Optical springs and optical dilution for gravitational wave detector enhancement

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Abstract.

Gravitational wave detectors operate at optimal sensitivities such that they are limited by the quantum nature of light. At high frequencies (>500 Hz) detector sensitivity degrades due to detuning from arm cavity resonance caused by increased fluctuations in photon arrival time, shot noise. Mitigation is theoretically possible by installing a negative dispersion filter cavity (white light cavity) into the existing signal recycling system upgrading to white light signal recycling. This enhances the existing detuned narrow-band high sensitivity setup to broadband with high sensitivity by making more frequencies resonant when recycling. For the white light cavity to provide this enhancement the filter requires sufficiently low thermal noise that has currently only been demonstrated between 1 - 4 K. A proposed solution for higher temperature operation is to use a optomechanically driven resonator that utilises the near lossless mechanical nature of an optical spring to achieve optical dilution, diluting the material mechanical loss. This proposal discusses the theoretical background behind using an optomechanical AlGaAs/GaAs micro-resonator to increase the sensitivity of current detectors, the process of reaching the thermal noise optimised lolly-pop design, and the research plan to build a cavity to demonstrate an optical spring >100 kHz with moderate optical dilution.

1. Introduction

The first detection of gravitational waves was made by Advanced LIGO (aLIGO) in 2015, using ground-base Michelson-type interferometry, during the first joint observation run (O1) with Advanced Virgo (aVirgo) [1–5]. This discovery, made possible by the unprecedented spacetime strain sensitivity of such detectors, not only proved the existence of gravitational waves, supporting the general theory of relativity, but also their detect-ability. Since then detector sensitivity has been crucial in increasing the detection rate of both black hole and neutron star coalescent events.

Currently detectors have highest strain sensitivity (m $\mathrm{Hz}^{-1/2}$), a ratio measurement equivalent to the smallest measurable gravitational strain in space-time, in the frequency band from 35 - 500 Hz, $\approx 5 \times 10^{-}23$ m $\mathrm{Hz}^{-1/2}$ for both a aLIGO detectors during O3 [6,7]. Outside this band sensitivity deteriorates due to increasing fundamental quantum noise [8, 9]. This noise consists of two phenomena: shot noise, due to fluctuations in photon arrival time at the interferometer's output port (mainly dominating high frequency region >500 Hz), and quantum radiation pressure noise (QRPN), due to varying photon momentum exchange between the interferometer's mirrors (dominating at lower frequencies <100 Hz) [6,8,10–13]. Squeezed state injection is one quantum noise mitigation technique that was first implemented in current detectors for O3 with further enhancement (frequency dependent squeezing) being installed for observation run 4 (O4 proposed March 2023) [14–16]. This technique allows for a broadband sensitivity improvement however it does not change the sensitivity slope of the high frequency region (>500 Hz) [9].

Neutron star mergers, whilst currently detectable, display strong structure in the shot noise under the current sensitivity slope in the 1 - 4 kHz frequency range [17, 18]. Enhancing this band's sensitivity for a future observation run would allow for the the neutron star equation of state as well as the dynamics of collapse to a black hole to be determined. High frequency sensitivity improvement also allows for quasinormal mode measurements for lower mass black holes which would give the ability to make precise calculations of lower mass black hole mass and spin [19]. Thus, both cases stipulate the importance of a high frequency sensitivity enhancement to current gravitational wave detectors.

Both aLIGO and aVirgo currently use signal recycling to increase the signal to noise ratio by re-injecting the interferometer output signal back into the arms, using a signal recycling mirror (SRM), for cavity resonant enhancement. The system has two setups either detuned for a narrow band enhancement with high optical quality factor (Q_{opt}) or tuned for a broadband with low Q_{opt}. This can be upgraded to broadband with high Q_{opt} by installing a sufficiently low-loss negative dispersion filter cavity creating a white light signal recycler (WLSR) scheme. The theoretical improvement to sensitivity, with both current tuned and detuned recycling sensitivities, is shown in Fig. 1.

