Wound Solenoid





The field distribution is similar to that of a bar magnet.

As the length of the solenoid increases,

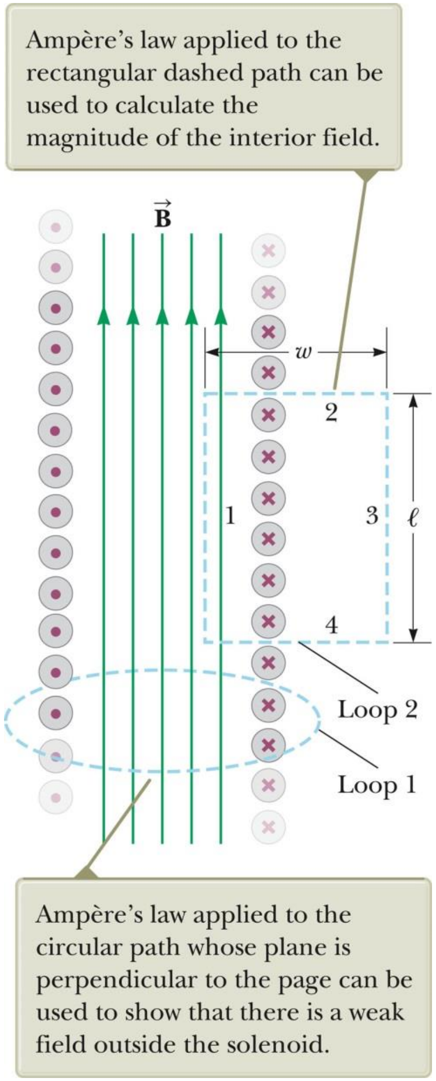
- The interior field becomes more uniform.
- The exterior field becomes weaker.

Ideal Solenoid – Characteristics

An ideal solenoid is approached when:

The turns are closely spaced.

The length is much greater than the radius of the turns.

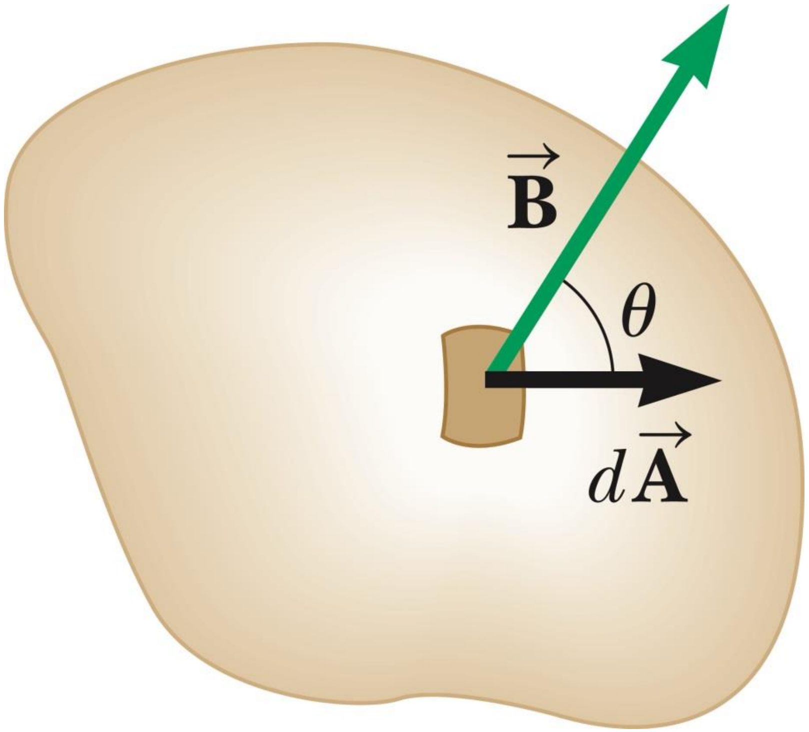


Magnetic Flux

The magnetic flux associated with any magnetic field is defined in a way similar to electric flux.

Consider an area element dA on an arbitrarily shaped surface. The magnetic field in this element is $\vec{\mathbf{B}}$

$d\vec{\mathbf{A}}$ is a vector perpendicular to the surface and has a magnitude equal to the area *dA.*



Magnetic Flux, cont.

