

# Lecture 11 - Phase Transitions

Eg of combined quantities in mixtures:

$$V = \sum_{i=1}^k \bar{V}_i n_i$$

if ideal,  $\bar{V}_i = \bar{V}_i^* = 1/\rho$

$$\rho = \frac{n}{V}$$

$$E_{\text{tot}} = \sum \bar{\epsilon}_i n_i$$

$$\left[ P = \rho R T \right. \\ \left. (\text{ideal gas}) \right]$$

\*  $G_{\text{total}} = \sum \bar{G}_i n_i = \sum_i G_i n_i$

Aside

## Partial pressure

$$P_i = X_i P_{\text{total}} \quad (\text{definition})$$

$$X_i = \frac{n_i}{\sum n_i}$$

for ideal gasses  $P_i = \text{ideal gas pressure}$

$$(\text{Dalton's}) \quad P_{\text{total}} = \frac{RT}{V} \sum_i n_i = \frac{n_i}{V} RT$$

Non ideal gases: (cf Van-der Waal's eqn of state)

2 contributions to interactions:

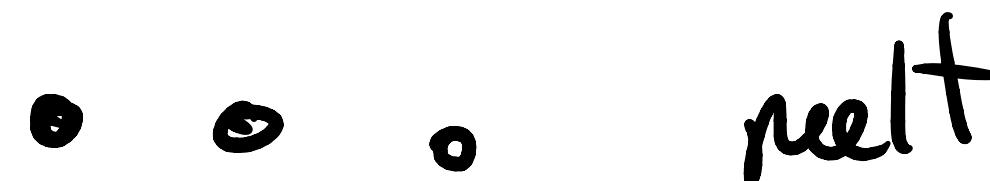
- "volume exclusion"  $\leftarrow$  pressure  $\uparrow$  (relative to an ideal gas)
- attractions  $\rightarrow$  pressure  $\downarrow$  (relatively) (gas)



# Phase transitions

"Phases of matter": solids, liquids, gasses

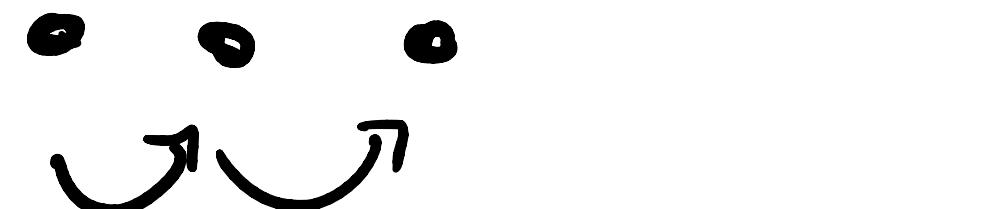
Difference: is about Symmetry (order)



melt



Interactions / packing  
set local structure



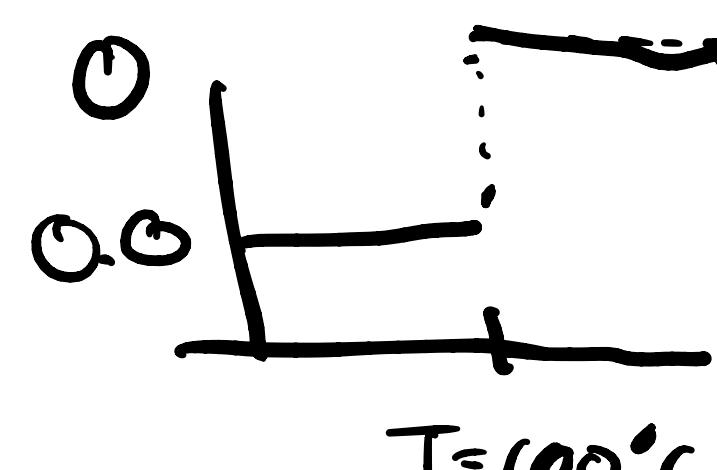
long range order  
"very ordered"

