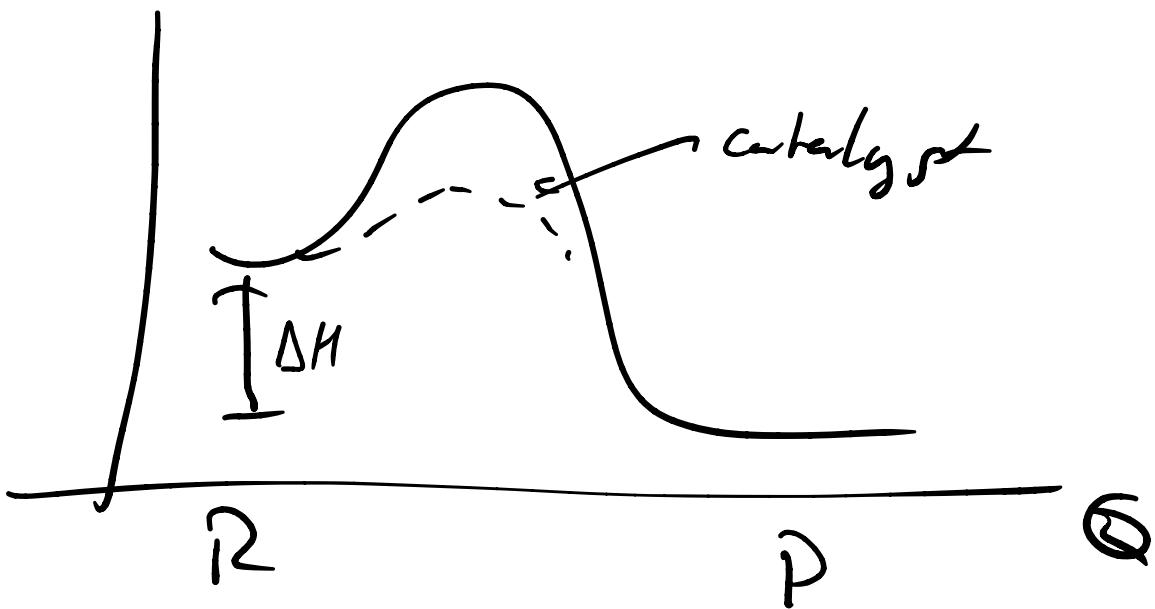


# Catalysts



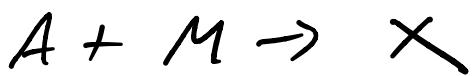
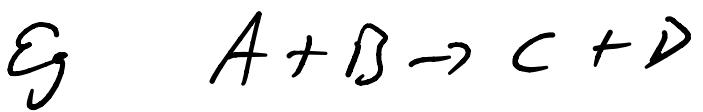
Important - different reaction paths!

Homogeneous catalysis -  
in solution mixed in

Heterogeneous catalysis (different phase)

Eg metal surface

Catalyst takes place in reaction  
but is not used up

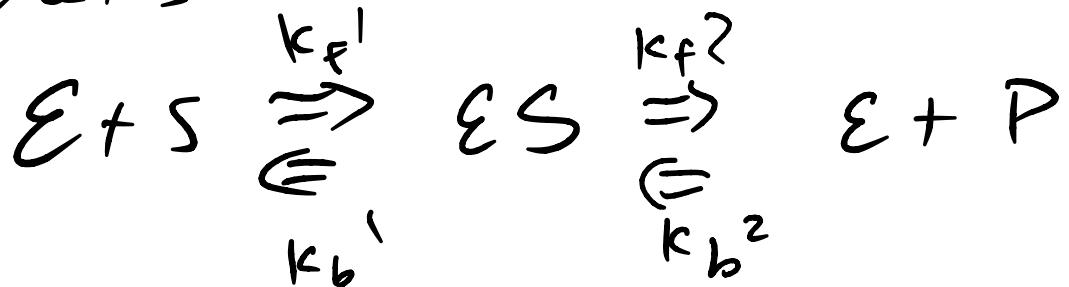


Enzymes are biological catalysts

Greatly accelerate reactions

and can be very specific & stereospecific

Michaelis - Menten Scheme



$$\frac{d[S]}{dt} = -k_f^1 [E][S] + k_b^1 [ES] \quad (1)$$

$$\frac{d[ES]}{dt} = k_f^1 [E][S] + k_b^2 [E][P] - (k_b^1 + k_f^2) [ES] \quad (2)$$

