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The Scalable Software Infrastructure Project http://www.ssisc.org/

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0 Changes from Version 1.1 2 1 Introduction 3 2 Installation 4 2.1 System Requirements 4 2.2 Extracting Archive 4 2.3 Installing on UNIX and Compatible Systems 4 2.3.1 Configuring Source Tree 4 2.3.2 Compiling 5 2.3.3 Installing on Windows Systems 8 2.4 Installing on Windows Systems 8 2.5 Testing 9 2.5.1 test1 9 2.5.2 test2 9 2.5.3 test3 9 2.5.4 test4 9 2.5.5 test5 10 2.5.5 etest1 10 2.5.5 etest3 10 2.5.5 etest4 10 2.5.1 spmvtest 11 2.5.10 ctest5 11 2.5.11 spmvtest1 11 2.5.12 spmvtest3 12 2.5.13 spmvtest3 12 2.5.15 spmvtest4 12 2.5.15 spmvtest5 12 2.6 Restrictions 12 3 Basic Operations	\mathbf{C}	ont	tents	
2 Installation	0	Cha	anges from Version 1.1	2
2.1 System Requirements 4 2.2 Extracting Archive 4 2.3 Installing on UNIX and Compatible Systems 4 2.3.1 Configuring Source Tree 4 2.3.2 Compiling 8 2.3 Installing on Windows Systems 8 2.5 Testing 9 2.5.1 testing 9 2.5.1 test 9 2.5.1 test 9 2.5.2 test2 9 2.5.1 test 9 2.5.2 test2 9 2.5.5 test4 9 2.5.6 etest1 10 2.5.6 etest2 10 2.5.9 etest3 10 2.5.9 etest4 10 2.5.10 etest5 11 2.5.11 spmvtest1 11 2.5.12 spmvtest3 12 2.5.13 spmvtest4 12 2.5.15 spmvtest4 12 2.5.15 spmvtest5 12 2.	1	Intr	roduction	3
2.1 System Requirements 4 2.2 Extracting Archive 4 2.3 Installing on UNIX and Compatible Systems 4 2.3.1 Configuring Source Tree 4 2.3.2 Compiling 8 2.3 Installing on Windows Systems 8 2.5 Testing 9 2.5.1 testing 9 2.5.1 test 9 2.5.1 test 9 2.5.2 test2 9 2.5.1 test 9 2.5.2 test2 9 2.5.5 test4 9 2.5.6 etest1 10 2.5.6 etest2 10 2.5.9 etest3 10 2.5.9 etest4 10 2.5.10 etest5 11 2.5.11 spmvtest1 11 2.5.12 spmvtest3 12 2.5.13 spmvtest4 12 2.5.15 spmvtest4 12 2.5.15 spmvtest5 12 2.	2	Inst	tallation	4
2.2 Extracting Archive	_			
2.3 Installing on UNIX and Compatible Systems 4 2.3.1 Configuring Source Tree 4 2.3.2 Compiling 5 2.3.3 Installing on Windows Systems 8 2.4 Installing on Windows Systems 8 2.5 Testing 9 2.5.1 test1 9 2.5.2 test2 9 2.5.3 test3 9 2.5.4 test4 9 2.5.5 test5 10 2.5.6 etest1 10 2.5.7 ctst2 10 2.5.8 test3 10 2.5.9 etest4 10 2.5.10 etest5 11 2.5.11 spmvtest1 11 2.5.12 spmvtest2 11 2.5.13 spmvtest3 12 2.5.14 spmvtest4 12 2.5.15 spmvtest5 12 2.6 Restrictions 12 3 Basic Operating 14 3.2 Operating Wectors 15 3.3 Operating Matrices 15 3.4 Solving Linear Equations 26 3.5 Solving Eigenvalue Problems 26 3.6 Writing Programs 29 3.7 Compiling and Linkin		2.2	v -	4
2.3.2 Compiling		2.3	Installing on UNIX and Compatible Systems	4
2.3.3 Installing 8 2.4 Installing on Windows Systems 8 2.5 Testing 9 2.5.1 test1 9 2.5.2 test2 9 2.5.3 test3 9 2.5.4 test4 9 2.5.5 test5 10 2.5.7 etest2 10 2.5.8 etest3 10 2.5.9 etest4 10 2.5.10 etest5 11 2.5.11 spmvtest1 11 2.5.12 spmvtest2 11 2.5.13 spmvtest3 12 2.5.15 spmvtest4 12 2.5.15 spmvtest5 12 2.6 Restrictions 12 3 Basic Operations 14 3.1 Initializing and Finalizing 14 3.2 Operating Vectors 15 3.3 Operating Matrices 15 3.4 Solving Linear Equations 23 3.5 Solving Eigenvalue Problems 26 3.6 Writing Programs 29 3.7 Compiling and Linking 31 3.8 Running 31 4 Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operation			2.3.1 Configuring Source Tree	4
2.4 Installing on Windows Systems 8 2.5 Testing 9 2.5.1 test1 9 2.5.2 test2 9 2.5.3 test3 9 2.5.4 test4 9 2.5.5 test5 10 2.5.6 etest1 10 2.5.7 etest2 10 2.5.8 etest3 10 2.5.9 ctest4 10 2.5.10 etest5 11 2.5.11 spmvtest1 11 2.5.12 spmvtest2 11 2.5.13 spmvtest3 12 2.5.14 spmvtest4 12 2.5.15 spmvtest5 12 2.6 Restrictions 12 3 Basic Operations 14 3.1 Initializing and Finalizing 14 3.2 Operating Vectors 15 3.3 Operating Matrices 17 3.4 Solving Linear Equations 23 3.5 Solving Eigenvalue Problems 26 3.6 Writing Programs 29 3.7 Compiling and Linking 31 3.8 Running 34 4 Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations </td <td></td> <td></td> <td>1 0</td> <td></td>			1 0	
2.5. Testing 9 2.5.1 test1 9 2.5.2 test2 9 2.5.3 test3 9 2.5.4 test4 9 2.5.5 test5 10 2.5.6 etest1 10 2.5.7 etest2 10 2.5.9 etest3 10 2.5.9 etest4 10 2.5.10 etest5 11 2.5.11 spmvtest1 11 2.5.12 spmvtest2 11 2.5.13 spmvtest3 12 2.5.14 spmvtest4 12 2.5.15 spmvtest5 12 2.6 Restrictions 12 3 Basic Operations 14 3.1 Initializing and Finalizing 14 3.2 Operating Vectors 15 3.3 Operating Matrices 17 3.4 Solving Linear Equations 23 3.5 Solving Eigenvalue Problems 26 3.6 Writing Programs 29 3.7 Compiling and Linking 31 3.8 Running 31 4 Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations 34 4.1 Using Quadruple Precision Ope			9	
2.5.1 test1 9 2.5.2 test2 9 2.5.3 test3 9 2.5.4 test4 9 2.5.5 test5 10 2.5.6 etest1 10 2.5.7 etest2 10 2.5.8 etest3 10 2.5.9 etest4 10 2.5.10 etest5 11 2.5.11 spmvtest1 11 2.5.12 spmvtest2 11 2.5.13 spmvtest3 12 2.5.14 spmvtest4 12 2.5.15 spmvtest5 12 2.6 Restrictions 12 3 Basic Operations 14 3.1 Initializing and Finalizing 14 3.2 Operating Vectors 15 3.3 Operating Matrices 17 3.4 Solving Linear Equations 23 3.5 Solving Eigenvalue Problems 26 3.6 Writing Programs 29 3.7 Compiling and Linking 31 3.8 Running 34 4 Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations 34 5.1 Compressed Row Storage (CRS) 36 5.1.1 Creati			· · · · · · · · · · · · · · · · · · ·	
2.5.2 test2 9 2.5.3 test3 9 2.5.4 test4 9 2.5.5 test5 10 2.5.6 ctest1 10 2.5.7 etest2 10 2.5.8 etest3 10 2.5.9 etest4 10 2.5.10 etest5 11 2.5.11 spmvtest1 11 2.5.12 spmvtest2 11 2.5.13 spmvtest3 12 2.5.14 spmvtest4 12 2.5.15 spmvtest5 12 2.6 Restrictions 12 3 Basic Operations 14 3.1 Initializing and Finalizing 14 3.2 Operating Vectors 15 3.3 Operating Vectors 15 3.4 Solving Linear Equations 23 3.5 Solving Eigenvalue Problems 26 3.6 Writing Programs 29 3.7 Compiling and Linking 31 3.8 Running 33 4 Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations 34 5.1.2 Creating Matrices (for Serial and Multithreaded Environments) 36 5.1.3 Associating Arrays 3		2.5		
2.5.3 test3 9 2.5.4 test4 9 2.5.5 test5 10 2.5.6 etest1 10 2.5.7 etest2 10 2.5.8 etest3 10 2.5.9 etest4 10 2.5.10 etest5 11 2.5.11 spmvtest1 11 2.5.12 spmvtest2 11 2.5.13 spmvtest3 12 2.5.14 spmvtest4 12 2.5.15 spmvtest5 12 2.6 Restrictions 12 3.1 Initializing and Finalizing 14 3.2 Operating Vectors 15 3.3 Operating Matrices 17 3.4 Solving Linear Equations 23 3.5 Solving Eigenvalue Problems 26 3.6 Writing Programs 29 3.7 Compiling and Linking 31 3.8 Running 33 4 Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations 34 5.1 Compressed Row Storage (CRS) 36 5.1.1 Creating Matrices (for Serial and Multithreaded Environments) 36 5.1.2 Creating Matrices (for Serial and Multithreaded Environments) 36 <td></td> <td></td> <td></td> <td></td>				
2.5.4 test4 9 2.5.5 test5 10 2.5.6 etest1 10 2.5.7 etest2 10 2.5.8 etest3 10 2.5.9 etest4 10 2.5.10 etest5 11 2.5.11 spmvtest1 11 2.5.12 spmvtest2 11 2.5.13 spmvtest3 12 2.5.14 spmvtest4 12 2.5.15 spmvtest5 12 2.6 Restrictions 12 3 Basic Operations 14 3.1 Initializing and Finalizing 14 3.2 Operating Vectors 15 3.3 Operating Vectors 15 3.4 Solving Linear Equations 23 3.5 Solving Eigenvalue Problems 26 3.6 Writing Programs 29 3.7 Compiling and Linking 31 3.8 Running 34 4 Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations 34 5.1 Compressed Row Storage (CRS) 36 5.1.2 Creating Matrices (for Serial and Multithreaded Environments) 36 5.1.3 Associating Arrays 37 5.2 C				
2.5.5 test5 10 2.5.6 test1 10 2.5.7 test2 10 2.5.8 test3 10 2.5.9 test4 10 2.5.10 test5 11 2.5.11 spmvtest1 11 2.5.12 spmvtest2 11 2.5.13 spmvtest3 12 2.5.14 spmvtest4 12 2.5.15 spmvtest5 12 2.6 Restrictions 12 3 Basic Operations 14 3.1 Initializing and Finalizing 14 3.2 Operating Vectors 15 3.3 Operating Matrices 17 3.4 Solving Linear Equations 23 3.5 Solving Eigenvalue Problems 26 3.6 Writing Programs 29 3.7 Compiling and Linking 31 3.8 Running 33 4 Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations 36 5.1.1 Creating Matrices (for Serial and Multithreaded Environments) 36 5.1.2 Creating Matrices (for Serial and Multithreaded Environments) 36 5.2.2 Creating Matrices (for Multiprocessing Environment) 38 5.2.				
2.5.6 etest1 10 2.5.7 etest2 10 2.5.8 etest3 10 2.5.9 etest4 10 2.5.10 etest5 11 2.5.11 spmvtest1 11 2.5.12 spmvtest2 11 2.5.13 spmvtest3 12 2.5.14 spmvtest4 12 2.5.15 spmvtest5 12 2.6 Restrictions 12 3 Initializing and Finalizing 14 3.1 Initializing and Finalizing 14 3.1 Initializing and Finalizing 14 3.2 Operating Vectors 15 3.3 Operating Processor 15 3.3 Operating Programs 23 3.5 Solving Eigenvalue Problems 26 3.6 Writing Programs 29 3.7 Compiling and Linking 31 3.8 Running 31 4 Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations 36 <td></td> <td></td> <td></td> <td></td>				
2.5.7 etest2 10 2.5.8 etest3 10 2.5.9 etest4 10 2.5.10 etest5 11 2.5.11 spmvtest1 11 2.5.12 spmvtest2 11 2.5.13 spmvtest3 12 2.5.14 spmvtest4 12 2.5.15 spmvtest5 12 2.6 Restrictions 12 3 Basic Operations 14 3.1 Initializing and Finalizing 14 3.2 Operating Vectors 15 3.3 Operating Matrices 17 3.4 Solving Linear Equations 23 3.5 Solving Eigenvalue Problems 26 3.6 Writing Programs 29 3.7 Compiling and Linking 31 3.8 Running 31 4 Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations 34 5.1.1 Creating Matrices (for Serial and Multithreaded Environments) 36 5.1.2 Creating Matrices (for Serial and Multithreaded Environments) 36 5.2.1 Creating Matrices (for Serial and Multithreaded Environments) 38 5.2.2 Creating Matrices (for Serial and Multithreaded Environments) 38 <t< td=""><td></td><td></td><td></td><td></td></t<>				
2.5.8 etest3 10 2.5.9 etest4 10 2.5.10 etest5 11 2.5.11 spmvtest1 11 2.5.12 spmvtest2 11 2.5.13 spmvtest3 12 2.5.14 spmvtest4 12 2.5.15 spmvtest5 12 2.6 Restrictions 12 3 Basic Operations 14 3.1 Initializing and Finalizing 14 3.2 Operating Vectors 15 3.3 Operating Matrices 17 3.4 Solving Linear Equations 23 3.5 Solving Eigenvalue Problems 26 3.6 Writing Programs 29 3.7 Compiling and Linking 31 3.8 Running 31 4 Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations 34 5.1 Compressed Row Storage (CRS) 36 5.1.1 Creating Matrices (for Serial and Multithreaded Environments) 36 5.1.2 Creating Matrices (for Serial and Multithreaded Environments) 37 5.2 Compressed Column Storage (CCS) 38 5.2.1 Creating Matrices (for Serial and Multithreaded Environments) 38 5.2.2 Crea				
2.5.9 etest4 10 2.5.10 etest5 11 2.5.11 spmvtest1 11 2.5.12 spmvtest2 11 2.5.13 spmvtest3 12 2.5.14 spmvtest4 12 2.5.15 spmvtest5 12 2.6 Restrictions 12 3 Basic Operations 14 3.1 Initializing and Finalizing 14 3.2 Operating Vectors 15 3.3 Operating Matrices 17 3.4 Solving Linear Equations 23 3.5 Solving Eigenvalue Problems 26 3.6 Writing Programs 29 3.7 Compiling and Linking 31 3.8 Running 31 4 Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations 34 5.1.1 Creating Matrices (for Serial and Multithreaded Environments) 36 5.1.2 Creating Matrices (for Serial and Multithreaded Environments) 37 5.2 Compressed Column Storage (CCS) 38 5.2.1 Creating Matrices (for Serial and Multithreaded Environments) 38 5.2.1 Creating Matrices (for Serial and Multithreaded Environments)				-
2.5.10 etest5 11 2.5.11 spmvtest1 11 2.5.12 spmvtest2 11 2.5.13 spmvtest3 12 2.5.14 spmvtest4 12 2.5.15 spmvtest5 12 2.6 Restrictions 12 3 Basic Operations 14 3.1 Initializing and Finalizing 14 3.2 Operating Vectors 15 3.3 Operating Matrices 17 3.4 Solving Linear Equations 23 3.5 Solving Eigenvalue Problems 26 3.6 Writing Programs 29 3.7 Compiling and Linking 31 3.8 Running 31 4 Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations 34 5.1.1 Creating Matrices (for Serial and Multithreaded Environments) 36 5.1.2 Creating Matrices (for Serial and Multithreaded Environments) 37 5.2 Compressed Column Storage (CCS) 38 5.2.1 Creating Matrices (for Serial and Multithreaded Environments) 38 5.2.1 Creating Matrices (for Serial and Multithreaded Environments) 38 5.2.2 Creati				-
2.5.11 spmvtest1 11 2.5.12 spmvtest2 11 2.5.13 spmvtest3 12 2.5.14 spmvtest4 12 2.5.15 spmvtest5 12 2.6 Restrictions 12 3 Basic Operations 12 3.1 Initializing and Finalizing 14 3.2 Operating Vectors 15 3.3 Operating Matrices 17 3.4 Solving Linear Equations 23 3.5 Solving Eigenvalue Problems 26 3.6 Writing Programs 29 3.7 Compiling and Linking 31 3.8 Running 31 4 Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations 34 5.1.1 Creating Matrices (for Serial and Multithreaded Environments) 36 5.1.2 Creating Matrices (for Serial and Multithreaded Environments) 37 5.2 Compressed Column Storage (CCS) 38 5.2.1 Creating Matrices (for Serial and Multithreaded Environments) 38 5.2.2 Creating Matrices (for Serial and Multithreaded Environments) 38 5.2.2 Creating Matrices (for Multiprocessing Environment) 39 <td></td> <td></td> <td></td> <td>-</td>				-
2.5.12 spmvtest2 11 2.5.13 spmvtest3 12 2.5.14 spmvtest4 12 2.5.15 spmvtest5 12 2.6 Restrictions 12 3.1 Initializing and Finalizing 14 3.1 Operating Vectors 15 3.3 Operating Matrices 15 3.4 Solving Linear Equations 23 3.5 Solving Eigenvalue Problems 26 3.6 Writing Programs 29 3.7 Compiling and Linking 31 3.8 Running 31 4 Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations 34 5.1 Compressed Row Storage (CRS) 36 5.1.1 Creating Matrices (for Serial and Multithreaded Environments) 36 5.1.2 Creating Matrices (for Multiprocessing Environment) 37 5.2 Compressed Column Storage (CCS) 38 5.2.1 Creating Matrices (for Serial and Multithreaded Environments) 38 5.2.2 Creating Matrices (for Multiprocessing Environment) 39				
2.5.13 spmvtest3 12 2.5.14 spmvtest4 12 2.5.15 spmvtest5 12 2.6 Restrictions 12 3 Basic Operations 14 3.1 Initializing and Finalizing 14 3.2 Operating Vectors 15 3.3 Operating Matrices 17 3.4 Solving Linear Equations 23 3.5 Solving Eigenvalue Problems 26 3.6 Writing Programs 26 3.7 Compiling and Linking 31 3.8 Running 33 4 Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations 34 5.1 Compressed Row Storage (CRS) 36 5.1.1 Creating Matrices (for Serial and Multithreaded Environments) 36 5.1.2 Creating Matrices (for Multiprocessing Environment) 37 5.1.2 Compressed Column Storage (CCS) 38 5.2.1 Creating Matrices (for Serial and Multithreaded Environments) 38 5.2.2 Creating Matrices (for Serial and Multithreaded Environments) 38 5.2.2 Creating Matrices (for Multiprocessing Environment) 39 36 39				
2.5.14 spmvtest4 12 2.5.15 spmvtest5 12 2.6 Restrictions 12 3 Basic Operations 14 3.1 Initializing and Finalizing 14 3.2 Operating Vectors 15 3.3 Operating Matrices 17 3.4 Solving Linear Equations 23 3.5 Solving Eigenvalue Problems 26 3.6 Writing Programs 26 3.7 Compiling and Linking 31 3.8 Running 31 3.8 Running 33 4 Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations 34 5 Matrix Storage Formats 36 5.1.1 Creating Matrices (for Serial and Multithreaded Environments) 36 5.1.2 Creating Matrices (for Multiprocessing Environment) 37 5.1.2 Compressed Column Storage (CCS) 38 5.2.1 Creating Matrices (for Serial and Multithreaded Environments) 38 5.2.2 Creating Matrices (for Serial and Multithreaded Environments) 38 5.2.2 Creating Matrices (for Multiprocessing Environment) 39			•	12
2.5.15 spmvtest5 12 2.6 Restrictions 12 3 Basic Operations 14 3.1 Initializing and Finalizing 14 3.2 Operating Vectors 15 3.3 Operating Matrices 17 3.4 Solving Linear Equations 23 3.5 Solving Eigenvalue Problems 26 3.6 Writing Programs 29 3.7 Compiling and Linking 31 3.8 Running 33 4 Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations 36 5.1.1 Creating Matrices (for Serial and Multithreaded Environments) 36 5.1.2 Creating Matrices (for Multiprocessing Environment) 37 5.1.3 Associating Arrays 37 5.2 Compressed Column Storage (CCS) 38 5.2.1 Creating Matrices (for Serial and Multithreaded Environments) 38 5.2.2 Creating Matrices (for Serial and Multithreaded Environments) 38 5.2.2 Creating Matrices (for Multiprocessing Environment) 39			•	12
3 Basic Operations 14 3.1 Initializing and Finalizing 14 3.2 Operating Vectors 15 3.3 Operating Matrices 17 3.4 Solving Linear Equations 23 3.5 Solving Eigenvalue Problems 26 3.6 Writing Programs 29 3.7 Compiling and Linking 31 3.8 Running 31 4 Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations 34 5.1 Compressed Row Storage (CRS) 36 5.1.1 Creating Matrices (for Serial and Multithreaded Environments) 36 5.1.2 Creating Matrices (for Multiprocessing Environment) 37 5.2 Compressed Column Storage (CCS) 38 5.2.1 Creating Matrices (for Serial and Multithreaded Environments) 38 5.2.1 Creating Matrices (for Serial and Multithreaded Environments) 38 5.2.2 Creating Matrices (for Serial and Multithreaded Environments) 38 5.2.2 Creating Matrices (for Multiprocessing Environment) 39			•	12
3.1 Initializing and Finalizing 14 3.2 Operating Vectors 15 3.3 Operating Matrices 17 3.4 Solving Linear Equations 23 3.5 Solving Eigenvalue Problems 26 3.6 Writing Programs 29 3.7 Compiling and Linking 31 3.8 Running 33 4 Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations 34 5 Matrix Storage Formats 36 5.1 Compressed Row Storage (CRS) 36 5.1.1 Creating Matrices (for Serial and Multithreaded Environments) 36 5.1.2 Creating Matrices (for Multiprocessing Environment) 37 5.1.3 Associating Arrays 37 5.2 Compressed Column Storage (CCS) 38 5.2.1 Creating Matrices (for Serial and Multithreaded Environments) 38 5.2.2 Creating Matrices (for Serial and Multithreaded Environments) 39 5.2.2 Creating Matrices (for Multiprocessing Environment) 39		2.6	Restrictions	12
3.1 Initializing and Finalizing 14 3.2 Operating Vectors 15 3.3 Operating Matrices 17 3.4 Solving Linear Equations 23 3.5 Solving Eigenvalue Problems 26 3.6 Writing Programs 29 3.7 Compiling and Linking 31 3.8 Running 33 4 Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations 34 5 Matrix Storage Formats 36 5.1 Compressed Row Storage (CRS) 36 5.1.1 Creating Matrices (for Serial and Multithreaded Environments) 36 5.1.2 Creating Matrices (for Multiprocessing Environment) 37 5.1.3 Associating Arrays 37 5.2 Compressed Column Storage (CCS) 38 5.2.1 Creating Matrices (for Serial and Multithreaded Environments) 38 5.2.2 Creating Matrices (for Serial and Multithreaded Environments) 39 5.2.2 Creating Matrices (for Multiprocessing Environment) 39	9	D	:- O	11
3.2 Operating Vectors 15 3.3 Operating Matrices 17 3.4 Solving Linear Equations 23 3.5 Solving Eigenvalue Problems 26 3.6 Writing Programs 29 3.7 Compiling and Linking 31 3.8 Running 33 4 Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations 34 5 Matrix Storage Formats 36 5.1 Compressed Row Storage (CRS) 36 5.1.1 Creating Matrices (for Serial and Multithreaded Environments) 36 5.1.2 Creating Matrices (for Multiprocessing Environment) 37 5.2 Compressed Column Storage (CCS) 38 5.2.1 Creating Matrices (for Serial and Multithreaded Environments) 38 5.2.2 Creating Matrices (for Multiprocessing Environment) 39	3			
3.3 Operating Matrices 17 3.4 Solving Linear Equations 23 3.5 Solving Eigenvalue Problems 26 3.6 Writing Programs 29 3.7 Compiling and Linking 31 3.8 Running 33 4 Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations 34 5.1 Compressed Row Storage (CRS) 36 5.1.1 Creating Matrices (for Serial and Multithreaded Environments) 36 5.1.2 Creating Matrices (for Multiprocessing Environment) 37 5.1.3 Associating Arrays 37 5.2 Compressed Column Storage (CCS) 38 5.2.1 Creating Matrices (for Serial and Multithreaded Environments) 38 5.2.2 Creating Matrices (for Serial and Multithreaded Environments) 38 5.2.2 Creating Matrices (for Multiprocessing Environment) 39 39				
3.4 Solving Linear Equations 23 3.5 Solving Eigenvalue Problems 26 3.6 Writing Programs 29 3.7 Compiling and Linking 31 3.8 Running 33 4 Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations 34 5 Matrix Storage Formats 36 5.1 Compressed Row Storage (CRS) 36 5.1.1 Creating Matrices (for Serial and Multithreaded Environments) 36 5.1.2 Creating Matrices (for Multiprocessing Environment) 37 5.2 Compressed Column Storage (CCS) 38 5.2.1 Creating Matrices (for Serial and Multithreaded Environments) 38 5.2.2 Creating Matrices (for Multiprocessing Environment) 39			. 0	
3.5 Solving Eigenvalue Problems 26 3.6 Writing Programs 29 3.7 Compiling and Linking 31 3.8 Running 33 4 Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations 34 5 Matrix Storage Formats 36 5.1 Compressed Row Storage (CRS) 36 5.1.1 Creating Matrices (for Serial and Multithreaded Environments) 36 5.1.2 Creating Matrices (for Multiprocessing Environment) 37 5.2 Compressed Column Storage (CCS) 38 5.2.1 Creating Matrices (for Serial and Multithreaded Environments) 38 5.2.2 Creating Matrices (for Multiprocessing Environment) 39			. 0	
3.6 Writing Programs 29 3.7 Compiling and Linking 31 3.8 Running 33 4 Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations 34 5 Matrix Storage Formats 36 5.1 Compressed Row Storage (CRS) 36 5.1.1 Creating Matrices (for Serial and Multithreaded Environments) 36 5.1.2 Creating Matrices (for Multiprocessing Environment) 37 5.1.3 Associating Arrays 37 5.2 Compressed Column Storage (CCS) 38 5.2.1 Creating Matrices (for Serial and Multithreaded Environments) 38 5.2.2 Creating Matrices (for Multiprocessing Environment) 39				
3.7 Compiling and Linking 31 3.8 Running 33 4 Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations 34 5 Matrix Storage Formats 36 5.1 Compressed Row Storage (CRS) 36 5.1.1 Creating Matrices (for Serial and Multithreaded Environments) 36 5.1.2 Creating Matrices (for Multiprocessing Environment) 37 5.1.3 Associating Arrays 37 5.2 Compressed Column Storage (CCS) 38 5.2.1 Creating Matrices (for Serial and Multithreaded Environments) 38 5.2.2 Creating Matrices (for Multiprocessing Environment) 39			8 8 4 4 4	
3.8 Running 33 4 Quadruple Precision Operations 34 4.1 Using Quadruple Precision Operations 34 5 Matrix Storage Formats 36 5.1 Compressed Row Storage (CRS) 36 5.1.1 Creating Matrices (for Serial and Multithreaded Environments) 36 5.1.2 Creating Matrices (for Multiprocessing Environment) 37 5.1.3 Associating Arrays 37 5.2 Compressed Column Storage (CCS) 38 5.2.1 Creating Matrices (for Serial and Multithreaded Environments) 38 5.2.2 Creating Matrices (for Multiprocessing Environment) 39				
4.1 Using Quadruple Precision Operations				
4.1 Using Quadruple Precision Operations		_		0.4
5 Matrix Storage Formats 36 5.1 Compressed Row Storage (CRS) 36 5.1.1 Creating Matrices (for Serial and Multithreaded Environments) 36 5.1.2 Creating Matrices (for Multiprocessing Environment) 37 5.1.3 Associating Arrays 37 5.2 Compressed Column Storage (CCS) 38 5.2.1 Creating Matrices (for Serial and Multithreaded Environments) 38 5.2.2 Creating Matrices (for Multiprocessing Environment) 39	4			
5.1 Compressed Row Storage (CRS)		7.1	Using Quadruple I recision Operations	OΊ
5.1.1 Creating Matrices (for Serial and Multithreaded Environments) 36 5.1.2 Creating Matrices (for Multiprocessing Environment) 37 5.1.3 Associating Arrays 37 5.2 Compressed Column Storage (CCS) 38 5.2.1 Creating Matrices (for Serial and Multithreaded Environments) 38 5.2.2 Creating Matrices (for Multiprocessing Environment) 39	5		-	
5.1.2 Creating Matrices (for Multiprocessing Environment)		5.1		
5.1.3 Associating Arrays				
5.2 Compressed Column Storage (CCS)				
5.2.1 Creating Matrices (for Serial and Multithreaded Environments)		F 0		
5.2.2 Creating Matrices (for Multiprocessing Environment)		5.2		
- ,				
			- ,	