less order  
local order

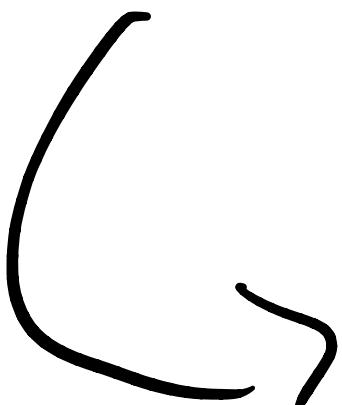
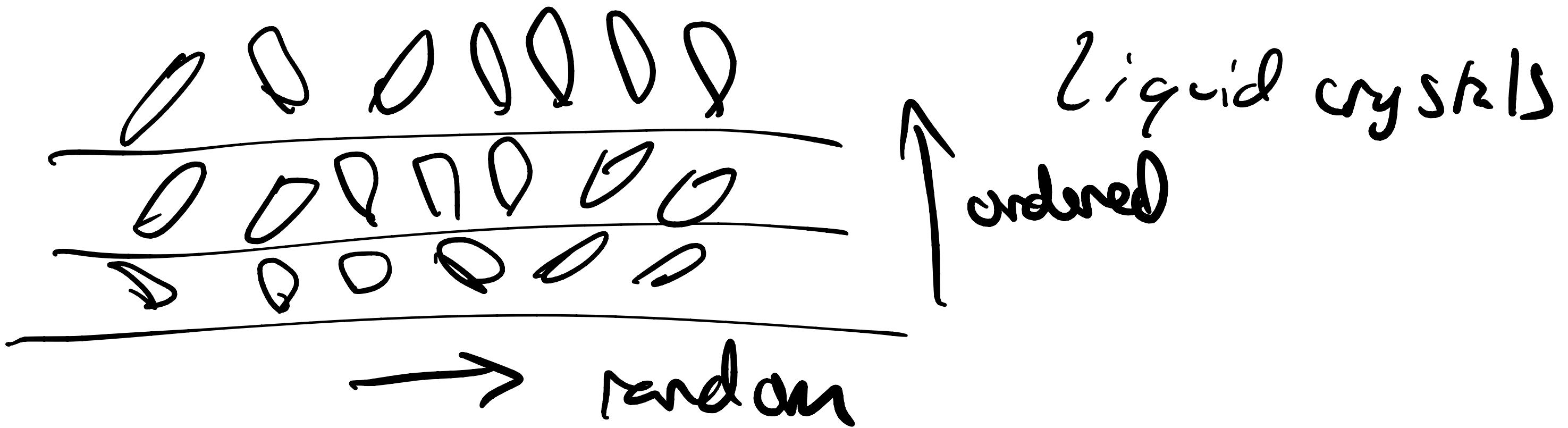
gas