$$\frac{d[P]}{dt} = k_f^2 [ES] - k_b^2 [E][P] \quad (3)$$

Enzyme always conserved

$$[\mathcal{E}]_0 = [\mathcal{ES}] + [\mathcal{E}]$$

$$\text{so } [\mathcal{E}] = [\mathcal{E}]_0 - [\mathcal{ES}]$$

② becomes  $\frac{d[\mathcal{ES}]}{dt} = k_f' ([\mathcal{E}]_0 - [\mathcal{ES}]) [S]$   
 $+ k_b^2 ([\mathcal{E}]_0 - [\mathcal{ES}]) [P] - (k_b' + k_f^2) [\mathcal{ES}]$   
 $= -[\mathcal{ES}] \left[ k_f' [S] + k_b^2 [P] + k_b' + k_f^2 \right]$   
 $+ k_f' [S] [\mathcal{E}]_0 + k_b^2 [P] [\mathcal{E}]_0$

This reaches steady state since

$$[S] \gg [\mathcal{E}]$$

$$\Rightarrow [\mathcal{ES}] = \frac{k_f' [S] + k_b^2 [P]}{k_f' [S] + k_b' + k_b^2 [P] + k_f^2} [\mathcal{E}]_0$$

$$\begin{aligned} v = -\frac{d[S]}{dt} &= k_f' ([\mathcal{E}]_0 - [\mathcal{ES}]) [S] - k_b' [\mathcal{ES}] \\ &= k_f' [\mathcal{E}] [S] - [\mathcal{ES}] (k_f' [S] - k_b') \end{aligned}$$

$$v = k_f' [\epsilon]_0 [S] - [\epsilon S] (k_f' [S] + k_b')$$

and

$$[\epsilon S] = \frac{k_f' [S] + k_b^2 [P]}{k_f^2 [S] + k_b^2 + k_b^2 [P] + k_f^2} [\epsilon]_0$$

$$\begin{aligned} &= k_f' [\epsilon]_0 [S] \left( \cancel{k_f' [S]}^{\textcircled{1}} + \cancel{k_b'}^{\textcircled{2}} + \cancel{k_b^2 [P]}^{\textcircled{3}} + k_f^2 \right) \\ &\quad - [\epsilon]_0 \left( \cancel{k_f' k_f' [S]^2}^{\textcircled{1}} + k_f' k_b^2 [P] + \cancel{k_f' k_b'}^{\textcircled{2}} + \cancel{k_b' k_b^2 [P]}^{\textcircled{3}} \right) \\ &\quad \text{denom} \end{aligned}$$

$$\begin{aligned} &= \frac{(k_f' k_f^2 [S] - k_b' k_b^2 [P]) [\epsilon]_0}{k_f' [S] + k_b' + k_b^2 [P] + k_f^2} \quad \text{*} \end{aligned}$$

initial rate!  $[S] \approx [S]_0, [P] \approx 0$

$$\Rightarrow \frac{k_f' k_f^2 [\epsilon]_0 [S]_0}{k_f' [S]_0 + k_f^2 + k_b'} = \frac{k_f^2 [\epsilon]_0 [S]_0}{[S]_0 + k_m}$$

$\epsilon$  factor out

$$k_m = \frac{k_f^2 + k_b'}{k_f'} \quad \begin{matrix} \text{Michelis} \\ \text{const} \end{matrix}$$

At early time

$$v = \frac{k_2 [S]_0 [E]_0}{[S]_0 + k_m} \quad k_2 \text{ also called } \underline{\underline{k_{cat}}}$$

If substrate conc is low,  $[S]_0 \ll k_m$

$v$  first order in  $[S]_0$

but 0th order when  $[S]_0 \gg k_m$

Here  $v = k_2 [E]_0 = v_{max}$

all enzymes bound substrate

$$v = \frac{v_{max} [S]_0}{[S]_0 + k_m} \quad \text{if } k_m = [S]_0$$
$$v = v_{max}/2$$

