

Last time

$$M_i = \left(\frac{\partial G}{\partial n_i} \right)_{T, P, n_j \neq i}$$

Molecules convert from high μ state
to a low μ state, until $\mu_i = \mu_j$

At const T & P

$$M_i = \bar{G}_i = G_i / n_i$$

$$G = \sum n_i \mu_i$$

how does μ depend on T & P?

Consider within one phase

$$\mu = \bar{G} = \bar{H} - T \bar{S}$$

$G = H - TS$ How does H & S depend
on T & P

$$dH = d(E + PV) = dE + PdV + VdP$$

$$dE = dq - \underbrace{PdV}_{+} \quad = \quad \overline{\dot{Q}}$$

@ const T & P

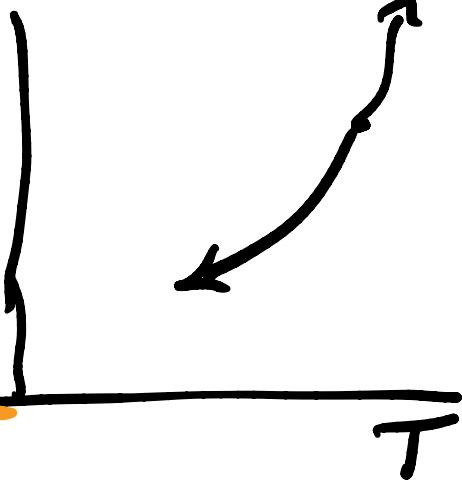
$$dH = dq_f = C_p dT$$

$$TdS = dq \quad \text{?}$$

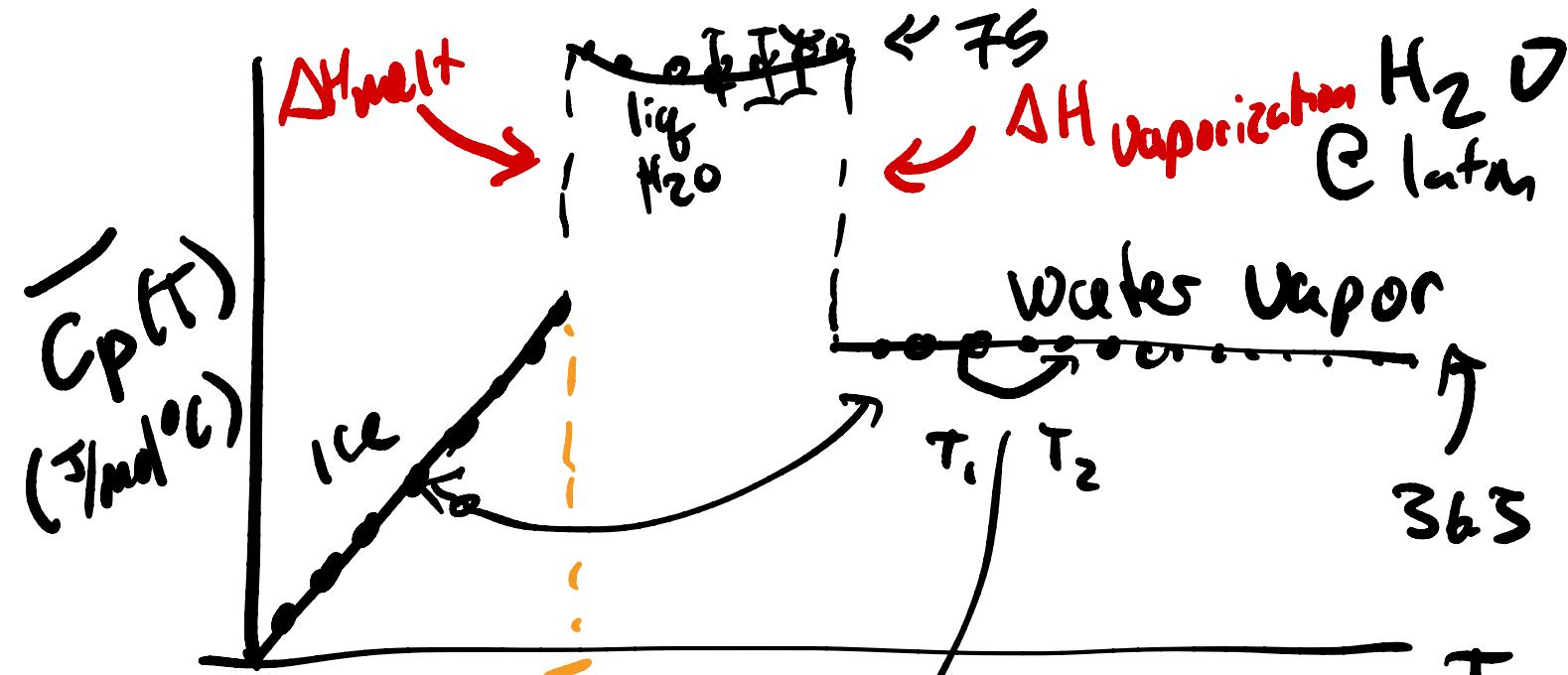
$$dH = C_p dT$$

$$dS = C_p/T dT$$

need
to integrate



need to know C_p



$$\Delta H = \int_{T_1}^{T_2} dH = \int_{T_1}^{T_2} C_p(T) dT \approx \overset{\text{gas}}{C_p^{\text{gas}}} \Delta T$$