Define as "order parameter"  
that distinguishes different phases

Eg liquid/gas  $\alpha(\rho) = \frac{\rho - \rho_L}{\rho_L}$

$$\alpha(\bar{V}) = \frac{\bar{V} - \bar{V}_L}{\bar{V}_L}$$


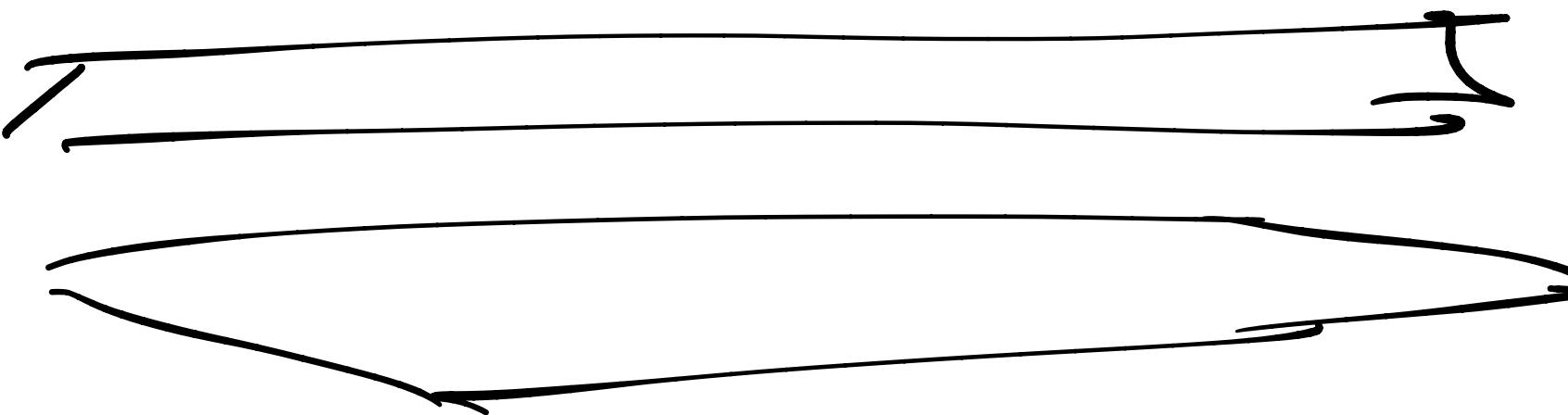
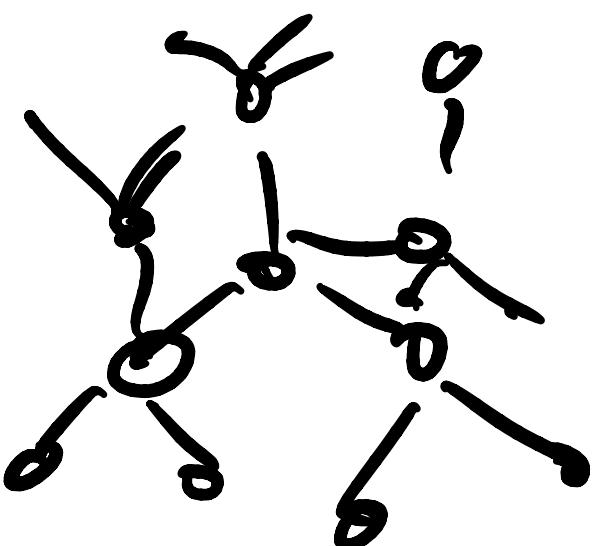
eg



Differences: density, compressibility  
between phases heat capacity, conductivity

