IBM Composer Files Link:
quantum-computing.ibm.com/composer/files/new?initial=N4IgdghgtgpiBcICcA2AjEgLADgPoEEBnQgSwHMxYwAXAWgCZcBFNAYxABoQATGQ1gE4kADtRIB7MAhCcQARwiEo0gPIAFAKIA5JvgDKAWQAE9AHQAGANwAdMCTCsANgFdeR6-JiOSAIzSn7Vg8bMFs5ARgyIzkAbXoAXRDBSKNWOMTbWwEATwAKNAAqYRIASmiY8wywAQ

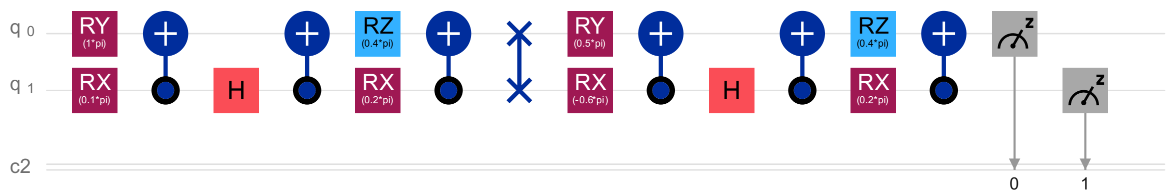
QUESTION 1

a. https://qui.science.unimelb.edu.au/circuits/60b462067b095e0049101fcc
Heavy states are $|01\rangle$ and $|10\rangle$

b. $h_2 = 0.489 + 0.441 = 0.93$

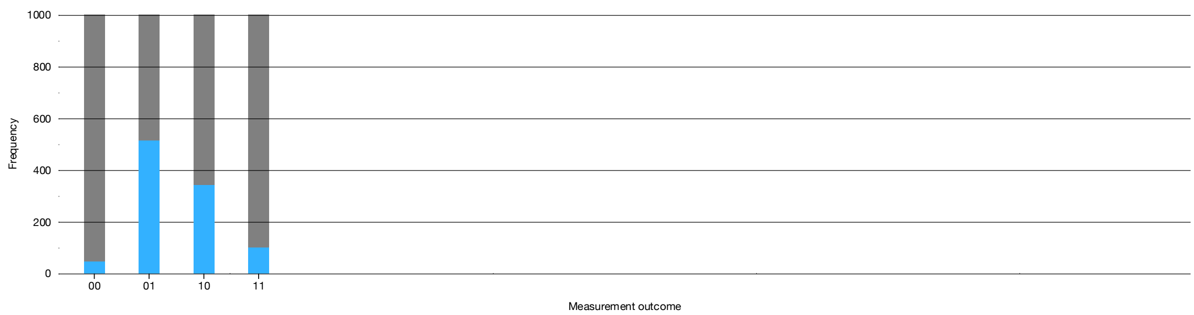
c. State probabilities $= 0.046|00\rangle + 0.513|01\rangle + 0.341|10\rangle + 0.1|11\rangle$

Qiskit Code : Q1c.py
Device : ibmq_manila
Circuit Image :



Transpiled Circuit Image:

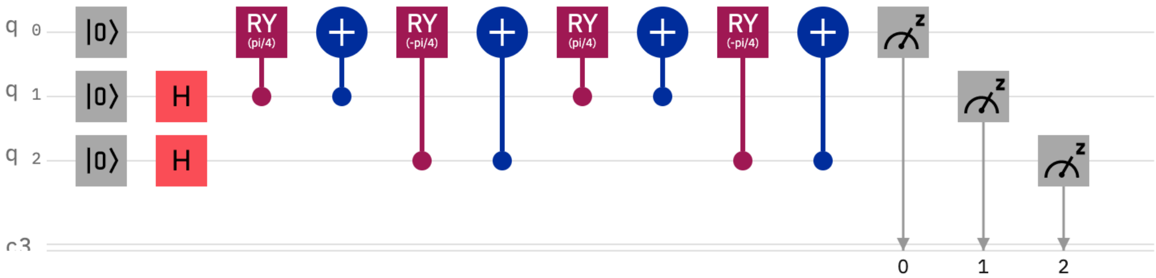
Results :