A proposed technique to achieve WLSR is to use the optical spring effect. First discussed for gravitational wave detector enhancement by Braginsky [21], the technique takes advantage of cavity optomechanics, the interaction between electromagnetic radiation and mechanical motion [22], to cyclically drive a suspended mirror (a resonator), producing spring action. Installing the optomechanically driven resonator into a cavity as a end mirror provides the required negative dispersion, discussed more in Sec. 1.1. However for the system to provide detector enhancement rather than sensitivity degradation the resonator must meet a stringent thermal noise requirement, proportional to $T \cdot Q_{mech}^{-1} < 6 \cdot 10^{-9} K$ (where T is temperature and Q_{mech} is the mechanical quality factor (Q-factor)). Hence for temperatures from 1 - 4 K the resonator needs to have a high Q_{mech} of $\approx 10^{10}$ [23, 24]. This has already been achieved but

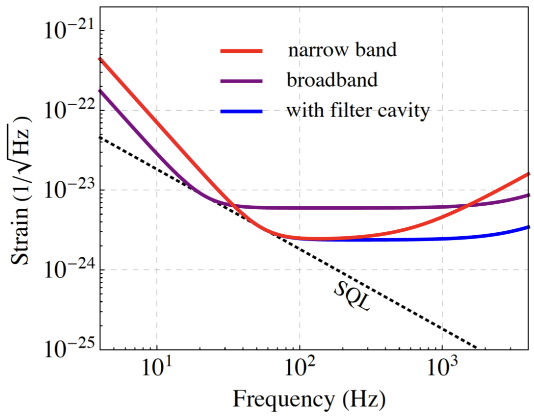

Figure 1. Detector strain sensitivity for recycling with either tuned broadband resonance at lower sensitivity (purple) or detuned narrow band resonance at high sensitivity (red). The blue line represents the improvement to high frequency sensitivity when the negative dispersion filter cavity is installed within the existing signal recycling scheme achieving broadband resonance but at detuned recycling sensitivity [20].

degrading Q_{mech} arises when operating at temperatures that exceed 4 K. The optical spring, due to its nearly lossless nature, can be implemented to make the thermal noise requirement possible at higher temperatures by using optical dilution to dilute the mechanical loss [9]. This technique will be explored in more detail in Sec. 1.1.

Upgrading to a WLSR system using an optical driven micro-resonator when combined with frequency-dependent squeezing allows for a detection volume increase by a factor of 500 at 2 kHz according to Page *et al.* [9]. Realising this system involves developing optically diluted AlGaAs/GaAs micro-resonators to create an ultra-low loss cavity. In this proposal I will review the theoretical background and previous work done developing the WLSR using micro-resonators to provide negative dispersion with optical springs and how optical dilution can be used to operate at higher temperatures (>4 K). In Sec. 2 I will review previous simulation work done during a OzGrav summer internship with results and the finalised design to demonstrate a >100 kHz optical spring. A research plan is presented in Sec. 3 with methods, intended outcomes and time management followed by a progress report, in Sec. 4, with already met objectives and results.

1.1. Background

Current gravitational wave detectors measure space-time strains using electromagnetic radiation in two orthogonal arms [8]. As a gravitational wave passes through the detector a differential change in arm length is induced, this adds a phase difference to the two orthogonal fields which recombined at a beamsplitter and pass to a photo detector. The beams are locked to initially destructively interfere but the phase difference causes an increase in optical power indicating a gravitational wave detection [6].

Signal recycling is one sensitivity enhancement technique currently being used in both aLIGO and aVirgo. For detuned recycling, narrow band high Q_{mech}, the gravitational wave carrier signal is isolated for enhancement with gravitational induced side bands cancelled out [6]. A proposed improvement to this isolation technique is to use a WLSR to achieve broadband high sensitivity cavity resonance by using a negative dispersion filter to cancel the side-band frequency dependent phase delay [9,25], shown in Fig. 2. Negative dispersion in the WLSR system was first proposed to use a atomic media by Wicht *et al.* in 1997 and 2007 [26–28] but was later replaced by a optomechanically driven resonator by Miao *et al.* [29] due to gravitational wave detector thermal noise requirements [30]. Fig. 3 shows the simplified optical layout of a Fabry-Perot interferometer with this optomechanical filter cavity implemented to create a WLSR system.