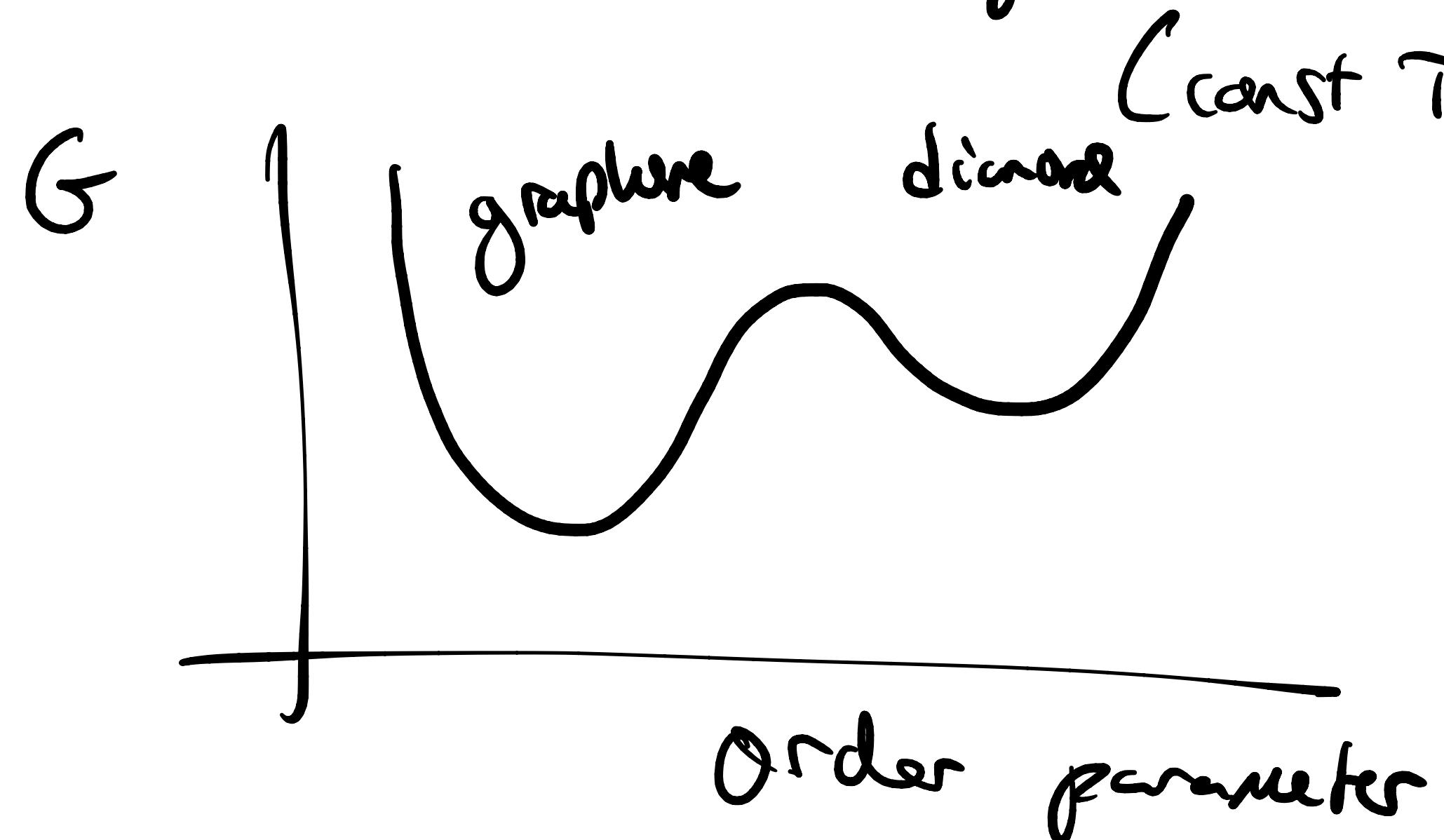
Other kind of "phase": different arrangements of same atoms

Carbon: Diamond, graphite,  $\rightarrow$  C<sub>60</sub> ...  
 $\uparrow$  more stable