$$\Delta \bar{H} = \bar{C}_p^{\text{gas}} \Delta T$$

$$\bar{H}(T) = \bar{H}(T_{ref}) + \int_{T_{ref}}^T \bar{C}_p dT$$

$$\bar{S}(T) = \bar{S}(T_{ref}) + \int_{T_{ref}}^T \frac{\bar{C}_p}{T} dT$$

T_{ref} for H is T_{melt} , $\bar{H}(T_m) = 0$ \downarrow
solid
phase

T_{ref} for S , $S(T_{ref}=0) = 0$

ΔS_{melt} ΔS_{vap}

$$\Delta \bar{G} = \bar{G}(\text{liq}, T_m) - \bar{G}(\text{solid}, T_m) = 0$$

@ T_m liq & solid are in equilibrium

$$\mu_{\text{liq}} = \mu_{\text{solid}}$$

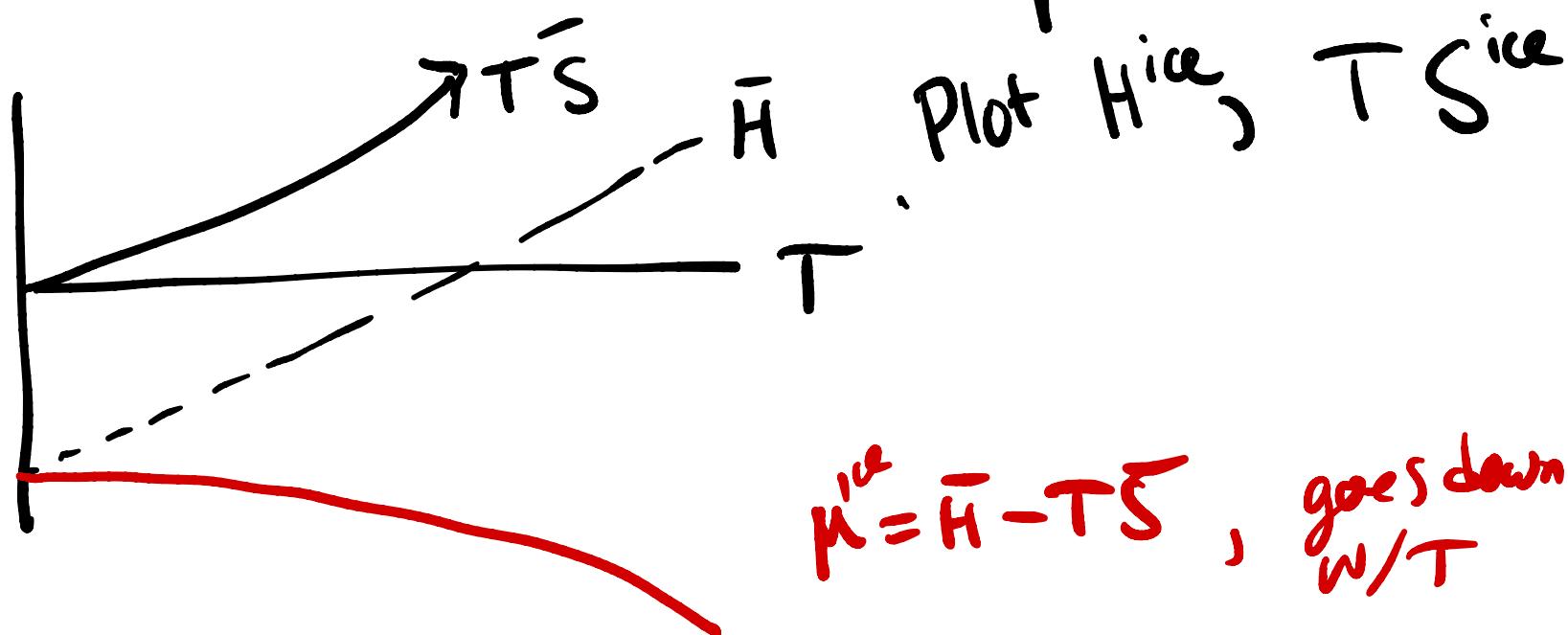
$$\Delta H = \Delta \bar{H}_{\text{melt}} - T_m \Delta \bar{S}_{\text{melt}}$$

$$\bar{\Delta S}_{\text{melt}} = \frac{\Delta \bar{H}_{\text{melt}}}{T_m}$$

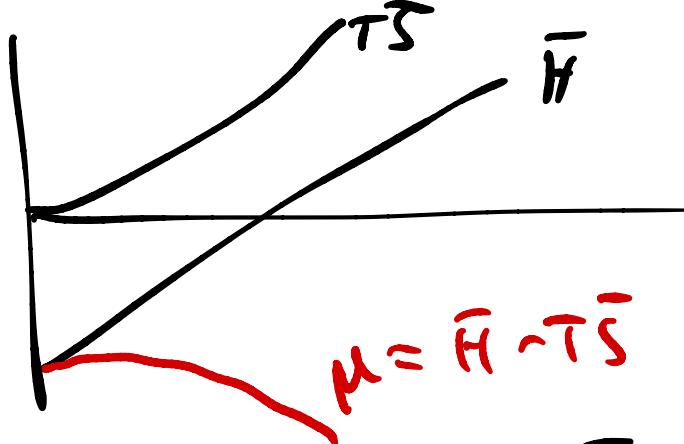
↑ Kelvin

$$\Delta \bar{H}_m = \bar{H}^{\text{liq}}(T_m) - \cancel{\bar{H}^{\text{sol}}(T_m)}$$

