MULT90063 Introduction to Quantum Computing

Assignment 2

Due: 5pm, Week 12, Friday 28th May, 2021

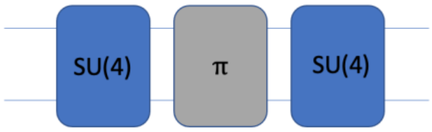
Welcome to Assignment 2 for MULT90063 *Introduction to Quantum Computing*.

Instructions: Work on your own, attempt all questions. Hand in your completed written work on or before the due date as per instructions above (standard late penalties will apply). The QUI circuits you create for this assignment must be shared (to do this you must enable sharing through the file menu) and saved with the indicated filenames. Please submit your assignment online via LMS. To do this convert your answers to a PDF

Total marks = 40

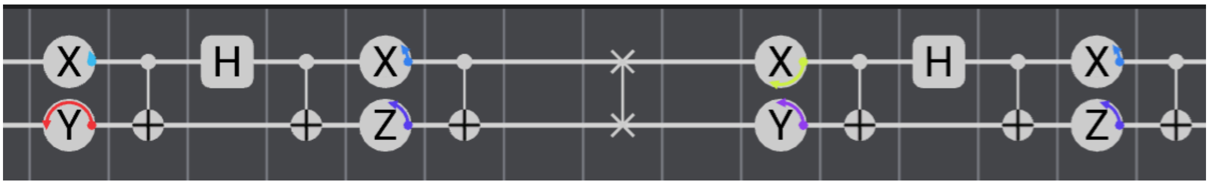
(1) Quantum Volume [9 marks = 2, 2, 4, 1]

One metric which is currently considered as an indicator of the utility of quantum computers is the "quantum volume". In order to determine the quantum volume, you need to run a quantum circuit and examine the outcome. Let us implement a typical circuit for two qubits:



Here SU(4) indicates a *random* two qubit gate, and π indicates a random permutation of the two qubits (ie. randomly either applying a swap gate or not). For the purposes of this question, however, we will choose specific gates to apply.

Implement the following gates on QUI:



Here the angles (reading top to bottom, left to right) are, 0.1 π, π, 0.2 π, 0.4 π and -0.6 π, 0.5 π, 0.2 π, 0.4π.

(a) Implement this circuit in QUI and determine (recording in your assignment) the states with the two largest probabilities. These states are known as the "heavy" outputs. |01> & |10>

(b) What is the total combined probability (h_2) of measuring either of these two states? 0.489 + 0.441 = 0.93

(c) Implement the same circuit on an IBM quantum device of your choice. Record the probabilities of measuring each state after the application of this circuit. |00>, |01>, |10>, |11>
0.016, 0.489, 0.411, 0.053

In actual experiments to determine the quantum volume, this would be repeated for many different choices of random unitary operation, and random permutations, for different numbers of qubits (d) and depth of circuit (with the same d). For each we would calculate h_d the total probability of “heavy” outputs (with a probability greater than the median). The quantum volume is then given by

$$V = 2^d$$

the largest size, d, for which we are confident that $h_d > 2/3$.

(d) For the case above, what is the total combined probability of measuring the heavy outputs, h_2? Is the total probability measuring a heavy output, h_2, greater than 2/3?

0.441 + 0.489 = 0.93 > 2/3

(2) Toffoli gate on IBM Quantum device [11 marks = 4, 3, 4]

We have seen that many quantum algorithms rely on the Toffoli gate. In practice, most quantum computer architectures do not implement a 3-qubit gate directly and so the Toffoli gate must be constructed from (compiled into) a decomposition involving 1 and 2-qubit gates.

a) Using any IBM 5-qubit quantum computing device, implement the best Toffoli gate you can (averaging over 1000 instances). Provide full details of the including your Qiskit code, device and circuit image, and a summary of the results in the for each input in the computational basis. Your circuit must work on an arbitrary input state, and you will be judged on the circuit optimisation and final fidelity obtained.

b) Assume the main source of error in a) is an effective Y-rotation error. Using the QUI, find the continuous error model (including mean and standard deviation) that most closely matches the distribution of probabilities most closely matching your results in a) for the 110 input state. Save your circuit as “Student number_Assignment-2_Q2b”.

c) The Toffoli gate can be used in a three qubit Grover search. Choose an IBM Quantum device and implement one iteration of Grover’s search which searches for the state 110. Provide full details of the program (text code and compiled circuit image), and the results (probability histogram) of the outcome. Did your circuit amplify the correct state? Discuss.