	5.3	Modified Compressed Sparse Row (MSR)	40
		5.3.1 Creating Matrices (for Serial and Multithreaded Environments)	40
		5.3.2 Creating Matrices (for Multiprocessing Environment)	41
		5.3.3 Associating Arrays	41
	5.4	Diagonal (DIA)	42
		5.4.1 Creating Matrices (for Serial Environment)	42
		5.4.2 Creating Matrices (for Multithreaded Environment)	43
		5.4.3 Creating Matrices (for Multiprocessing Environment)	44
		5.4.4 Associating Arrays	44
	5.5	Ellpack-Itpack Generalized Diagonal (ELL)	45
	0.0	5.5.1 Creating Matrices (for Serial and Multithreaded Environments)	45
		5.5.2 Creating Matrices (for Multiprocessing Environment)	46
	- 0	5.5.3 Associating Arrays	46
	5.6	Jagged Diagonal (JDS)	47
		5.6.1 Creating Matrices (for Serial Environment)	48
		5.6.2 Creating Matrices (for Multithreaded Environment)	49
		5.6.3 Creating Matrices (for Multiprocessing Environment)	50
		5.6.4 Associating Arrays	50
	5.7	Block Sparse Row (BSR)	51
		5.7.1 Creating Matrices (for Serial and Multithreaded Environments)	51
		5.7.2 Creating Matrices (for Multiprocessing Environment)	52
		5.7.3 Associating Arrays	52
	5.8	Block Sparse Column (BSC)	53
		5.8.1 Creating Matrices (for Serial and Multithreaded Environments)	53
		5.8.2 Creating Matrices (for Multiprocessing Environment)	54
		5.8.3 Associating Arrays	54
	5.9	Variable Block Row (VBR)	55
	0.0	5.9.1 Creating Matrices (for Serial and Multithreaded Environments)	56
		5.9.2 Creating Matrices (for Multiprocessing Environment)	57
		5.9.3 Associating Arrays	58
	F 10		59
	5.10	Coordinate (COO)	
		5.10.1 Creating Matrices (for Serial and Multithreaded Environments)	59
		5.10.2 Creating Matrices (for Multiprocessing Environment)	60
		5.10.3 Associating Arrays	60
	5.11	Dense (DNS)	61
		5.11.1 Creating Matrices (for Serial and Multithreaded Environments)	61
		5.11.2 Creating Matrices (for Multiprocessing Environment)	62
		5.11.3 Associating Arrays	62
_	_		
6		ctions	63
	6.1	Operating Vector Elements	64
		6.1.1 lis_vector_create	64
		6.1.2 lis_vector_destroy	64
		6.1.3 lis_vector_duplicate	65
		6.1.4 lis_vector_set_size	65
		6.1.5 lis_vector_get_size	66
		6.1.6 lis_vector_get_range	66
		6.1.7 lis_vector_set_value	67
		6.1.8 lis_vector_get_value	67
		6.1.9 lis_vector_set_values	68
		6.1.10 lis_vector_get_values	69
		6.1.11 lis_vector_scatter	69
		6.1.12 lis_vector_gather	70
		0.1.12 no-100001-800no1	10

	6.1.13	lis_vector_copy	. 70
	6.1.14	lis_vector_set_all	. 70
6.2	Operat	ting Matrix Elements	. 71
	6.2.1	lis_matrix_create	
	6.2.2	lis_matrix_destroy	
	6.2.3	lis_matrix_duplicate	
	6.2.4	lis_matrix_malloc	
	6.2.5	lis_matrix_set_value	
	6.2.6	lis_matrix_assemble	
	6.2.7	lis_matrix_set_size	
	6.2.8	lis_matrix_get_size	
	6.2.9	<u> </u>	
		lis_matrix_get_range	
		lis_matrix_set_type	
		lis_matrix_get_type	
		lis_matrix_set_blocksize	
		lis_matrix_convert	
		lis_matrix_copy	
		lis_matrix_get_diagonal	
		lis_matrix_set_crs	
	6.2.17	lis_matrix_set_ccs	. 79
	6.2.18	lis_matrix_set_msr	. 80
	6.2.19	lis_matrix_set_dia	. 80
	6.2.20	lis_matrix_set_ell	. 81
	6.2.21	lis_matrix_set_jds	. 81
	6.2.22	lis_matrix_set_bsr	. 82
		lis_matrix_set_bsc	
		lis_matrix_set_vbr	
		lis_matrix_set_coo	
		lis_matrix_set_dns	
6.3		ting Vectors and Matrices	
0.0	6.3.1	lis_vector_scale	
	6.3.2	lis_vector_dot	
	6.3.2	lis_vector_nrm1	
	6.3.4		
	6.3.4	lis_vector_nrm2	
		lis_vector_nrmi	
	6.3.6	lis_vector_axpy	
	6.3.7	lis_vector_xpay	
	6.3.8	lis_vector_axpyz	. 88
	6.3.9	lis_matrix_scaling	
		lis_matvec	
		lis_matvect	
6.4	-	g Linear Equations	
	6.4.1	lis_solver_create	
	6.4.2	lis_solver_destroy	. 90
	6.4.3	lis_solver_set_option	. 91
	6.4.4	lis_solver_set_optionC	. 94
	6.4.5	lis_solve	. 94
	6.4.6	lis_solve_kernel	. 95
	6.4.7	lis_solver_get_status	
	6.4.8	lis_solver_get_iters	
	6.4.9	lis_solver_get_itersex	
		lis_solver_get_time	
		lis solver get timeer	. 00

		6.4.12	lis_solver_get_residualnorm	98
		6.4.13	lis_solver_get_rhistory	99
		6.4.14	lis_solver_get_solver	00
		6.4.15	lis_solver_get_precon	01
		6.4.16	lis_get_solvername	02
		6.4.17	lis_get_preconname	02
	6.5	Solving	g Eigenvalue Problems	03
		6.5.1	lis_esolver_create	03
		6.5.2	lis_esolver_destroy	03
		6.5.3	lis_esolver_set_option	04
		6.5.4	lis_esolver_set_optionC	07
		6.5.5	lis_esolve	07
		6.5.6	lis_esolver_get_status	08
		6.5.7	lis_esolver_get_iters	08
		6.5.8	lis_esolver_get_itersex	09
		6.5.9	lis_esolver_get_time	
		6.5.10	· ·	10
		6.5.11	lis_esolver_get_residualnorm	10
		6.5.12	lis_esolver_get_rhistory	11
		6.5.13	lis_esolver_get_evalues	12
				12
		6.5.15	lis_esolver_get_esolver	13
		6.5.16	lis_get_esolvername	13
	6.6	Operat	ing External Files	14
		6.6.1	lis_input	14
		6.6.2	lis_input_vector	14
		6.6.3	lis_input_matrix	15
		6.6.4	lis_output	15
		6.6.5	lis_output_vector	16
		6.6.6	lis_output_matrix	16
	6.7	Other	Functions	17
		6.7.1	lis_initialize	17
		6.7.2	lis_finalize	17
		6.7.3	lis_wtime	17
	_			
Re	eferei	ices	17	18
A		Forma		20
	A.1	Extend	led Matrix Market Format	20
			ll-Boeing Format	
			led Matrix Market Format for Vectors	
			Format for Vectors	

0 Changes from Version 1.1

- 1. Added the support for the eigensolvers.
- 2. Changed the specifications of the following functions:
 - (a) Changed the names of lis_output_residual_history() and lis_get_residual_history() to lis_solver_output_rhistory() and lis_solver_get_rhistory(), respectively.
 - (b) Changed the origin of the Fortran interfaces lis_vector_set_value() and lis_vector_get_value() to 1
 - (c) Changed the origin of the Fortran interface lis_vector_set_size() to 1.
 - (d) Changed the name of the precision flag -precision to -f.
- 3. Changed the specifications of the integer types:
 - (a) Replaced the type of integer in the C programs with LIS_INT, which is equivalent with int by default. If the preprossor macro _LONGLONG is defined, it is replaced with long long int.
 - (b) Replaced the type of integer in the Fortran programs with LIS_INTEGER, which is equivalent with integer by default. If the preprossor macro LONGLONG is defined, it is replaced with integer*8.

1 Introduction

Lis, a Library of Iterative Solvers for linear systems, is a parallel numerical library for solving the linear equations

$$Ax = b$$

and the standard eigenvalue problems

$$Ax = \lambda x$$

with real sparse matrices using the iterative methods. The solvers available in Lis are listed in Table 1 and 2, and the preconditioners are listed in Table 3. The supported matrix storage formats are listed in Table 4.

Table 1: Linear Solvers

Table 1: Linear Solvers					
CG	CR				
BiCG	BiCR[2]				
CGS	CRS[3]				
BiCGSTAB	BiCRSTAB[3]				
GPBiCG	GPBiCR[3]				
BiCGSafe[1]	BiCRSafe[4]				
BiCGSTAB(l)	TFQMR				
Jacobi	Orthomin(m)				
Gauss-Seidel	GMRES(m)				
SOR	FGMRES(m)[5]				
IDR(s)[13]	MINRES[14]				

Table 2: Eigensolvers

8
Power
Inverse
Approximate Inverse
Rayleigh Quotient
Subspace
Lanczos
CG[18, 19]
CR[20]

Table 3: Preconditioners

Table 4: Matrix Storage Formats

able 5. I reconditioners	Table 4. Matrix biorage Pormats	
Jacobi	Compressed Row Storage	(CRS)
SSOR	Compressed Column Storage	(CCS)
ILU(k)	Modified Compressed Sparse Row	(MSR)
ILUT[6, 7]	Diagonal	(DIA)
Crout ILU[8, 7]	Ellpack-Itpack Generalized Diagonal	(ELL)
I+S[9]	Jagged Diagonal	(JDS)
SA-AMG[10]	Block Sparse Row	(BSR)
Hybrid[11]	Block Sparse Column	(BSC)
SAINV[12]	Variable Block Row	(VBR)
Additive Schwarz	Dense	(DNS)
User defined	Coordinate	(COO)

2 Installation

This section describes the instructions for installing and testing Lis. We assume Lis being installed on a Linux cluster.

2.1 System Requirements

Installation of Lis requires a C compiler. The Fortran interface requires a Fortran compiler. The algebraic multigrid preconditioner requires a Fortran 90 compiler. For parallel computing environments, the OpenMP library or the MPI-1 library is used. Lis has been tested on the environments shown in Table 5 (see also Table 7).

C Compilers OS Intel C/C++ Compiler 7.0, 8.0, 9.1, 10.1, 11.1, Linux Intel C++ Composer XE Windows IBM XL C/C++ V7.0, 9.0 AIX Linux Sun WorkShop 6, Sun ONE Studio 7, Solaris Sun Studio 11, 12 PGI C++ 6.0, 7.1, 10.5 Linux gcc 3.3, 4.3 Linux Mac OS X Windows Microsoft Visual C++ 2008, 2010, 2012RC Windows Fortran Compilers (Optional) OS Intel Fortran Compiler 8.1, 9.1, 10.1, 11.1, Linux Intel Fortran Composer XE Windows AIX IBM XL Fortran V9.1, 11.1 Linux Sun WorkShop 6, Sun ONE Studio 7, Solaris Sun Studio 11, 12 PGI Fortran 6.0, 7.1, 10.5 Linux g77 3.3 Linux Mac OS X gfortran 4.3, 4.4

Windows

Table 5: Major Tested Platforms

2.2 Extracting Archive

Enter the following command to extract the archive, where (\$VERSION) represents the version: >gunzip -c lis-(\$VERSION).tar.gz | tar xvf
It creates a directory lis-(\$VERSION) along with its subfolders as shown in Figure 1.

2.3 Installing on UNIX and Compatible Systems

2.3.1 Configuring Source Tree

Run the following script to configure the source tree:

g95 0.91

- default: >./configure
- specify the installation destination: >./configure --prefix=<install-dir>

Figure 1: Files contained in lis-(\$VERSION).tar.gz

Table 6 shows the major options which can be specified for the configuration. Table 7 shows the major computing environments which can be specified by TARGET.

Table 6: Major Configuration Options (see ./configure --help for the complete list)

enable-omp	Enable the OpenMP library
enable-mpi	Enable the MPI library
enable-fortran	Enable the Fortran interface
enable-saamg	Enable the SA-AMG preconditioner
enable-quad	Enable the quadruple precision operations
enable-longlong	Enable the 64bit integer
enable-shared	Enable the dynamic linking
enable-gprof	Enable the GNU profiler
prefix= <install-dir></install-dir>	Specify the installation destination
TARGET= <target></target>	Specify the computing environment
CC= <c_compiler></c_compiler>	Specify the C compiler
CFLAGS= <c_flags></c_flags>	Specify the options for the C compiler
FC= <fortran90_compiler></fortran90_compiler>	Specify the Fortran 90 compiler
FCFLAGS= <fc_flags></fc_flags>	Specify the options for the Fortran 90 compiler
LDFLAGS= <ld_flags></ld_flags>	Specify the link options

2.3.2 Compiling

In the directory lis-(\$VERSION), run the following command to generate executable files: >make

To ensure that the library has been successfully built, enter as follows in lis-(\$VERSION):

>make check

It runs a test script using the executable files created in lis-(\$VERSION)/test, which reads the data of the coefficient matrix and the right hand side vector from the file test/testmat.mtx and writes the solution of the linear equation Ax = b obtained by the BiCG method into test/sol.txt, and the residual history into test/res.txt. If the values of the elements of the solution are 1, then the result is correct. The result on the SGI Altix 3700 is shown below.

Table 7: Examples of Targets (see lis-(\$VERSION)/configure.in for details)

<target></target>	Equivalent options
cray_xt3_cross	./configure CC=cc FC=ftn CFLAGS="-03 -B -fastsse -tp k8-64"
cray_xt3_cross	FCFLAGS="-03 -fastsse -tp k8-64 -Mpreprocess" FCLDFLAGS="-Mnomain"
	ac_cv_sizeof_void_p=8 cross_compiling=yesenable-mpi
	ax_f77_mangling="lower case, no underscore, extra underscore"
fujitsu_fx10_cross	./configure CC=fccpx FC=frtpx CFLAGS="-Kfast,ocl,preex -w"
	FCFLAGS="-Kfast,ocl,preex -Cpp -fs" FCLDFLAGS="-mlcmain=main"
	ac_cv_sizeof_void_p=8 cross_compiling=yes
	ax_f77_mangling="lower case, underscore, no extra underscore"
hitachi_sr16k	./configure CC=cc FC=f90 CFLAGS="-Os -noparallel"
	FCFLAGS="-Oss -noparallel" FCLDFLAGS="-1f90s"
	ac_cv_sizeof_void_p=8
	ax_f77_mangling="lower case, underscore, no extra underscore"
ibm_bgl_cross	./configure CC=blrts_xlc FC=blrts_xlf90
	CFLAGS="-03 -qarch=440d -qtune=440 -qstrict
	-I/bgl/BlueLight/ppcfloor/bglsys/include"
	FCFLAGS="-03 -qarch=440d -qtune=440 -qsuffix=cpp=F90 -w
	-I/bgl/BlueLight/ppcfloor/bglsys/include"
	ac_cv_sizeof_void_p=4 cross_compiling=yesenable-mpi
	ax_f77_mangling="lower case, no underscore, no extra underscore"
nec_es_cross	./configure CC=esmpic++ FC=esmpif90 AR=esar RANLIB=true
	ac_cv_sizeof_void_p=8 ax_vector_machine=yes cross_compiling=yes
	enable-mpienable-omp
	ax_f77_mangling="lower case, no underscore, extra underscore"
nec_sx9_cross	./configure CC=sxmpic++ FC=sxmpif90 AR=sxar RANLIB=true
	ac_cv_sizeof_void_p=8 ax_vector_machine=yes cross_compiling=yes
	ax_f77_mangling="lower case, no underscore, extra underscore"

```
matrix size = 100 x 100 (460 nonzero entries)
initial vector x = 0
precision: double
solver: BiCG 2
precon: none
storage: CRS
lis_solve: normal end

BiCG: number of iterations = 15 (double = 15, quad = 0)
BiCG: elapsed time = 5.178690e-03 sec.
BiCG: preconditioner = 1.277685e-03 sec.
BiCG: matrix creation = 1.254797e-03 sec.
BiCG: linear solver = 3.901005e-03 sec.
BiCG: relative residual 2-norm = 6.327297e-15
```

```
max number of threads = 32
number of threads = 2
matrix size = 100 x 100 (460 nonzero entries)
initial vector x = 0
precision : double
solver : BiCG 2
precon : none
storage : CRS
lis_solve : normal end

BiCG: number of iterations = 15 (double = 15, quad = 0)
BiCG: elapsed time = 8.960009e-03 sec.
BiCG: preconditioner = 2.297878e-03 sec.
BiCG: matrix creation = 2.072096e-03 sec.
BiCG: linear solver = 6.662130e-03 sec.
BiCG: relative residual 2-norm = 6.221213e-15
```

```
--enable-mpi -
number of processes = 2
matrix size = 100 x 100 (460 nonzero entries)
initial vector x = 0
precision : double
solver
          : BiCG 2
precon
          : none
         : CRS
storage
lis_solve : normal end
BiCG: number of iterations
                                = 15 \text{ (double } = 15, \text{ quad } = 0)
BiCG: elapsed time
                                = 2.911400e-03 sec.
BiCG:
        preconditioner
                                = 1.560780e-04 sec.
BiCG:
          matrix creation
                                = 1.459997e-04 sec.
BiCG:
        linear solver
                                = 2.755322e-03 \text{ sec.}
BiCG: relative residual 2-norm = 6.221213e-15
```

2.3.3 Installing

In the directory lis-(\$VERSION), enter as follows:

>make install

It copies the files to the destination directory as follows:

```
$(INSTALLDIR)
+include
| +lis_config.h lis.h lisf.h
+lib
| +liblis.a
+share
+doc/lis examples/lis
```

lis_config.h is the header file required to build the library, and lis.h and lisf.h are the header files required by the C and Fortran compilers, respectively. liblis.a is the library file. To ensure that the library has been successfully installed, enter as follows in lis-(\$VERSION):

>make installcheck

It runs a test script using the executable files installed in examples/lis.

2.4 Installing on Windows Systems

Use one of the solution files or project files for the Microsoft Visual Studio in the directory lis-(\$VERSION)/win32. lis_with_fortran.sln is the solution file to be used with the Intel Visual Fortran Compiler.

lis_with_fortran_mpi.sln is the solution file to be used with the Visual Fortran and MPICH2. The header files are located in lis-(\$VERSION)/include. lis_config_win32.h is the header file required to build the library. lis.h and lisf.h are the header files required by the C and Fortran compilers, respectively. The library files are generated in lis-(\$VERSION)/lib. The executable files of the test programs are generated in lis-(\$VERSION)/test.

2.5 Testing

2.5.1 test1

Usage: test1 matrix_filename rhs_setting solution_filename residual_filename [options]

This program inputs the data of the coefficient matrix from matrix_filename and solves the linear equation Ax = b with the solver specified by options. It outputs the solution to solution_filename and the residual history to residual_filename. The extended Matrix Market format, which is extended to allow vector data, is supported (see Appendix). One of the following values can be specified by rhs_setting:

0 Use the right hand side vector b included in the data file

1 Use $b = (1, ..., 1)^T$

Use $b = A \times (1, \dots, 1)^T$

rhs_filename The filename for the right hand side vector

The PLAIN and Matrix Market formats are supported for rhs_filename. test1f.F is the Fortran version of test1.c.

2.5.2 test2

Usage: test2 m n matrix_type solution_filename residual_filename [options]

This program solves a discretized two dimensional Poisson equation Ax = b using the five point central difference scheme, with the coefficient matrix A of size mn in the storage format specified by matrix_type and the solver specified by options. It outputs the solution to solution_filename and the residual history to residual_filename. The right hand side vector is set to make the values of the elements of the solution to be 1. The values m and n represent the numbers of the grid points in each dimension.

2.5.3 test3

Usage: test3 1 m n matrix_type solution_filename residual_filename [options]

This program solves a discretized three dimensional Poisson equation Ax = b using the seven point central difference scheme, with the coefficient matrix A of size lmn in the storage format specified by matrix_type and the solver specified by options. It outputs the solution to solution_filename and the residual history to residual_filename. The right hand side vector is set to make the values of the elements of the solution to be 1. The values 1, m and n represent the numbers of the grid points in each dimension.

2.5.4 test4

This program solves the linear equation Ax = b with a specified solver and a preconditioner, where A is a tridiagonal matrix

$$\begin{pmatrix}
2 & -1 & & & & \\
-1 & 2 & -1 & & & & \\
& \ddots & \ddots & \ddots & & \\
& & -1 & 2 & -1 & \\
& & & -1 & 2
\end{pmatrix}$$

of size 12. The right hand side vector b is set to make the values of the elements of the solution x to be 1. test4f.F is the Fortran version of test4.c.

2.5.5 test5

Usage: test5 n gamma [options]

This program solves a linear equation Ax = b, where A is a Toeplitz matrix

$$\begin{pmatrix} 2 & 1 & & & & \\ 0 & 2 & 1 & & & \\ \gamma & 0 & 2 & 1 & & \\ & \ddots & \ddots & \ddots & \ddots & \\ & & \gamma & 0 & 2 & 1 \\ & & & \gamma & 0 & 2 \end{pmatrix}$$

of size n, with the solver specified by options. Note that the right hand vector is set to make the values of the elements of the solution to be 1.

2.5.6 etest1

Usage: etest1 matrix_filename solution_filename residual_filename [options]

This program inputs the matrix data from matrix_filename and solves the eigenvalue problem $Ax = \lambda x$ with the solver specified by options. It outputs the associated eigenvector to solution_filename and the residual history to residual_filename. The Matrix Market format is supported. etest1.F is the Fortran version of etest1.c.

2.5.7 etest2

Usage: etest2 m n matrix_type solution_filename residual_filename [options]

This program solves the eigenvalue problem $Ax = \lambda x$, where the coefficient matrix A of size mn is derived from a discretized two dimensional Helmholtz equation using the five point central difference scheme, with the coefficient matrix in the storage format specified by matrix_type and the solver specified by options. It outputs the associated eigenvector to solution_filename and the residual history to residual_filename. The values m and m represent the numbers of the grid points in each dimension.

2.5.8 etest3

Usage: etest3 1 m n matrix_type solution_filename residual_filename [options]

This program solves the eigenvalue problem $Ax = \lambda x$, where the coefficient matrix A of size lmn is derived from a discretized three dimensional Helmholtz equation using the seven point central difference scheme, with the coefficient matrix in the storage format specified by matrix_type and the solver specified by options. It outputs the associated eigenvector to solution_filename and the residual history to residual_filename. The values 1, m and n represent the numbers of the grid points in each dimension.

2.5.9 etest4

Usage: etest4 n [options]

This program solves the eigenvalue problem $Ax = \lambda x$ with a specified solver, where A is a tridiagonal

matrix

$$A = \begin{pmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ & & & -1 & 2 \end{pmatrix}$$

of size $n \times n$, etest4f.F is the Fortran version of etest4.c

2.5.10 etest5

Usage: etest5 evalue_filename evector_filename

This program solves the eigenvalue problem $Ax = \lambda x$ with the Subspace method, where A is a tridiagonal matrix

$$A = \begin{pmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ & & & -1 & 2 \end{pmatrix}$$

of size 12×12 . It outputs 2 extreme eigenvalues of the smallest magnitude to evalue_filename and the associated eigenvectors to evector_filename in the extended Matrix Market format (see Appendix).

2.5.11 spmvtest1

Usage: spmvtest1 n iter [matrix_type]

This program computes the multiply of a tridiagonal matrix

$$\begin{pmatrix}
2 & -1 & & & & \\
-1 & 2 & -1 & & & & \\
& \ddots & \ddots & \ddots & & \\
& & -1 & 2 & -1 & \\
& & & -1 & 2
\end{pmatrix}$$

of size n, derived from a discretized one dimensional Poisson equation using the three point central difference scheme, and a vector $(1, ..., 1)^T$. The FLOPS performance is measured as the average of iter iterations. If necessary, one of the following values can be specified by matrix_type:

0 Measure the performance for the available matrix storage formats

1-11 The number of the matrix storage format

2.5.12 spmvtest2

Usage: spmvtest2 m n iter [matrix_type]

This program computes the multiply of a sparse matrix, derived from a discretized two dimensional Poisson equation using the five point central difference scheme, and a vector $(1, ..., 1)^T$. The FLOPS performance is measured as the average of iter iterations. If necessary, one of the following values can be specified by matrix_type:

0 Measure the performance for the available matrix storage formats

1-11 The number of the matrix storage format

The values m and n represent the numbers of the grid points in each dimension.

2.5.13 spmvtest3

Usage: spmvtest3 1 m n iter [matrix_type]

This program computes the multiply of a sparse matrix, derived from a discretized three dimensional Poisson equation using the seven point central difference scheme, and a vector $(1, ..., 1)^T$. The values 1, m and n represent the numbers of the grid points in each dimension. The FLOPS performance is measured as the average of iter iterations. If necessary, one of the following values can be specified by matrix_type:

0 Measure the performance for the available matrix storage formats

1-11 The number of the matrix storage format

2.5.14 spmvtest4

Usage: spmvtest4 matrix_filename_list iter [block]

This program inputs the matrix data from the files listed in matrix_filename_list, and computes the multiplies of matrices in available matrix storage formats and a vector $(1, ..., 1)^T$. The FLOPS performance is measured as the average of iter iterations. If necessary, the block size of the BSR and BSC can be specified by block.