Turnover number =  $v_{max} / [\text{Enzyme active sites}]$

$$[\text{active sites}] = [E]_0 \times n_{sites}$$

$$= k_{cat} \text{ if } 1 \text{ site}^{-1} \text{ (in } 1/\text{sec})$$

Catalytic efficiency is  
a measure of how effectively  
enzyme converts substrate to product

$$E = k_{cat}/k_m \text{ in } \frac{1}{M} \cdot \frac{1}{s}$$

Range from  $1 - 10^{10} M^{-1} s^{-1}$

$10^8 - 10^{10}$  is diffusion limited

$$\frac{k_{cat}}{k_m} = \frac{k_{cat}}{\underbrace{(k_{cat} + k_b')}_{k_p'}} = \frac{k_f'}{1 + \frac{k_b'}{k_{cat}}}$$

so maximized for  $k_f'$  big,  $k_b'$  small

# Introduction to Statistical thermodynamics (Browne ch 9)

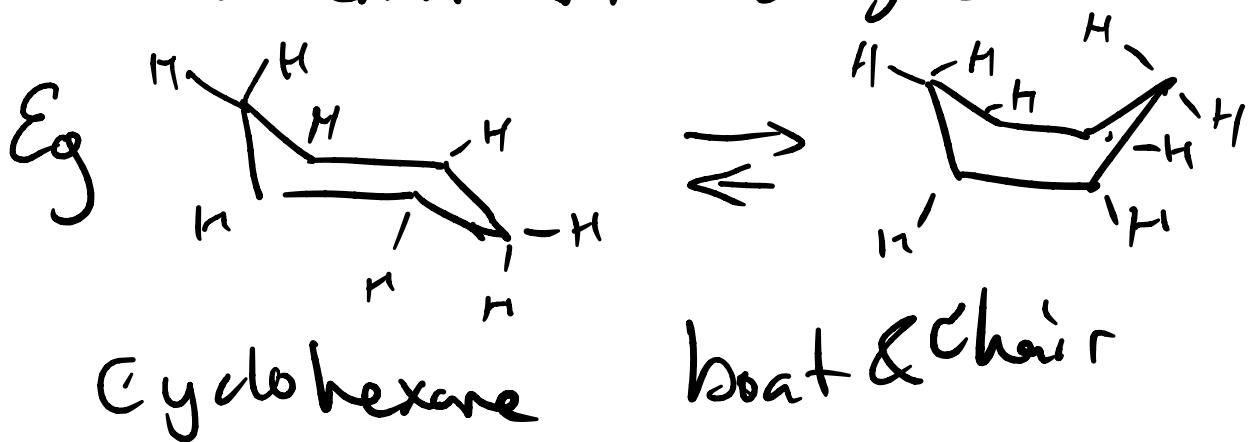
We previously made a connection between rates, eq constants, & free energy

$$\frac{k_{on}}{k_{off}} = k_{eq} = e^{-\Delta G^\circ / RT}$$

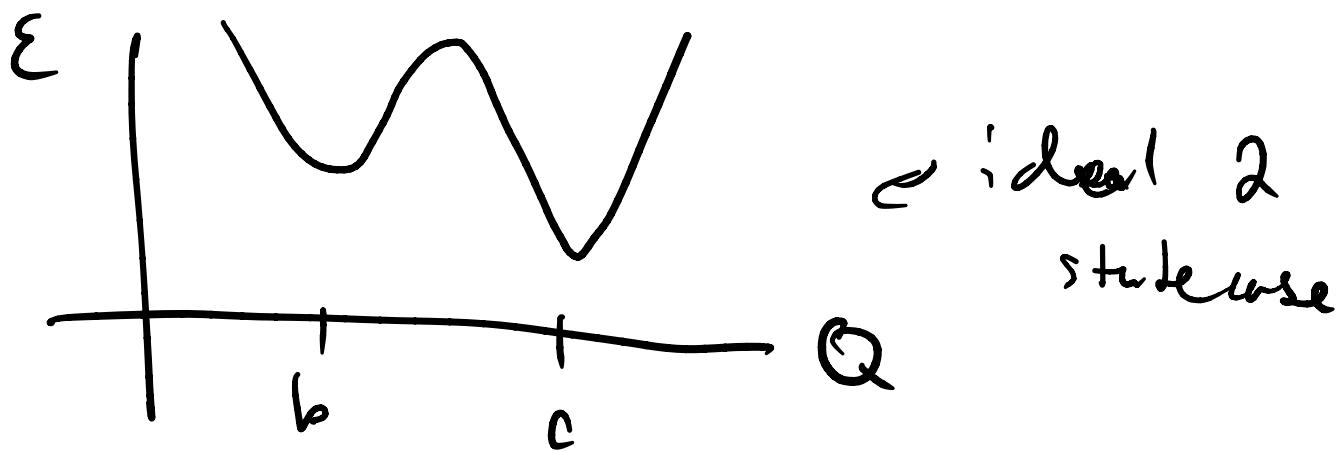
But we did not see how these properties arise from considering different configurations of sets of molecules