have functions fit for $\bar{c}_p(T)$



$$\mu' = \bar{H} - T\bar{S}, \text{ goes down w/T}$$

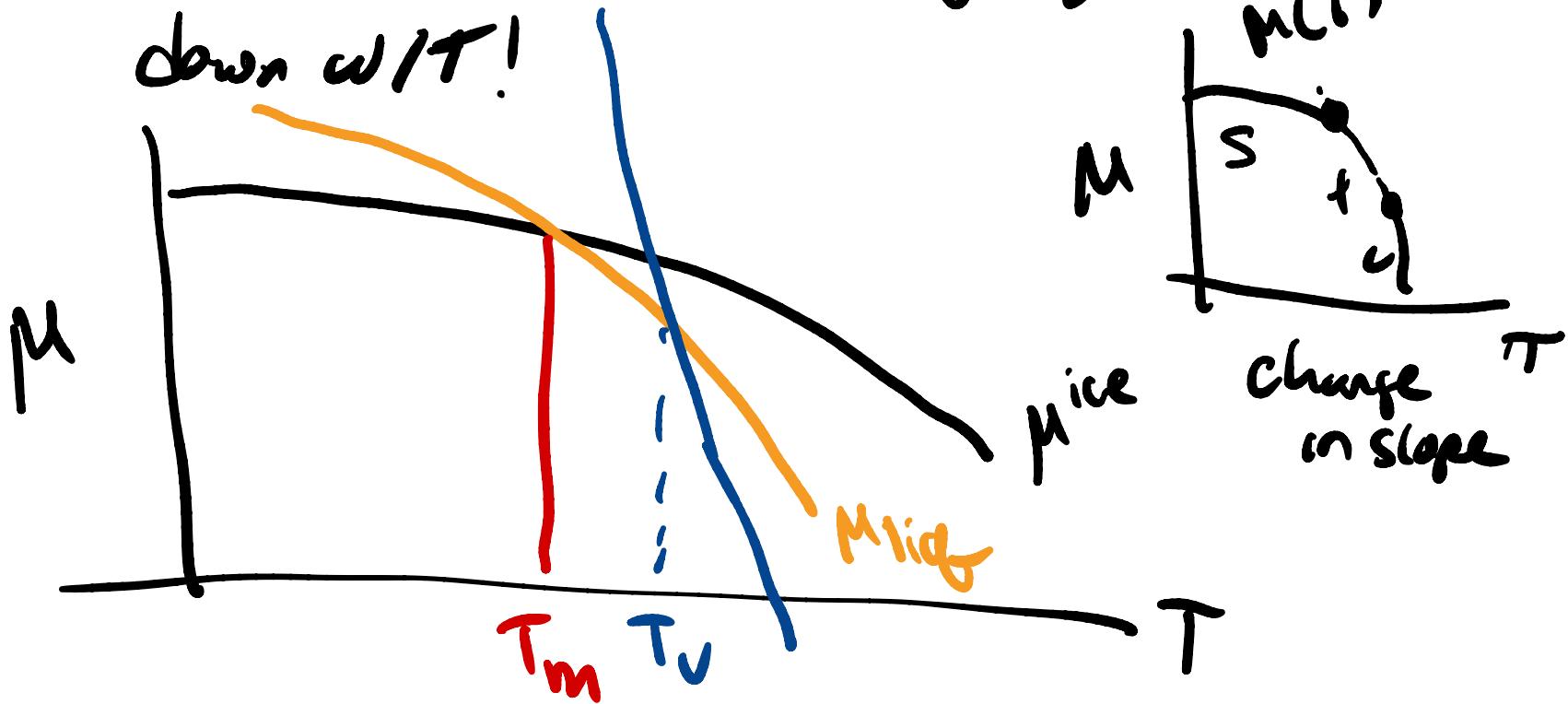


④ $\left(\frac{\partial \mu}{\partial T}\right)_P = -S$
decreasing

- ① @ T=0 $\mu = \bar{H}$
- ② $\bar{C}_P^{\text{ice}} = \left(\frac{\partial \bar{H}^{\text{ice}}}{\partial T}\right)_P > 0$ every where
- ③ $\left(\frac{\partial S}{\partial T}\right)_P = \bar{C}_P/T > 0$ S increases with T

Ice chemical potential going

down w/T!



If small (nano scale) system
no free tree phase transitions

$$G^{\text{system}} = \frac{n_{\text{solid}} \mu_{\text{solid}}}{n_{\text{tot}}} + \frac{n_{\text{liq}} \mu_{\text{liq}}}{n_{\text{tot}}} + \frac{n_{\text{vap}} \mu_{\text{vap}}}{n_{\text{tot}}}$$

$$= \sum X^{\pi} \mu^{\pi}$$

π - solid, liq, vapor?