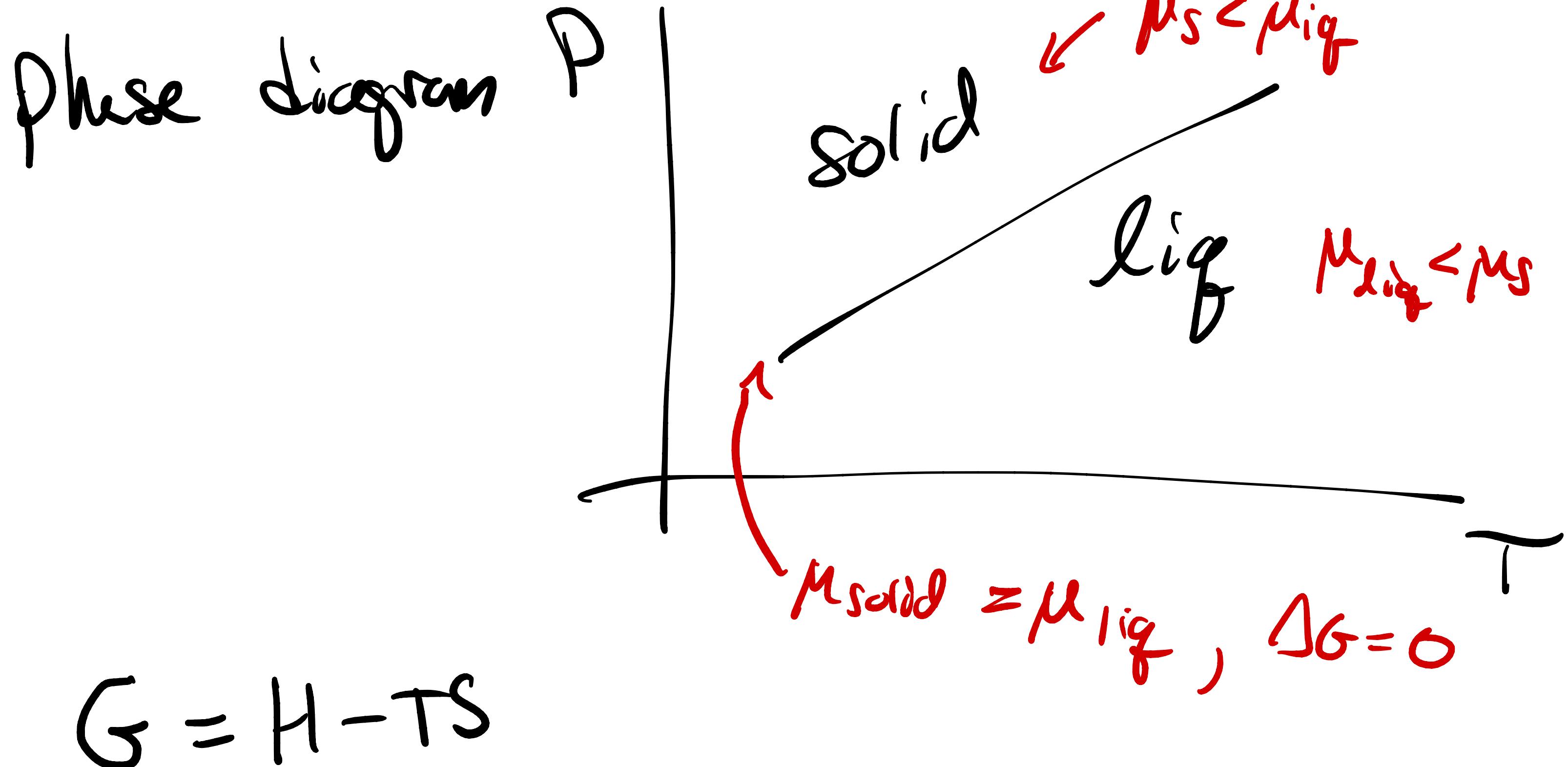
(TMDS, magic angle graphene)

Stability



system is in a particular phase because  $G$  is lower

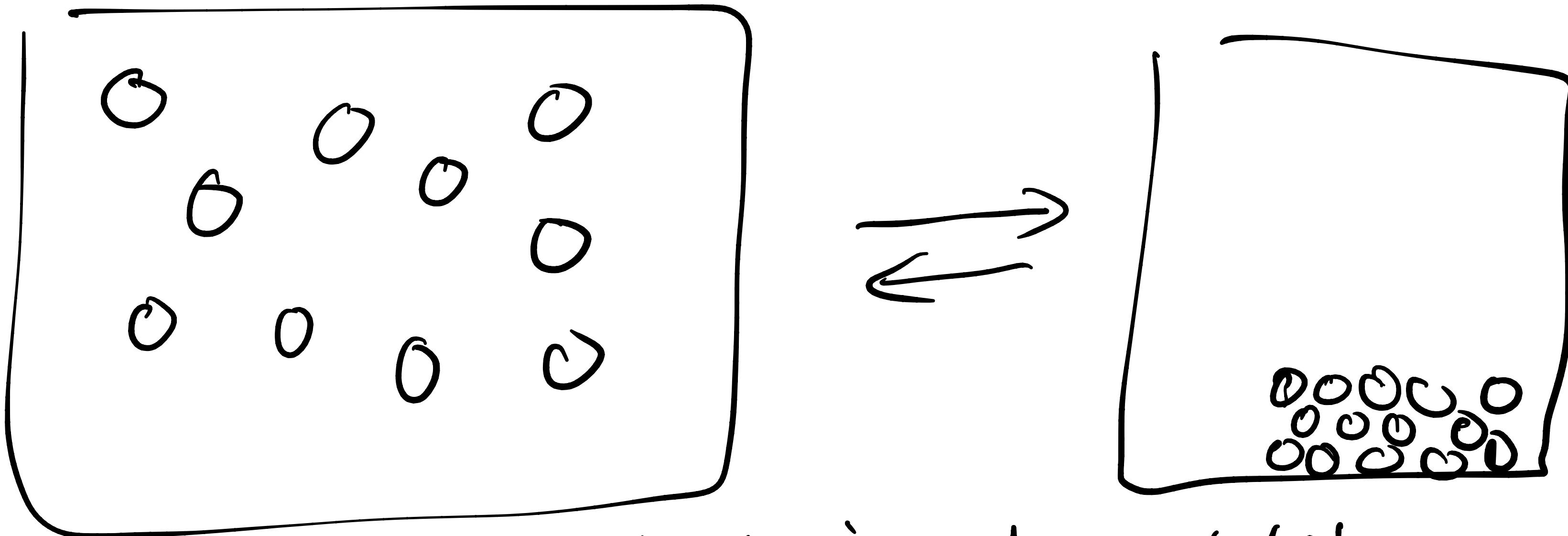
(const  $T$  &  $P$ )