Statistical thermodynamics connects average properties of sets of molecules with macroscopic thermodynamic quantities

Imagine a molecule that can transition between two states of different energies

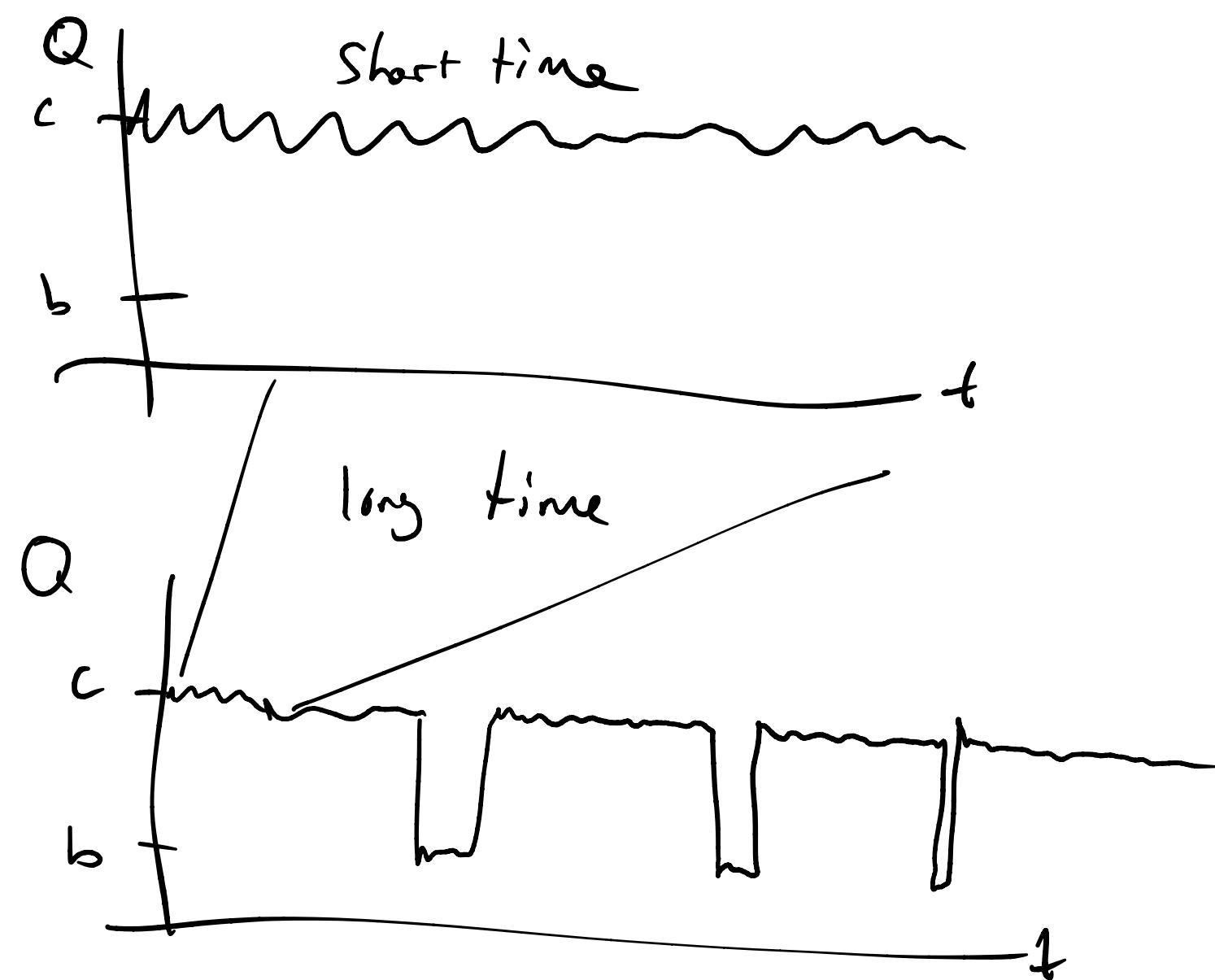


(Turns out more - populated in reality)



Suppose we put one in a box of water and watch for a long time,

and suppose we knew the conformation



See stable conformations, with transitions every so often

How often? We can get an idea of the lifetime of a state by computing something called an auto correlation function

$C(\tau)$  should be 1 at short time -  
configuration is correlated over time  $t$

0 at long times, not correlated

$Q = b$  at time 0 does not  
tell you what  $\delta(t_0 \text{ days})$  is