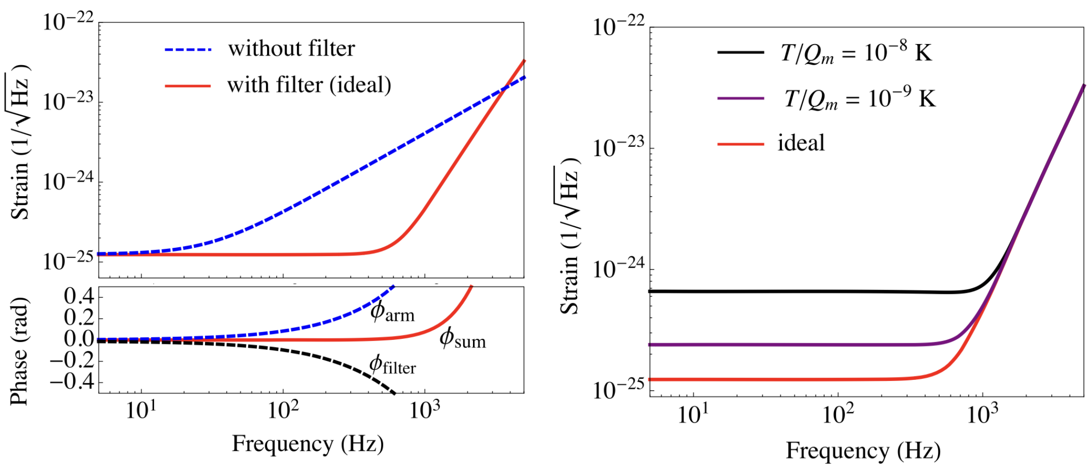

Figure 2. Top left: Interferometer strain sensitivity improvement with filter cavity when operating ideally with detuned signal recycling. The plot shows the slope change in high frequency mentioned by Page *et al.* [9]. Bottom left: Resulting phase correction to gravitational wave signal due to negative dispersion filter cavity. Right: Theoretical sensitivities that can be achieved proportional to the negative dispersion filter thermal noise $T \cdot Q_{mech}$ (where T is temperature and Q_{mech}) [9,25].

As mentioned in Sec. 1 the challenge in implementing a WLSR into existing detectors are stringent requirements on the the thermal noise of the resonator, proportional to $T \cdot Q_{mech}$. According to Page *et al.* a Q-factor for the optomechanically driven resonator of 10^{10} between 1 - 4 K is required to prevent sensitivity degradation, shown in Fig. 2. An alternative solution, to meet the $T \cdot Q_{mech}^{-1} < 6 \cdot 10^{-9} K$ requirement at temperatures above 4 K, is to use a resonator that is optimised to have its mechanical restoring force replaced with the modulating radiation pressure force, creating an optical spring [31, 32]. The total spring constant of this system is given by $k_{total} = k_m + k_o$ where k_m is the mechanical spring constant and k_o is the optical spring constant. As the optical spring is nearly lossless the total effective mechanical Q factor becomes