(3) Three qubit K-state [10 marks = 4, 3, 1, 2]

A three-qubit K-state given by

1/3-1/6+1/2

$$|K\rangle = \frac{\sqrt{2}|100\rangle - e^{i3\pi/4}|010\rangle + \sqrt{3}|001\rangle}{\sqrt{6}}$$

(a) Using QUI, implement a circuit to create the three-qubit K-state. Save your circuit as “<Student_number>_Assignment-2_Q3” and briefly explain the reasoning you used to create this circuit.

(b) On an IBM-Q device of your choice, make a choice of qubits and implement this circuit, attempting to achieve the highest fidelity state possible. Describe which device, and which qubits you chose, and any additional optimization which you made.

(c) Measure each of the qubits in the Z-basis, and record the results. Plot the measurement results. Comment on the results you obtained, including any errors.

(d) Apply a Hadamard gate to every qubit directly before the measurement and measure the result. What is the resulting probability of measuring the $|000\rangle$ state. What should the probability be ideally? Show your working.

(4) Variational Quantum Eigensolver [10 marks = 3, 3, 4]

Consider a problem for which the Hamiltonian can be written as:

$$\mathrm{H} = \mathrm{c_0 I + c_1 Z_1 + c_2 Z_2 + c_3 Z_1 Z_2 + c_4 Y_1 Y_2 + c_5 X_1 X_2 + c_6 X_1 Y_2}$$

The trial wavefunction, from which we will start our search for the lowest energy configuration, can be shown to have the form

$$|\varphi(\theta)\rangle = (\cos(\theta)\, I - i \sin(\theta)\, X_1 Y_2)|01\rangle$$

a) The circuit to construct $|\varphi(\theta)\rangle$ differs from the circuit to get *ZZ*-couplings in the QAOA applications we have considered so far. Based on the circuit for a *ZZ* coupling (e.g. shown in 5d), and the fact that $X = H\,Z\,H$ and $Y = \left(\sqrt{X}\right)^{\dagger} Z\sqrt{X}$ (where H are Hadamard operations), construct a circuit in the QUI that produces $|\varphi(\theta)\rangle$. Save your circuit as "Student number_Assignment-2_Q4".

b) In order to minimize the energy of the Hamiltonian, θ must be varied to minimize

$$\langle H\rangle = c_0 + c_1\langle Z_1\rangle + c_2\langle Z_2\rangle + c_3\langle Z_1 Z_2\rangle + c_4\langle Y_1 Y_2\rangle + c_5\langle X_1 X_2\rangle + c_6\langle X_1 Y_2\rangle$$

where for each observable $\mathcal{O}$ we must measure the quantity $\langle\mathcal{O}\rangle = \langle\varphi(\theta)|\mathcal{O}|\,\varphi(\theta)\rangle$. Note that measuring in the X or Y bases is equivalent to measuring in Z with a suitable basis-change gate. Modify your circuit to measure the expectation values required to determine $\langle H\rangle$ for a given value of θ. Rather than submit all the circuits, describe how you are programming the measurement of each term in $\langle H\rangle$.

c) Consider the case where the c's are given by:

$c_0 = -1.470$
$c_1 = 0.442$
$c_2 = -0.246$
$c_3 = 0.373$
$c_4 = c_5 = 0.191$
$c_6 = 0.010$

By trying out different values of θ, find the minimum energy for the Hamiltonian.