2.5.15 spmvtest5

Usage: spmvtest5 matrix_filename matrix_type iter [block]

This program inputs the matrix data from matrix_filename and compute the multiply of the matrix with matrix_type and a vector $(1, ..., 1)^T$. The FLOPS performance is measured as the average of iter iterations. If necessary, the block size of the BSR and BSC can be specified by block.

2.6 Restrictions

The current version has the following restrictions:

- Preconditioners
 - If a preconditioner other than the Jacobi or SSOR is selected and the matrix A is not in the CRS format, a new matrix is created in the CRS format for preconditioning.
 - The SA-AMG preconditioner does not support the BiCG method.
 - The SA-AMG preconditioner does not support the multithreaded environment.
 - The assembly of the matrices in the SAINV preconditioner is not parallelized.
- Quadruple precision operations
 - The Jacobi, Gauss-Seidel, SOR, and IDR(s) methods do not support the quadruple precision operations.
 - The CG and CR methods for the eigenvalue problems do not support the quadruple precision operations.
 - The Jacobi, Gauss-Seidel and SOR methods in the hybrid preconditioner do not support the quadruple precision operations.
 - The I+S and SA-AMG preconditioners do not support the quadruple precision operations.
- Matrix storage formats

arrays.			

13

– In the multiprocessing environment, the CRS is the only accepted format for the user defined

3 Basic Operations

This section describes how to use the library. A program requires the following statements:

- Initialization
- Matrix creation
- Vector creation
- Solver creation
- Value assignment for matrices and vectors
- Solver assignment
- Solver execution
- Finalization

In addition, it must include one of the following include statements:

```
• C #include "lis.h"
```

• Fortran #include "lisf.h"

When Lis is installed in \$(INSTALLDIR), lis.h and lisf.h are located in \$(INSTALLDIR)/include.

3.1 Initializing and Finalizing

The functions for initializing and finalizing the execution environment must be called at the top and bottom of the program, respectively, as follows:

```
1: #include "lis.h"
2: LIS_INT main(LIS_INT argc, char* argv[])
3: {
4: lis_initialize(&argc, &argv);
5: ...
6: lis_finalize();
7: }
```

```
Fortran

1: #include "lisf.h"

2: call lis_initialize(ierr)

3: ...

4: call lis_finalize(ierr)
```

Initializing

For initializing, the following functions are used:

- C lis_initialize(LIS_INT* argc, char** argv[])
- Fortran subroutine lis_initialize(LIS_INTEGER ierr)

This function initializes the MPI execution environment, and specifies the options on the command line. The default type of integer in the C programs is LIS_INT, which is equivalent with int. If the preprossor macro _LONGLONG is defined, it is replaced with long long int. The default type of integer in the Fortran programs is LIS_INTEGER, which is equivalent with integer. If the preprossor macro LONGLONG

is defined, it is replaced with integer*8.

Finalizing

For finalizing, the following functions are used:

- C LIS_INT lis_finalize()
- Fortran subroutine lis_finalize(LIS_INTEGER ierr)

3.2 Operating Vectors

Assume that the size of the vector v is $global_n$, and the size of each partial vector stored on nprocs processing elements is $local_n$. If $global_n$ is divisible, then $local_n$ is equal to $global_n / nprocs$. For example, when the vector v is stored on two processing elements, as shown in Equation (3.1), $global_n$ and $local_n$ are 4 and 2, respectively.

$$v = \begin{pmatrix} 0 \\ \frac{1}{2} \\ 3 \end{pmatrix} \text{ PE1}$$
(3.1)

In the case of creating the vector v in Equation (3.1), the vector v itself is created for the serial and multithreaded environments, while the partial vectors are created and stored on a given number of processing elements for the multiprocessing environment.

Programs to create the vector v are as follows, where the number of the processing elements for the multiprocessing environment is assumed to be two:

```
C (for serial and multithreaded environments) —
1: LIS_INT
                  i,n;
2: LIS_VECTOR
                  v;
3: n = 4;
4: lis_vector_create(0,&v);
5: lis_vector_set_size(v,0,n);
                                            /* or lis_vector_set_size(v,n,0); */
6:
7: for(i=0;i<n;i++)
8: {
9:
        lis_vector_set_value(LIS_INS_VALUE,i,(double)i,v);
10:
    }
```

```
C (for multiprocessing environment) —
 1: LIS_INT
                  i,n,is,ie;
                                             /* or LIS_INT i,ln,is,ie; */
 2: LIS_VECTOR
                                             /* ln = 2; */
3: n = 4;
4: lis_vector_create(MPI_COMM_WORLD,&v);
5: lis_vector_set_size(v,0,n);
                                             /* lis_vector_set_size(v,ln,0); */
6: lis_vector_get_range(v,&is,&ie);
7: for(i=is;i<ie;i++)
8: {
        lis_vector_set_value(LIS_INS_VALUE,i,(double)i,v);
9:
10:
    }
```

· Fortran (for serial and multithreaded environments) -

```
1: LIS_INTEGER i,n
2: LIS_VECTOR v
3: n = 4
4: call lis_vector_create(0,v,ierr)
5: call lis_vector_set_size(v,0,n,ierr)
6:
7: do i=1,n
9: call lis_vector_set_value(LIS_INS_VALUE,i,DBLE(i),v,ierr)
10: enddo
```

Fortran (for multiprocessing environment) -

```
1: LIS_INTEGER i,n,is,ie
2: LIS_VECTOR v
3: n = 4
4: call lis_vector_create(MPI_COMM_WORLD,v,ierr)
5: call lis_vector_set_size(v,0,n,ierr)
6: call lis_vector_get_range(v,is,ie,ierr)
7: do i=is,ie-1
8: call lis_vector_set_value(LIS_INS_VALUE,i,DBLE(i),v,ierr);
9: enddo
```

Declaring Variables

As the second line shows, the declaration is stated as follows:

```
LIS_VECTOR v;
```

Creating Vectors

To create the vector v, the following functions are used:

- C LIS_INT lis_vector_create(LIS_Comm comm, LIS_VECTOR *v)
- Fortran subroutine lis_vector_create(LIS_Comm comm, LIS_VECTOr v, LIS_INTEGER ierr)

For the example program above, comm must be replaced with the MPI communicator. For the serial and multithreaded environments, the value of comm is ignored.

Assigning Sizes

To assign a size to the vector v, the following functions are used:

- C LIS_INT lis_vector_set_size(LIS_VECTOR v, LIS_INT local_n, LIS_INT global_n)
- Fortran subroutine lis_vector_set_size(LIS_VECTOR v, LIS_INTEGER local_n, LIS_INTEGER global_n, LIS_INTEGER ierr)

Either $local_n$ or $global_n$ must be provided.

In the case of the serial and multithreaded environments, $local_n$ is equal to $global_n$. Therefore, both $lis_vector_set_size(v,n,0)$ and $lis_vector_set_size(v,0,n)$ create a vector of size n.

For the multiprocessing environment, $lis_{vector_set_size(v,n,0)}$ creates a partial vector of size n on each processing element. On the other hand, $lis_{vector_set_size(v,0,n)}$ creates a partial vector of size m_p on the processing element p. The values of m_p are determined by the library.

Assigning Values

To assign a value to the i-th element of the vector v, the following functions are used:

- C LIS_INT lis_vector_set(LIS_INT flag, LIS_INT i, LIS_SCALAR value, LIS_VECTOR v)
- Fortran subroutine lis_vector_set_value(LIS_INT flag, LIS_INT i, LIS_SCALAR value, LIS_VECTOR v, LIS_INTEGER ierr)

For the multiprocessing environment, the i-th row of the global vector must be specified. Either

LIS_INS_VALUE : v[i] = value, or

 $LIS_ADD_VALUE : v[i] = v[i] + value$

must be provided for flag.

Duplicating Vectors

To create a vector which has the same information as the existing vector, the following functions are used:

- C LIS_INT lis_vector_duplicate(LIS_VECTOR vin, LIS_VECTOR *vout)
- Fortran subroutine lis_vector_duplicate(LIS_VECTOR vin, LIS_VECTOR vout, LIS_INTEGER ierr)

This function does not copy the values of the vector. To copy the values as well, the following functions must be called after the above functions:

- C LIS_INT lis_vector_copy(LIS_VECTOR vsrc, LIS_VECTOR vdst)
- Fortran subroutine lis_vector_copy(LIS_VECTOR vsrc, LIS_VECTOR vdst, LIS_INTEGER ierr)

Destroying Vectors

To destroy the vector, the following functions are used:

- C LIS_INT lis_vector_destroy(LIS_VECTOR v)
- Fortran subroutine lis_vector_destroy(LIS_VECTOR v, LIS_INTEGER ierr)

3.3 Operating Matrices

Assume that the size of the matrix A is $global_n \times global_n$, and that the size of each row block of the matrix A stored on nprocs processing elements is $local_n \times global_n$. If $global_n$ is divisible, then $local_n$ is equal to $global_n / nprocs$. For example, when the row block of the matrix A is stored on two processing elements, as shown in Equation (3.2), $global_n$ and $local_n$ are 4 and 2, respectively.

$$A = \begin{pmatrix} 2 & 1 & & \\ 1 & 2 & 1 & & \\ \hline & 1 & 2 & 1 & \\ & & 1 & 2 & \end{pmatrix}$$
PE0
PE1
(3.2)

A matrix in a specific storage format can be created in one of the following three ways:

Method 1: Define Arrays in a Specific Storage Format with Library Functions

In the case of creating the matrix A in Equation (3.2) in the CRS format, the matrix A itself is created for the serial and multithreaded environments, while the partial matrices are created and stored on the given number of processing elements for the multiprocessing environment.

Programs to create the matrix A in the CRS format are as follows, where the number of the processing elements for the multiprocessing environment is assumed to be two:

```
- C (for serial and multithreaded environments) -
 1: LIS_INT
                  i,n;
 2: LIS_MATRIX
                  A:
3: n = 4;
 4: lis_matrix_create(0,&A);
5: lis_matrix_set_size(A,0,n);
                                            /* or lis_matrix_set_size(A,n,0); */
 6: for(i=0;i<n;i++) {
        if( i>0 ) lis_matrix_set_value(LIS_INS_VALUE,i,i-1,1.0,A);
7:
8:
        if( i<n-1 ) lis_matrix_set_value(LIS_INS_VALUE,i,i+1,1.0,A);</pre>
9:
        lis_matrix_set_value(LIS_INS_VALUE,i,i,2.0,A);
10: }
11: lis_matrix_set_type(A,LIS_MATRIX_CRS);
12: lis_matrix_assemble(A);
```

```
- C (for multiprocessing environment) —
 1: LIS_INT
                  i,n,gn,is,ie;
2: LIS_MATRIX
                  Α;
3: gn = 4;
                                             /* or n=2 */
4: lis_matrix_create(MPI_COMM_WORLD,&A);
5: lis_matrix_set_size(A,0,gn);
                                             /* lis_matrix_set_size(A,n,0); */
 6: lis_matrix_get_size(A,&n,&gn);
7: lis_matrix_get_range(A,&is,&ie);
8: for(i=is;i<ie;i++) {
9:
        if( i>0 ) lis_matrix_set_value(LIS_INS_VALUE,i,i-1,1.0,A);
10:
        if( i<gn-1 ) lis_matrix_set_value(LIS_INS_VALUE,i,i+1,1.0,A);</pre>
11:
        lis_matrix_set_value(LIS_INS_VALUE,i,i,2.0,A);
12: }
13: lis_matrix_set_type(A,LIS_MATRIX_CRS);
14: lis_matrix_assemble(A);
```

```
- Fortran (for serial and multithreaded environments) —
 1: LIS_INTEGER
                  i,n
2: LIS_MATRIX
                  Α
3: n = 4
4: call lis_matrix_create(0,A,ierr)
5: call lis_matrix_set_size(A,0,n,ierr)
 6: do i=1,n
        if( i>1 ) call lis_matrix_set_value(LIS_INS_VALUE,i,i-1,1.0d0,A,ierr)
7:
        if( i<n ) call lis_matrix_set_value(LIS_INS_VALUE,i,i+1,1.0d0,A,ierr)</pre>
 8:
        call lis_matrix_set_value(LIS_INS_VALUE,i,i,2.0d0,A,ierr)
9:
10: enddo
11: call lis_matrix_set_type(A,LIS_MATRIX_CRS,ierr)
12: call lis_matrix_assemble(A,ierr)
```

```
Fortran (for multiprocessing environment)
 1: LIS_INTEGER
                  i,n,gn,is,ie
 2: LIS_MATRIX
                  Α
3: gn = 4
4: call lis_matrix_create(MPI_COMM_WORLD,A,ierr)
5: call lis_matrix_set_size(A,0,gn,ierr)
 6: call lis_matrix_get_size(A,n,gn,ierr)
7: call lis_matrix_get_range(A,is,ie,ierr)
8: do i=is,ie-1
        if( i>1 ) call lis_matrix_set_value(LIS_INS_VALUE,i,i-1,1.0d0,A,ierr)
9:
10:
        if( i<gn ) call lis_matrix_set_value(LIS_INS_VALUE,i,i+1,1.0d0,A,ierr)</pre>
        call lis_matrix_set_value(LIS_INS_VALUE,i,i,2.0d0,A,ierr)
11:
12:
    enddo
    call lis_matrix_set_type(A,LIS_MATRIX_CRS,ierr)
13:
    call lis_matrix_assemble(A,ierr)
```

Declaring Variables

As the second line shows, the declaration is stated as follows:

```
LIS_MATRIX A;
```

Creating Matrices

To create the matrix A, the following functions are used:

- C LIS_INT lis_matrix_create(LIS_Comm comm, LIS_MATRIX *A)
- Fortran subroutine lis_matrix_create(LIS_Comm comm, LIS_MATRIX A, LIS_INTEGER ierr)

comm must be replaced with the MPI communicator. For the serial and multithreaded environments, the value of comm is ignored.

Assigning Sizes

To assign a size to the matrix A, the following functions are used:

- C LIS_INT lis_matrix_set_size(LIS_MATRIX A, LIS_INT local_n, LIS_INT global_n)
- Fortran subroutine lis_matrix_set_size(LIS_MATRIX A, LIS_INTEGER local_n, LIS_INTEGER global_n, LIS_INTEGER ierr)

Either $local_n$ or $global_n$ must be provided.

In the case of the serial and multithreaded environments, $local_n$ is equal to $global_n$. Therefore, both $lis_matrix_set_size(A,n,0)$ and $lis_matrix_set_size(A,0,n)$ create a matrix of size $n \times n$.

For the multiprocessing environment, $lis_{matrix_set_size}(A,n,0)$ creates a partial matrix of size $n \times N$ on each processing element, where N is the total sum of n. On the other hand, $lis_{matrix_set_size}(A,0,n)$ creates a partial matrix of size $m_p \times n$ on the processing element p. The values of m_p are determined by the library.

Assigning Values

To assign a value to the element at the i-th row and the j-th column of the matrix A, the following functions are used:

- C LIS_INT lis_matrix_set_value(LIS_INT flag, LIS_INT i, LIS_INT j, LIS_SCALAR value, LIS_MATRIX A)
- Fortran subroutine lis_matrix_set_value(LIS_INTEGER flag, LIS_INTEGER i, LIS_INTEGER j, LIS_SCALAR value, LIS_MATRIX A, LIS_INTEGER ierr)

For the multiprocessing environment, the i-th row and the j-th column of the global matrix must be specified. Either

LIS_INS_VALUE : A(i, j) = value, or

LIS_ADD_VALUE : A(i, j) = A(i, j) + value

must be provided for the parameter flag.

Assigning Storage Formats

To assign a storage format to the matrix A, the following functions are used:

- C LIS_INT lis_matrix_set_type(LIS_MATRIX A, LIS_INT matrix_type)
- Fortran subroutine lis_matrix_set_type(LIS_MATRIX A, LIS_INT matrix_type, LIS_INTEGER ierr)

 $\mathtt{matrix_type}$ of A is $\mathtt{LIS_MATRIX_CRS}$ when the matrix is created. The following storage formats are supported:

Storage format		matrix_type
Compressed Row Storage	(CRS)	{LIS_MATRIX_CRS 1}
Compressed Column Storage	(CCS)	{LIS_MATRIX_CCS 2}
Modified Compressed Sparse Row	(MSR)	{LIS_MATRIX_MSR 3}
Diagonal	(DIA)	{LIS_MATRIX_DIA 4}
Ellpack-Itpack Generalized Diagonal	(ELL)	{LIS_MATRIX_ELL 5}
Jagged Diagonal	(JDS)	{LIS_MATRIX_JDS 6}
Block Sparse Row	(BSR)	{LIS_MATRIX_BSR 7}
Block Sparse Column	(BSC)	{LIS_MATRIX_BSC 8}
Variable Block Row	(VBR)	{LIS_MATRIX_VBR 9}
Dense	(DNS)	{LIS_MATRIX_DNS 10}
Coordinate	(COO)	{LIS_MATRIX_COO 11}

Assembling Matrices

After assigning values and storage formats, the following functions must be called:

- C LIS_INT lis_matrix_assemble(LIS_MATRIX A)
- Fortran subroutine lis_matrix_assemble(LIS_MATRIX A, LIS_INTEGER ierr)

 $lis_matrix_assemble assembles A into the storage format specified by <math>lis_matrix_set_type$.

Destroying Matrices

To destroy the matrix, the following functions are used:

- C LIS_INT lis_matrix_destroy(LIS_MATRIX A)
- Fortran subroutine lis_matrix_destroy(LIS_MATRIX A, LIS_INTEGER ierr)

Method 2: Define Arrays in a Specific Storage Format Directly

In the case of creating the matrix A in Equation (3.2) in the CRS format, the matrix A itself is created for the serial and multithreaded environments, while the partial matrices are created and stored on the given number of processing elements for the multiprocessing environment.

Programs to create the matrix A in the CRS format are as follows, where the number of the processing elements for the multiprocessing environment is assumed to be two:

```
- C (for serial and multithreaded environments) —
 1: LIS_INT
                  i,k,n,nnz;
2: LIS_INT
                  *ptr,*index;
3: LIS_SCALAR
                  *value;
4: LIS_MATRIX
                 A;
5: n = 4; nnz = 10; k = 0;
6: lis_matrix_malloc_crs(n,nnz,&ptr,&index,&value);
7: lis_matrix_create(0,&A);
                                           /* or lis_matrix_set_size(A,n,0); */
8: lis_matrix_set_size(A,0,n);
9:
10: for(i=0;i<n;i++)
11: {
12:
        if( i>0 ) {index[k] = i-1; value[k] = 1; k++;}
13:
        index[k] = i; value[k] = 2; k++;
14:
        if( i < n-1 ) {index[k] = i+1; value[k] = 1; k++;}
15:
        ptr[i+1] = k;
16: }
17:
    ptr[0] = 0;
18:
    lis_matrix_set_crs(nnz,ptr,index,value,A);
19: lis_matrix_assemble(A);
```

```
- C (for multiprocessing environment) -
 1: LIS_INT
                  i,k,n,nnz,is,ie;
 2: LIS_INT
                  *ptr,*index;
3: LIS_SCALAR
                  *value;
 4: LIS_MATRIX
                 A;
 5: n = 2; nnz = 5; k = 0;
 6: lis_matrix_malloc_crs(n,nnz,&ptr,&index,&value);
 7: lis_matrix_create(MPI_COMM_WORLD,&A);
8: lis_matrix_set_size(A,n,0);
9: lis_matrix_get_range(A,&is,&ie);
10: for(i=is;i<ie;i++)</pre>
11: {
        if( i>0 ) {index[k] = i-1; value[k] = 1; k++;}
12:
        index[k] = i; value[k] = 2; k++;
        if( i < n-1 ) {index[k] = i+1; value[k] = 1; k++;}
14:
15:
        ptr[i-is+1] = k;
16: }
17: ptr[0] = 0;
18: lis_matrix_set_crs(nnz,ptr,index,value,A);
19: lis_matrix_assemble(A);
```

Associating Arrays

To associate the arrays in the CRS format with the matrix A, the following functions are used:

- C LIS_INT lis_matrix_set_crs(LIS_INT nnz, LIS_INT row[], LIS_INT index[], LIS_SCALAR value[], LIS_MATRIX A)
- Fortran subroutine lis_matrix_set_crs(LIS_INTEGER nnz, LIS_INTEGER row(), LIS_INTEGER index(), LIS_SCALAR value(), LIS_MATRIX A, LIS_INTEGER ierr)

Method 3: Read Matrix and Vector Data from External Files

Programs to read the matrix A in Equation (3.2) in the CRS format and vector b in Equation (3.1) from an external file are as follows:

```
C (for serial, multithreaded and multiprocessing environments)

1: LIS_MATRIX A;
2: LIS_VECTOR b,x;
3: lis_matrix_create(LIS_COMM_WORLD,&A);
4: lis_vector_create(LIS_COMM_WORLD,&b);
5: lis_vector_create(LIS_COMM_WORLD,&x);
6: lis_matrix_set_type(A,LIS_MATRIX_CRS);
7: lis_input(A,b,x,"matvec.mtx");
```

```
Fortran (for serial, multithreaded and multiprocessing environments)

1: LIS_MATRIX A

2: LIS_VECTOR b,x

3: call lis_matrix_create(LIS_COMM_WORLD,A,ierr)

4: call lis_vector_create(LIS_COMM_WORLD,b,ierr)

5: call lis_vector_create(LIS_COMM_WORLD,x,ierr)

6: call lis_matrix_set_type(A,LIS_MATRIX_CRS,ierr)

7: call lis_input(A,b,x,'matvec.mtx',ierr)
```

The content of the destination file matvec.mtx is as follows:

```
%%MatrixMarket matrix coordinate real general
```

```
4 4 10 1 0
1 2 1.0e+00
1 1 2.0e+00
2 3 1.0e+00
2 1 1.0e+00
2 2 2.0e+00
3 4 1.0e+00
3 2 1.0e+00
4 4 2.0e+00
4 3 1.0e+00
1 0.0e+00
2 1.0e+00
3 2.0e+00
4 3 3.0e+00
```

Reading from External Files

To input the matrix data for A from an external file, the following functions are used:

```
• C LIS_INT lis_input_matrix(LIS_MATRIX A, char *filename)
```

```
• Fortran subroutine lis_input(LIS_MATRIX A, character filename, LIS_INTEGER ierr)
```

filename must be replaced with the file path. The following file formats are supported:

- Matrix Market format
- Harwell-Boeing format

To read the data for the matrix A and vectors b and x from external files, the following functions are used:

- C LIS_INT lis_input(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x, char *filename)
- Fortran subroutine lis_input(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x, character filename, LIS_INTEGER ierr)

filename must be replaced with the file path. The following file formats are supported:

- Extended Matrix Market format (extended to allow vector data)
- Harwell-Boeing format

3.4 Solving Linear Equations

A program to solve the linear equation Ax = b with a specified solver is as follows:

```
C (for serial, multithreaded and multiprocessing environments)

1: LIS_MATRIX A;
2: LIS_VECTOR b,x;
3: LIS_SOLVER solver;
4:
5: /* Create matrix and vector */
6:
7: lis_solver_create(&solver);
8: lis_solver_set_option("-i bicg -p none",solver);
9: lis_solver_set_option("-tol 1.0e-12",solver);
10: lis_solver(A,b,x,solver);
```

```
Fortran (for serial, multithreaded and multiprocessing environments)

1: LIS_MATRIX A

2: LIS_VECTOR b,x

3: LIS_SOLVER solver

4:

5: /* Create matrix and vector */

6:

7: call lis_solver_create(solver,ierr)

8: call lis_solver_set_option('-i bicg -p none',solver,ierr)

9: call lis_solver_set_option('-tol 1.0e-12',solver,ierr)

10: call lis_solver(A,b,x,solver,ierr)
```

Creating Solvers

To create a solver, the following functions are used:

- C LIS_INT lis_solver_create(LIS_SOLVER *solver)
- Fortran subroutine lis_solver_create(LIS_SOLVER solver, LIS_INTEGER ierr)

Specifying Options

To specify options, the following functions are used:

- C LIS_INT lis_solver_set_option(char *text, LIS_SOLVER solver)
- Fortran subroutine lis_solver_set_option(character text, LIS_SOLVER solver, LIS_INTEGER ierr)

or

- C LIS_INT lis_solver_set_optionC(LIS_SOLVER solver)
- Fortran subroutine lis_solver_set_optionC(LIS_SOLVER solver, LIS_INTEGER ierr)

lis_solver_set_optionC is a function which sets the options specified on the command line, and pass them to solver when the program is run.

The table below shows the available command line options, where $-i \{cg|1\}$ means -i cg or -i 1 and -maxiter [1000] indicates that -maxiter defaults to 1,000.