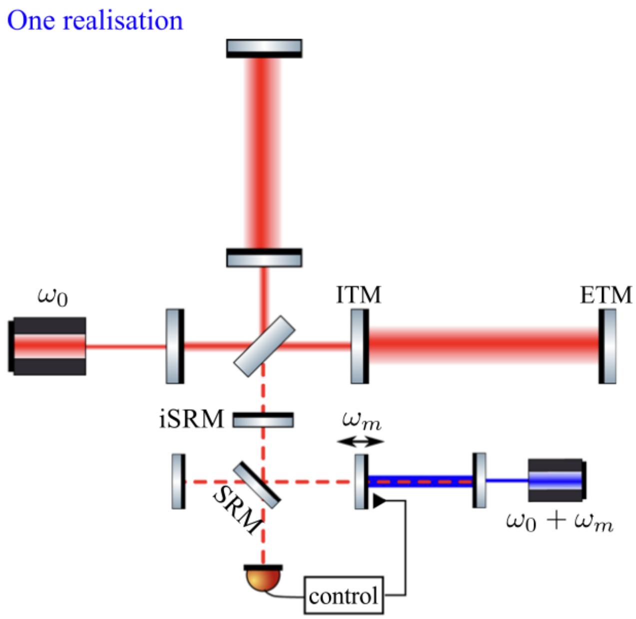

Figure 3. Gravitational wave detector optical layout with negative dispersion filter cavity using optomechanically driven resonator with mechanical frequency ω_m. iSRM and SRM are the internal and external signal recycling mirrors respectively. The resonator creates negative dispersion by coupling, via radiation pressure, to the interferometer output field (dashed red) of frequency ω_0 induced by beating with the pump field (blue). The result is then re-injected back into the interferometer output port and into the ITM-ETM Fabry-Perot arm cavities for resonant enhancement. The global feedback loop is used to stabilise the filter cavity to attain optimal enhancement [25].

$Q_{eff} = Q_{mech}(k_o/k_m)$ where if $k_o >> k_m$ the mechanical Q-factor is optically diluted so that $T \cdot Q_{eff}^{-1} < 6 \cdot 10^{-9} K$ is achievable > 4 K.

Page *et al.* [9] explored the resonator parameter space which to achieve sufficiently high micro-resonator mechanical Q-factors using optical dilution with expected results as high as 10^{14} for a square mounted mirror with rectangular cantilever, the cat-flap micro-resonator shown in Fig. 4 [24]. They concluded that a 10 - 100 ng AlGaAs micro-resonator (micro scale), with a GaAs cantilever, would be required for a optically diluted negative dispersion filter cavity, smaller than previously suggested [9].

Another promising micro-resonator design that lies within this parameter space with a mass of 50 ng, designed by Cripe *et al.* [8], is the "lolly-pop" design. This design, also shown in Fig. 4, consists of a circular 70 μm diameter mirror 3.3 μm thick attached to a 55 μm long cantilever ribbon 220 nm thick and 8 μm wide with a mechanical quality factor at room temperature of 16 000. The group modelled the thermal noise spectrum of this micro-resonator from 100 Hz - 100 kHz, See appendix A1, using finite element modelling with the aim to minimise noise to make a quantum radiation pressure noise measurement [8]. This noise model allows for building and confirming of a similar finite element analysis simulation but with the intention of optimising the dimensions for an optical spring cavity [9]. This will be discussed further in Sec. 2.

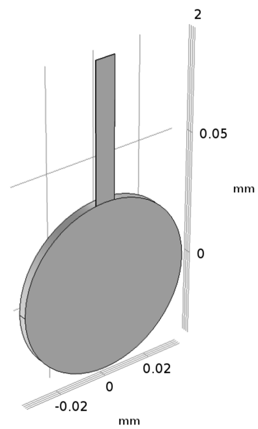

Figure 4. Left: Lolly-pop micro-resonator COMSOL finite element anaylsis model based on the Cripe *et al.* design [8]. Right: Photo of the cat-flap micro-resonator mentioned in [9].

1.2. Significance

Implementing an optical spring driven micro-resonator with optical dilution has the ability to produce a low loss negative dispersion cavity to upgrade the current gravitational wave detector detuned narrow-band recycling scheme to broadband white light signal recycling. This can theoretically increase the sensitivity in the high frequency region (>500 Hz) currently limited by quantum shot noise. The importance of increasing high frequency sensitivity is evident by currently undetectable predicted signals including: the signatures of merged neutron stars and the energy dissipation modes of lower-mass black hole mergers [17–19]. By installing the lolly-pop micro-resonator into an optical spring cavity I intend to demonstrate a >100 kHz optical spring with moderate optical dilution to mature the negative dispersion filter cavity.

2. Previous work

This section contains a review of work I completed from December 2020 to January 2021 during a summer internship with OzGrav [33]. During this time I conducted thermal noise finite element analysis modelling using COMSOL of the lolly-pop micro-resonator [8] with the aim of determining the optimal dimensions for optical spring cavity operation. This was done by building a copy of the Cripe *et al.* micro-resonator starting with the parameters in Appendix. B1 and then altering it [8]. Note that all simulated micro-resonators must've remained within the 10 - 100 ng parameter space determined by Page *et al.* [9].