$$\langle C(\tau) \rangle = \frac{\langle Q(t_i + \tau) - \langle Q \rangle \rangle \langle Q(t_i) - \langle Q \rangle \rangle}{\langle Q(t_i) - \langle Q \rangle \rangle^2}$$

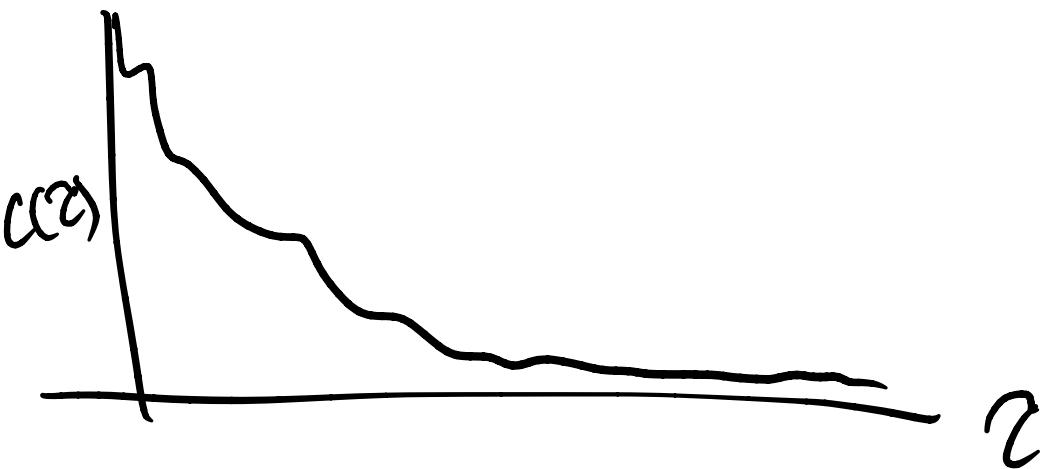
top & bottom  
averaged over measurements at  
time  $t_i$

$$SQ_i(\tau) = Q(t_i + \tau) - \langle Q \rangle$$

$$\text{then } C(\tau) = \frac{1}{n} \frac{\sum SQ_i(\tau) SQ_i(0)}{\frac{1}{n} \sum (SQ_i(0))^2}$$

at  $\tau = 0$ , this = 1

at  $\tau \rightarrow \infty$ , uncorrelated so top  $\rightarrow 0$



average transition time is

defined as

$$\int_0^{\infty} C(z) dz \equiv \tau_{rxn}$$

if  $C(z) \approx e^{-az^2}$  then  $a = 1/\tau_{rxn}$

the exponential decay time

To predict this from microscopic principles, can do computer simulations of an accurate model for a very long time, but this is actually quite hard

And impossible as  $N \rightarrow 10^{23} \approx \infty$  or  $t \rightarrow \infty$