Options for Linear Solvers (Default: -i bicg)

Options for Linear Solvers (Default: -1 blcg)							
Solver	Option	Auxiliary Options					
CG	-i {cg 1}						
BiCG	-i {bicg 2}						
CGS	-i {cgs 3}						
BiCGSTAB	-i {bicgstab 4}						
BiCGSTAB(1)	-i {bicgstabl 5}	-ell [2]	The degree l				
GPBiCG	-i {gpbicg 6}						
TFQMR	-i {tfqmr 7}						
Orthomin(m)	-i {orthomin 8}	-restart [40]	The restart value m				
GMRES(m)	-i {gmres 9}	-restart [40]	The restart value m				
Jacobi	-i {jacobi 10}						
Gauss-Seidel	-i {gs 11}						
SOR	-i {sor 12}	-omega [1.9]	The relaxation coefficient ω (0 < ω < 2)				
BiCGSafe	-i {bicgsafe 13}						
CR	-i {cr 14}						
BiCR	-i {bicr 15}						
CRS	-i {crs 16}						
BiCRSTAB	-i {bicrstab 17}						
GPBiCR	-i {gpbicr 18}						
BiCRSafe	-i {bicrsafe 19}						
FGMRES(m)	-i {fgmres 20}	-restart [40]	The restart value m				
IDR(s)	-i {idrs 21}	-irestart [2]	The restart value s				
MINRES	-i {minres 22}						

Options for Preconditioners (Default: -p none)

Preconditioner	Option	Auxiliary Options	
None	-p {none 0}		
Jacobi	-p {jacobi 1}		
ILU(k)	-p {ilu 2}	-ilu_fill [0]	The fill level k
SSOR	-p {ssor 3}	-ssor_w [1.0]	The relaxation coefficient ω (0 < ω < 2)
Hybrid	-p {hybrid 4}	-hybrid_i [sor]	The linear solver
		-hybrid_maxiter [25]	The maximum number of the iterations
		-hybrid_tol [1.0e-3]	The convergence criterion
		-hybrid_w [1.5]	The relaxation coefficient ω of the SOR $(0 < \omega < 2)$
		-hybrid_ell [2]	The degree l of the BiCGSTAB(l)
		-hybrid_restart [40]	The restart values of the GMRES
		-	and Orthomin
I+S	-p {is 5}	-is_alpha [1.0]	The parameter α of the preconditioner
			of the $I + \alpha S^{(m)}$ type
		-is_m [3]	The parameter m of the preconditioner
			of the $I + \alpha S^{(m)}$ type
SAINV	-p {sainv 6}	-sainv_drop [0.05]	The drop criterion
SA-AMG	-p {saamg 7}	-saamg_unsym [false]	Selects the unsymmetric version
			(The matrix structure must be symmetric)
		-saamg_theta [0.05 0.12]	The drop criterion $a_{ij}^2 \leq \theta^2 a_{ii} a_{jj} $
			(symmetric or unsymmetric)
Crout ILU	-p {iluc 8}	-iluc_drop [0.05]	The drop criterion
		-iluc_rate [5.0]	The ratio of the maximum fill-in
ILUT	-p {ilut 9}	-ilut_drop [0.05]	The drop criterion
		-ilut_rate [5.0]	The ratio of the maximum fill-in
Additive Schwarz	-adds true	-adds_iter [1]	The number of the iterations

Other Options

Option			
-maxiter [1000]	The maximum number of the iterations		
-tol [1.0e-12]	The convergence criterion		
-print [0]	The display of the residual		
	-print {none 0}	None	
	-print {mem 1}	Save the residual history	
	-print {out 2}	Display the residual history	
	-print {all 3}	Save the residual history and display it on the screen	
-scale [0]	The scaling		
	(The result will overwrite the original matrix and vectors)		
	-scale {none 0}		
	-scale {jacobi 1}	The Jacobi scaling $D^{-1}Ax = D^{-1}b$	
		(D represents the diagonal of $A = (a_{ij})$)	
	-scale {symm_diag 2} The diagonal scaling $D^{-1/2}AD^{-1/2}x = D^{-1/2}b$		
		$(D^{-1/2}$ represents the diagonal matrix with $1/\sqrt{a_{ii}}$	
		as the diagonal)	
-initx_zeros [true]	[true] The behavior of the initial vector x_0		
	-initx_zeros {false	(10) Given values	
	-initx_zeros {true 1} All values are set to 0		
-omp_num_threads [t]	The number of the threads		
	(t represents the maximum number of the threads)		
-storage [0]	The matrix storage format		
-storage_block [2]	The block size of the BSR and BSC		
-f [0]	The precision of the linear solvers		
	-f {double 0} Double precision		
	-f {quad 1}	Quadruple precision	

Solving Linear Equations

To solve the linear equation Ax = b, the following functions are used:

- C LIS_INT lis_solve(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x, LIS_SOLVER solver)
- Fortran subroutine lis_solve(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x, LIS_SOLVER solver, LIS_INTEGER ierr)

3.5 Solving Eigenvalue Problems

A program to solve the eigenvalue problem $Ax = \lambda x$ with a specified solver is as follows:

```
C (for serial, multithreaded and multiprocessing environments)

1: LIS_MATRIX A;

2: LIS_VECTOR x;

3: LIS_REAL evalue;

4: LIS_ESOLVER esolver;

5:

6: /* Create matrix and vector */

7:

8: lis_esolver_create(&esolver);

9: lis_esolver_set_option("-e ii -i bicg -p none",esolver);

10: lis_esolver_set_option("-etol 1.0e-12 -tol 1.0e-12",esolver);

11: lis_esolve(A,x,evalue,esolver);
```

```
Fortran (for serial, multithreaded and multiprocessing environments)

1: LIS_MATRIX A

2: LIS_VECTOR x

3: LIS_REAL evalue

4: LIS_ESOLVER esolver

5:

6: /* Create matrix and vector */

7:

8: call lis_esolver_create(esolver,ierr)

9: call lis_esolver_set_option('-e ii -i bicg -p none',esolver,ierr)

10: call lis_esolver_set_option('-etol 1.0e-12 -tol 1.0e-12',esolver,ierr)

11: call lis_esolve(A,x,evalue,esolver,ierr)
```

Creating Eigensolvers

To create an eigensolver, the following functions are used:

- C LIS_INT lis_esolver_create(LIS_ESOLVER *esolver)
- Fortran subroutine lis_esolver_create(LIS_ESOLVER esolver, LIS_INTEGER ierr)

Specifying Options

To specify options, the following functions are used:

- C LIS_INT lis_esolver_set_option(char *text, LIS_ESOLVER esolver)
- Fortran subroutine lis_esolver_set_option(character text, LIS_ESOLVER esolver, LIS_INTEGER ierr)

or

- C LIS_INT lis_esolver_set_optionC(LIS_ESOLVER esolver)
- Fortran subroutine lis_esolver_set_optionC(LIS_ESOLVER esolver, LIS_INTEGER ierr)

lis_esolver_set_optionC is a function which sets the options specified on the command line, and pass them to esolver when the program is run.

The table below shows the available command line options, where -e {pi|1} means -e pi or -e 1 and -emaxiter [1000] indicates that -emaxiter defaults to 1,000.

Ontions	for Fig	rensolvers	(Default		ni)	
Obtions	TOP PAIS	rensoivers	Петани	· –e	וומ	

Eigensolver	Option	Auxiliary Options	
Power	-e {pi 1}		
Inverse	-e {ii 2}	-i [bicg]	The linear solver
Approximate Inverse	-e {aii 3}		
Rayleigh Quotient	-e {rqi 4}	-i [bicg]	The linear solver
Subspace	-e {si 5}	-ss [2]	The size of the subspace
		-m [O]	The mode number
Lanczos	-e {li 6}	-ss [2]	The size of the subspace
		-m [O]	The mode number
CG	-e {cg 7} -e {cr 8}		
CR	-e {cr 8}		

Options for Preconditioners (Default: -p none)

Preconditioner	Option	Auxiliary Options	
None	-p {none 0}		
Jacobi	-p {jacobi 1}		
ILU(k)	-p {ilu 2}	-ilu_fill [0]	The fill level k
SSOR	-p {ssor 3}	-ssor_w [1.0]	The relaxation coefficient ω (0 < ω < 2)
Hybrid	-p {hybrid 4}	-hybrid_i [sor]	The linear solver
		-hybrid_maxiter [25]	The maximum number of the iterations
		-hybrid_tol [1.0e-3]	The convergence criterion
		-hybrid_w [1.5]	The relaxation coefficient ω of the SOR
			$(0 < \omega < 2)$
		-hybrid_ell [2]	The degree l of the BiCGSTAB(l)
		-hybrid_restart [40]	The restart values of the GMRES
			and Orthomin
I+S	-p {is 5}	-is_alpha [1.0]	The parameter α of the preconditioner
			of the $I + \alpha S^{(m)}$ type
		-is_m [3]	The parameter m of the preconditioner
			of the $I + \alpha S^{(m)}$ type
SAINV	-p {sainv 6}	-sainv_drop [0.05]	The drop criterion
SA-AMG	-p {saamg 7}	-saamg_unsym [false]	Selects the unsymmetric version
			(The matrix structure must be symmetric)
		-saamg_theta [0.05 0.12]	The drop criterion $a_{ij}^2 \le \theta^2 a_{ii} a_{jj} $
			(symmetric or unsymmetric)
Crout ILU	-p {iluc 8}	-iluc_drop [0.05]	The drop criterion
		-iluc_rate [5.0]	The ratio of the maximum fill-in
ILUT	-p {ilut 9}	-ilut_drop [0.05]	The drop criterion
		-ilut_rate [5.0]	The ratio of the maximum fill-in
Additive Schwarz	-adds true	-adds_iter [1]	The number of the iterations

Other Options

Option			
-emaxiter [1000]	The maximum number of the iterations		
-etol [1.0e-12]	The convergence criterion		
-eprint [0]	The display of the residual		
	-eprint {none 0}	None	
	-eprint {mem 1}	Save the residual history	
	-eprint {out 2}	Display the residual history	
	-eprint {all 3}	Save the residual history and display it on the screen	
-ie [ii]	The inner eigensolver us	sed in the Lanczos and Subspace	
	-ie {pi 1}	The Power (the Subspace only)	
	-ie {ii 2}	The Inverse	
	-ie {aii 3}	The Approximate Inverse	
	-ie {rqi 4}	The Rayleigh Quotient	
-shift [0.0]	The amount of the shift		
-initx_ones [true]	The behavior of the initial vector x_0		
	-initx_ones {false 0} Given values		
	-initx_ones {true 1}	All values are set to 1	
<pre>-omp_num_threads [t]</pre>	The number of the threads		
	(t represents the maximum number of the threads)		
-estorage [0]	The matrix storage format		
-estorage_block [2]	The block size of the BSR and BSC		
-ef [0]	The precision of the eigensolvers		
	-ef {double 0} Double precision		
	-ef {quad 1}	Quadruple precision	

Solving Eivenvalue Problems

To solve the eigenvalue problem $Ax = \lambda x$, the following functions are used:

- C LIS_INT lis_esolve(LIS_MATRIX A, LIS_VECTOR x, LIS_REAL evalue, LIS_ESOLVER esolver)
- Fortran subroutine lis_esolve(LIS_MATRIX A, LIS_VECTOR x, LIS_REAL evalue, LIS_ESOLVER esolver, LIS_INTEGER ierr)

3.6 Writing Programs

The following are the programs for solving the linear equation Ax = b, where the matrix A is a tridiagonal matrix

$$\begin{pmatrix}
2 & -1 & & & \\
-1 & 2 & -1 & & & \\
& \ddots & \ddots & \ddots & \\
& & -1 & 2 & -1 \\
& & & -1 & 2
\end{pmatrix}$$

of size 12. The the right hand side vector b is set to make the values of the elements of the solution x is 1. The program is located in the directory lis-(\$VERSION)/test.

```
- Test program: test4.c -
 1: #include <stdio.h>
 2: #include "lis.h"
 3: main(LIS_INT argc, char *argv[])
 4: {
5:
        LIS_INT i,n,gn,is,ie,iter;
        LIS_MATRIX A;
 6:
        LIS_VECTOR b,x,u;
7:
        LIS_SOLVER solver;
8:
9:
        n = 12;
10:
        lis_initialize(&argc,&argv);
11:
        lis_matrix_create(LIS_COMM_WORLD,&A);
12:
        lis_matrix_set_size(A,0,n);
13:
        lis_matrix_get_size(A,&n,&gn)
14:
        lis_matrix_get_range(A,&is,&ie)
        for(i=is;i<ie;i++)</pre>
15:
16:
17:
            if( i>0 ) lis_matrix_set_value(LIS_INS_VALUE,i,i-1,-1.0,A);
18:
            if( i<gn-1 ) lis_matrix_set_value(LIS_INS_VALUE,i,i+1,-1.0,A);</pre>
19:
            lis_matrix_set_value(LIS_INS_VALUE,i,i,2.0,A);
20:
        lis_matrix_set_type(A,LIS_MATRIX_CRS);
21:
        lis_matrix_assemble(A);
22:
23:
24:
        lis_vector_duplicate(A,&u);
25:
        lis_vector_duplicate(A,&b);
26:
        lis_vector_duplicate(A,&x);
27:
        lis_vector_set_all(1.0,u);
28:
        lis_matvec(A,u,b);
29:
30:
        lis_solver_create(&solver);
31:
        lis_solver_set_optionC(solver);
32:
        lis_solve(A,b,x,solver);
33:
        lis_solver_get_iters(solver,&iter);
34:
        printf("iter = %d\n",iter);
35:
        lis_vector_print(x);
36:
        lis_matrix_destroy(A);
37:
        lis_vector_destroy(u);
38:
        lis_vector_destroy(b);
39:
        lis_vector_destroy(x);
40:
        lis_solver_destroy(solver);
41:
        lis_finalize();
42:
        return 0;
43: }
}
```

```
Test program: test4f.F -
 1:
         implicit none
 3:#include "lisf.h"
 4:
 5:
         LIS_INTEGER
                           i,n,gn,is,ie,iter,ierr
         LIS_MATRIX
 6:
                           Α
         LIS_VECTOR
7:
                           b,x,u
         LIS_SOLVER
 8٠
                            solver
         n = 12
9:
10:
         call lis_initialize(ierr)
11:
         call lis_matrix_create(LIS_COMM_WORLD, A, ierr)
12:
         call lis_matrix_set_size(A,0,n,ierr)
13:
         call lis_matrix_get_size(A,n,gn,ierr)
14:
         call lis_matrix_get_range(A,is,ie,ierr)
15:
         do i=is,ie-1
           if( i>1 ) call lis_matrix_set_value(LIS_INS_VALUE,i,i-1,-1.0d0,
16:
17:
                                                   A,ierr)
           if( i<gn ) call lis_matrix_set_value(LIS_INS_VALUE,i,i+1,-1.0d0,</pre>
18:
19:
                                                   A,ierr)
           call lis_matrix_set_value(LIS_INS_VALUE,i,i,2.0d0,A,ierr)
20:
21:
         enddo
22:
         call lis_matrix_set_type(A,LIS_MATRIX_CRS,ierr)
23:
         call lis_matrix_assemble(A,ierr)
24:
25:
         call lis_vector_duplicate(A,u,ierr)
26:
         call lis_vector_duplicate(A,b,ierr)
27:
         call lis_vector_duplicate(A,x,ierr)
28:
         call lis_vector_set_all(1.0d0,u,ierr)
29:
         call lis_matvec(A,u,b,ierr)
30:
31:
         call lis_solver_create(solver,ierr)
32:
         call lis_solver_set_optionC(solver,ierr)
33:
         call lis_solve(A,b,x,solver,ierr)
34:
         call lis_solver_get_iters(solver,iter,ierr)
35:
         write(*,*) 'iter = ',iter
36:
         call lis_vector_print(x,ierr)
37:
         call lis_matrix_destroy(A,ierr)
38:
         call lis_vector_destroy(b,ierr)
         call lis_vector_destroy(x,ierr)
39:
40:
         call lis_vector_destroy(u,ierr)
         call lis_solver_destroy(solver,ierr)
41:
42:
         call lis_finalize(ierr)
43:
44:
         stop
45:
         end
```

3.7 Compiling and Linking

Provided below is an example test4.c located in the directory lis-(\$VERSION)/test, compiled on the SGI Altix 3700 using the Intel C/C++ Compiler 8.1 (icc). Since the library includes some Fortran 90 codes when the SA-AMG preconditioner is selected, a Fortran 90 compiler must be used for the linking. The preprocessor macro USE_MPI must be defined for the multiprocessing environment.

```
Compiling

>icc -c -I$(INSTALLDIR)/include test4.c

Linking

>icc -o test4 test4.o -llis

Linking (with SA-AMG)

>ifort -nofor_main -o test4 test4.o -llis

For multithreaded environment

Compiling
```

```
Compiling

>icc -c -openmp -I$(INSTALLDIR)/include test4.c

Linking

>icc -openmp -o test4 test4.o -llis

Linking (with SA-AMG)

>ifort -nofor_main -openmp -o test4 test4.o -llis
```

· For multiprocessing environment –

```
Compiling

>icc -c -DUSE_MPI -I$(INSTALLDIR)/include test4.c

Linking

>icc -o test4 test4.o -llis -lmpi

Linking (with SA-AMG)

>ifort -nofor_main -o test4 test4.o -llis -lmpi
```

- For multithreaded and multiprocessing environments —

```
Compiling

>icc -c -openmp -DUSE_MPI -I$(INSTALLDIR)/include test4.c

Linking

>icc -openmp -o test4 test4.o -llis -lmpi

Linking (with SA-AMG)

>ifort -nofor_main -openmp -o test4 test4.o -llis -lmpi
```

Provided below is an example test4f.F located in the directory lis-(\$VERSION)/test, compiled on the SGI Altix 3700 using the Intel Fortran Compiler 8.1 (ifort). Since an include statement is used in the program, the compiler option -fpp is specified to use the preprocessor.

```
For serial environment

Compiling

>ifort -c -fpp -I$(INSTALLDIR)/include test4f.F

Linking

>ifort -o test4 test4.o -llis
```

```
- For multithreaded environment

Compiling

>ifort -c -fpp -openmp -I$(INSTALLDIR)/include test4f.F

Linking

>ifort -openmp -o test4 test4.o -llis
```

3.8 Running

The test programs test4 and test4f in the directory lis-(\$VERSION)/test are run as follows:

```
For serial environment
```

>./test4 -i bicgstab

For multithreaded environment

>env OMP_NUM_THREADS=2 ./test4 -i bicgstab

For multiprocessing environment

>mpirun -np 2 ./test4 -i bicgstab

For multithreaded and multiprocessing environment

>mpirun -np 2 env OMP_NUM_THREADS=2 ./test4 -i bicgstab

The following results will be returned:

precision : double solver : BiCGSTAB 4 precon : none storage : CRS

lis_solve : normal end

iter = 6

- 0 1.000000e+000
- 1 1.000000e+000
- 2 1.000000e+000
- 3 1.000000e+000
- 4 1.000000e+000
- 5 1.000000e+000
- 6 1.000000e+000 7 1.000000e+000
- 8 1.000000e+000
- 9 1.000000e+000
- 10 1.000000e+000
- 11 1.000000e+000

4 Quadruple Precision Operations

Double precision operations sometimes require a large number of iterations because of the rounding error. Lis supports "double-double", or quadruple precision operations by combining two double precision floating point numbers[15, 16]. To use the quadruple precision with the same interface as the double precision operations, both the matrix and vectors are assumed to be double precision. Lis also supports the performance acceleration of the quadruple precision operations with the SIMD instructions, such as Intel's Streaming SIMD Extensions (SSE)[24].

4.1 Using Quadruple Precision Operations

The test program test5.c solves a linear equation Ax = b, where A is a Toeplitz matrix

$$\begin{pmatrix} 2 & 1 & & & & \\ 0 & 2 & 1 & & & \\ \gamma & 0 & 2 & 1 & & \\ & \ddots & \ddots & \ddots & \ddots & \\ & & \gamma & 0 & 2 & 1 \\ & & & \gamma & 0 & 2 \end{pmatrix}.$$

The right hand vector is set to make the values of the elements of the solution to be 1. The value n is the size of the matrix A. test5 with -f option is run as follows:

Double precision

```
By entering >./test5 200 2.0 -f double the following results will be returned:
```

```
n = 200, gamma = 2.000000
initial vector x = 0
precision : double
solver
          : BiCG 2
precon
          : none
          : CRS
storage
lis_solve : LIS_MAXITER(code=4)
BiCG: number of iterations
                                 = 1001 \text{ (double } = 1001, \text{ quad } = 0)
BiCG: elapsed time
                                  = 2.044368e-02 sec.
BiCG:
        preconditioner
                                 = 4.768372e-06 \text{ sec.}
                                 = 4.768372e-06 \text{ sec.}
BiCG:
          matrix creation
BiCG:
        linear solver
                                  = 2.043891e-02 sec.
BiCG: relative residual 2-norm = 8.917591e+01
```

Quadruple precision

By entering >./test5 200 2.0 -f quad the following results will be returned:

```
n = 200, gamma = 2.000000
initial vector x = 0
precision : quad
solver : BiCG 2
precon : none
storage : CRS
lis_solve : normal end
```

BiCG: number of iterations = 230 (double = 230, quad = 0)
BiCG: elapsed time = 2.267408e-02 sec.
BiCG: preconditioner = 4.549026e-04 sec.
BiCG: matrix creation = 5.006790e-06 sec.
BiCG: linear solver = 2.221918e-02 sec.

5 Matrix Storage Formats

This section describes the matrix storage formats supported by the library. Assume that the matrix row (column) number begins with 0 and that the number of the nonzero elements of the matrix A of size $n \times n$ is nnz.

5.1 Compressed Row Storage (CRS)

The CRS format uses three arrays ptr, index and value to store data.

- value is a double precision array with a length of nnz, which stores the nonzero elements of the matrix A along the row.
- index is an integer array with a length of nnz, which stores the column numbers of the nonzero elements stored in the array value.
- ptr is an integer array with a length of n + 1, which stores the starting points of the rows of the arrays value and index.

5.1.1 Creating Matrices (for Serial and Multithreaded Environments)

The right diagram in Figure 2 shows how the matrix A in Figure 2 is stored in the CRS format. A program to create the matrix in the CRS format is as follows:

$$A = \begin{pmatrix} 11 & & & & \\ 21 & 22 & & & \\ & 32 & 33 & \\ 41 & & 43 & 44 \end{pmatrix} \begin{pmatrix} 0 & 1 & 3 & 5 & 8 \\ \hline 0 & 0 & 1 & 1 & 2 & 0 & 2 & 3 \\ \hline 11 & 21 & 22 & 32 & 33 & 41 & 43 & 44 \end{pmatrix} & \text{A.ptr}$$

$$A \cdot \text{ptr}$$

Figure 2: Data structure of CRS format (for serial and multithreaded environments).

```
For serial and multithreaded environments
 1: LIS_INT
                  n,nnz;
 2: LIS_INT
                  *ptr,*index;
 3: LIS_SCALAR
                  *value;
 4: LIS_MATRIX
 5: n = 4; nnz = 8;
        = (LIS_INT *)malloc( (n+1)*sizeof(LIS_INT) );
7: index = (LIS_INT *)malloc( nnz*sizeof(LIS_INT) );
8: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
9: lis_matrix_create(0,&A);
10: lis_matrix_set_size(A,0,n);
12: ptr[0] = 0; ptr[1] = 1; ptr[2] = 3; ptr[3] = 5; ptr[4] = 8;
13: index[0] = 0; index[1] = 0; index[2] = 1; index[3] = 1;
14: index[4] = 2; index[5] = 0; index[6] = 2; index[7] = 3;
15: value[0] = 11; value[1] = 21; value[2] = 22; value[3] = 32;
16: value[4] = 33; value[5] = 41; value[6] = 43; value[7] = 44;
17:
18: lis_matrix_set_crs(nnz,ptr,index,value,A);
19: lis_matrix_assemble(A);
```

5.1.2 Creating Matrices (for Multiprocessing Environment)

Figure 3 shows how the matrix A in Figure 2 is stored in the CRS format on two processing elements. A program to create the matrix in the CRS format on two processing elements is as follows:

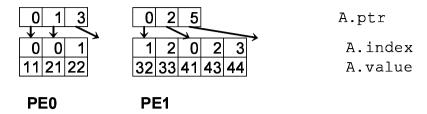


Figure 3: Data structure of CRS format (for multiprocessing environment).

```
For multiprocessing environment -
 1: LIS_INT
                  i,k,n,nnz,my_rank;
 2: LIS_INT
                  *ptr,*index;
 3: LIS_SCALAR
                  *value;
 4: LIS_MATRIX
                  Α;
 5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
 6: if( my_rank==0 ) {n = 2; nnz = 3;}
                     {n = 2; nnz = 5;}
7: else
8: ptr
          = (LIS_INT *)malloc( (n+1)*sizeof(LIS_INT) );
9: index = (LIS_INT *)malloc( nnz*sizeof(LIS_INT) );
10: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
11: lis_matrix_create(MPI_COMM_WORLD,&A);
12: lis_matrix_set_size(A,n,0);
13: if( my_rank==0 ) {
        ptr[0] = 0; ptr[1] = 1; ptr[2] = 3;
14:
15:
        index[0] = 0; index[1] = 0; index[2] = 1;
16:
        value[0] = 11; value[1] = 21; value[2] = 22;}
17: else {
        ptr[0] = 0; ptr[1] = 2; ptr[2] = 5;
18:
        index[0] = 1; index[1] = 2; index[2] = 0; index[3] = 2; index[4] = 3;
19:
        value[0] = 32; value[1] = 33; value[2] = 41; value[3] = 43; value[4] = 44;}
20:
21: lis_matrix_set_crs(nnz,ptr,index,value,A);
    lis_matrix_assemble(A);
```

5.1.3 Associating Arrays

To associate the arrays in the CRS format with the matrix A, the following functions are used:

- C LIS_INT lis_matrix_set_crs(LIS_INT nnz, LIS_INT row[], LIS_INT index[], LIS_SCALAR value[], LIS_MATRIX A)
- Fortran subroutine lis_matrix_set_crs(LIS_INTEGER nnz, LIS_INTEGER row(), LIS_INTEGER index(), LIS_SCALAR value(), LIS_MATRIX A, LIS_INTEGER ierr)

5.2 Compressed Column Storage (CCS)

The CSS format uses three arrays ptr, index and value to store data.

- value is a double precision array with a length of nnz, which stores the values of the nonzero elements of the matrix A along the column.
- index is an integer array with a length of nnz, which stores the row numbers of the nonzero elements stored in the array value.
- ptr is an integer array with a length of n + 1, which stores the starting points of the rows of the arrays value and index.

5.2.1 Creating Matrices (for Serial and Multithreaded Environments)

The right diagram in Figure 4 shows how the matrix A in Figure 4 is stored in the CCS format. A program to create the matrix in the CCS format is as follows:

Figure 4: Data structure of CCS format (for serial and multithreaded environments).

```
For serial and multithreaded environments –
 1: LIS_INT
                 n,nnz;
 2: LIS_INT
                  *ptr,*index;
                  *value;
3: LIS_SCALAR
 4: LIS_MATRIX
                  Α;
5: n = 4; nnz = 8;
6: ptr = (LIS_INT *)malloc( (n+1)*sizeof(LIS_INT) );
7: index = (LIS_INT *)malloc( nnz*sizeof(LIS_INT) );
8: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
9: lis_matrix_create(0,&A);
10: lis_matrix_set_size(A,0,n);
11:
12: ptr[0] = 0; ptr[1] = 3; ptr[2] = 5; ptr[3] = 7; ptr[4] = 8;
13: index[0] = 0; index[1] = 1; index[2] = 3; index[3] = 1;
14: index[4] = 2; index[5] = 2; index[6] = 3; index[7] = 3;
15: value[0] = 11; value[1] = 21; value[2] = 41; value[3] = 22;
16: value[4] = 32; value[5] = 33; value[6] = 43; value[7] = 44;
17:
18:
    lis_matrix_set_ccs(nnz,ptr,index,value,A);
    lis_matrix_assemble(A);
```

5.2.2 Creating Matrices (for Multiprocessing Environment)

Figure 5 shows how the matrix A in Figure 4 is stored on two processing elements. A program to create the matrix in the CCS format on two processing elements is as follows:

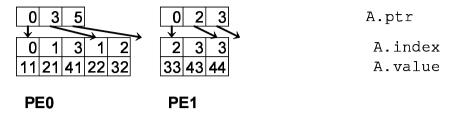


Figure 5: Data structure of CCS format (for multiprocessing environment).

```
For multiprocessing environment -
 1: LIS_INT
                  i,k,n,nnz,my_rank;
 2: LIS_INT
                  *ptr,*index;
 3: LIS_SCALAR
                  *value;
 4: LIS_MATRIX
                  Α;
 5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
 6: if( my_rank==0 ) {n = 2; nnz = 3;}
                     {n = 2; nnz = 5;}
7: else
          = (LIS_INT *)malloc( (n+1)*sizeof(LIS_INT) );
8: ptr
9: index = (LIS_INT *)malloc( nnz*sizeof(LIS_INT) );
10: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
11: lis_matrix_create(MPI_COMM_WORLD,&A);
12: lis_matrix_set_size(A,n,0);
13: if( my_rank==0 ) {
        ptr[0] = 0; ptr[1] = 3; ptr[2] = 5;
14:
15:
        index[0] = 0; index[1] = 1; index[2] = 3; index[3] = 1; index[4] = 2;
        value[0] = 11; value[1] = 21; value[2] = 41; value[3] = 22; value[4] = 32}
16:
17: else {
        ptr[0] = 0; ptr[1] = 2; ptr[2] = 3;
18:
19:
        index[0] = 2; index[1] = 3; index[2] = 3;
        value[0] = 33; value[1] = 43; value[2] = 44;}
20:
    lis_matrix_set_ccs(nnz,ptr,index,value,A);
21:
    lis_matrix_assemble(A);
```

5.2.3 Associating Arrays

To associate the arrays in the CCS format with the matrix A, the following functions are used:

- C LIS_INT lis_matrix_set_ccs(LIS_INT nnz, LIS_INT row[], LIS_INT index[], LIS_SCALAR value[], LIS_MATRIX A)
- Fortran subroutine lis_matrix_set_ccs(LIS_INTEGER nnz, LIS_INTEGER row(), LIS_INTEGER index(), LIS_SCALAR value(), LIS_MATRIX A, LIS_INTEGER ierr)

5.3 Modified Compressed Sparse Row (MSR)

The MSR format uses two arrays index and value to store data. Assume that ndz represents the number of the zero elements of the diagonal.