Once built a gaussian radiation pressure, with optical power level α of 10^{-8} and a spot size equal to a third of the mirror diameter, was applied to the resonator to act as the optical driving force. The model was then run through a solid-state frequency domain study, sweeping from 100 Hz to 1 MHz at high resolution, to determine its elastic strain energy (J) data. This data was then exported into Matlab, divided by α for normalisation, and calculated into thermal noise amplitude using eq. 1 for comparison to the Cripe *et al.* data, Appendix A1 [8], and optimisation.

$$S = \sqrt{\frac{4k_B T}{\pi f}} U \cdot l \tag{1}$$

In the above equation S is the thermal displacement noise (m Hz$^{-1/2}$), U is the elastic strain energy (J), f is the corresponding frequency (Hz), l is a unit-less loss factor assigned to the material (6.25×10^{-5}), T is the temperature of the system (273 K) and k_B is Boltzmann's constant (J K^{-1}) [34].

Finding the optimal lolly-pop resonator involved reducing the thermal noise, keeping the first mode close to 1 kHz and higher order modes at high frequencies [9]. The finalised dimensions, determined from Appendix B2, were a 50 μm diameter mirror 3.3μm thick with a 10 μm long cantilever 60 μm wide and 220 nm thick. This design was sent for fabrication within windowed silicon wafer and arrived in August 2022 ready for experimental testing.

3. Research Plan

Reiterating the aim of my research, I seek to make a high precision measurement of lolly-pop micro-resonator thermal nosie and Q_{mech} followed by demonstrating an optical spring >100 kHz with moderate optical dilution. This measurement will take place in the UWA Physics B16 optical lab with the micro-resonator installed onto a seismic isolated breadboard, currently under development (more detail in Sec. 3.1.1 and 4), inside a vacuum tank. My main focus is to mature the optical layout for making micro-resonator optical dilution measurements that in the future can be used for other designs.

3.1. Project Methods

To achieve my aim my research methods include: the use of finite element analysis for simulating a seismic isolation system, signal analysis for testing this isolation system and optics to characterise the micro-resonator and demonstrate an optical spring with moderate optical dilution.

3.1.1. Seismic Isolation Finite Element Analysis A main limitation to conducting micro-resonator experiments is seismic noise contamination, noise due to 3 - 100 Hz ground acoustics. Whilst the optical tables inside B16 already have primitive seismic noise isolation, air cushion suppression, a matured system is needed to before taking both characterisation and optical dilution measurements. The system, proposed by John Winterflood, is to suspend the optics breadboard within the vacuum tank using four constant radius of curvature spring steel flexures designed to achieve a solely vertical motion and noise suppression within the seismic noise frequency band.

Developing this system requires an optimised blade design that meets the following criteria: has high internal modes, provides only vertical motion when a specified weight is applied to its tip, is mounted vertically, in bent position has stress that is within safety range, is within fabrication limits and fits within the vacuum tank allowing space for mounted optics while not obstructing any optical ports. The optimised mounted model was attained using a combination of finite element analysis, mathematica and solid works.

The outcome of this simulation work is maturing a vacuum tank seismic isolation design into the testing phase for the high precision micro-resonator measurements. Another benefit is future implementation the simulation can be used to adapt the system to various tanks thus providing a matured universal seismic isolation design. This research component was completed during semester 2 of 2022 with finalised results to be discussed in Sec. 4.

3.1.2. Seismic Isolation Testing Continuing from the previous method the aim now is to measure the blade suspension frequency response for the tip suspended mass when driving its base. This requires the use of two geophones, to convert vertical motion to voltage, and a spectrum analyser to determine the displacement transfer function. Once measured the modal positions will be compared to the optimised blade simulation to confirm its validity and determine the level of seismic isolation achieved.

A crucial consideration for this scheme is the need for system robustness, so that the suspension can be used with for future work without the need for constant maintenance. During this testing the design needs to be developed concerning: blade mounting, wire connections, locking schemes and passive magnetic damping while also considering ease of use for future researchers. Potential limitations currently undergoing consideration include: the precision of testing solely vertical motion, blade deformation due to twisting once mounted into the vacuum tank and wire shearing from sharp edges.