The magnetic flux Φ_B is:

$$\Phi_B \equiv \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

The unit of magnetic flux is T·m^2 = Wb

Where: Wb is a weber

Magnetic Flux Through a Plane, 1

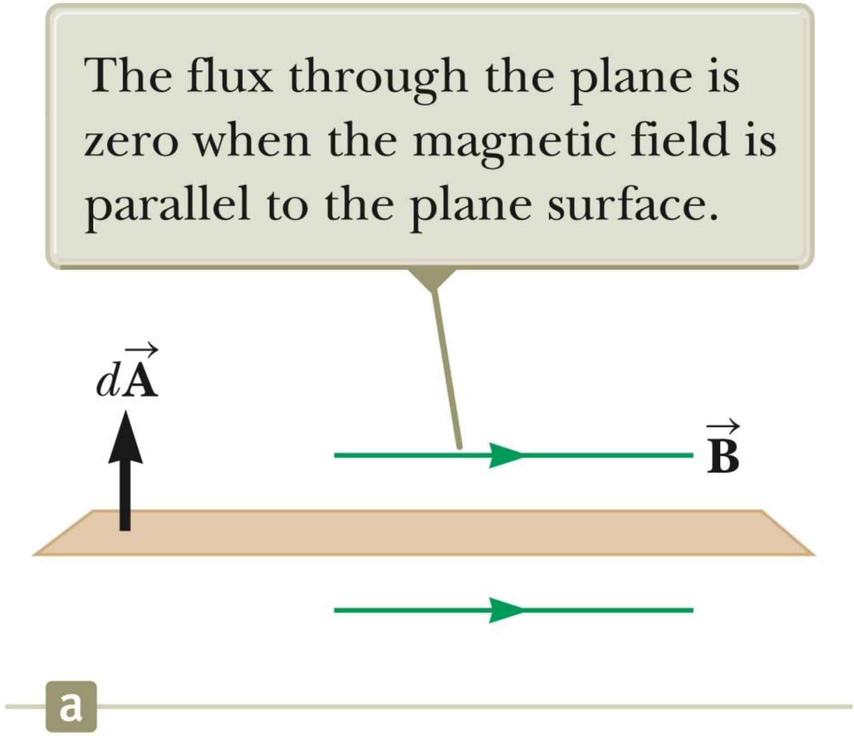
A special case is when a plane of area A makes an angle θ with $d\vec{\mathbf{A}}$.

The magnetic flux is:

$$\Phi_B = BA \cos \theta$$

In this case, the field is parallel to the plane and

$$\Phi_B = 0$$



Magnetic Flux Through A Plane, 2

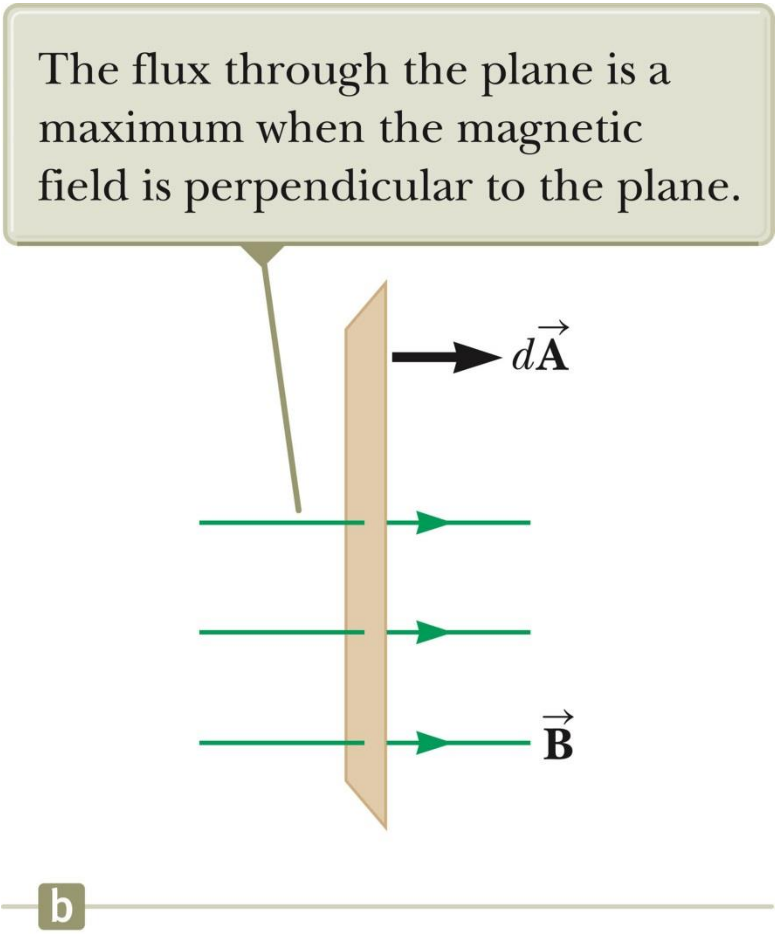
The magnetic flux is:

$$\Phi_B = BA \cos \theta$$

In this case, the field is now perpendicular to the plane so:

$$\Phi = BA$$

This is the maximum value of the flux.



Gauss' Law in Magnetism

Magnetic fields do not begin or end at any point.

Magnetic field lines are continuous and form closed loops.

The number of lines entering a surface equals the number of lines leaving the surface.

Gauss' law in magnetism says the magnetic flux through any closed surface is always zero:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

Gauss’ Law in Magnetism

This indicates that isolated magnetic poles (monopoles) have never been detected.

Perhaps they do not exist?

Certain theories do suggest the possible existence of magnetic monopoles.