- value is a double precision array with a length of nnz + ndz + 1, which stores the diagonal of the matrix A down to the *n*-th element. The n + 1-th element is not used. For the n + 2-th and after, the values of the nonzero elements except the diagonal of the matrix A are stored along the row.
- index is an integer array with a length of nnz + ndz + 1, which stores the starting points of the rows of the off-diagonal elements of the matrix A down to the n + 1-th element. For the n + 2-th and after, it stores the row numbers of the off-diagonal elements of the matrix A stored in the array value.

5.3.1 Creating Matrices (for Serial and Multithreaded Environments)

The right diagram in Figure 6 shows how matrix A is stored in the MSR format. A program to create the matrix in the MSR format is as follows:

Figure 6: Data structure of MSR format (for serial and multithreaded environments).

```
For serial and multithreaded environments
 1: LIS_INT
                  n,nnz,ndz;
 2: LIS_INT
                  *index;
 3: LIS_SCALAR
                  *value;
 4: LIS_MATRIX
                  A;
 5: n = 4; nnz = 8; ndz = 0;
 6: index = (LIS_INT *)malloc( (nnz+ndz+1)*sizeof(LIS_INT) );
7: value = (LIS_SCALAR *)malloc( (nnz+ndz+1)*sizeof(LIS_SCALAR) );
8: lis_matrix_create(0,&A);
9: lis_matrix_set_size(A,0,n);
10:
11: index[0] = 5; index[1] = 5; index[2] = 6; index[3] = 7;
12: index[4] = 9; index[5] = 0; index[6] = 1; index[7] = 0; index[8] =
13: value[0] = 11; value[1] = 22; value[2] = 33; value[3] = 44;
14: value[4] = 0; value[5] = 21; value[6] = 32; value[7] = 41; value[8] = 43;
15:
16:
    lis_matrix_set_msr(nnz,ndz,index,value,A);
17: lis_matrix_assemble(A);
```

5.3.2 Creating Matrices (for Multiprocessing Environment)

Figure 7 shows how the matrix A in Figure 6 is stored in the MSR format on two processing elements. A program to create the matrix in the MSR format on two processing element is as follows:

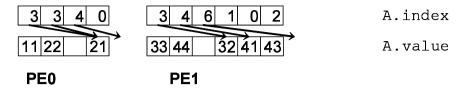


Figure 7: Data structure of MSR format (for multiprocessing environment).

```
For multiprocessing environment -
 1: LIS_INT
                  i,k,n,nnz,ndz,my_rank;
 2: LIS_INT
                  *index;
 3: LIS_SCALAR
                  *value;
 4: LIS_MATRIX
                  A;
 5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
 6: if( my_rank==0 ) {n = 2; nnz = 3; ndz = 0;}
                     {n = 2; nnz = 5; ndz = 0;}
7: else
8: index = (LIS_INT *)malloc( (nnz+ndz+1)*sizeof(LIS_INT) );
9: value = (LIS_SCALAR *)malloc( (nnz+ndz+1)*sizeof(LIS_SCALAR) );
10: lis_matrix_create(MPI_COMM_WORLD,&A);
11: lis_matrix_set_size(A,n,0);
12: if( my_rank==0 ) {
13:
        index[0] = 3; index[1] = 3; index[2] = 4; index[3] = 0;
        value[0] = 11; value[1] = 22; value[2] = 0; value[3] = 21;}
14:
15: else {
        index[0] = 3; index[1] = 4; index[2] = 6; index[3] = 1;
16:
        index[4] = 0; index[5] = 2;
17:
18:
        value[0] = 33; value[1] = 44; value[2] = 0; value[3] = 32;
        value[4] = 41; value[5] = 43;}
19:
    lis_matrix_set_msr(nnz,ndz,index,value,A);
20:
    lis_matrix_assemble(A);
```

5.3.3 Associating Arrays

To associate the arrays in the MSR format with the matrix A, the following functions are used:

- C LIS_INT lis_matrix_set_msr(LIS_INT nnz, LIS_INT ndz, LIS_INT index[], LIS_SCALAR value[], LIS_MATRIX A)
- Fortran subroutine lis_matrix_set_msr(LIS_INTEGER nnz, LIS_INTEGER ndz, LIS_INTEGER index(), LIS_SCALAR value(), LIS_MATRIX A, LIS_INTEGER ierr)

Diagonal (DIA)

The DIA format uses two arrays index and value to store data. Assume that nnd represents the number of the nonzero diagonal elements of the matrix A.

- value is a double precision array with a length of $nnd \times n$, which stores the values of the nonzero diagonal elements of the matrix A.
- index is an integer array with a length of nnd, which stores the offsets from the main diagonal.

For the multithreaded environment, the following modifications have been made: the format uses two arrays index and value to store data. Assume that nprocs represents the number of the threads. nnd_n is the number of the nonzero diagonal elements of the partial matrix into which the row block of the matrix A is divided. maxnnd is the maximum value nnd_n .

- value is a double precision array with a length of $maxnnd \times n$, which stores the values of the nonzero diagonal elements of the matrix A.
- index is an integer array with a length of $nprocs \times maxnnd$, which stores the offsets from the main diagonal.

5.4.1 Creating Matrices (for Serial Environment)

The right diagram in Figure 8 shows how the matrix A in Figure 8 is stored in the DIA format. A program to create the matrix in the DIA format is as follows:

Figure 8: Data structure of DIA format (for serial environment).

```
For serial environment -
 1: LIS_INT
                  n, nnd;
 2: LIS_INT
                  *index;
 3: LIS_SCALAR
                  *value:
 4: LIS_MATRIX
                  A;
 5: n = 4; nnd = 3;
 6: index = (LIS_INT *)malloc( nnd*sizeof(LIS_INT) );
7: value = (LIS_SCALAR *)malloc( n*nnd*sizeof(LIS_SCALAR) );
8: lis_matrix_create(0,&A);
9: lis_matrix_set_size(A,0,n);
10:
11: index[0] = -3; index[1] = -1; index[2] = 0;
12: value[0] = 0; value[1] = 0; value[2] = 0; value[3] = 41;
13: value[4] = 0; value[5] = 21; value[6] = 32; value[7] = 43;
14: value[8] = 11; value[9] = 22; value[10] = 33; value[11] = 44;
15:
     lis_matrix_set_dia(nnd,index,value,A);
16:
    lis_matrix_assemble(A);
```

5.4.2 Creating Matrices (for Multithreaded Environment)

Figure 9 shows how the matrix A in Figure 8 is stored in the DIA format on two threads. A program to create the matrix in the DIA format on two threads is as follows:

-1	0		-3	-	0							A.index
0	21	11	22			0	41	32	43	33	44	A.value

Figure 9: Data structure of DIA format (for multithreaded environment).

```
For multithreaded environment —
 1: LIS_INT
                 n, maxnnd, nprocs;
 2: LIS_INT
                  *index;
3: LIS_SCALAR
                  *value;
4: LIS_MATRIX
                 Α;
5: n = 4; maxnnd = 3; nprocs = 2;
6: index = (LIS_INT *)malloc( maxnnd*sizeof(LIS_INT) );
7: value = (LIS_SCALAR *)malloc( n*maxnnd*sizeof(LIS_SCALAR) );
8: lis_matrix_create(0,&A);
9: lis_matrix_set_size(A,0,n);
10:
11: index[0] = -1; index[1] = 0; index[2] = 0; index[3] = -3; index[4] = -1; index[5] = 0;
12: value[0] = 0; value[1] = 21; value[2] = 11; value[3] = 22; value[4] = 0; value[5] = 0;
13: value[6] = 0; value[7] = 41; value[8] = 32; value[9] = 43; value[10] = 33; value[11] = 44;
14:
15: lis_matrix_set_dia(maxnnd,index,value,A);
16: lis_matrix_assemble(A);
```

5.4.3 Creating Matrices (for Multiprocessing Environment)

Figure 10 shows how the matrix A in Figure 8 is stored in the DIA format on two processing elements. A program to create the matrix in the DIA format on two processing elements is as follows:



Figure 10: Data structure of DIA format (for multiprocessing environment).

```
For multiprocessing environment —
 1: LIS_INT
                  i,n,nnd,my_rank;
 2: LIS_INT
                  *index;
3: LIS_SCALAR
                  *value;
 4: LIS_MATRIX
                  A:
 5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
 6: if( my_rank==0 ) {n = 2; nnd = 2;}
                     {n = 2; nnd = 3;}
8: index = (LIS_INT *)malloc( nnd*sizeof(LIS_INT) );
9: value = (LIS_SCALAR *)malloc( n*nnd*sizeof(LIS_SCALAR) );
10: lis_matrix_create(MPI_COMM_WORLD,&A);
11: lis_matrix_set_size(A,n,0);
12: if( my_rank==0 ) {
13:
        index[0] = -1; index[1] = 0;
        value[0] = 0; value[1] = 21; value[2] = 11; value[3] = 22;}
14:
15: else {
        index[0] = -3; index[1] = -1; index[2] = 0;
16:
17:
        value[0] = 0; value[1] = 41; value[2] = 32; value[3] = 43; value[4] = 33;
18:
        value[5] = 44;
19:
     lis_matrix_set_dia(nnd,index,value,A);
20:
     lis_matrix_assemble(A);
```

5.4.4 Associating Arrays

To associate the arrays in the DIA format with the matrix A, the following functions are used:

- C LIS_INT lis_matrix_set_dia(LIS_INT nnd, LIS_INT index[], LIS_SCALAR value[], LIS_MATRIX A)
- Fortran subroutine lis_matrix_set_dia(LIS_INTEGER nnd, LIS_INTEGER index(), LIS_SCALAR value(), LIS_MATRIX A, LIS_INTEGER ierr)

5.5 Ellpack-Itpack Generalized Diagonal (ELL)

The ELL format uses two arrays index and value to store data. Assume that maxnzr is the maximum value of the number of the nonzero elements in the rows of the matrix A.

- value is a double precision array with a length of $maxnzr \times n$, which stores the values of the nonzero elements of the rows of the matrix A along the column. The first column consists of the first nonzero elements of each row. If there is no nonzero elements to be stored, then 0 is stored.
- index is an integer array with a length of $maxnzr \times n$, which stores the column numbers of the nonzero elements stored in the array value. If the number of the nonzero elements in the *i*-th row is nnz, then index $[nnz \times n + i]$ stores row number *i*.

5.5.1 Creating Matrices (for Serial and Multithreaded Environments)

The right diagram in Figure 11 shows how the matrix A in Figure 11 is stored in the ELL format. A program to create the matrix in the ELL format is as follows:

Figure 11: Data structure of ELL format (for serial and multithreaded environments).

```
For serial and multithreaded environments -
 1: LIS_INT
                    n,maxnzr;
 2: LIS_INT
                    *index;
 3: LIS_SCALAR
                    *value;
 4: LIS_MATRIX
                    A;
 5: n = 4; maxnzr = 3;
 6: index = (LIS_INT *)malloc( n*maxnzr*sizeof(LIS_INT) );
 7: value = (LIS_SCALAR *)malloc( n*maxnzr*sizeof(LIS_SCALAR) );
 8: lis_matrix_create(0,&A);
 9: lis_matrix_set_size(A,0,n);
11: index[0] = 0; index[1] = 0; index[2] = 1; index[3] = 0; index[4] = 0; index[5] = 1; 12: index[6] = 2; index[7] = 2; index[8] = 0; index[9] = 1; index[10] = 2; index[11] = 3;
13: value[0] = 11; value[1] = 21; value[2] = 32; value[3] = 41; value[4] = 0; value[5] = 22;
14: value[6] = 33; value[7] = 43; value[8] = 0; value[9] = 0; value[10] = 0; value[11] = 44;
15:
16:
     lis_matrix_set_ell(maxnzr,index,value,A);
     lis_matrix_assemble(A);
```

5.5.2 Creating Matrices (for Multiprocessing Environment)

Figure 12 shows how the matrix A in Figure 11 is stored in the ELL format. A program to create the matrix in the ELL format on two processing elements is as follows:

0 0 0 1	1 0 2 2 3	A.index
11 21 0 22	32 41 33 43 0 44	A.value
PE0	PE1	

Figure 12: Data structure of ELL format (for multiprocessing environment).

```
For multiprocessing environment —
 1: LIS_INT
                  i,n,maxnzr,my_rank;
 2: LIS_INT
                  *index;
3: LIS_SCALAR
                  *value;
4: LIS_MATRIX
                  A:
 5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
 6: if( my_rank==0 ) {n = 2; maxnzr = 2;}
                     {n = 2; maxnzr = 3;}
8: index = (LIS_INT *)malloc( n*maxnzr*sizeof(LIS_INT) );
9: value = (LIS_SCALAR *)malloc( n*maxnzr*sizeof(LIS_SCALAR) );
10: lis_matrix_create(MPI_COMM_WORLD,&A);
11: lis_matrix_set_size(A,n,0);
12: if( my_rank==0 ) {
13:
        index[0] = 0; index[1] = 0; index[2] = 0; index[3] = 1;
        value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 22;}
14:
15: else {
        index[0] = 1; index[1] = 0; index[2] = 2; index[3] = 2; index[4] = 2;
16:
17:
        index[5] = 3;
18:
        value[0] = 32; value[1] = 41; value[2] = 33; value[3] = 43; value[4] = 0;
19:
        value[5] = 44;}
20:
    lis_matrix_set_ell(maxnzr,index,value,A);
21:
    lis_matrix_assemble(A);
```

5.5.3 Associating Arrays

To associate an array required by the ELL format with the matrix A, the following functions are used:

- C LIS_INT lis_matrix_set_ell(LIS_INT maxnzr, LIS_INT index[], LIS_SCALAR value[], LIS_MATRIX A)
- Fortran subroutine lis_matrix_set_ell(LIS_INTEGER maxnzr, LIS_INTEGER index(), LIS_SCALAR value(), LIS_MATRIX A, LIS_INTEGER ierr)

5.6 Jagged Diagonal (JDS)

The JDS format first sorts the nonzero elements of the rows in decreasing order of size, and then stores them along the column. The JDS format uses four arrays perm, ptr, index and value to store data. Assume that maxnzr represents the maximum value of the number of the nonzero elements of the matrix A.

- perm is an integer array with a length of n, which stores the sorted row numbers.
- value is a double precision array with a length of nnz, which stores the values of the jagged diagonal elements of the sorted matrix A. The first jagged diagonal consists of the values of the first nonzero elements of each row. The next jagged diagonal consists of the values of the second nonzero elements, and so on.
- index is an integer array with a length of nnz, which stores the row numbers of the nonzero elements stored in the array value.
- ptr is an integer array with a length of maxnzr + 1, which stores the starting points of the jagged diagonal elements.

For the multithreaded environment, the following modifications have been made: the format uses four arrays perm, ptr, index and value to store data. Assume that nprocs is the number of the threads. $maxnzr_p$ is the number of the nonzero diagonal elements of the partial matrix into which the row block of the matrix A is divided. maxmaxnzr is the maximum value of $maxnzr_p$.

- perm is an integer array with a length of n, which stores the sorted row numbers.
- value is a double precision array with a length of nnz, which stores the values of the jagged diagonal elements of the sorted matrix A. The first jagged diagonal consists of the values of the first nonzero elements of each row. The next jagged diagonal consist of the values of the second nonzero elements of each row, and so on.
- index is an integer array with a length of nnz, which stores the row numbers of the nonzero elements stored in the array value.
- ptr is an integer array with a length of $nprocs \times (maxmaxnzr+1)$, which stores the starting points of the jagged diagonal elements.

5.6.1 Creating Matrices (for Serial Environment)

The right diagram in Figure 13 shows how the matrix A in Figure 13 is stored in the JDS format. A program to create the matrix in the JDS format is as follows:

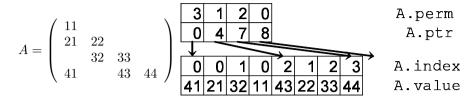


Figure 13: Data structure of JDS format (for serial environment).

```
For serial environment -
 1: LIS_INT
                 n,nnz,maxnzr;
 2: LIS_INT
                  *perm,*ptr,*index;
3: LIS_SCALAR
                  *value;
4: LIS_MATRIX
                 A;
5: n = 4; nnz = 8; maxnzr = 3;
 6: perm = (LIS_INT *)malloc( n*sizeof(LIS_INT) );
7: ptr = (LIS_INT *)malloc( (maxnzr+1)*sizeof(LIS_INT) );
8: index = (LIS_INT *)malloc( nnz*sizeof(LIS_INT) );
9: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
10: lis_matrix_create(0,&A);
11: lis_matrix_set_size(A,0,n);
13: perm[0] = 3; perm[1] = 1; perm[2] = 2; perm[3] = 0;
14: ptr[0] = 0; ptr[1] = 4; ptr[2] = 7; ptr[3] = 8;
15: index[0] = 0; index[1] = 0; index[2] = 1; index[3] = 0;
16: index[4] = 2; index[5] = 1; index[6] = 2; index[7] = 3;
17: value[0] = 41; value[1] = 21; value[2] = 32; value[3] = 11;
18: value[4] = 43; value[5] = 22; value[6] = 33; value[7] = 44;
19:
20: lis_matrix_set_jds(nnz,maxnzr,perm,ptr,index,value,A);
21: lis_matrix_assemble(A);
```

5.6.2 Creating Matrices (for Multithreaded Environment)

Figure 14 shows how the matrix A in Figure 13 is stored in the JDS format on two threads. A program to create the matrix in the JDS format on two threads is as follows:

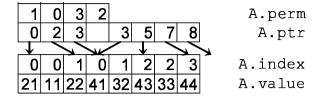


Figure 14: Data structure of JDS format (for multithreaded environment).

```
For multithreaded environment —
 1: LIS INT
                 n,nnz,maxmaxnzr,nprocs;
 2: LIS_INT
                  *perm,*ptr,*index;
3: LIS_SCALAR
                  *value;
4: LIS_MATRIX
                 A;
5: n = 4; nnz = 8; maxmaxnzr = 3; nprocs = 2;
 6: perm = (LIS_INT *)malloc( n*sizeof(LIS_INT) );
7: ptr = (LIS_INT *)malloc( nprocs*(maxmaxnzr+1)*sizeof(LIS_INT) );
8: index = (LIS_INT *)malloc( nnz*sizeof(LIS_INT) );
9: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
10: lis_matrix_create(0,&A);
11: lis_matrix_set_size(A,0,n);
13: perm[0] = 1; perm[1] = 0; perm[2] = 3; perm[3] = 2;
14: ptr[0] = 0; ptr[1] = 2; ptr[2] = 3; ptr[3] = 0;
15: ptr[4] = 3; ptr[5] = 5; ptr[6] = 7; ptr[7] = 8;
16: index[0] = 0; index[1] = 0; index[2] = 1; index[3] = 0;
17: index[4] = 1; index[5] = 2; index[6] = 2; index[7] = 3;
18: value[0] = 21; value[1] = 11; value[2] = 22; value[3] = 41;
19: value[4] = 32; value[5] = 43; value[6] = 33; value[7] = 44;
20:
21: lis_matrix_set_jds(nnz,maxmaxnzr,perm,ptr,index,value,A);
22: lis_matrix_assemble(A);
```

5.6.3 Creating Matrices (for Multiprocessing Environment)

Figure 15 shows how the matrix A in Figure 13 is stored in the JDS format on two processing elements. A program to create the matrix in the JDS format on two processing elements is as follows:

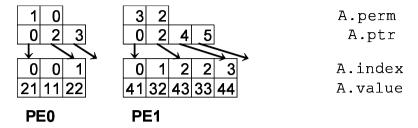


Figure 15: Data structure of JDS format (for multiprocessing environment).

```
For multiprocessing environment -
 1: LIS_INT
                  i,n,nnz,maxnzr,my_rank;
 2: LIS_INT
                  *perm,*ptr,*index;
3: LIS_SCALAR
                  *value:
 4: LIS_MATRIX
                  Α;
5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
 6: if( my_rank==0 ) {n = 2; nnz = 3; maxnzr = 2;}
7: else
                     {n = 2; nnz = 5; maxnzr = 3;}
8: perm = (LIS_INT *)malloc( n*sizeof(LIS_INT) );
9: ptr = (LIS_INT *)malloc( (maxnzr+1)*sizeof(LIS_INT) );
10: index = (LIS_INT *)malloc( nnz*sizeof(LIS_INT) );
11: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
12: lis_matrix_create(MPI_COMM_WORLD,&A);
13: lis_matrix_set_size(A,n,0);
14: if( my_rank==0 ) {
        perm[0] = 1; perm[1] = 0;
15:
        ptr[0] = 0; ptr[1] = 2; ptr[2] = 3;
16:
17:
        index[0] = 0; index[1] = 0; index[2] = 1;
18:
        value[0] = 21; value[1] = 11; value[2] = 22;}
19: else {
20:
        perm[0] = 3; perm[1] = 2;
        ptr[0] = 0; ptr[1] = 2; ptr[2] = 4; ptr[3] = 5;
21:
        index[0] = 0; index[1] = 1; index[2] = 2; index[3] = 2; index[4] = 3;
22:
        value[0] = 41; value[1] = 32; value[2] = 43; value[3] = 33; value[4] = 44;}
23:
24:
    lis_matrix_set_jds(nnz,maxnzr,perm,ptr,index,value,A);
25:
    lis_matrix_assemble(A);
```

5.6.4 Associating Arrays

To associate an array required by the JDS format with the matrix A, the following functions are used:

- C LIS_INT lis_matrix_set_jds(LIS_INT nnz, LIS_INT maxnzr, LIS_INT perm[], LIS_INT ptr[], LIS_INT index[], LIS_SCALAR value[], LIS_MATRIX A)
- Fortran subroutine lis_matrix_set_jds(LIS_INTEGER nnz, LIS_INTEGER maxnzr, LIS_INTEGER ptr(),integer index(), LIS_SCALAR value(), LIS_MATRIX A, LIS_INTEGER ierr)

5.7 Block Sparse Row (BSR)

The BSR format breaks down the matrix A into partial matrices called blocks, with a size of $r \times c$. The BSR format stores the nonzero blocks, in which at least one nonzero element exists, with the similar format as the CRS. Assume that nr = n/r and nnzb are the numbers of the nonzero blocks of A. The BSR format uses three arrays bptr, bindex and value to store data.

- value is a double precision array with a length of $nnzb \times r \times c$, which stores the values of the elements of the nonzero blocks.
- bindex is an integer array with a length of nnzb, which stores the block column numbers of the nonzero blocks.
- bptr is an integer array with a length of nr + 1, which stores the starting points of the block rows in the array bindex.

5.7.1 Creating Matrices (for Serial and Multithreaded Environments)

The right diagram in Figure 16 shows how the matrix A in Figure 16 is stored in the BSR format. A program to create the matrix in the BSR format is as follows:

$$A = \begin{pmatrix} 11 & & & & \\ 21 & 22 & & & \\ \hline & 32 & 33 & \\ 41 & & 43 & 44 \end{pmatrix} \begin{pmatrix} 0 & 1 & 3 & & & \\ \hline & 0 & 0 & 1 & & \\ \hline & 1 & 21 & 0 & 22 & 0 & 41 & 32 & 0 & 33 & 43 & 0 & 44 \\ \hline & 11 & 21 & 0 & 22 & 0 & 41 & 32 & 0 & 33 & 43 & 0 & 44 \\ \hline \end{pmatrix} & A. \text{bindex}$$

Figure 16: Data structure of BSR format (for serial and multithreaded environments).

```
For serial and multithreaded environments -
 1: LIS_INT
                  n,bnr,bnc,nr,nc,bnnz;
 2: LIS_INT
                  *bptr,*bindex;
 3: LIS_SCALAR
                  *value;
 4: LIS_MATRIX
                  Α;
 5: n = 4; bnr = 2; bnc = 2; bnnz = 3; nr = (n-1)/bnr+1; nc = (n-1)/bnc+1;
          = (LIS_INT *)malloc( (nr+1)*sizeof(LIS_INT) );
 7: bindex = (LIS_INT *)malloc( bnnz*sizeof(LIS_INT) );
 8: value = (LIS_SCALAR *)malloc( bnr*bnc*bnnz*sizeof(LIS_SCALAR) );
9: lis_matrix_create(0,&A);
10: lis_matrix_set_size(A,0,n);
11:
12: bptr[0] = 0; bptr[1] = 1; bptr[2] = 3;
13: bindex[0] = 0; bindex[1] = 0; bindex[2] = 1;
14: value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 22;
15: value[4] = 0; value[5] = 41; value[6] = 32; value[7] = 0;
16: value[8] = 33; value[9] = 43; value[10] = 0; value[11] = 44;
17:
    lis_matrix_set_bsr(bnr,bnc,bnnz,bptr,bindex,value,A);
18:
19:
    lis_matrix_assemble(A);
```

5.7.2 Creating Matrices (for Multiprocessing Environment)

Figure 17 shows how the matrix A in Figure 16 is stored in the BSR format on two processing elements. A program to create the matrix in the BSR format on two processing elements is as follows:

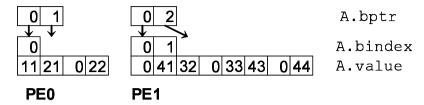


Figure 17: Data structure of BSR format (for multiprocessing environment).

```
For multiprocessing environment -
 1: LIS_INT
                  n,bnr,bnc,nr,nc,bnnz,my_rank;
 2: LIS_INT
                  *bptr,*bindex;
 3: LIS_SCALAR
                  *value;
 4: LIS_MATRIX
                  Α;
5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
 6: if( my_rank==0 ) {n = 2; bnr = 2; bnc = 2; bnnz = 1; nr = (n-1)/bnr+1; nc = (n-1)/bnc+1;}
                     {n = 2; bnr = 2; bnc = 2; bnnz = 2; nr = (n-1)/bnr+1; nc = (n-1)/bnc+1;}
7: else
           = (LIS_INT *)malloc( (nr+1)*sizeof(LIS_INT) );
9: bindex = (LIS_INT *)malloc( bnnz*sizeof(LIS_INT) );
10: value = (LIS_SCALAR *)malloc( bnr*bnc*bnnz*sizeof(LIS_SCALAR) );
11: lis_matrix_create(MPI_COMM_WORLD,&A);
12: lis_matrix_set_size(A,n,0);
13: if( my_rank==0 ) {
14:
        bptr[0] = 0; bptr[1] = 1;
15:
        bindex[0] = 0;
        value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 22;}
16:
17: else {
18:
        bptr[0] = 0; bptr[1] = 2;
        bindex[0] = 0; bindex[1] = 1;
19:
        value[0] = 0; value[1] = 41; value[2] = 32; value[3] = 0;
20:
21:
        value[4] = 33; value[5] = 43; value[6] = 0; value[7] = 44;}
22:
    lis_matrix_set_bsr(bnr,bnc,bnnz,bptr,bindex,value,A);
23:
    lis_matrix_assemble(A);
```

5.7.3 Associating Arrays

To associate the arrays in the BSR format with the matrix A, the following functions are used:

- C LIS_INT lis_matrix_set_bsr(LIS_INT bnr, LIS_INT bnc, LIS_INT bnnz, LIS_INT bptr[], LIS_INT bindex[], LIS_SCALAR value[], LIS_MATRIX A)
- Fortran subroutine lis_matrix_set_bsr(LIS_INTEGER bnr, LIS_INTEGER bnc, LIS_INTEGER bnnz, LIS_INTEGER bptr(), LIS_INTEGER bindex(), LIS_SCALAR value(), LIS_MATRIX A, LIS_INTEGER ierr)

5.8 Block Sparse Column (BSC)

The BSC format breaks down the matrix A into partial matrices called blocks, with a size of $r \times c$. The BSC format stores the nonzero blocks, in which at least one nonzero element exists, in the similar format as the CCS. Assume that nc = n/c and nnzb are the numbers of the nonzero blocks of A. The BSC format uses three arrays bptr, bindex and value to store data.