At the end of this stage the group will have a seismic isolation system with passive damping and a locking scheme for the new vacuum tank that will be used in the next stage of micro-resonator research. Testing began as of week 8 of semester 2 2022 and is intended to be installed into the vacuum tank before 2023. Current progress will be discussed in Sec. 4.

3.1.3. Optomechanic Measurement Once the vacuum tank seismic isolation system is installed the micro-resonator wafer will be mounted into a picomotor controller, used for optical alignment, inside the tank. Care needs to be taken not to accelerate the wafer during mounting as the thin cantilever is very fragile. Another problem that could occur is any contamination when installing the picomotor into the tank, clean room procedures need to be up held. Once installed the first step is to characterise the lolly-pop micro-resonator thermal noise and mechanical quality factor by measuring

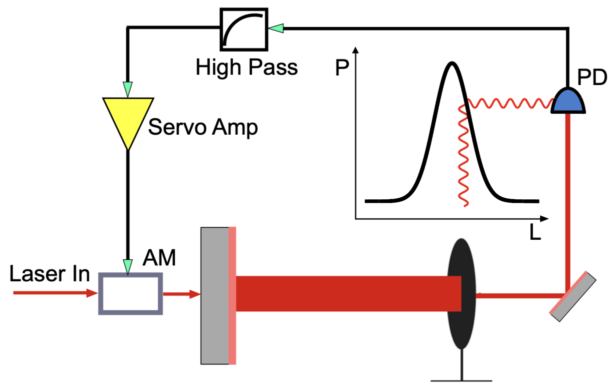

Figure 5. Schematic for optical spring cavity setup from Cripe *et al.* [22]. The laser field is passed through a amplitude modulator, for intensity modulation, and then into the detuned micro-resonator cavity. Once the field exits the cavity it is passed to a photo-detector (PD) which is connected to a control feedback loop with a high pass filter and servo control amplifier to obtain an error signal. This signal is then fed back to the amplitude modulator to control the light intensity entering the cavity to achieve a stable radiation pressure. The graph shows the distance from resonance with transmission optical power on the vertical axis and cavity length on the horizontal [22].

a modulating incident laser beam's back reflection with a photo-detector. The aim is to characterise the effective mechanical quality factor without optical dilution for comparison once dilution has been achieved in a later setup. The optical layout for characterisation is currently under development with building intended to take place in semester 1 2023.

Once characterisation of the lolly-pop micro-resonator is complete the next step is to build a cavity with the micro-resonator installed as one of the mirrors, shown in Fig. 5. Crucial to operation is stabilising the optical spring cavity using a feedback loop with amplitude modulator (AM). This locks the cavity in a detuned state that is close to resonance such that a cyclically modulating laser pressure on the micro-resonator is achieved, producing an optical spring. The photo-detector (PD) readout allows for the optical spring frequency and the effective mechanical Q-factor to be determined. The optical dilution level can then be calculated using the characterised Q_{mech}. Building and Modifying the optical setup in Fig. 5 is intended to take place is semester 2 2023. Finalised results intend to show that an optical spring > 100 kHz can be achieved for the lolly-pop micro-resonator with a moderate optical dilution level.

For future work the two optical layouts implemented in this section will be used for other micro-resonator designs that have the ability to achieve higher optical dilution necessary for the WLSR.

3.2. Project Management

I intend to work at least 20 hours a week on my research during semesters 1 and 2 of 2023 reporting on my progress weekly in OzGrav instrumentation meetings to my supervisors and peers.

4. Current Progress and Preliminary Results

This semester I have focused on implementing a matured vacuum tank seismic isolation system for future micro-resonator measurements. The optimised blade suspension system was achieved by combining mathematica equation modelling and COMSOL finite element analysis. Simulation requirements included: the first internal mode frequency was below 3 Hz, higher order modes had the highest possible frequencies, the blade stress with suspended mass was within spring steel's safety range $\approx 1 \times 10^9$ Pa, dimensions were available to order, the blade shape provided solely vertical motion with 6 kg on the breadboard and the system needed to fit within the vacuum tank without obstructing any ports. The result was a 11.73 cm blade with a base width of 4.52 cm and a constant thickness of 560 μm with an ideal radius of curvature of 5.03 cm. The simulated blade's first two eigenfrequencies are at 1.2631 Hz and 350.21 hz meeting requirements for being outside the 3 - 100 Hz frequency band. Before fabrication a extended base and a tip block was added for the pillar and wire mounting respectively.