X



How does pressure affect μ

P biggest effect on gasses

Gibb-Duhem

$$\hookrightarrow d\mu^\pi = -\bar{S}^\pi dT + \bar{V}^\pi dP$$

@ const T

$$= \bar{V}^\pi dP$$

π

how does \bar{V} in $\underbrace{\text{liq, sol. or gas}}_{\text{change}}$

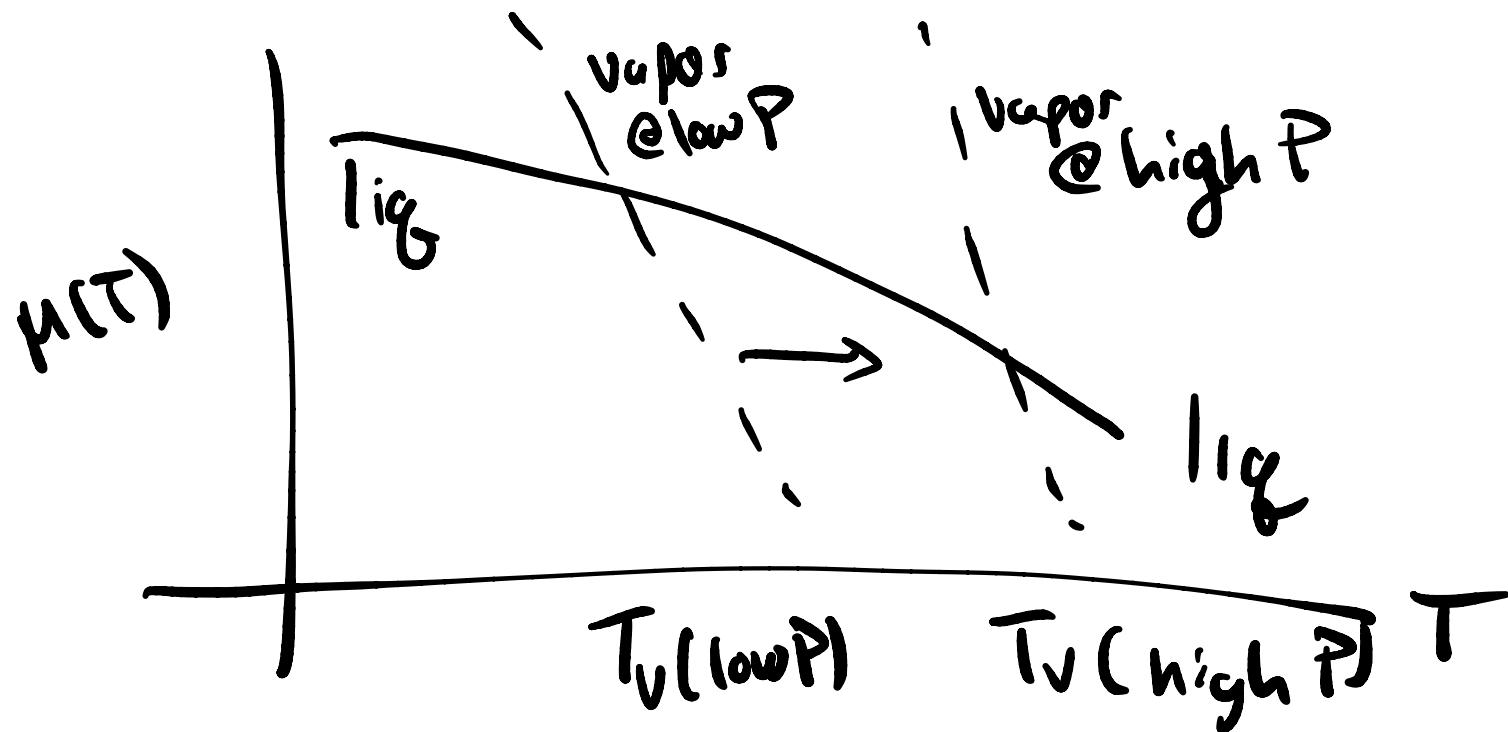
$$d\mu_{gas} = \bar{v} dp \approx \frac{RT}{P} dP$$