- value is a double precision array with a length of $nnzb \times r \times c$, which stores the values of the elements of the nonzero blocks.
- bindex is an integer array with a length of nnzb, which stores the block row numbers of the nonzero blocks.
- bptr is an integer array with a length of nc+1, which stores the starting points of the block columns in the array bindex.

5.8.1 Creating Matrices (for Serial and Multithreaded Environments)

The right diagram in Figure 18 shows how the matrix A in Figure 18 is stored in the BSC format. A program to create the matrix in the BSC format is as follows:

Figure 18: Data structure of BSC format (for serial and multithreaded environments).

```
For serial and multithreaded environments:
 1: LIS_INT
                  n,bnr,bnc,nr,nc,bnnz;
 2: LIS_INT
                  *bptr,*bindex;
 3: LIS_SCALAR
                  *value;
 4: LIS_MATRIX
                  Α;
 5: n = 4; bnr = 2; bnc = 2; bnnz = 3; nr = (n-1)/bnr+1; nc = (n-1)/bnc+1;
          = (LIS_INT *)malloc( (nc+1)*sizeof(LIS_INT) );
 7: bindex = (LIS_INT *)malloc( bnnz*sizeof(LIS_INT) );
 8: value = (LIS_SCALAR *)malloc( bnr*bnc*bnnz*sizeof(LIS_SCALAR) );
9: lis_matrix_create(0,&A);
10: lis_matrix_set_size(A,0,n);
11:
12: bptr[0] = 0; bptr[1] = 1; bptr[2] = 3;
13: bindex[0] = 0; bindex[1] = 1; bindex[2] = 1;
14: value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 22;
15: value[4] = 0; value[5] = 41; value[6] = 32; value[7] = 0;
16: value[8] = 33; value[9] = 43; value[10] = 0; value[11] = 44;
17:
    lis_matrix_set_bsc(bnr,bnc,bnnz,bptr,bindex,value,A);
18:
    lis_matrix_assemble(A);
```

5.8.2 Creating Matrices (for Multiprocessing Environment)

Figure 19 shows how the matrix A in Figure 18 is stored in the BSC format on two processing elements. A program to create the matrix in the BSC format on two processing elements is as follows:

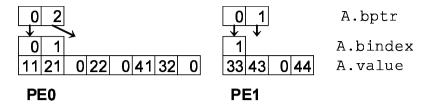


Figure 19: Data structure of BSC format (for multiprocessing environment).

```
For multiprocessing environment -
 1: LIS_INT
                  n,bnr,bnc,nr,nc,bnnz,my_rank;
 2: LIS_INT
                  *bptr,*bindex;
 3: LIS_SCALAR
                  *value;
 4: LIS_MATRIX
                  Α;
5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
 6: if( my_rank==0 ) {n = 2; bnr = 2; bnc = 2; bnnz = 2; nr = (n-1)/bnr+1; nc = (n-1)/bnc+1;}
                     {n = 2; bnr = 2; bnc = 2; bnnz = 1; nr = (n-1)/bnr+1; nc = (n-1)/bnc+1;}
7: else
           = (LIS_INT *)malloc( (nr+1)*sizeof(LIS_INT) );
9: bindex = (LIS_INT *)malloc( bnnz*sizeof(LIS_INT) );
10: value = (LIS_SCALAR *)malloc( bnr*bnc*bnnz*sizeof(LIS_SCALAR) );
11: lis_matrix_create(MPI_COMM_WORLD,&A);
12: lis_matrix_set_size(A,n,0);
13: if( my_rank==0 ) {
        bptr[0] = 0; bptr[1] = 2;
14:
        bindex[0] = 0; bindex[1] = 1;
15:
        value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 22;
16:
17:
        value[4] = 0; value[5] = 41; value[6] = 32; value[7] = 0;}
18: else {
        bptr[0] = 0; bptr[1] = 1;
19:
20:
        bindex[0] = 1;
        value[0] = 33; value[1] = 43; value[2] = 0; value[3] = 44;}
21:
22:
    lis_matrix_set_bsc(bnr,bnc,bnnz,bptr,bindex,value,A);
23:
    lis_matrix_assemble(A);
```

5.8.3 Associating Arrays

To associate the arrays in the BSC format with the matrix A, the following functions are used:

- C LIS_INT lis_matrix_set_bsc(LIS_INT bnr, LIS_INT bnc, LIS_INT bnnz, LIS_INT bptr[], LIS_INT bindex[], LIS_SCALAR value[], LIS_MATRIX A)
- Fortran subroutine lis_matrix_set_bsc(LIS_INTEGER bnr, LIS_INTEGER bnc, LIS_INTEGER bnnz, LIS_INTEGER bptr(), LIS_INTEGER bindex(), LIS_SCALAR value(), LIS_MATRIX A, LIS_INTEGER ierr)

5.9 Variable Block Row (VBR)

The VBR format is the generalized version of the BSR format. The division points of the rows and columns are given by the arrays row and col. The VBR format stores the nonzero blocks (the blocks in which at least one nonzero element exists) in the similar format as the CRS. Assume that nr and nc are the numbers of row and column divisions, respectively, and that nnzb denotes the number of the nonzero blocks of A, and nnz denotes the total number of the elements of the nonzero blocks. The VBR format uses six arrays bptr, bindex, row, col, ptr and value to store data.

- row is an integer array with a length of nr + 1, which stores the starting row number of the block rows.
- col is an integer array with a length of nc + 1, which stores the starting column number of the block columns.
- bindex is an integer array with a length of nnzb, which stores the block column numbers of the nonzero blocks.
- bptr is an integer array with a length of nr + 1, which stores the starting points of the block rows in the array bindex.
- \bullet value is a double precision array with a length of nnz, which stores the values of the elements of the nonzero blocks.
- ptr is an integer array with a length of nnzb + 1, which stores the starting points of the nonzero blocks in the array value.

5.9.1 Creating Matrices (for Serial and Multithreaded Environments)

The right diagram in Figure 20 shows how the matrix A in Figure 20 is stored in the VBR format. A program to create the matrix in the VBR format is as follows:

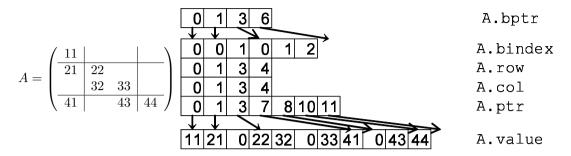


Figure 20: Data structure of VBR format (for serial and multithreaded environments).

```
For serial and multithreaded environments ·
 1: LIS_INT
                  n,nnz,nr,nc,bnnz;
 2: LIS_INT
                  *row,*col,*ptr,*bptr,*bindex;
 3: LIS_SCALAR
                  *value;
 4: LIS_MATRIX
                  Α;
 5: n = 4; nnz = 11; bnnz = 6; nr = 3; nc = 3;
           = (LIS_INT *)malloc( (nr+1)*sizeof(LIS_INT) );
           = (LIS_INT *)malloc( (nr+1)*sizeof(LIS_INT) );
           = (LIS_INT *)malloc( (nc+1)*sizeof(LIS_INT) );
9: ptr
           = (LIS_INT *)malloc( (bnnz+1)*sizeof(LIS_INT) );
10: bindex = (LIS_INT *)malloc( bnnz*sizeof(LIS_INT) );
11: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
12: lis_matrix_create(0,&A);
13: lis_matrix_set_size(A,0,n);
15: bptr[0] = 0; bptr[1] = 1; bptr[2] = 3; bptr[3] = 6;
16: row[0] = 0; row[1] = 1; row[2] = 3; row[3] = 4;
17: col[0] = 0; col[1] = 1; col[2] = 3; col[3] = 4;
18: bindex[0] = 0; bindex[1] = 0; bindex[2] = 1; bindex[3] = 0;
19: bindex[4] = 1; bindex[5] = 2;
20: ptr[0]
              = 0; ptr[1]
                               = 1; ptr[2]
                                                = 3; ptr[3]
21: ptr[4]
              = 8; ptr[5]
                               = 10; ptr[6]
                                                = 11;
22: value[0] = 11; value[1] = 21; value[2] = 0; value[3]
23: value[4] = 32; value[5] = 0; value[6] = 33; value[7]
24: value[8] = 0; value[9] = 43; value[10] = 44;
25:
26:
     lis_matrix_set_vbr(nnz,nr,nc,bnnz,row,col,ptr,bptr,bindex,value,A);
   lis_matrix_assemble(A);
```

5.9.2 Creating Matrices (for Multiprocessing Environment)

Figure 21 shows how the matrix A in Figure 20 is stored in the VBR format on two processing elements. A program to create the matrix in the VBR format on two processing elements is as follows:

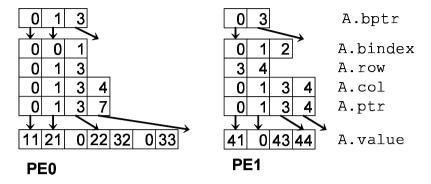


Figure 21: Data structure of VBR format (for multiprocessing environment).

```
For multiprocessing environment -
 1: LIS_INT
                 n,nnz,nr,nc,bnnz,my_rank;
 2: LIS_INT
                  *row,*col,*ptr,*bptr,*bindex;
3: LIS_SCALAR
                  *value;
4: LIS_MATRIX
                 A;
 5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
 6: if( my_rank==0 ) {n = 2; nnz = 7; bnnz = 3; nr = 2; nc = 3;}
                     {n = 2; nnz = 4; bnnz = 3; nr = 1; nc = 3;}
7: else
          = (LIS_INT *)malloc( (nr+1)*sizeof(LIS_INT) );
8: bptr
          = (LIS_INT *)malloc( (nr+1)*sizeof(LIS_INT) );
9: row
10: col
          = (LIS_INT *)malloc( (nc+1)*sizeof(LIS_INT) );
          = (LIS_INT *)malloc( (bnnz+1)*sizeof(LIS_INT) );
11: ptr
12: bindex = (LIS_INT *)malloc( bnnz*sizeof(LIS_INT) );
13: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
14: lis_matrix_create(MPI_COMM_WORLD,&A);
15: lis_matrix_set_size(A,n,0);
16: if( my_rank==0 ) {
17:
        bptr[0] = 0; bptr[1] = 1; bptr[2] = 3;
        row[0] = 0; row[1] = 1; row[2] = 3;
18:
        col[0] = 0; col[1] = 1; col[2] = 3; col[3] = 4;
19:
20:
        bindex[0] = 0; bindex[1] = 0; bindex[2] = 1;
                 = 0; ptr[1]
                               = 1; ptr[2]
21:
                                                = 3; ptr[3]
        value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 22;
22:
23:
        value[4] = 32; value[5] = 0; value[6] = 33;}
24: else {
        bptr[0] = 0; bptr[1] = 3;
25:
        row[0] = 3; row[1] = 4;
26:
        col[0] = 0; col[1] = 1; col[2] = 3; col[3] = 4;
27:
28:
        bindex[0] = 0; bindex[1] = 1; bindex[2] = 2;
29:
        ptr[0]
                = 0; ptr[1]
                               = 1; ptr[2]
                                               = 3; ptr[3]
30:
        value[0] = 41; value[1] = 0; value[2] = 43; value[3] = 44;}
31:
    lis_matrix_set_vbr(nnz,nr,nc,bnnz,row,col,ptr,bptr,bindex,value,A);
    lis_matrix_assemble(A);
```

5.9.3 Associating Arrays

To associate the arrays in the VBR format with the matrix A, the following functions are used:

- C LIS_INT lis_matrix_set_vbr(LIS_INT nnz, LIS_INT nr, LIS_INT nc, LIS_INT bnnz, LIS_INT row[], LIS_INT col[], LIS_INT ptr[], LIS_INT bptr[], LIS_INT bindex[], LIS_SCALAR value[], LIS_MATRIX A)
- Fortran subroutine lis_matrix_set_vbr(LIS_INTEGER nnz, LIS_INTEGER nr, LIS_INTEGER nc, LIS_INTEGER bnnz, LIS_INTEGER row(), LIS_INTEGER col(), LIS_INTEGER ptr(), LIS_INTEGER bptr(), LIS_INTEGER bindex(), LIS_SCALAR value(), LIS_MATRIX A, LIS_INTEGER ierr)

5.10 Coordinate (COO)

The COO format uses three arrays row, col and value to store data.

- \bullet value is a double precision array with a length of nnz, which stores the values of the nonzero elements.
- row is an integer array with a length of nnz, which stores the row numbers of the nonzero elements.
- \bullet col is an integer array with a length of nnz, which stores the column numbers of the nonzero elements.

5.10.1 Creating Matrices (for Serial and Multithreaded Environments)

The right diagram in Figure 22 shows how the matrix A in Figure 22 is stored in the COO format. A program to create the matrix in the COO format is as follows:

Figure 22: Data structure of COO format (for serial and multithreaded environments).

```
For serial and multithreaded environments -
 1: LIS INT
                  n,nnz;
 2: LIS_INT
                  *row,*col;
 3: LIS_SCALAR
                  *value;
 4: LIS_MATRIX
                  A:
 5: n = 4; nnz = 8;
         = (LIS_INT *)malloc( nnz*sizeof(LIS_INT) );
7: col = (LIS_INT *)malloc( nnz*sizeof(LIS_INT) );
8: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
9: lis_matrix_create(0,&A);
10: lis_matrix_set_size(A,0,n);
11:
12: row[0] = 0; row[1] = 1; row[2] = 3; row[3] = 1;
13: row[4] = 2; row[5] = 2; row[6] = 3; row[7] = 3;
14: col[0] = 0; col[1] = 0; col[2] = 0; col[3] = 1;
15: col[4] = 1; col[5] = 2; col[6] = 2; col[7] = 3;
16: value[0] = 11; value[1] = 21; value[2] = 41; value[3] = 22;
17: value[4] = 32; value[5] = 33; value[6] = 43; value[7] = 44;
18:
19:
     lis_matrix_set_coo(nnz,row,col,value,A);
20:
    lis_matrix_assemble(A);
```

5.10.2 Creating Matrices (for Multiprocessing Environment)

Figure 23 shows how the matrix A in Figure 22 is stored in the COO format on two processing elements. A program to create the matrix in the COO format on two processing elements is as follows:

0	1	1	3	2	2	3	3	A.row
0	0	1	0	1	2	2	3	A.col
11	21	22	41	32	33	43	44	A.value
PE	Ξ0		PI	≣1				

Figure 23: Data structure of COO format (for multiprocessing environment).

```
For multiprocessing environment -
 1: LIS_INT
                  n,nnz,my_rank;
 2: LIS_INT
                  *row,*col;
3: LIS_SCALAR
                  *value;
4: LIS_MATRIX
                  Α;
5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
6: if( my_rank==0 ) {n = 2; nnz = 3;}
7: else
                     {n = 2; nnz = 5;}
8: row
          = (LIS_INT *)malloc( nnz*sizeof(LIS_INT) );
9: col = (LIS_INT *)malloc( nnz*sizeof(LIS_INT) );
10: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
11: lis_matrix_create(MPI_COMM_WORLD,&A);
12: lis_matrix_set_size(A,n,0);
13: if( my_rank==0 ) {
14:
        row[0] = 0; row[1] = 1; row[2] = 1;
        col[0] = 0; col[1] = 0; col[2] = 1;
15:
        value[0] = 11; value[1] = 21; value[2] = 22;}
16:
17: else {
        row[0] = 3; row[1] = 2; row[2] = 2; row[3] = 3; row[4] = 3;
18:
19:
        col[0] = 0; col[1] = 1; col[2] = 2; col[3] = 2; col[4] = 3;
20:
        value[0] = 41; value[1] = 32; value[2] = 33; value[3] = 43; value[4] = 44;}
21:
    lis_matrix_set_coo(nnz,row,col,value,A);
    lis_matrix_assemble(A);
```

5.10.3 Associating Arrays

To associate the arrays in the COO format with the matrix A, the following functions are used:

- C LIS_INT lis_matrix_set_coo(LIS_INT nnz, LIS_INT row[], LIS_INT col[], LIS_SCALAR value[], LIS_MATRIX A)
- Fortran subroutine lis_matrix_set_coo(LIS_INTEGER nnz, LIS_INTEGER row(), LIS_INTEGER col(), LIS_SCALAR value(), LIS_MATRIX A, LIS_INTEGER ierr)

5.11 Dense (DNS)

The DNS format uses one array value to store data.

• value is a double precision array with a length of $n \times n$, which stores the values of the elements with priority given to the columns.

5.11.1 Creating Matrices (for Serial and Multithreaded Environments)

The right diagram in Figure 24 shows how the matrix A in Figure 24 is stored in the DNS format. A program to create the matrix in the DNS format is as follows:

Figure 24: Data structure of DNS format (for serial and multithreaded environments).

```
For serial and multithreaded environments —
 1: LIS_INT
 2: LIS_SCALAR
                  *value;
3: LIS_MATRIX
4: n = 4;
5: value = (LIS_SCALAR *)malloc( n*n*sizeof(LIS_SCALAR) );
6: lis_matrix_create(0,&A);
7: lis_matrix_set_size(A,0,n);
9: value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 41;
10: value[4] = 0; value[5] = 22; value[6] = 32; value[7] = 0;
11: value[8] = 0; value[9] = 0; value[10] = 33; value[11] = 43;
12: value[12] = 0; value[13] = 0; value[14] = 0; value[15] = 44;
13:
14:
     lis_matrix_set_dns(value,A);
     lis_matrix_assemble(A);
```

5.11.2 Creating Matrices (for Multiprocessing Environment)

Figure 25 shows how the matrix A in Figure 24 is stored in the DNS format on two processing elements. A program to create the matrix in the DNS format on two processing elements is as follows:

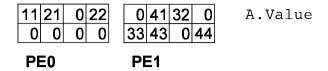


Figure 25: Data structure of DNS format (for multiprocessing environment).

```
For multiprocessing environment -
 1: LIS_INT
                  n,my_rank;
 2: LIS_SCALAR
                  *value;
3: LIS_MATRIX
                  A;
 4: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
5: if( my_rank==0 ) {n = 2;}
                     {n = 2;}
 6: else
7: value = (LIS_SCALAR *)malloc( n*n*sizeof(LIS_SCALAR) );
8: lis_matrix_create(MPI_COMM_WORLD,&A);
9: lis_matrix_set_size(A,n,0);
10: if( my_rank==0 ) {
        value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 22;
11:
12:
        value[4] = 0; value[5] = 0; value[6] = 0; value[7] = 0;}
13: else {
        value[0] = 0; value[1] = 41; value[2] = 32; value[3] = 0;
14:
        value[4] = 33; value[5] = 43; value[6] = 0; value[7] = 44;}
15:
16: lis_matrix_set_dns(value,A);
17: lis_matrix_assemble(A);
```

5.11.3 Associating Arrays

To associate the arrays in the DNS format with the matrix A, the following functions are used:

- C LIS_INT lis_matrix_set_dns(LIS_SCALAR value[], LIS_MATRIX A)
- Fortran subroutine lis_matrix_set_dns(LIS_SCALAR value(), LIS_MATRIX A, LIS_INTEGER ierr)

6 Functions

This section describes the functions which can be employed by the user. The return codes of the functions in C and the values of ierr in Fortran are as follows:

LIS_SUCCESS(0) Normal termination

LIS_ILL_OPTION(1) Illegal option

LIS_BREAKDOWN(2) Breakdown

LIS_OUT_OF_MEMORY(3) Out of working memory

LIS_MAXITER(4) Maximum number of iterations

LIS_NOT_IMPLEMENTED(5) Not implemented

LIS_ERR_FILE_IO(6) File I/O error

6.1 Operating Vector Elements

Assume that the size of the vector v is $global_n$ and that the size of the partial vectors stored on nprocs processing elements is $local_n$. $global_n$ and $local_n$ are called the global size and the local size, respectively.

6.1.1 lis_vector_create

C LIS_INT lis_vector_create(LIS_Comm comm, LIS_VECTOR *v)
Fortran subroutine lis_vector_create(LIS_Comm comm, LIS_VECTOR v, LIS_INTEGER ierr)

Description

Create the vector v

Input

LIS_Comm The MPI communicator

Output

v The vector

ierr The return code

Note

For the serial and multithreaded environments, the value of comm is ignored.

6.1.2 lis_vector_destroy

```
C LIS_INT lis_vector_destroy(LIS_VECTOR v)
Fortran subroutine lis_vector_destroy(LIS_VECTOR v, LIS_INTEGER ierr)
```

Description

Destroy the vector v

Input

v The vector to be destroyed

Output

ierr The return code

6.1.3 lis_vector_duplicate

```
C LIS_INT lis_vector_duplicate(void *vin, LIS_VECTOR *vout)
Fortran subroutine lis_vector_duplicate(LIS_VECTOR vin, LIS_VECTOR vout,
LIS_INTEGER ierr)
```

Description

Create the vector v_{out} which has the same information as v_{in}

Input

vin The source vector

Output

vout The destination vector

ierr The return code

Note

The function lis_vector_duplicate does not copy the values, but only allocates the memory. To copy the values as well, the function lis_vector_copy must be called after this function.

6.1.4 lis_vector_set_size

Description

Assign the size of the vector v

Input

v The vector

local_n The size of the partial vector

global_n The size of the global vector

Output

ierr The return code

Note

Either *local_n* or *global_n* must be provided.

In the case of the serial and multithreaded environments, $local_n$ is equal to $global_n$. Therefore, both $lis_{vector_set_size(v,n,0)}$ and $lis_{vector_set_size(v,0,n)}$ create a vector of size n.

For the multiprocessing environment, $lis_vector_set_size(v,n,0)$ creates a partial vector of size n on each processing element. On the other hand, $lis_vector_set_size(v,0,n)$ creates a partial vector of size m_p on the processing element p. The values of m_p are determined by the library.

6.1.5 lis_vector_get_size

```
C LIS_INT lis_vector_get_size(LIS_VECTOR v, LIS_INT *local_n,
C LIS_INT *global_n)
Fortran subroutine lis_vector_get_size(LIS_VECTOR v, LIS_INTEGER local_n,
LIS_INTEGER global_n, LIS_INTEGER ierr)
```

Description

Get the size of the vector v

Input

v The vector

Output

local_n The size of the partial vector global_n The size of the global vector

ierr The return code

Note

In the case of the serial and multithreaded environments, local_n is equal to global_n.

6.1.6 lis_vector_get_range

```
C LIS_INT lis_vector_get_range(LIS_VECTOR v, LIS_INT *is, LIS_INT *ie)
Fortran subroutine lis_vector_get_range(LIS_VECTOR v, LIS_INTEGER is,
LIS_INTEGER ie, LIS_INTEGER ierr)
```

Description

Get the location of the partial vector v in the global vector

Input

v The partial vector

Output

is The location where the partial vector v starts in the global vector

ie The next location where the partial vector v ends in the global

vector

ierr The return code

Note

For the serial and multithreaded environments, a vector of size n results in is = 0 and ie = n.

6.1.7 lis_vector_set_value

```
C LIS_INT lis_vector_set_value(LIS_INT flag, LIS_INT i, LIS_SCALAR value, LIS_VECTOR v)

Fortran subroutine lis_vector_set_value(LIS_INTEGER flag, LIS_INTEGER i, LIS_SCALAR value, LIS_VECTOR v, LIS_INTEGER ierr)
```

Description

Assign the scalar value to the i-th row of the vector v

Input

flag LIS_INS_VALUE : v[i] = value

LIS_ADD_VALUE : v[i] = v[i] + value

i The location where the value is assigned

value The scalar value to be assigned

v The destination vector

Output

v The vector with the scalar value assigned to the i-th row

ierr The return code

Note

For the multiprocessing environment, the i-th row of the global vector must be specified instead of the i-th row of the partial vector.

6.1.8 lis_vector_get_value

Description

Get the value of the i-th row of the vector v

Input

i The location where the value is assigned

v The destination vector

Output

value The value of the *i*-th row

ierr The return code

Note

For the multiprocessing environment, the i-th row of the global vector must be specified.

6.1.9 lis_vector_set_values

```
C LIS_INT lis_vector_set_values(LIS_INT flag, LIS_INT count,
LIS_INT index[], LIS_SCALAR value[], LIS_VECTOR v)

Fortran subroutine lis_vector_set_values(LIS_INTEGER flag, LIS_INTEGER count,
LIS_INTEGER index(), LIS_SCALAR value(), LIS_VECTOR v, LIS_INTEGER ierr)
```

Description

Assign the scalar values value [i] to the index [i]-th rows of the vector v

Input

flag
 LIS_INS_VALUE : v[index[i]] = value[i]

LIS_ADD_VALUE : v[index[i]] = v[index[i]] + value[i]

count The number of the elements of the array which stores the scalar

values to be assigned

index The array which stores the location where the scalar values are

assigned

value The array which stores the scalar values to be assigned

v The destination vector

Output

The vector with the scalar value[i] assigned to the index[i]-th

row

ierr The return code

Note

For the multiprocessing environment, the index[i]-th row of the global vector must be specified instead of the index[i]-th row of the partial vector.

6.1.10 lis_vector_get_values

```
C LIS_INT lis_vector_get_values(LIS_VECTOR v, LIS_INT start, LIS_INT count, LIS_SCALAR value[])
Fortran subroutine lis_vector_get_values(LIS_VECTOR v, LIS_INTEGER start, LIS_INTEGER count, LIS_SCALAR value(), LIS_INTEGER ierr)
```

Description

Get the scalar values of the start + i-th row of the vector v, where i = 0, 1, ..., count - 1

Input

start The starting location

count The number of the values to get

v The destination vector

Output

value The vector to store the scalar values

ierr The return code

Note

For the multiprocessing environment, the start + i-th row of the global vector must be specified.