d. $h_2 = 0.513 + 0.341 = 0.854 > \frac{2}{3}$

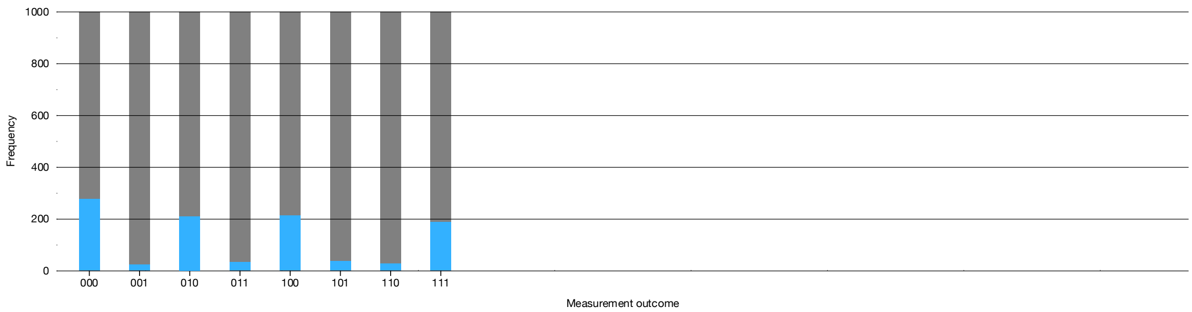
QUESTION 2

a. Qiskit Code : Q2a.py
Device : ibmq_manila
Circuit Image :



Transpiled Circuit Image:

Results :

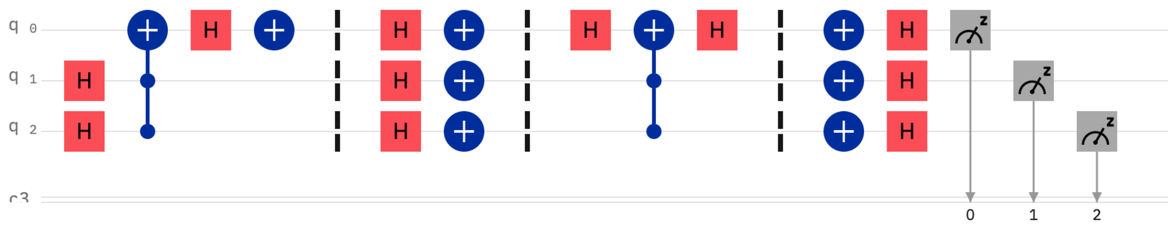


b. https://qui.science.unimelb.edu.au/circuits/60b4621d088e4a008b256bde

c. Yes, the code successfully amplified the |110⟩ state, as shown in the results below. Having run 1000 shots, we can safely assume that this amplification of the |110⟩ state is not due errors.

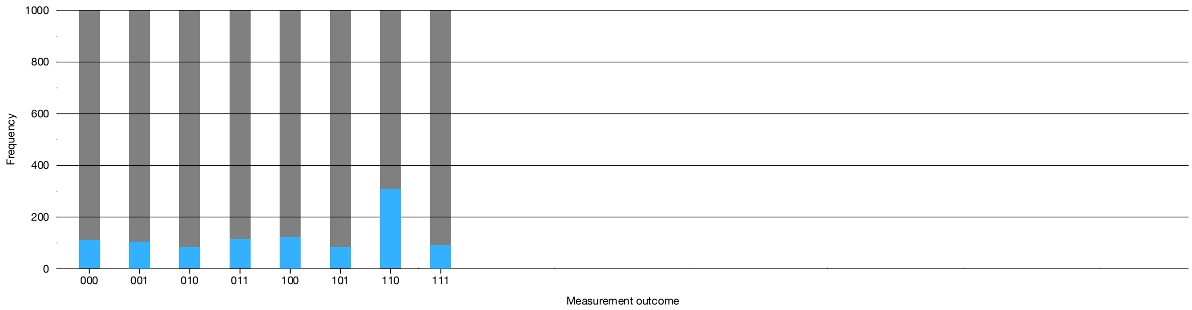
The way the circuit works is that it starts of with a Hadamard gate on all the three qubits, and then runs the oracle (before the first barrier) to flip the phase of the |110⟩ state. In this case, the oracle is made up of a Toffoli gate controlling on the first two qubits and then two more Hadamard gates surrounding the target qubit. Combined, this flips the phase of only the |110⟩ state. This Toffoli and Hadamard phase-flip combination can also be used during the mean-reversion stage (middle of the second and third barriers) since IBM does not allow for multi-control qubits. We can also optimise away the two adjacent Hadamard gates at the very start of the circuit.