$$PV = nRT$$

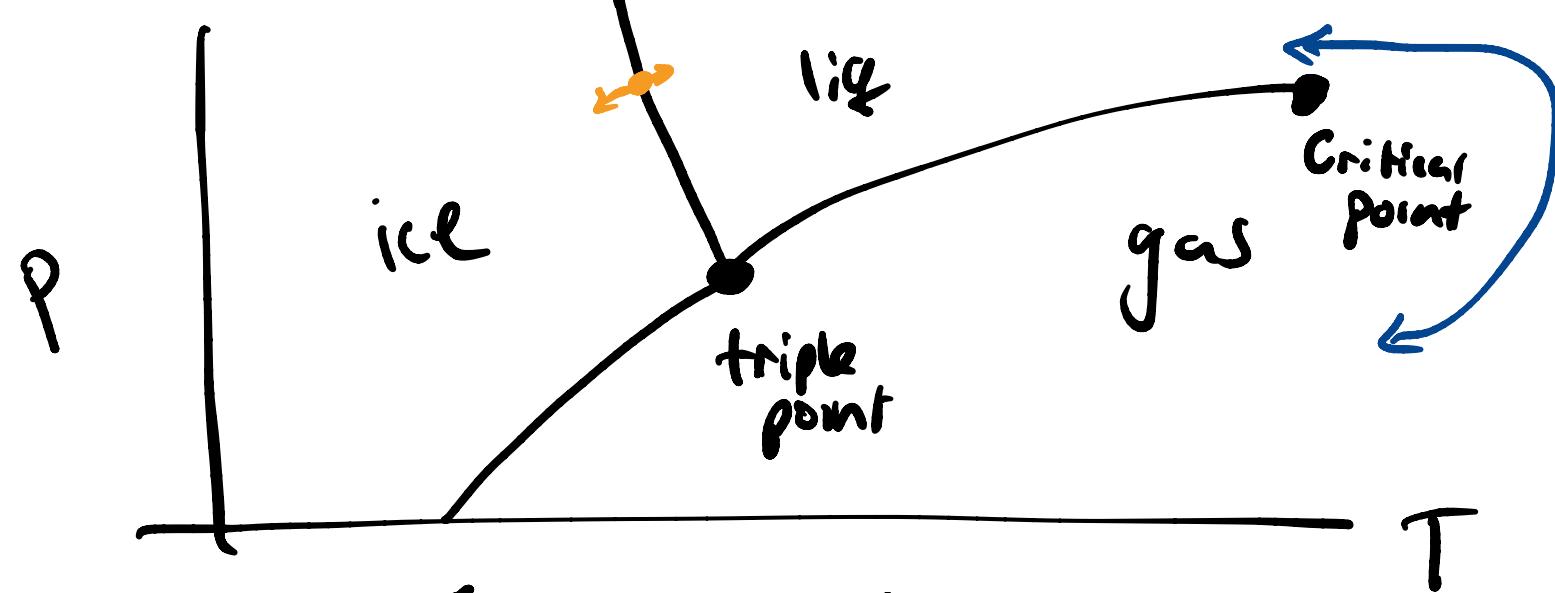
$$V_n = \frac{RT}{P}$$

$$\Delta \mu = \int_{1\text{ atm}}^P d\mu = \int_{1\text{ atm}}^P \frac{RT}{P} dP$$

