

Lecture 11 - Phase Equilibria

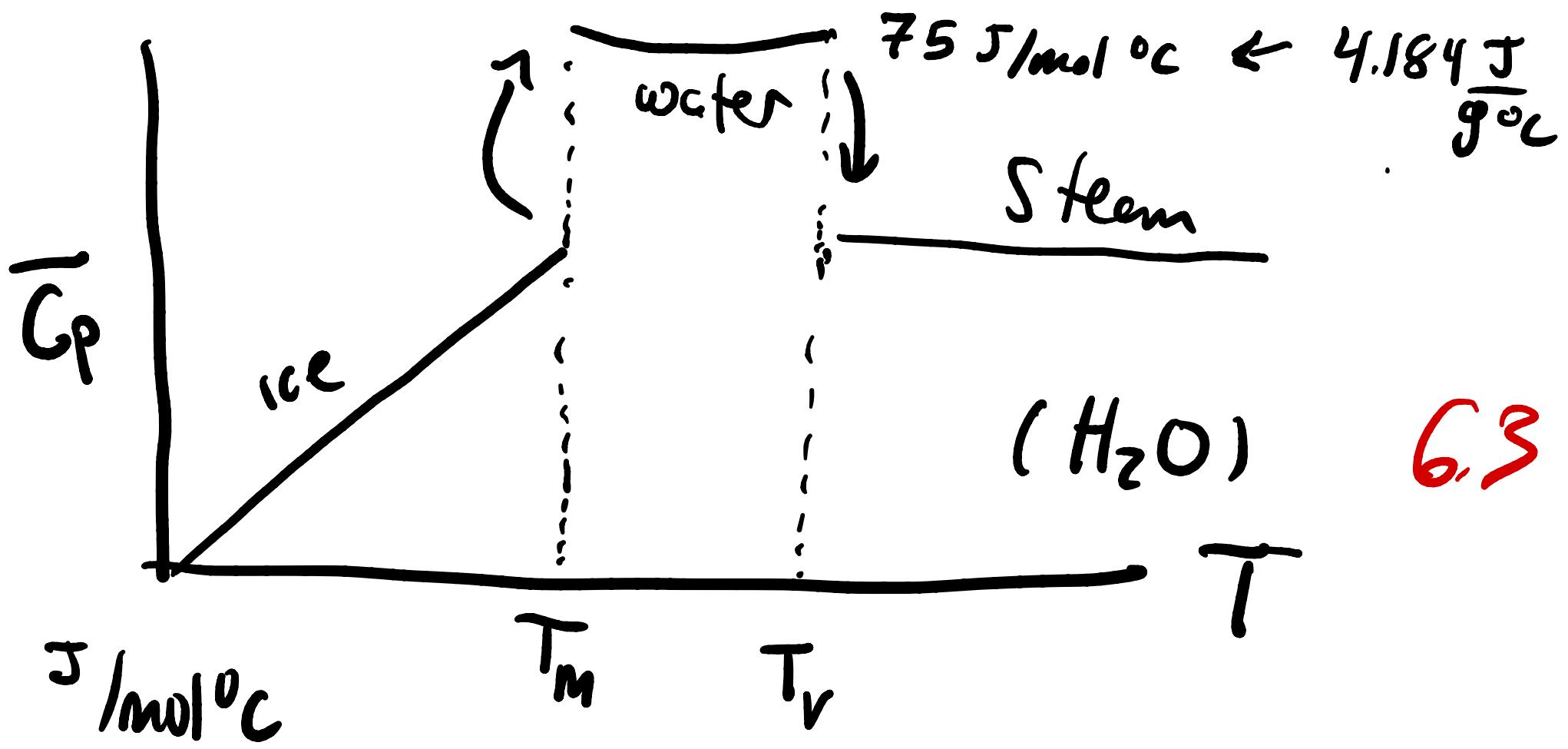
Why 1 phase over another

How does μ change with
 T & P

For each phase

$$\mu^\pi = \frac{G^\pi}{n^\pi} = \bar{H}^\pi - T \bar{S}^\pi$$

Heat capacity - depends on substance



? (low T heat capacity of solids)

$$C_P^{\text{steam}} = 36.3$$

$$C_P^{\lambda} = a + bT + cT^2 = 101 - 0.163T$$

$$C_P^S = a \cancel{T} / b + T = \frac{228.3T}{1387 + T} + 2.56 \times 10^{-4} T^2$$

$$\text{Want } \mu^\pi = \bar{H}^\pi - T \bar{S}^\pi$$

$$\begin{aligned} dH &= d(E + PV) = dE + PdV + Vdp \\ &= (dq - PdV) + PdV + VdP \\ &= dq_f + VdP = TdS + VdP \end{aligned}$$

@ const P

$$dH = \underline{dq} = TdS = \underline{C_P dT}$$

$$dH = C_P dT$$

$$dS = \frac{C_P}{T} dT$$

To do integral, start at some reference temperature

$$S(T=0) = 0$$

$$H(T_m) = 0$$

$$\begin{aligned} S(T) &= \int_0^{T_m} \frac{C_P^{\text{solid}}}{T'} dT' + \int_{T_m}^{T_V} \frac{C_P^{\text{liq}}}{T'} dT' \\ &\quad + \int_{T_V}^T \frac{C_P^{\text{gas}}}{T'} dT' + \Delta S^{\text{fus}} \\ &\quad + \Delta S^{\text{vap}} \end{aligned}$$