6.1.11 lis_vector_scatter

Description

Assign the scalar values of the *i*-th row of the vector v, where $i = 0, 1, ..., global_n - 1$

Input

value The array which stores the scalar values to be assigned

Output

v The destination vector

ierr The return code

Note

6.1.12 lis_vector_gather

Description

Get the scalar values of the *i*-th row of the vector v, where $i = 0, 1, ..., global_n - 1$

Input

v The source vector

Output

value The vector to store the scalar values

ierr The return code

Note

6.1.13 lis_vector_copy

```
C LIS_INT lis_vector_copy(LIS_VECTOR x, LIS_VECTOR y)
Fortran subroutine lis_vector_copy(LIS_VECTOR x, LIS_VECTOR y, LIS_INTEGER ierr)
```

Description

Copy the values of the vector elements

Input

x The source vector

Output

y The destination vector

ierr The return code

6.1.14 lis_vector_set_all

```
C LIS_INT lis_vector_set_all(LIS_SCALAR value, LIS_VECTOR x)
Fortran subroutine lis_vector_set_all(LIS_SCALAR value, LIS_VECTOR x,
LIS_INTEGER ierr)
```

Description

Assign the scalar value to the all elements of the vector v

Input

value The scalar value to be assigned

v The destination vector

Output

The vector with the *value* assigned to the all elements

6.2 Operating Matrix Elements

Assume that the size of the matrix A is $global_n \times global_n$ and that the size of each partial matrix stored on nprocs processing elements is $local_n \times global_n$. Here, $global_n$ and $local_n$ are called the number of the rows of the global matrix and the number of the rows of the partial matrix, respectively.

6.2.1 lis matrix create

C LIS_INT lis_matrix_create(LIS_Comm comm, LIS_MATRIX *A)
Fortran subroutine lis_matrix_create(LIS_Comm comm, LIS_MATRIX A, LIS_INTEGER ierr)

Description

Create the matrix A

Input

LIS_Comm The MPI communicator

Output

A The matrix

ierr The return code

Note

For the sequential and the multithreaded environments, the value of comm is ignored.

6.2.2 lis_matrix_destroy

C LIS_INT lis_matrix_destroy(LIS_MATRIX A)
Fortran subroutine lis_matrix_destroy(LIS_MATRIX A, LIS_INTEGER ierr)

Description

Destroy the matrix A

Input

A The matrix to be destroyed

Output

6.2.3 lis_matrix_duplicate

Description

Create the matrix A_{out} which has the same information as the original A_{in}

Input

Ain The source matrix

Output

Aout The destination matrix

ierr The return code

Note

The function <code>lis_matrix_duplicate</code> does not copy the values of the elements of the matrix, but only allocates the memory. To copy the values of the elements as well, the function <code>lis_matrix_copy</code> must be called after this function.

6.2.4 lis_matrix_malloc

Description

Allocate the memory for the matrix A

Input

A The matrix

nnz_row The average number of the nonzero elements

nnz The array of numbers of the nonzero elements in each row

Output

ierr The return code

Note

Either nnz_row or nnz must be provided.

6.2.5 lis_matrix_set_value

Description

Assign the scalar value to the (i, j)-th element of the matrix A

Input

 ${\tt LIS_INS_VALUE} \, : \, A(i,j) = value$

 $\texttt{LIS_ADD_VALUE} \, : \, A(i,j) = A(i,j) + value$

i The row number of the matrix

j The column number of the matrix

value The value to be assigned

A The matrix

Output

A The matrix

ierr The return code

Note

For the multiprocessing environment, the i-th row and the j-th column of the global matrix must be specified.

The function lis_matrix_set_value stores the assigned value in a temporary internal format. Therefore, after lis_matrix_set_value is called, the function lis_matrix_assemble must be called.

6.2.6 lis_matrix_assemble

```
C LIS_INT lis_matrix_assemble(LIS_MATRIX A)
Fortran subroutine lis_matrix_assemble(LIS_MATRIX A, LIS_INTEGER ierr)
```

Description

Assemble the matrix A into the specified storage format

Input

A The matrix

Output

A The matrix assembled into the specified storage format

6.2.7 lis_matrix_set_size

```
C LIS_INT lis_matrix_set_size(LIS_MATRIX A, LIS_INT local_n,
LIS_INT global_n)

Fortran subroutine lis_matrix_set_size(LIS_MATRIX A, LIS_INTEGER local_n,
LIS_INTEGER global_n, LIS_INTEGER ierr)
```

Description

Assign the size of the matrix A

Input

A The matrix

local_n The number of the rows of the partial matrix global_n The number of the rows of the global matrix

Output

ierr The return code

Note

Either $local_n$ or $global_n$ must be provided.

In the case of the serial and multithreaded environments, $local_n$ is equal to $global_n$. Therefore, both $lis_matrix_set_size(A,n,0)$ and $lis_matrix_set_size(A,0,n)$ create a matrix of size $n \times n$.

For the multiprocessing environment, $lis_{\mathtt{matrix_set_size}}(A,n,0)$ creates a partial matrix of size $n \times N$ on each processing element, where N is the total sum of n. On the other hand, $lis_{\mathtt{matrix_set_size}}(A,0,n)$ creates a partial matrix of size $m_p \times n$ on the processing element p. The values of m_p are determined by the library.

6.2.8 lis_matrix_get_size

Description

Get the size of the matrix A

Input

A The matrix

Output

local_n The number of the rows of the partial matrix
global_n The number of the rows of the global matrix

ierr The return code

Note

In case of the serial and multithreaded environments, local_n is equal to global_n.

6.2.9 lis_matrix_get_range

Description

Get the location of the partial matrix A in the global matrix

Input

A The partial matrix

Output

is The location where the partial matrix A starts in the global matrix

ie The next location where the partial matrix A ends in the global

matrix

ierr The return code

Note

For the serial and multithreaded environments, a matrix of $n \times n$ results in is = 0 and ie = n.

6.2.10 lis_matrix_set_type

Description

Assign the storage format

Input

A The matrix

matrix_type The storage format

Output

ierr The return code

Note

 $\mathtt{matrix_type}$ of A is LIS_MATRIX_CRS when the matrix is created. The table below shows the available storage formats for $\mathtt{matrix_type}$.

Storage format		matrix_type
Compressed Row Storage	(CRS)	{LIS_MATRIX_CRS 1}
Compressed Column Storage	(CCS)	{LIS_MATRIX_CCS 2}
Modified Compressed Sparse Row	(MSR)	{LIS_MATRIX_MSR 3}
Diagonal	(DIA)	{LIS_MATRIX_DIA 4}
Ellpack-Itpack Generalized Diagonal	(ELL)	{LIS_MATRIX_ELL 5}
Jagged Diagonal	(JDS)	{LIS_MATRIX_JDS 6}
Block Sparse Row	(BSR)	{LIS_MATRIX_BSR 7}
Block Sparse Column	(BSC)	{LIS_MATRIX_BSC 8}
Variable Block Row	(VBR)	{LIS_MATRIX_VBR 9}
Dense	(DNS)	{LIS_MATRIX_DNS 10}
Coordinate	(COO)	{LIS_MATRIX_COO 11}

6.2.11 lis_matrix_get_type

Description

Get the storage format

Input

A The matrix

Output

matrix_type The storage format ierr The return code

6.2.12 lis_matrix_set_blocksize

C LIS_INT lis_matrix_set_blocksize(LIS_MATRIX A, LIS_INT bnr, LIS_INT bnc, LIS_INT row[], LIS_INT col[])

Fortran subroutine lis_matrix_set_blocksize(LIS_MATRIX A, LIS_INTEGER bnr, LIS_INTEGER bnc, LIS_INTEGER row[], LIS_INTEGER col[], LIS_INTEGER ierr)

Description

Assign the block size of the BSR, BSC, and VBR

Input

A The matrix

bnr The row block size of the BSR (BSC) format or the number of the

row blocks of the VBR format

bnc The olumn block size of the BSR (BSC) format or the number of

the column blocks of the VBR format

row The array of the row division information about the VBR format

col The array of the column division information about the VBR for-

mat

Output

ierr The return code

6.2.13 lis_matrix_convert

Description

Convert the matrix A_{in} into A_{out} of the format specified by lis_matrix_set_type

Input

Ain The source matrix

Output

Aout The destination matrix

ierr The return code

Note

The storage format of the A_{out} is set by lis_matrix_set_type. The block size of the BSR, BSC, and VBR is set by lis_matrix_set_blocksize.

The conversions indicated by one in the table below are performed directly, and the other ones are performed via the indicated formats. The conversions with no indication are performed via the CRS format.

Src \Dst	CRS	CCS	MSR	DIA	ELL	JDS	BSR	BSC	VBR	DNS	COO
CRS		1	1	1	1	1	1	CCS	1	1	1
COO	1	1	1	CRS	CRS	CRS	CRS	CCS	CRS	CRS	

6.2.14 lis_matrix_copy

Description

Copy the values of the matrix elements

Input

Ain The source matrix

Output

Aout The destination matrix

ierr The return code

6.2.15 lis_matrix_get_diagonal

Description

Store the diagonal elements of the matrix A to the vector d

Input

A The matrix

Output

d The vector which stores the diagonal elements of the matrix

6.2.16 lis_matrix_set_crs

Description

Associate the arrays in the CRS format with the matrix A

Input

nnz The number of nonzero elements

ptr, index, value The rrays in the CRS format

A The matrix

Output

A The matrix associated with the arrays

Note

After lis_matrix_set_crs is called, the function lis_matrix_assemble must be called.

6.2.17 lis_matrix_set_ccs

Description

Associate the arrays in the CCS format with the matrix A

Input

nnz The number of the nonzero elements

ptr, index, value The arrays in the CCS format

A The matrix

Output

A The matrix associated with the arrays

Note

After lis_matrix_set_ccs is called, the function lis_matrix_assemble must be called.

6.2.18 lis_matrix_set_msr

Description

Associate the arrays in the MSR format with the matrix A

Input

nnz The number of the nonzero elements

ndz The number of the nonzero elements in the diagonal

index, value The arrays in the MSR format

A The matrix

Output

A The matrix associated with the arrays

Note

After lis_matrix_set_msr is called, the function lis_matrix_assemble must be called.

6.2.19 lis_matrix_set_dia

Description

Associate the arrays in the DIA format with the matrix A

Input

nnd The number of the nonzero diagonal elements

A The matrix

Output

A The matrix associated with the arrays

Note

After lis_matrix_set_dia is called, the function lis_matrix_assemble must be called.

6.2.20 lis_matrix_set_ell

Description

Associate the arrays in the ELL format with the matrix A

Input

maxnzr The maximum number of the nonzero elements in each row

index, value The arrays in the ELL format

A The matrix

Output

A The matrix associated with the arrays

Note

After lis_matrix_set_ell is called, the function lis_matrix_assemble must be called.

6.2.21 lis_matrix_set_jds

Description

Associate the arrays in the JDS format with the matrix A

Input

nnz The number of the nonzero elements

maxnzr The maximum number of the nonzero elements in each row

perm, ptr, index, value The arrays in the JDS format

A The matrix

Output

A The matrix associated with the arrays

Note

After lis_matrix_set_jds is called, the function lis_matrix_assemble must be called.

6.2.22 lis_matrix_set_bsr

Description

Associate the arrays in the BSR format with the matrix A

Input

bnr The row block size

bnc The column block size

bnnz The number of the nonzero blocks

bptr, bindex, value The arrays in the BSR format

A The matrix

Output

A The matrix associated with the arrays

Note

After lis_matrix_set_bsr is called, the function lis_matrix_assemble must be called.

6.2.23 lis_matrix_set_bsc

```
C LIS_INT lis_matrix_set_bsc(LIS_INT bnr, LIS_INT bnc, LIS_INT bnnz,
LIS_INT bptr[], LIS_INT bindex[], LIS_SCALAR value[], LIS_MATRIX A)

Fortran subroutine lis_matrix_set_bsc(LIS_INTEGER bnr, LIS_INTEGER bnc,
LIS_INTEGER bnnz, LIS_INTEGER bptr(), LIS_INTEGER bindex(),
LIS_SCALAR value(), LIS_MATRIX A, LIS_INTEGER ierr)
```

Description

Associate the arrays in the BSC format with the matrix A

Input

bnr The row block size

bnc The column block size

bnnz The number of the nonzero blocks

bptr, bindex, value The arrays in the BSC format

A The matrix

Output

A The matrix associated with the arrays

Note

After lis_matrix_set_bsc is called, the function lis_matrix_assemble must be called.

6.2.24 lis_matrix_set_vbr

```
C LIS_INT lis_matrix_set_vbr(LIS_INT nnz, LIS_INT nr, LIS_INT nc,
    LIS_INT bnnz, LIS_INT row[], LIS_INT col[], LIS_INT ptr[],
    LIS_INT bptr[], LIS_INT bindex[], LIS_SCALAR value[],
    LIS_MATRIX A)

Fortran subroutine lis_matrix_set_vbr(LIS_INTEGER nnz, LIS_INTEGER nr,
    LIS_INTEGER nc, LIS_INTEGER bnnz, LIS_INTEGER row(),
    LIS_INTEGER col(), LIS_INTEGER ptr(), LIS_INTEGER bptr(),
    LIS_INTEGER bindex(), LIS_SCALAR value(), LIS_MATRIX A,
    LIS_INTEGER ierr)
```

Description

Associate the arrays in the VBR format with the matrix A

Input

nnz The number of the all nonzero elements

nr The number of the row blocks

nc The number of the column blocks
bnnz The number of the nonzero blocks

row, col, ptr, bptr, bindex, value The arrays in the VBR format

A The matrix

Output

A The matrix associated with the arrays

Note

After lis_matrix_set_vbr is called, the function lis_matrix_assemble must be called.

6.2.25 lis_matrix_set_coo

Description

Associate the arrays in the COO format with the matrix A

Input

nnz The number of the nonzero elements

row, col, value The arrays in the COO format

A The matrix

Output

A The matrix associated with the arrays

Note

After lis_matrix_set_coo is called, the function lis_matrix_assemble must be called.

6.2.26 lis_matrix_set_dns

Description

Associate the array in the DNS format with the matrix A

Input

value The array in the DNS format

A The matrix

Output

A The matrix associated with the array

Note

After lis_matrix_set_dns is called, the function lis_matrix_assemble must be called.

6.3 Operating Vectors and Matrices

6.3.1 lis_vector_scale

```
C LIS_INT lis_vector_scale(LIS_SCALAR alpha, LIS_VECTOR x)
Fortran subroutine lis_vector_scale(LIS_SCALAR alpha, LIS_VECTOR x,
LIS_INTEGER ierr)
```

Description

Multiply the vector x by the scalar α

Input

alpha The scalar value α

x The vector to be multiplied

Output

x The vector multiplied by α

ierr The return code

6.3.2 lis_vector_dot

C LIS_INT lis_vector_dot(LIS_VECTOR x, LIS_VECTOR y, LIS_SCALAR *val)
Fortran subroutine lis_vector_dot(LIS_VECTOR x, LIS_VECTOR y, LIS_SCALAR val,
LIS_INTEGER ierr)

Description

Calculate the inner product x^Ty

Input

x The vector

y The vector

Output

val The inner product value

6.3.3 lis_vector_nrm1

C LIS_INT lis_vector_nrm1(LIS_VECTOR x, LIS_SCALAR *val)
Fortran subroutine lis_vector_nrm1(LIS_VECTOR x, LIS_SCALAR val, LIS_INTEGER ierr)

Description

Calculate the 1-norm of the vector x

Input

x The vector

Output

val The 1-norm of the vector

ierr The return code

6.3.4 lis_vector_nrm2

C LIS_INT lis_vector_nrm2(LIS_VECTOR x, LIS_SCALAR *val)
Fortran subroutine lis_vector_nrm2(LIS_VECTOR x, LIS_SCALAR val, LIS_INTEGER ierr)

Description

Calculate the 2-norm of the vector x

Input

x The vector

Output

val The 2-norm of the vector

ierr The return code

6.3.5 lis_vector_nrmi

C LIS_INT lis_vector_nrmi(LIS_VECTOR x, LIS_SCALAR *val)
Fortran subroutine lis_vector_nrmi(LIS_VECTOR x, LIS_SCALAR val, LIS_INTEGER ierr)

Description

Calculate the infinity norm of the vector x

Input

x The vector

Output

The infinity norm of the vector

6.3.6 lis_vector_axpy

C LIS_INT lis_vector_axpy(LIS_SCALAR alpha, LIS_VECTOR x, LIS_VECTOR y)
Fortran subroutine lis_vector_axpy(LIS_SCALAR alpha, LIS_VECTOR x, LIS_VECTOR y,
LIS_INTEGER ierr)

Description

Calculate the vector sum $y = \alpha x + y$

Input

alpha The scalar value

x, y The vectors

Output

y $\alpha x + y$ (the vector y is overwritten)

ierr The return code

6.3.7 lis_vector_xpay

C LIS_INT lis_vector_xpay(LIS_VECTOR x, LIS_SCALAR alpha, LIS_VECTOR y)
Fortran subroutine lis_vector_xpay(LIS_VECTOR x, LIS_SCALAR alpha, LIS_VECTOR y,
LIS_INTEGER ierr)

Description

Calculate the vector sum $y = x + \alpha y$

Input

alpha The scalar value

x, y The vectors

Output

y $x + \alpha y$ (the vector y is overwritten)

6.3.8 lis_vector_axpyz

C LIS_INT lis_vector_axpyz(LIS_SCALAR alpha, LIS_VECTOR x, LIS_VECTOR y, LIS_VECTOR z)

Fortran subroutine lis_vector_axpyz(LIS_SCALAR alpha, LIS_VECTOR x, LIS_VECTOR y, LIS_VECTOR z, LIS_INTEGER ierr)

Description

Calculate the vector sum $z = \alpha x + y$

Input

alpha The scalar value

x, y The vectors

Output

z $x + \alpha y$

ierr The return code

6.3.9 lis_matrix_scaling

C LIS_INT lis_matrix_scaling(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR d, LIS_INT action)

Fortran subroutine lis_matrix_scaling(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR d, LIS_INTEGER action, LIS_INTEGER ierr)

Description

Scale the matrix A

Input

A The matrix

b The vector

action LIS_SCALE_JACOBI : Jacobi scaling $D^{-1}Ax = D^{-1}b$, where D rep-

resents the diagonal of $A = (a_{ij})$

LIS_SCALE_SYMM_DIAG : Diagonal scaling $D^{-1/2}AD^{-1/2}x=D^{-1/2}b$, where $D^{-1/2}$ represents a diagonal matrix with $1/\sqrt{a_{ii}}$ as the

diagonal

Output

d The vector which stores the diagonal elements of D^{-1} or $D^{-1/2}$

6.3.10 lis_matvec

C void lis_matvec(LIS_MATRIX A, LIS_VECTOR x, LIS_VECTOR y)
Fortran subroutine lis_matvec(LIS_MATRIX A, LIS_VECTOR x, LIS_VECTOR y)

Description

Calculate the matrix vector product y = Ax

Input

A The matrix

x The vector

Output

 \mathbf{y} Ax

6.3.11 lis_matvect

C void lis_matvect(LIS_MATRIX A, LIS_VECTOR x, LIS_VECTOR y)
Fortran subroutine lis_matvect(LIS_MATRIX A, LIS_VECTOR x, LIS_VECTOR y)

Description

Calculate the transposed matrix vector product $y = A^T x$

Input

A The matrix

x The vector

Output

 \mathbf{y} A^Tx

6.4 Solving Linear Equations

6.4.1 lis_solver_create

C LIS_INT lis_solver_create(LIS_SOLVER *solver)
Fortran subroutine lis_solver_create(LIS_SOLVER solver, LIS_INTEGER ierr)

Description

Create the solver

Input

None

Output

solver The solver

ierr The return code

Note

solver has the information on the solver, the preconditioner, etc.

6.4.2 lis_solver_destroy

C LIS_INT lis_solver_destroy(LIS_SOLVER solver)
Fortran subroutine lis_solver_destroy(LIS_SOLVER solver, LIS_INTEGER ierr)

Description

Destroy the solver

Input

solver The solver to be destroyed

Output

6.4.3 lis_solver_set_option

Description

Set the options for the solver

Input

text The command line options

Output

solver The solver

ierr The return code

Note

The table below shows the available command line options, where $-i \{cg|1\}$ means -i cg or -i 1 and -maxiter [1000] indicates that -maxiter defaults to 1,000.

Options for Linear Solvers (Default: -i bicg)

Solver	Option	Auxiliary Options	
CG	-i {cg 1}		
BiCG	-i {bicg 2}		
CGS	-i {cgs 3}		
BiCGSTAB	-i {bicgstab 4}		
BiCGSTAB(1)	-i {bicgstabl 5}	-ell [2]	The degree l
GPBiCG	-i {gpbicg 6}		
TFQMR	-i {tfqmr 7}		
Orthomin(m)	-i {orthomin 8}	-restart [40]	The restart value m
GMRES(m)	-i {gmres 9}	-restart [40]	The restart value m
Jacobi	-i {jacobi 10}		
Gauss-Seidel	-i {gs 11}		
SOR	-i {sor 12}	-omega [1.9]	The relaxation coefficient ω (0 < ω < 2)
BiCGSafe	-i {bicgsafe 13}		
CR	-i {cr 14}		
BiCR	-i {bicr 15}		
CRS	-i {crs 16}		
BiCRSTAB	-i {bicrstab 17}		
GPBiCR	-i {gpbicr 18}		
BiCRSafe	-i {bicrsafe 19}		
FGMRES(m)	-i {fgmres 20}	-restart [40]	The restart value m
IDR(s)	-i {idrs 21}	-irestart [2]	The restart value s
MINRES	-i {minres 22}		

Options for Preconditioners (Default: -p none)

Preconditioner	Option	Auxiliary Options	
None	-p {none 0}		
Jacobi	-p {jacobi 1}		
ILU(k)	-p {ilu 2}	-ilu_fill [0]	The fill level k
SSOR	-p {ssor 3}	-ssor_w [1.0]	The relaxation coefficient ω (0 < ω < 2)
Hybrid	-p {hybrid 4}	-hybrid_i [sor]	The linear solver
		-hybrid_maxiter [25]	The maximum number of the iterations
		-hybrid_tol [1.0e-3]	The convergence criterion
		-hybrid_w [1.5]	The relaxation coefficient ω of the SOR $(0 < \omega < 2)$
		-hybrid_ell [2]	The degree l of the BiCGSTAB(l)
		-hybrid_restart [40]	The restart values of the GMRES
			and Orthomin
I+S	-p {is 5}	-is_alpha [1.0]	The parameter α of the preconditioner
			of the $I + \alpha S^{(m)}$ type
		-is_m [3]	The parameter m of the preconditioner
			of the $I + \alpha S^{(m)}$ type
SAINV	-p {sainv 6}	-sainv_drop [0.05]	The drop criterion
SA-AMG	-p {saamg 7}	-saamg_unsym [false]	Selects the unsymmetric version
			(The matrix structure must be symmetric)
		-saamg_theta [0.05 0.12]	The drop criterion $a_{ij}^2 \leq \theta^2 a_{ii} a_{jj} $
			(symmetric or unsymmetric)
Crout ILU	-p {iluc 8}	-iluc_drop [0.05]	The drop criterion
		-iluc_rate [5.0]	The ratio of the maximum fill-in
ILUT	-p {ilut 9}	-ilut_drop [0.05]	The drop criterion
		-ilut_rate [5.0]	The ratio of the maximum fill-in
Additive Schwarz	-adds true	-adds_iter [1]	The number of the iterations

Other Options

Option				
-maxiter [1000]	The maximum number of the iterations			
-tol [1.0e-12]	The convergence criterion			
-print [0]	The display of the residual			
	-print {none 0}	None		
	-print {mem 1}	Save the residual history		
	-print {out 2}	Display the residual history		
	-print {all 3}	Save the residual history and display it on the screen		
-scale [0]	The scaling			
	(The result will overwri	ite the original matrix and vectors)		
	-scale {none 0}			
	-scale {jacobi 1}	The Jacobi scaling $D^{-1}Ax = D^{-1}b$		
		(D represents the diagonal of $A = (a_{ij})$)		
	-scale {symm_diag 2}	ymm_diag 2} The diagonal scaling $D^{-1/2}AD^{-1/2}x = D^{-1/2}b$		
		$(D^{-1/2}$ represents the diagonal matrix with $1/\sqrt{a_{ii}}$		
		as the diagonal)		
-initx_zeros [true]	The behavior of the initial vector x_0			
	-initx_zeros {false	0) Given values		
	-initx_zeros {true 1	-initx_zeros {true 1} All values are set to 0		
-omp_num_threads [t]	The number of the threads			
	(t represents the maximum number of the threads)			
-storage [0]	The matrix storage format			
-storage_block [2]	The block size of the BSR and BSC			
-f [0]	The precision of the line	he precision of the linear solvers		
	-f {double 0}	Double precision		
	-f {quad 1}	Quadruple precision		

6.4.4 lis_solver_set_optionC

```
C LIS_INT lis_solver_set_optionC(LIS_SOLVER solver)
Fortran subroutine lis_solver_set_optionC(LIS_SOLVER solver, LIS_INTEGER ierr)
```

Description

Set the options for the solver on the command line

Input

None

Output

solver The solver

ierr The return code

6.4.5 lis_solve

```
C LIS_INT lis_solve(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x,
LIS_SOLVER solver)

Fortran subroutine lis_solve(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x,
LIS_SOLVER solver, LIS_INTEGER ierr)
```

Description

Solve the linear equation Ax = b with the specified solver

Input

A The coefficient matrix

b The right hand side vector

x The initial vector

solver The solver

Output

x The solution

The number of iterations, execution time, etc.

6.4.6 lis_solve_kernel

```
C LIS_INT lis_solve_kernel(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x, LIS_SOLVER solver, LIS_PRECON, precon)

Fortran subroutine lis_solve_kernel(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x, LIS_SOLVER solver, LIS_PRECON precon, LIS_INTEGER ierr)
```

Description

Solve the linear equation Ax = b with the specified solver and the predefined preconditioner

Input

A The coefficient matrix

b The right hand side vector

x The initial vector

solver The solver

precon The preconditioner

Output

x The solution

The number of the iterations, the execution time, etc.

ierr The return code

Note

See lis-(\$VERSION)/src/esolver/lis_esolver_ii.c, which computes the smallest eigenvalue by calling lis_solve_kernel multiple times, for example.