$$= RT \ln(P/\text{atm})$$



boiling point elevation

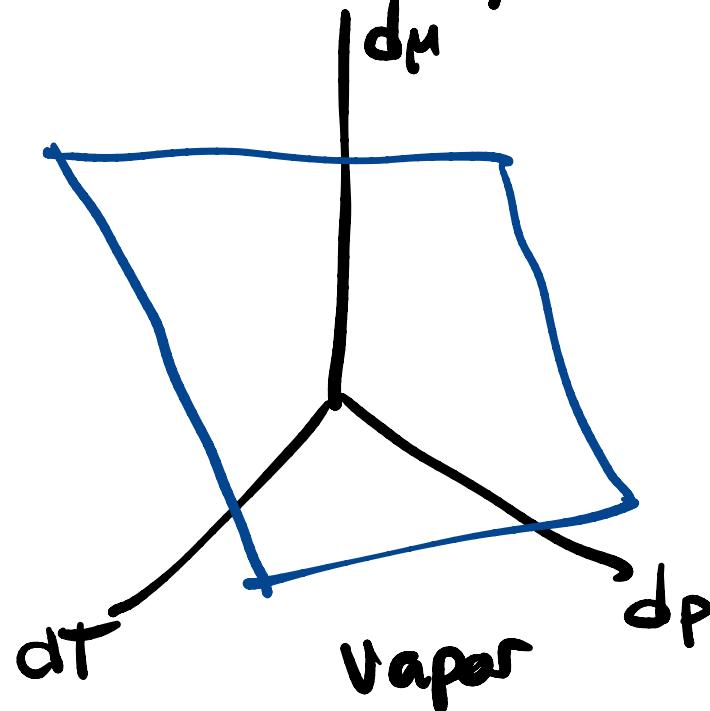
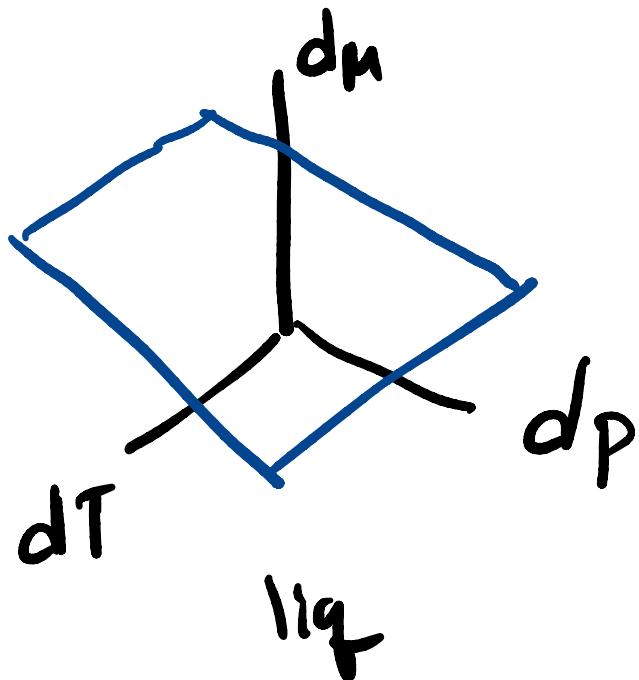


line: $\mu^{\pi_1}(T, p) = \mu^{\pi_2}(T, p)$

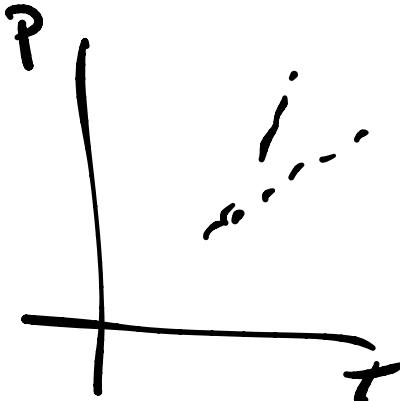
T.P. $\mu_{\text{ice}} = \mu_{\text{water}} = \mu_{\text{gas}}$, $5 \times 10^5 \text{ atm}$