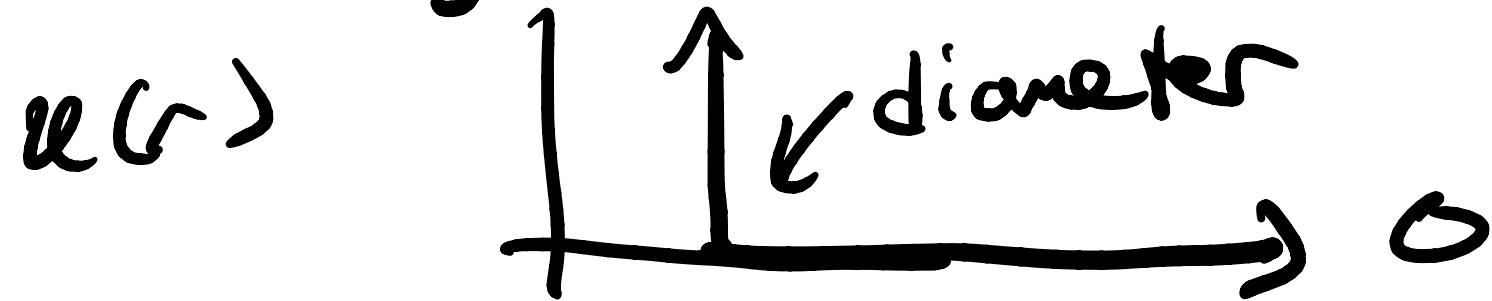
$$G = H - TS$$

$\hookrightarrow \mu = \bar{H} - \bar{T}\bar{S}$  @  $T, P$  in a certain phase

Old Question: Should hard spheres/disc  
crystallize



Hard: only interaction is volume exclusion



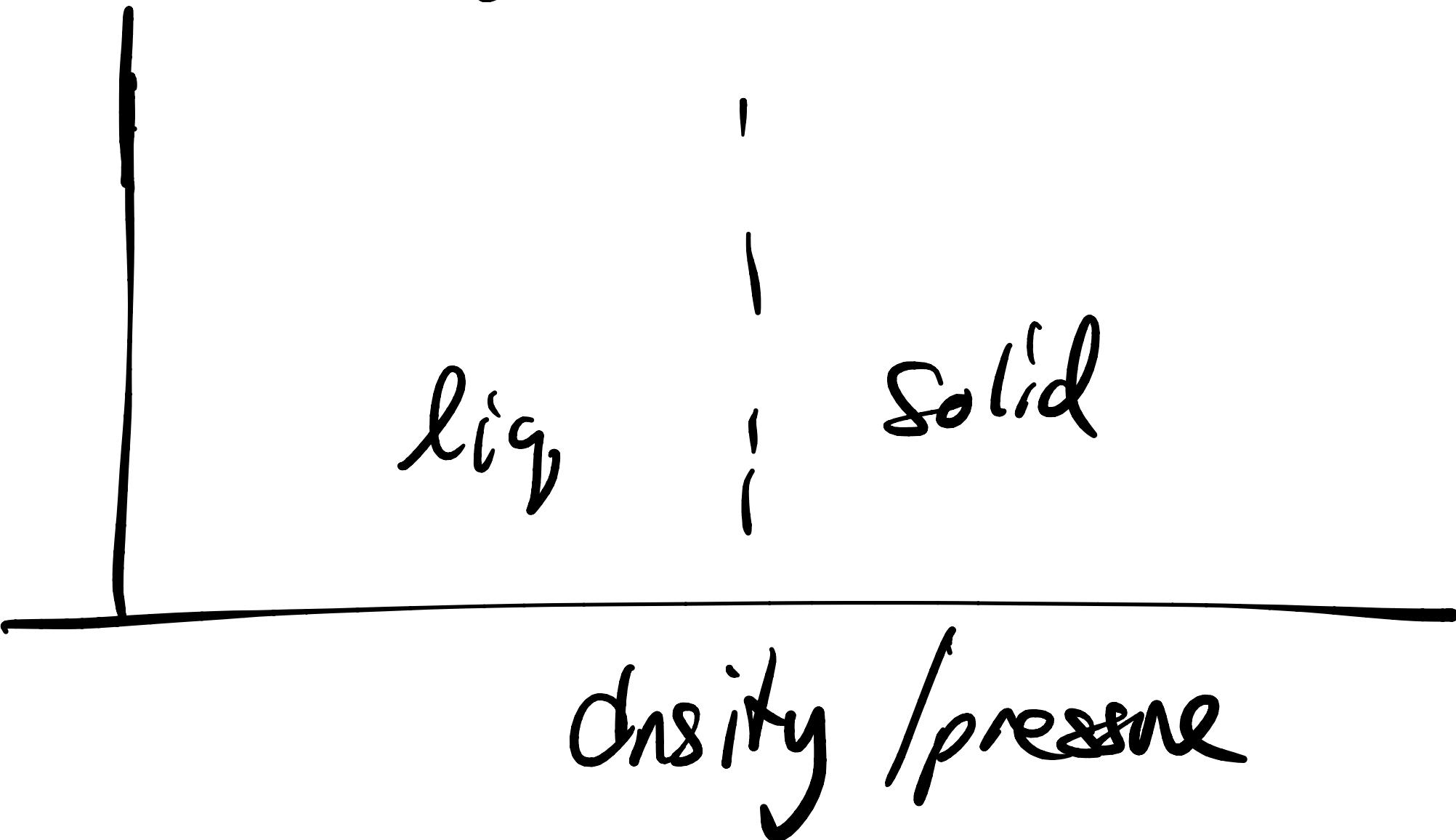
$$A = \epsilon - TS$$

for hard spheres,  $\epsilon = 0$

$$A = -TS$$

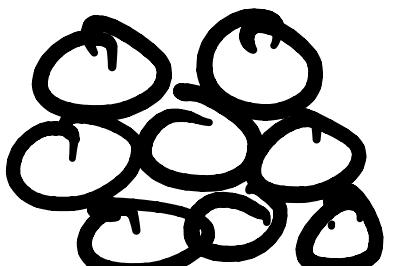
Crystallization means that the crystal has lower entropy

Hard particles crystallize at some density

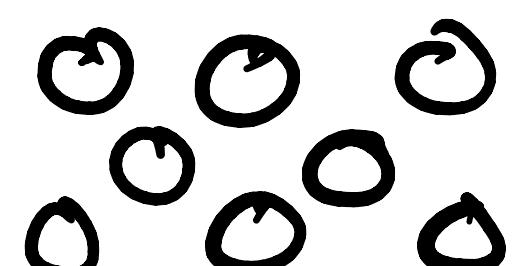


at high density : reach random state  
where every thing is touching

at some density



some density



higher entropy

Non-equilibrium "phase"

Can get stuck in a configuration  
that is not local free energy minimum

Change conditions "quickly", avoid phase  
change

Supercooled liquid-glass

Supercooled liquid

density

supercooling

cooling

liquid

(glass is  
still a liquid)

$T_m$