6.4.7 lis_solver_get_status

Description

Get the status from the solver

Input

solver The solver

Output

status The number of iterations

ierr The return code

6.4.8 lis_solver_get_iters

Description

Get the number of iterations from the solver

Input

solver The solver

Output

iters The number of iterations

6.4.9 lis_solver_get_itersex

Description

Get the number of iterations from the solver

Input

solver The solver

Output

iters The number of the iterations

iters_double The number of the double precision iterations

iters_quad The number of the quadruple precision iterations

ierr The return code

6.4.10 lis_solver_get_time

Description

Get the execution time from the solver

Input

solver The solver

Output

times The time in seconds of the execution

6.4.11 lis_solver_get_timeex

C LIS_INT lis_solver_get_timeex(LIS_SOLVER solver, double *times, double *times, double *p_c_times, double *p_i_times)

Fortran subroutine lis_solver_get_timeex(LIS_SOLVER solver, real*8 times, real*8 ptimes, real*8 p_c_times, real*8 p_i_times, LIS_INTEGER ierr)

Description

Get the execution time from the solver

Input

solver The solver

Output

times The total time in seconds

itimes The time in seconds of the iteration

ptimes The time in seconds of the preconditioning

p_c_times The time in seconds of the creation of the preconditioner

p_i_times The time in seconds of the iteration in the preconditioner

ierr The return code

6.4.12 lis_solver_get_residualnorm

Description

Calculate the relative redidual norm $||b - Ax||_2/||b||_2$ from the solution x

Input

solver The solver

Output

residual The relative residual norm $||b - Ax||_2/||b||_2$

${\bf 6.4.13 \quad lis_solver_get_rhistory}$

```
C LIS_INT lis_solver_get_rhistory(VECTOR v)
Fortran subroutine lis_solver_get_rhistory(LIS_VECTOR v, LIS_INTEGER ierr)
```

Description

Store the residual norm history of the solver

Input

None

Output

v The vector

ierr The return code

Note

The vector v must be created in advance with the function lis_vector_create. When the vector v is shorter than the residual history, it stores the residual history in order to the vector v.

6.4.14 lis_solver_get_solver

C LIS_INT lis_solver_get_solver(LIS_SOLVER solver, LIS_INT *nsol)
Fortran subroutine lis_solver_get_solver(LIS_SOLVER solver, LIS_INTEGER nsol,
LIS_INTEGER ierr)

Description

Get the solver number from the solver

Input

solver The solver

Output

nsol The solver number

ierr The return code

Note

The number of the solver is as follows:

Solver	Number	Solver	Number	
CG	1	SOR	12	
BiCG	2	BiCGSafe	13	
CGS	3	CR	14	
BiCGSTAB	4	BiCR	15	
BiCGSTAB(l)	5	CRS	16	
GPBiCG	6	BiCRSTAB	17	
TFQMR	7	GPBiCR	18	
Orthomin(m)	8	BiCRSafe	19	
GMRES(m)	9	FGMRES(m)	20	
Jacobi	10	IDR(s)	21	
Gauss-Seidel	11	MINRES	22	

6.4.15 lis_solver_get_precon

Description

Get the preconditioner number from the solver

Input

solver The solver

Output

precon_type
The preconditioner number

ierr The return code

Note

The number of the preconditioner is as follows:

Preconditioner	Number
none	0
Jacobi	1
ILU(k)	2
SSOR	3
Hybrid	4
I+S	5
SAINV	6
SA-AMG	7
Crout ILU	8
ILUT	9

6.4.16 lis_get_solvername

C LIS_INT lis_get_solvername(LIS_INT nsol, char *name)
Fortran subroutine lis_get_solvername(LIS_INTEGER nsol, character name,
LIS_INTEGER ierr)

Description

Get the solver name from the solver number

Input

nsol The solver number

Output

name The solver name ierr The return code

6.4.17 lis_get_preconname

Description

Get the preconditioner name from the preconditioner number

Input

precon_type The preconditioner number

Output

name The preconditioner name

6.5 Solving Eigenvalue Problems

${\bf 6.5.1} \quad {\bf lis_esolver_create}$

C LIS_INT lis_esolver_create(LIS_ESOLVER *esolver)
Fortran subroutine lis_esolver_create(LIS_ESOLVER esolver, LIS_INTEGER ierr)

Description

Create the eigensolver

Input

None

Output

esolver The eigensolver ierr The return code

Note

esolver has the information on the eigensolver, the preconditioner, etc.

6.5.2 lis_esolver_destroy

C LIS_INT lis_esolver_destroy(LIS_ESOLVER esolver)
Fortran subroutine lis_esolver_destroy(LIS_ESOLVER esolver, LIS_INTEGER ierr)

Description

Destroy the eigensolver

Input

esolver The eigensolver to be destoyed

Output

6.5.3 lis_esolver_set_option

Description

Set the options for the eigensolver

Input

text The command line options

Output

esolver The eigensolver ierr The return code

Note

The table below shows the available command line options, where -e {pi|1} means -e pi or -e 1 and -emaxiter [1000] indicates that -emaxiter defaults to 1,000.

Options for Eigensolvers (Default: -e pi)

Eigensolver	Option	Auxiliary Options	
		Tuxinary Options	
Power	-e {pi 1}		
Inverse	-e {ii 2}	-i [bicg]	The linear solver
Approximate Inverse	-e {aii 3}		
Rayleigh Quotient	-e {rqi 4}	-i [bicg]	The linear solver
Subspace	-e {si 5}	-ss [2]	The size of the subspace
		-m [O]	The mode number
Lanczos	-e {li 6}	-ss [2]	The size of the subspace
		-m [O]	The mode number
CG	-e {cg 7}		
CR	-e {cg 7} -e {cr 8}		

Options for Preconditioners (Default: -p none)

Preconditioner	Option	Auxiliary Options	
None	-p {none 0}		
Jacobi	-p {jacobi 1}		
ILU(k)	-p {ilu 2}	-ilu_fill [0]	The fill level k
SSOR	-p {ssor 3}	-ssor_w [1.0]	The relaxation coefficient ω (0 < ω < 2)
Hybrid	-p {hybrid 4}	-hybrid_i [sor]	The linear solver
		-hybrid_maxiter [25]	The maximum number of the iterations
		-hybrid_tol [1.0e-3]	The convergence criterion
		-hybrid_w [1.5]	The relaxation coefficient ω of the SOR
			$(0 < \omega < 2)$
		-hybrid_ell [2]	The degree l of the BiCGSTAB(l)
		-hybrid_restart [40]	The restart values of the GMRES
			and Orthomin
I+S	-p {is 5}	-is_alpha [1.0]	The parameter α of the preconditioner
			of the $I + \alpha S^{(m)}$ type
		-is_m [3]	The parameter m of the preconditioner
			of the $I + \alpha S^{(m)}$ type
SAINV	-p {sainv 6}	-sainv_drop [0.05]	The drop criterion
SA-AMG	-p {saamg 7}	-saamg_unsym [false]	Selects the unsymmetric version
			(The matrix structure must be symmetric)
		-saamg_theta [0.05 0.12]	The drop criterion $a_{ij}^2 \le \theta^2 a_{ii} a_{jj} $
			(symmetric or unsymmetric)
Crout ILU	-p {iluc 8}	-iluc_drop [0.05]	The drop criterion
		-iluc_rate [5.0]	The ratio of the maximum fill-in
ILUT	-p {ilut 9}	-ilut_drop [0.05]	The drop criterion
		-ilut_rate [5.0]	The ratio of the maximum fill-in
Additive Schwarz	-adds true	-adds_iter [1]	The number of the iterations

Other Options

Option			
-emaxiter [1000]	The maximum number of the iterations		
-etol [1.0e-12]	The convergence criterion		
-eprint [0]	The display of the residual		
	-eprint {none 0}	None	
	-eprint {mem 1}	Save the residual history	
	-eprint {out 2}	Display the residual history	
	-eprint {all 3}	Save the residual history and display it on the screen	
-ie [ii]	The inner eigensolver used in the Lanczos and Subspace		
	-ie {pi 1}	The Power (the Subspace only)	
	-ie {ii 2}	The Inverse	
	-ie {aii 3}	The Approximate Inverse	
	-ie {rqi 4}	The Rayleigh Quotient	
-shift [0.0]	The amount of the shift		
-initx_ones [true]	The behavior of the initial vector x_0		
	-initx_ones {false 0	Given values	
	-initx_ones {true 1}	All values are set to 1	
-omp_num_threads [t]	The number of the threads		
	(t represents the maximum number of the threads)		
-estorage [0]	The matrix storage format		
-estorage_block [2]	The block size of the BSR and BSC		
-ef [0]	The precision of the eigensolvers		
	-ef {double 0}	Double precision	
	-ef {quad 1}	Quadruple precision	

6.5.4 lis_esolver_set_optionC

```
C LIS_INT lis_esolver_set_optionC(LIS_ESOLVER esolver)
Fortran subroutine lis_esolver_set_optionC(LIS_ESOLVER esolver, LIS_INTEGER ierr)
```

Description

Set the options for the eigensolver on the command line

Input

None

Output

esolver The eigensolver ierr The return code

6.5.5 lis_esolve

```
C LIS_INT lis_esolve(LIS_MATRIX A, LIS_VECTOR x,
LIS_REAL evalue, LIS_ESOLVER esolver)

Fortran subroutine lis_esolve(LIS_MATRIX A, LIS_VECTOR x,
LIS_REAL evalue, LIS_ESOLVER esolver, LIS_INTEGER ierr)
```

Description

Solve the eigenvalue problem $Ax = \lambda x$ with the specified eigensolver

Input

A The matrix

x The initial vectoresolver The eigensolver

Output

evalue The eigenvalue of the mode specified by the -m [0] option

x The associated eigenvector

esolver The number of the iterations, the execution time, etc.

6.5.6 lis_esolver_get_status

C LIS_INT lis_esolver_get_status(LIS_ESOLVER esolver, LIS_INT *status)
Fortran subroutine lis_esolver_get_status(LIS_ESOLVER esolver, LIS_INTEGER status,
LIS_INTEGER ierr)

Description

Get the status from the eigensolver

Input

esolver The eigensolver

Output

status The number of the iterations

ierr The return code

6.5.7 lis_esolver_get_iters

Description

Get the number of iterations from the eigensolver

Input

esolver The eigensolver

Output

iters The number of the iterations

6.5.8 lis_esolver_get_itersex

Description

Get the number of iterations from the eigensolver

Input

esolver The eigensolver

Output

iters The number of the iterations

iters_double The number of the double precision iterations

iters_quad The number of the quadruple precision iterations

ierr The return code

6.5.9 lis_esolver_get_time

Description

Get the execution time from the eigensolver

Input

esolver The eigensolver

Output

times The time in seconds of the execution

6.5.10 lis_esolver_get_timeex

C LIS_INT lis_esolver_get_timeex(LIS_ESOLVER esolver, double *times, double *times, double *ptimes, double *p_c_times, double *p_i_times)

Fortran subroutine lis_esolver_get_timeex(LIS_ESOLVER esolver, real*8 times, real*8 itimes, real*8 ptimes, real*8 p_c_times, real*8 p_i_times, LIS_INTEGER ierr)

Description

Get the execution time from the eigensolver

Input

esolver The eigensolver

Output

times The total time in seconds

itimes The time in seconds of the iteration

ptimes The time in seconds of the preconditioning

p_c_times The time in seconds of the creation of the preconditioner

p_i_times The time in seconds of the iteration in the preconditioner

ierr The return code

6.5.11 lis_esolver_get_residualnorm

C LIS_INT lis_esolver_get_residualnorm(LIS_ESOLVER esolver,
LIS_REAL *residual)

Fortran subroutine lis_esolver_get_residualnorm(LIS_ESOLVER esolver,
LIS_REAL residual, LIS_INTEGER ierr)

Description

Calculate the relative residual norm $||\lambda x - Ax||_2/\lambda$ from eigenvector x

Input

esolver The eigensolver

Output

residual norm $||\lambda x - Ax||_2/\lambda$

$\bf 6.5.12 \quad lis_esolver_get_rhistory$

```
C LIS_INT lis_esolver_get_rhistory(VECTOR v)
Fortran subroutine lis_esolver_get_rhistory(LIS_VECTOR v, LIS_INTEGER ierr)
```

Description

Store the residual norm history of the eigensolver

Input

None

Output

v The vector

ierr The return code

Note

The vector v must be created in advance with the function lis_vector_create. When the vector v is shorter than the residual history, it stores the residual history in order to the vector v.

6.5.13 lis_esolver_get_evalues

Description

Store the eigenvalues in the vector v

Input

esolver The eigensolver

Output

v The vector which stores the eigenvalues

ierr The return code

Note

The vector v must be created in advance with the function lis_vector_create.

6.5.14 lis_esolver_get_evectors

C LIS_INT lis_esolver_get_evectors(LIS_ESOLVER esolver, LIS_MATRIX A)
Fortran subroutine lis_esolver_get_evectors(LIS_ESOLVER esolver,
LIS_MATRIX A, LIS_INTEGER ierr)

Description

Store the eigenvectors in the matrix A

Input

esolver The eigensolver

Output

A The matrix in the CRS format which stores the eigenvectors

ierr The return code

Note

The matrix A must be created in advance with the function lis_matrix_create.

${\bf 6.5.15} \quad {\bf lis_esolver_get_esolver}$

Description

Get the eigensolver number from the eigensolver

Input

esolver The eigensolver

Output

nesol The eigensolver number

ierr The return code

Note

The number of the eigensolver is as follows:

Eigensolver	Number
Power	1
Inverse	2
Approximate Inverse	3
Rayleigh Quotient	4
Subspace	5
Lanczos	6
CG	7
CR	8

6.5.16 lis_get_esolvername

Description

Get the eigensolver name from the eigensolver number

Input

nesol The eigensolver number

Output

name The eigensolver name

6.6 Operating External Files

6.6.1 lis_input

Description

Read the matrix and vector data from the external file

Input

filename The source file

Output

A The matrix in the specified storage format

b The right hand side vector

x The solution

ierr The return code

Note

The supported file formats are shown below:

- The Matrix Market format (extended to allow vector data)
- The Harwell-Boeing format

6.6.2 lis_input_vector

Description

Read the vector data from the external file

Input

filename The source file

Output

v The vector

ierr The return code

Note

The following formats are supported:

- The PLAIN format
- The Matrix Market format

6.6.3 lis_input_matrix

Description

Read the matrix data from the external file

Input

filename The source file

Output

A The matrix in the specified storage format

x The solution

ierr The return code

Note

The supported file formats are shown below:

- The Matrix Market format (extended to allow vector data)
- The Harwell-Boeing format

6.6.4 lis_output

```
C LIS_INT lis_output(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x,
LIS_INT format, char *filename)

Fortran subroutine lis_output(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x,
LIS_INTEGER format, character path, LIS_INTEGER ierr)
```

Description

Write the matrix and vector data into the external file

Input

A The Matrix

b The right hand side vector (If no vector is written to the external

file, then NULL must be input.)

x The solution (If no vector is written to the external file, then NULL

must be input.)

format The file format

LIS_FMT_MM The Matrix Market format

filename The destination file

Output

6.6.5 lis_output_vector

Description

Write the vector data into the external file

Input

v The vector

format The file format

LIS_FMT_PLAIN The PLAIN format

LIS_FMT_MM The Matrix Market format

filename The destination file

Output

ierr The return code

6.6.6 lis_output_matrix

Description

Write the matrix data into the external file

Input

A The matrix

format The file format

LIS_FMT_MM The Matrix Market format

filename The destination file

Output

6.7 Other Functions

6.7.1 lis_initialize

```
C LIS_INT lis_initialize(LIS_INT* argc, char** argv[])
Fortran subroutine lis_initialize(LIS_INTEGER ierr)
```

Description

Initialize the execution environment

Input

argc The number of the command line arguments

argv The command line argument

Output

ierr The return code

6.7.2 lis_finalize

```
C void lis_finalize()
Fortran subroutine lis_finalize(LIS_INTEGER ierr)
```

Description

Finalize the execution environment

Input

None

Output

ierr The return code

6.7.3 lis_wtime

```
C double lis_wtime()
Fortran function lis_wtime()
```

Description

Measure the elapsed time

Input

None

Output

The elapsed time in seconds from the given point is returned as the double precision number

Note

To measure the processing time, call <code>lis_wtime</code> to get the starting time, call it again to get the ending time, and calculate the difference.

References

- [1] S. Fujino, M. Fujiwara and M. Yoshida. BiCGSafe method based on minimization of associate residual (in Japanese). Transactions of JSCES, Paper No.20050028, 2005. http://save.k.utokyo.ac.jp/jsces/trans/trans2005/No20050028.pdf.
- [2] T. Sogabe, M. Sugihara and S. Zhang. An Extension of the Conjugate Residual Method for Solving Nonsymmetric Linear Systems(in Japanese). Transactions of the Japan Society for Industrial and Applied Mathematics, Vol. 15, No. 3, pp. 445–460, 2005.
- [3] K. Abe, T. Sogabe, S. Fujino and S. Zhang. A Product-type Krylov Subspace Method Based on Conjugate Residual Method for Nonsymmetric Coefficient Matrices (in Japanese). IPSJ Transactions on Advanced Computing Systems, Vol. 48, No. SIG8(ACS18), pp. 11–21, 2007.
- [4] S. Fujino and Y. Onoue. Estimation of BiCRSafe method based on residual of BiCR method (in Japanese). IPSJ SIG Technical Report, 2007-HPC-111, pp. 25–30, 2007.
- [5] Y. Saad. A Flexible Inner-outer Preconditioned GMRES Algorithm. SIAM J. Sci. Stat. Comput., Vol. 14, pp. 461–469, 1993.
- [6] Y. Saad. ILUT: a dual threshold incomplete LU factorization. Numerical linear algebra with applications, Vol. 1, No. 4, pp. 387–402, 1994.
- [7] ITSOL: ITERATIVE SOLVERS package http://www-users.cs.umn.edu/~saad/software/ITSOL/index.html.
- [8] N. Li, Y. Saad and E. Chow. Crout version of ILU for general sparse matrices. SIAM J. Sci. Comput., Vol. 25, pp. 716–728, 2003.
- [9] T. Kohno, H. Kotakemori and H. Niki. Improving the Modified Gauss-Seidel Method for Z-matrices. Linear Algebra and its Applications, Vol. 267, pp. 113–123, 1997.
- [10] A. Fujii, A. Nishida, and Y. Oyanagi. Evaluation of Parallel Aggregate Creation Orders: Smoothed Aggregation Algebraic Multigrid Method. High Performance Computational Science And Engineering, pp. 99–122, Springer, 2005.
- [11] K. Abe, S. Zhang, H. Hasegawa and R. Himeno. A SOR-base Variable Preconditioned CGR Method (in Japanese). Trans. JSIAM, Vol. 11, No. 4, pp. 157–170, 2001.
- [12] R. Bridson and W.-P. Tang. Refining an approximate inverse. J. Comput. Appl. Math., Vol. 123, pp. 293–306, 2000.
- [13] P. Sonnerveld and M. B. van Gijzen. IDR(s): a family of simple and fast algorithms for solving large nonsymmetric systems of linear equations. SIAM J. Sci. Comput., Vol. 31, Issue 2, pp. 1035–1062, 2008.
- [14] A. Greenbaum. Iterative Methods for Solving Linear Systems. SIAM, 1997.
- [15] D. H. Bailey. A fortran-90 double-double library. http://www.nersc.gov/~dhbailey/mpdist/mpdist.html.
- [16] Y. Hida, X. S. Li and D. H. Bailey. Algorithms for quad-double precision floating point arithmetic. Proceedings of the 15th Symposium on Computer Arithmetic, pp.155–162, 2001.
- [17] T. Dekker. A floating-point technique for extending the available precision. Numerische Mathematik, vol.18 pp. 224–242, 1971.
- [18] A. V. Knyazev. Toward the Optimal Preconditioned Eigensolver: Locally Optimal Block Preconditioned Conjugate Gradient Method. SIAM J. Sci. Comput., Vol. 23, No. 2, pp. 517–541, 2001.

- [19] A. Nishida. Experience in Developing an Open Source Scalable Software Infrastructure in Japan. Lecture Notes in Computer Science 6017, Springer, pp. 87-98, 2010.
- [20] E. Suetomi and H. Sekimoto. Conjugate gradient like methods and their application to eigenvalue problems for neutron diffusion equation. Annals of Nuclear Energy, Vol. 18, No. 4, pp. 205–227, 1991.
- [21] D. E. Knuth. The Art of Computer Programming: Seminumerical Algorithms, vol.2. Addison-Wesley, 1969.
- [22] D. H. Bailey. High-Precision Floating-Point Arithmetic in Scientific Computation. Computing in Science and Engineering, Volume 7, Issue 3, pp. 54–61, IEEE, 2005.
- [23] Intel Fortran Compiler User's Guide Vol I.
- [24] H. Kotakemori, A. Fujii, H. Hasegawa and A. Nishida. Implementation of Fast Quad Precision Operation and Acceleration with SSE2 for Iterative Solver Library (in Japanese). IPSJ Transactions on Advanced Computing Systems, Vol. 1, No. 1, pp. 73–84, 2008.
- [25] R. Barrett, et al. Templates for the Solution of Linear Systems: Building Blocks for Iterative Methods. SIAM, 1994.
- [26] Z. Bai, et al. Templates for the Solution of Algebraic Eigenvalue Problems. SIAM, 2000.
- [27] Y. Saad. SPARSKIT: a basic tool kit for sparse matrix computations, version 2, June 1994. http://www.cs.umn.edu/~saad/software/SPARSKIT/sparskit.html.
- [28] S. Balay, et al. PETSc users manual. Technical Report ANL-95/11, Argonne National Laboratory, August 2004.
- [29] R. S. Tuminaro, et al. Official Aztec user's guide, version 2.1. Technical Report SAND99-8801J, Sandia National Laboratories, November 1999.
- [30] R. B. Lehoucq, D. C. Sorensen, and C. Yang. ARPACK Users' Guide: Solution of Large-scale Eigenvalue Problems with implicitly-restarted Arnoldi Methods. SIAM, 1998.
- [31] R. Bramley and X. Wang. SPLIB: A library of iterative methods for sparse linear system. Technical report, Indiana University–Bloomington, 1995.
- [32] Matrix Market. http://math.nist.gov/MatrixMarket.

A File Formats

This section describes the file formats available for the library.

A.1 Extended Matrix Market Format

The Matrix Market format [32] does not support the vector data. The extended Matrix Market format is the extension of the Matrix Market format to handle the matrix and vector data. Assume that the number of the nonzero elements of the matrix $A = (a_{ij})$ of size $M \times N$ is L and that $a_{ij} = A(I, J)$. The format is as follows:

```
%%MatrixMarket matrix coordinate real general <-- Header
                                                <-+
%
                                                  | Comment lines with 0 or more lines
%
                                                <-+
MNLBX
                                                <-- Numbers of rows, columns, and
I1 J1 A(I1,J1)
                                                     nonzero elements (0 or 1) (0 or 1)
I2 J2 A(I2,J2)
                                                  | Row and column number values
                                                  | The index is one origin
IL JL A(IL, JL)
I1 B(I1)
I2 B(I2)
                                                  | Exists only when B=1
                                                  | Row number value
IM B(IM)
I1 X(I1)
I2 X(I2)
                                                  | Exists only when X=1
                                                  | Row number value
 . . .
IM X(IM)
```

The extended Matrix Market format for the matrix A and the vector b in Equation (A.1) is as follows:

$$A = \begin{pmatrix} 2 & 1 & & \\ 1 & 2 & 1 & & \\ & 1 & 2 & 1 \\ & & 1 & 2 \end{pmatrix} \qquad b = \begin{pmatrix} 0 & & \\ 1 & & \\ 2 & & \\ 3 & & \end{pmatrix}$$
 (A.1)

%%MatrixMarket matrix coordinate real general

```
4 4 10 1 0
1 2 1.00e+00
1 1 2.00e+00
2 3 1.00e+00
2 1 1.00e+00
2 2
    2.00e+00
3 4 1.00e+00
3 2 1.00e+00
3 3 2.00e+00
4 4 2.00e+00
4 3 1.00e+00
1 0.00e+00
2 1.00e+00
3 2.00e+00
4 3.00e+00
```

A.2 Harwell-Boeing Format

The Harwell-Boeing format inputs and outputs the matrix in the CCS storage format. Assume that the array value stores the values of the nonzero elements of the matrix A, the array index stores the row

indices of the nonzero elements and the array ptr stores pointers to the top of each column in the arrays value and index. The format is as follows:

```
Line 1 (A72,A8)
  1 - 72 Title
  73 - 80 Key
Line 2 (5I14)
  1 - 14 Total number of lines excluding header
  15 - 28 Number of lines for ptr
  29 - 42 Number of lines for index
  43 - 56 Number of lines for value
  57 - 70 Number of lines for right hand side vectors
Line 3 (A3,11X,4I14)
   1 - 3 Matrix type
            Col.1: R Real matrix
                   C Complex matrix (Not supported)
                   P Pattern only (Not supported)
            Col.2: S Symmetric
                   U Unsymmetric
                   H Hermitian (Not supported)
                   Z Skew symmetric (Not supported)
                   R Rectangular (Not supported)
            Col.3: A Assembled
                   E Elemental matrices (Not supported)
   4 - 14 Blank space
  15 - 28 Number of rows
  29 - 42 Number of columns
  43 - 56 Number of nonzero elements
  57 - 70 0
Line 4 (2A16,2A20)
  1 - 16 Format for ptr
  17 - 32 Format for index
  33 - 52 Format for value
  53 - 72 Format for right hand side vectors
Line 5 (A3,11X,2I14) Only presents if there are right hand side vectors
          Right hand side vector type
            F for full storage
            M for same format as matrix (Not supported)
          G if a starting vector is supplied
          X if an exact solution is supplied
   4 - 14 Blank space
  15 - 28 Number of right hand side vectors
  29 - 42 Number of nonzero elements
```

The Harwell-Boeing format for the matrix A and the vector b in Equation (A.1) is as follows:

```
1------50------60------70------80
Harwell-Boeing format sample
                                                          Lis
                                            4
                     1
                                 1
                                                        2
          8
RUA
                      4
                                4
                                           10
                                                        4
(11i7)
             (13i6)
                          (3e26.18)
                                          (3e26.18)
                      1
                       3
 2.000000000000000000E+00 1.000000000000000E+00 1.000000000000000E+00
 2.0000000000000000E+00 1.00000000000000E+00 1.000000000000E+00
 2.00000000000000000E+00 1.00000000000000E+00 1.000000000000E+00
 2.0000000000000000E+00
```

A.3 Extended Matrix Market Format for Vectors

The extended Matrix Market format for vectors is the extension of the Matrix Market format to handle the vector data. Assume that the vector $b = (b_i)$ is a vector of size N and that $b_i = B(I)$. The format is as follows:

```
      %//MatrixMarket vector coordinate real general
      <-- Header</td>

      %
      <-+</td>

      %
      | Comment lines with 0 or more lines

      %
      <-+</td>

      N
      <-- Number of rows</td>

      I1 B(I1)
      <-+</td>

      I2 B(I2)
      | Row number value

      . . .
      | The index is one origin

      IN B(IN)
      <-+</td>
```

The extended Matrix Market format for the vector b in Equation (A.1) is as follows:

```
%%MatrixMarket vector coordinate real general
4
1 0.00e+00
2 1.00e+00
3 2.00e+00
4 3.00e+00
```

A.4 PLAIN Format for Vectors

The PLAIN format for vectors is designed to write vector values in order. Assume that the vector $b = (b_i)$ is a vector of size N and that b_i is equal to B(I). The format is as follows:

The PLAIN format for the vector b in Equation (A.1) is as follows:

0.00e+00 1.00e+00 2.00e+00 3.00e+00