Qiskit Code : Q2c.py
Device : ibmq_manila
Circuit Image :



Transpiled Circuit Image:

Results :



QUESTION 3

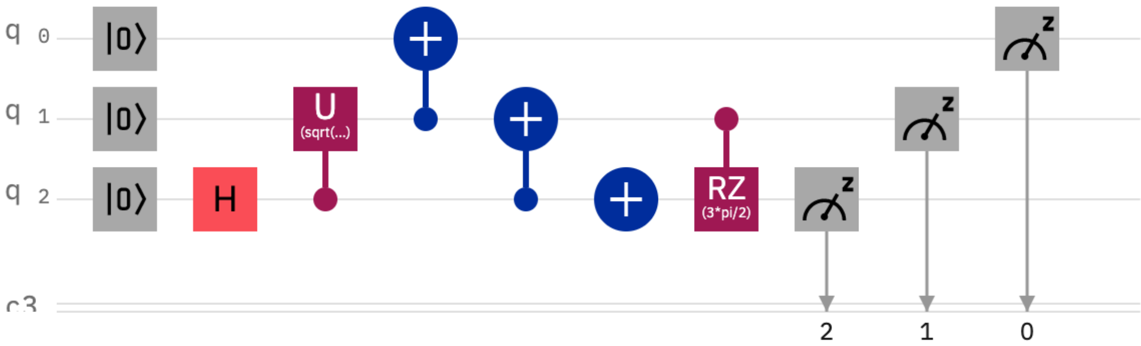
a. https://qui.science.unimelb.edu.au/circuits/60b3d0ca2a724f006a69d178

Using concepts that I used in the first assignment, my first goal was to create the W-3 state, which was the $\frac{|001\rangle+|010\rangle+|100\rangle}{\sqrt{3}}$ state and then modifying the magnitudes and phases. The creation of the W-3 state made up the first five qubits of the circuit.

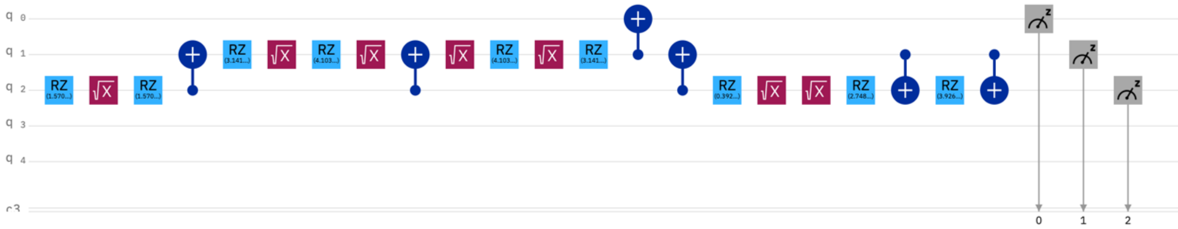
Upon calculating the probability from the magnitudes in the given K-state, we can see that the $|100\rangle$ state has a 0.5 probability while the $|001\rangle$ and $|010\rangle$ states make up $\frac{2}{3}$ and $\frac{1}{3}$ of the remaining 0.5. Hence, the first qubit was to create the two 0.5 probability distributions and the following R gate was used to split one of the 0.5 probabilities into $\frac{2}{3}$ and $\frac{1}{3}$ of itself. Ignoring the phases that was applied, the other CNOT and X gates were to flip the bits around till we get the final three $001\rangle, |010\rangle$, and $|100\rangle$ states that we want. The last step is to flip the phases according to the rotations specified in the K-state, which is the role of the final Z gate controlled on the other two gates (since we only need to alter the phase of the $|010\rangle$ state).

b. Following the code above, I replicated the same gates apart from the introduction of a U-gate and some changes to the final RZ angle rotation. Due to IBM not allowing a rotation gate with a custom axis, I used the U-gate that most closely replicated the custom axis.

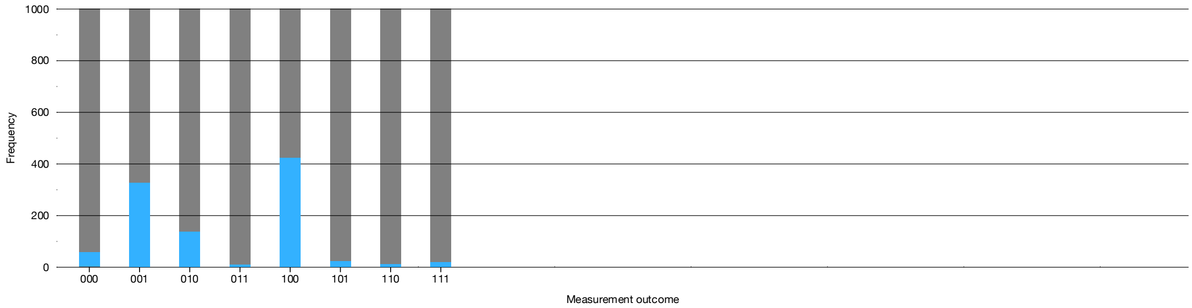
Qiskit code : Q3.py
Device : ibmq_manila
Circuit Image :