Preliminary testing is currently ongoing with improvements, mentioned in Sec. 3.1.2, to be added before a displacement transfer function is measured. Once complete four blades will be installed into the vacuum tank and the micro-resonator wafer can be mounted for characterisation as described in Sec. 3.1.3.

5. Summary and Outlook

Upgrading existing gravitational wave detectors to a white light signal recycling scheme has the ability to provide broadband enhancement at high frequency allowing for observation of currently undetectable phenomena. One candidate in realising this system is a negative dispersion filter cavity with a optomechanically driven resonator optimised for optical dilution. Previous simulation results indicate a micro-resonator design, lolly-pop, achieves sufficiently low noise to demonstrate an optical spring. While the resonator isn't expected to meet thermal noise requirements for the white light signal recyler it can be useful in achieving a > 100 kHz optical spring with moderate optical dilution. However before these measurements can take place past micro-resonator experiments imply the need for matured seismic isolation system to reduce ground vibration contamination which is currently under development for early 2023 installation.

Future work will involve using the micro-resonator optical cavity to measure optical springs and optical dilution for other designs that do meet requirements for the white light signal recycler maturing the technique toward current detector installation.

Appendix A

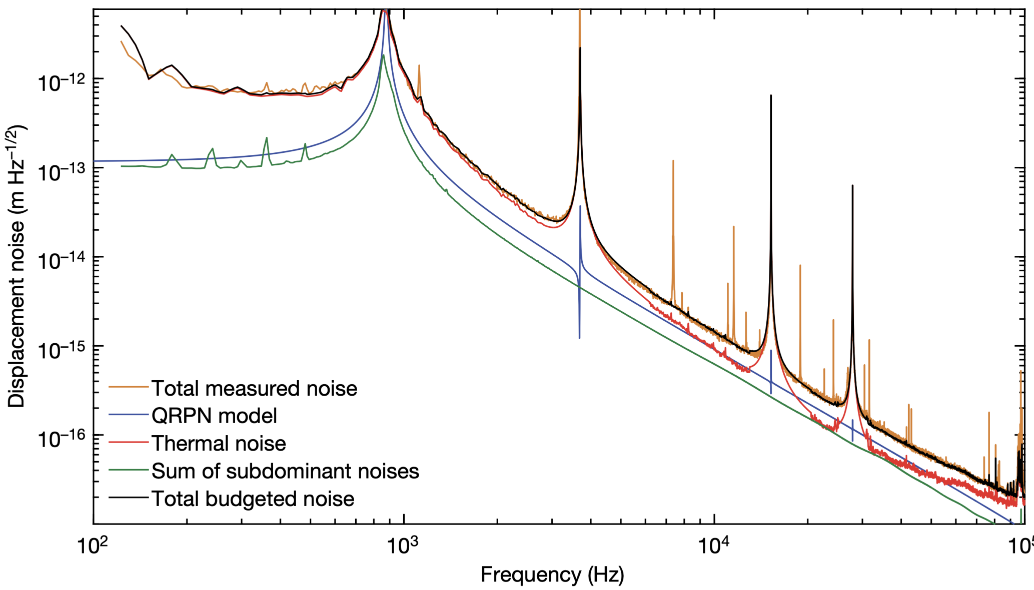

Figure A1. The measured and budgeted thermal noise from the Cripe *et al.* quantum radiation pressure noise study using lolly-pop micro-resonator [8]. Only the thermal noise finite element analysis data (red) was used in comparison to confirm the optimisation COMSOL model, Sec. 2 [33]

Appendix B

Properties	Microresonator Dimensions and Material Properties					
	Length/Diameter (um)	Width (um)	Thickness (um)	Density (kgm^{-3})	Young's Modulus (Pa)	Poisson's Ratio
Mirror	70	-	3.3	4329	15E10	0.31
Cantilever	55	8	0.22	5317.6	8.268E10	0.31

Figure B1. Cripe *et al.* [8] lolly-pop micro-resonator parameters used in COMSOL thermal noise finite element analysis modelling for confirmation comparison.