$$GD \text{ eqn} \quad d\mu + \bar{S}dT - \bar{V}dp = 0$$

one equation for μ in each phase



2 planes of $\mu(T, P)$
intersect along a line



Can Show

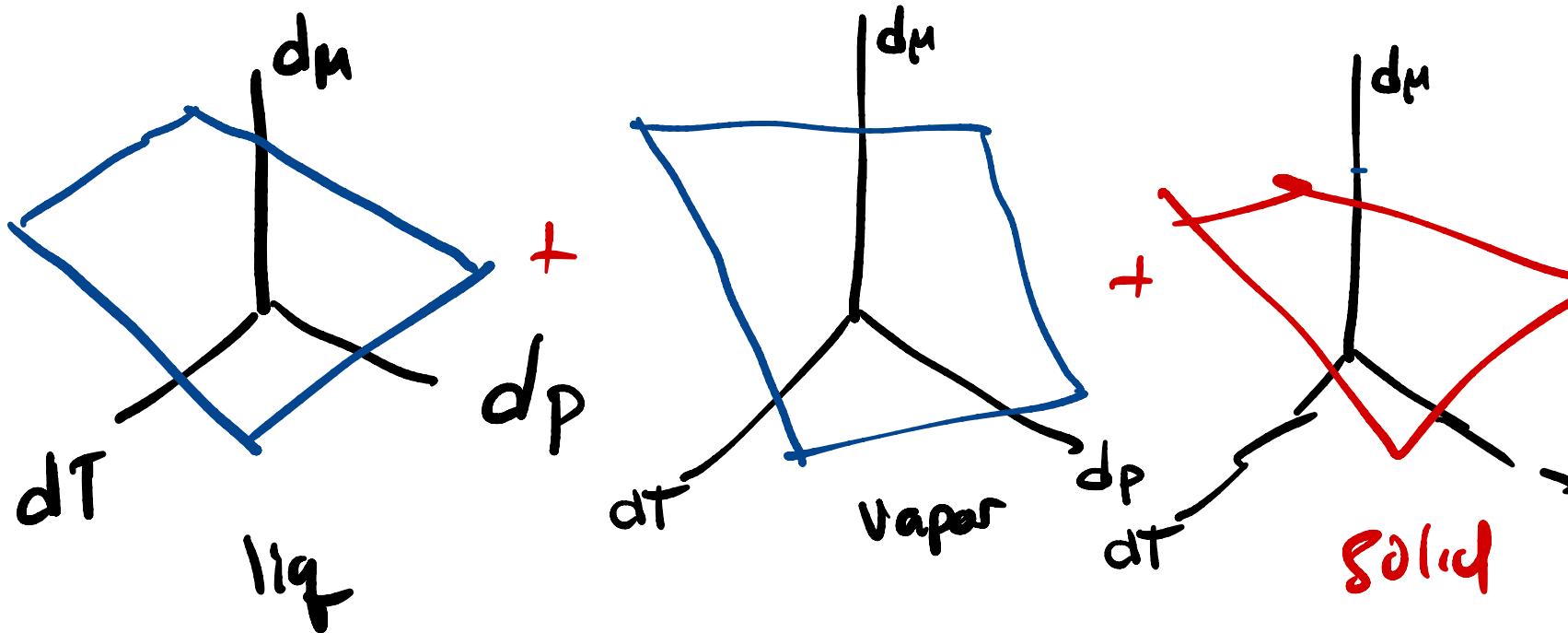
$$\left(\frac{\partial P}{\partial T}\right)_{\mu_{\text{phase}}} = \frac{\Delta S^{1 \rightarrow 2}}{\Delta V^{1 \rightarrow 2}} > \text{for almost anything}$$

μ_{phase}
 $= \mu_{\text{phase}}$

eg $l \rightarrow v_{\text{ap}}$

Claussius - Clapeyron equation

H₂O exception



5 planes can intersect only at a point

Gibbs Phase rule

$$\text{d.o.f.} = \# \text{ components} - \# \text{ coexisting phases} + 2$$

things you can change & still coexist

#	d.o.f.	
1 phase	2	P, T
2 phases	1	Change T also change P