Transpiled Circuit Image:

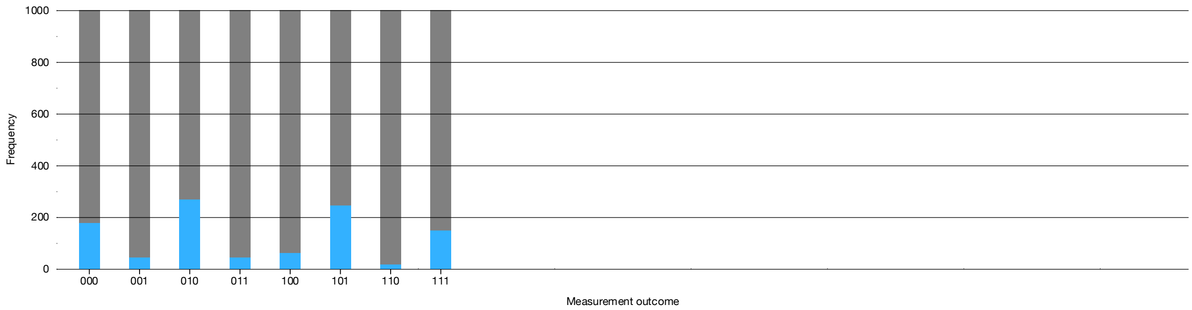


c. Results :



As shown above, upon running 1000 shots, our circuit successfully returned a frequency that approximately replicated the error-free probabilities we got from QUI. That said, there were some minor errors that resulted in our circuit reading the other states. One error that stands out is the $|000\rangle$ state. This could come in the form of an X- or Y rotation-error in one of the $|1\rangle$ qubits.

d. Results:



Resulting probability of measuring $|000\rangle = 0.176$
Ideally, it should be 0.134

$$\frac{\sqrt{2}}{\sqrt{6}}|100\rangle \xrightarrow{H_1H_2H_3} \frac{\sqrt{2}}{\sqrt{6}}\left(\frac{1}{2\sqrt{2}}\right)(|0\rangle - |1\rangle)(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)$$

$$-\frac{e^{i\frac{3}{4}\pi}}{\sqrt{6}}|010\rangle \xrightarrow{H_1H_2H_3} -\frac{e^{i\frac{3}{4}\pi}}{\sqrt{6}}\left(\frac{1}{2\sqrt{2}}\right)(|0\rangle + |1\rangle)(|0\rangle - |1\rangle)(|0\rangle + |1\rangle)$$

$$\frac{\sqrt{3}}{\sqrt{6}}|001\rangle \xrightarrow{H_1H_2H_3} \frac{\sqrt{3}}{\sqrt{6}}\left(\frac{1}{2\sqrt{2}}\right)(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)(|0\rangle - |1\rangle)$$

Adding the $|000\rangle$ terms, we get a magnitude of $|a| = 0.366$ and thus $|a|^2 = 0.134$

QUESTION 4

a. https://qui.science.unimelb.edu.au/circuits/60b43ebcf25ca0003e5fe604

b. With the single $\langle Z_i \rangle$ terms, all we need to do is apply a measure gate as it defaults to measuring in the Z basis.

   With the $\langle X_i \rangle$ terms, what we need to do is change the basis. Since the X gate is essentially a HZH gate, this means all we need to do is apply the Hadamard gate before measuring, then measure it in the Z-basis with the measurement gate, and after measuring, we can change it back to the original version by applying a second Hadamard gate.

   Similarly, with the $\langle Y_i \rangle$ terms, since the Y gate is essentially a $\sqrt{X}^\dagger Z\sqrt{X}$ gate, this means all we need to do is apply the $\sqrt{X}$ gate before measuring, then measure it in the Z-basis with the measurement gate, and after measuring, we can change it back to the original version by applying a $\sqrt{X}^\dagger$ gate.

   With the double measurement $\langle U_1 U_2 \rangle$ terms, we can just measure them separately using the appropriate methods as defined above and multiply them both.

c. Minimum $E = -1.47 + (+1)0.442 - (-1)0.246 + (-1)0.373 + (-1)0.191 + (-1)0.191 + (+1)0.01 = -2.019$