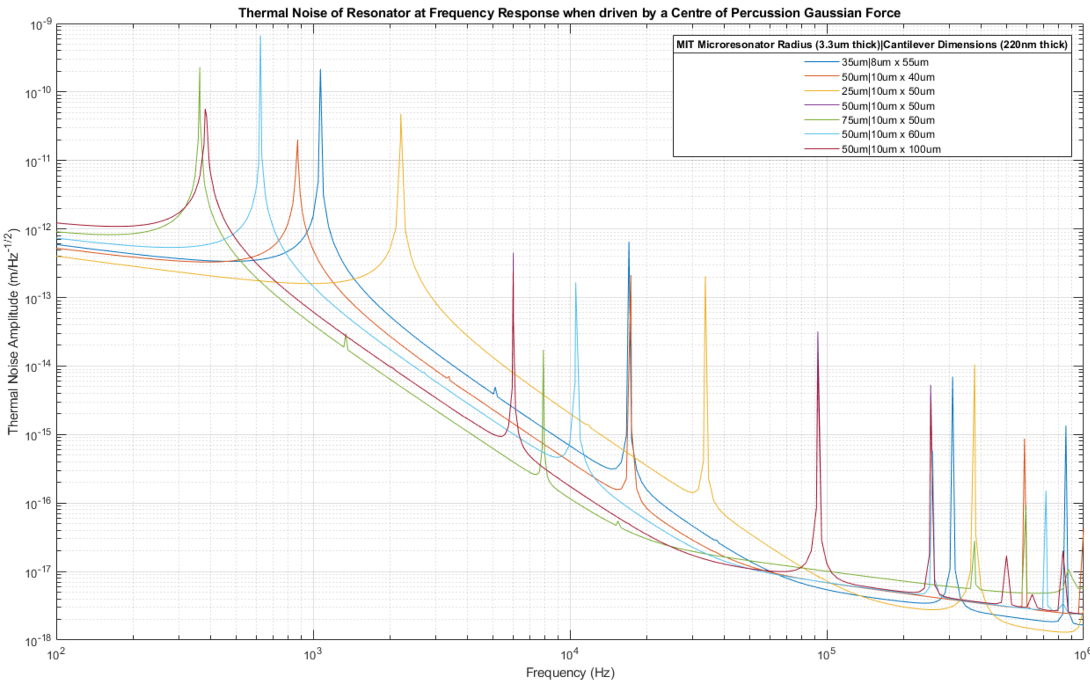

Figure B2. Final results of the thermal noise simulation for multiple lolly-pop micro-resonators. Many of these simulations were done to reach the finalised micro-resonator ready for fabrication, shown in light blue. The original Cripe *et al.* design (dark blue), with parameters shown in Appendix. B1, was compared to the red thermal noise spectrum in Appendix. A1 to confirm the simulation. [8,33]

Appendix C

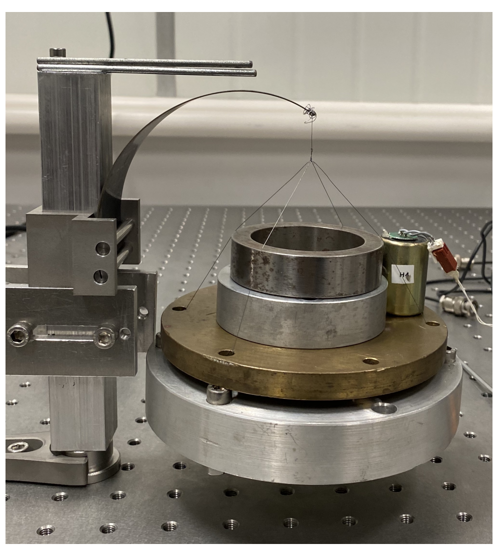

Figure C1. Mounted blade suspension for transfer function measurement with ideal mass for solely vertical motion. Aim to confirm location of internal modes to compare with simulation and then determine noise suppression factor.

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