@ phase transition

$$\Delta f^{\text{fus}} = 0 = \Delta H^{\text{fus}} - T_m \Delta S^{\text{fus}}$$

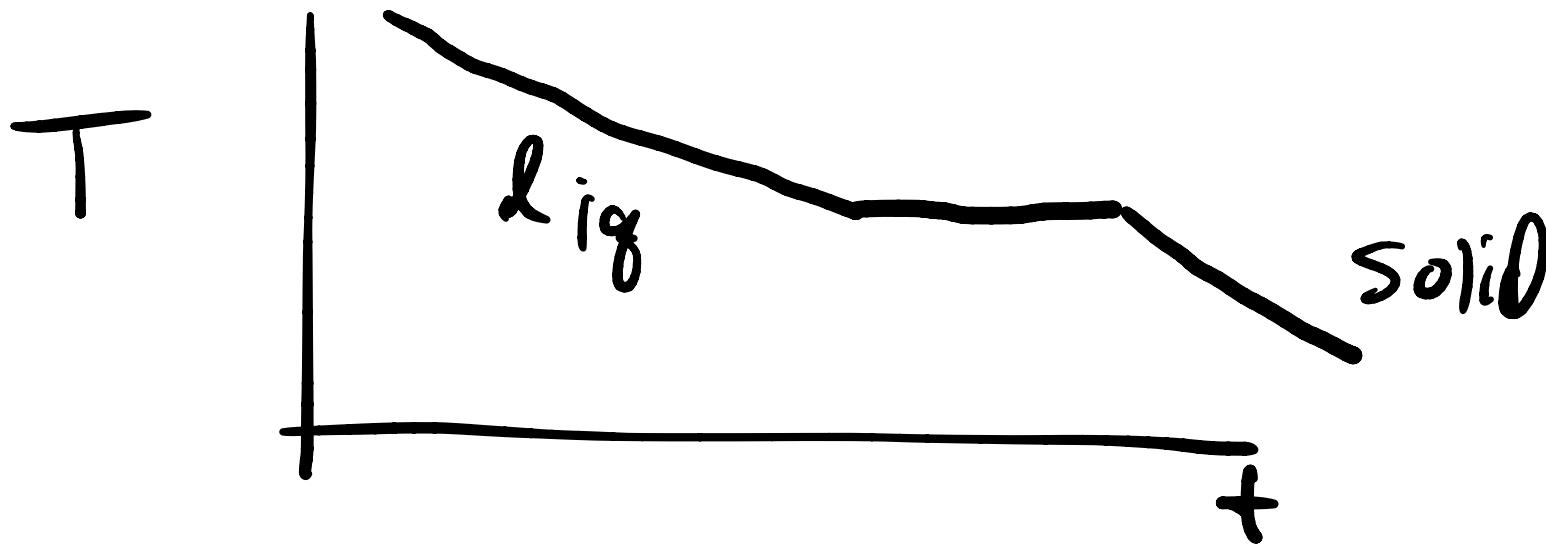
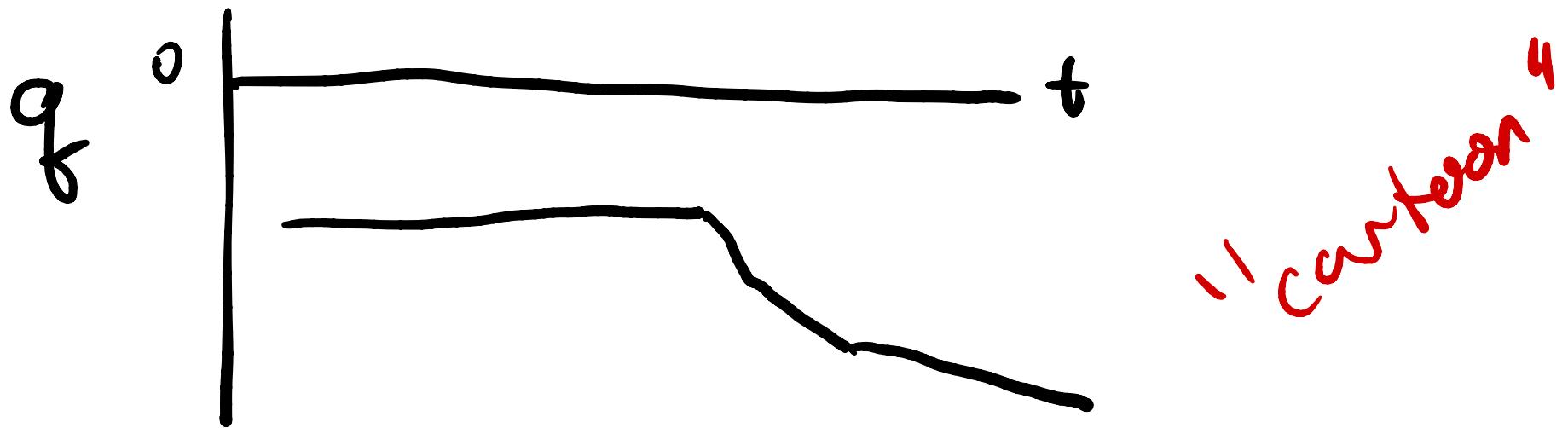
$$\Delta S^{\text{fus}} = \Delta H^{\text{fus}} / T_m$$

$$\Delta S^{\text{vap}} = \Delta H^{\text{vap}} / T_v$$

T_v & T_m come from discontinuities

$$\Delta H^{\text{vap}} = g^{\text{vap}}$$

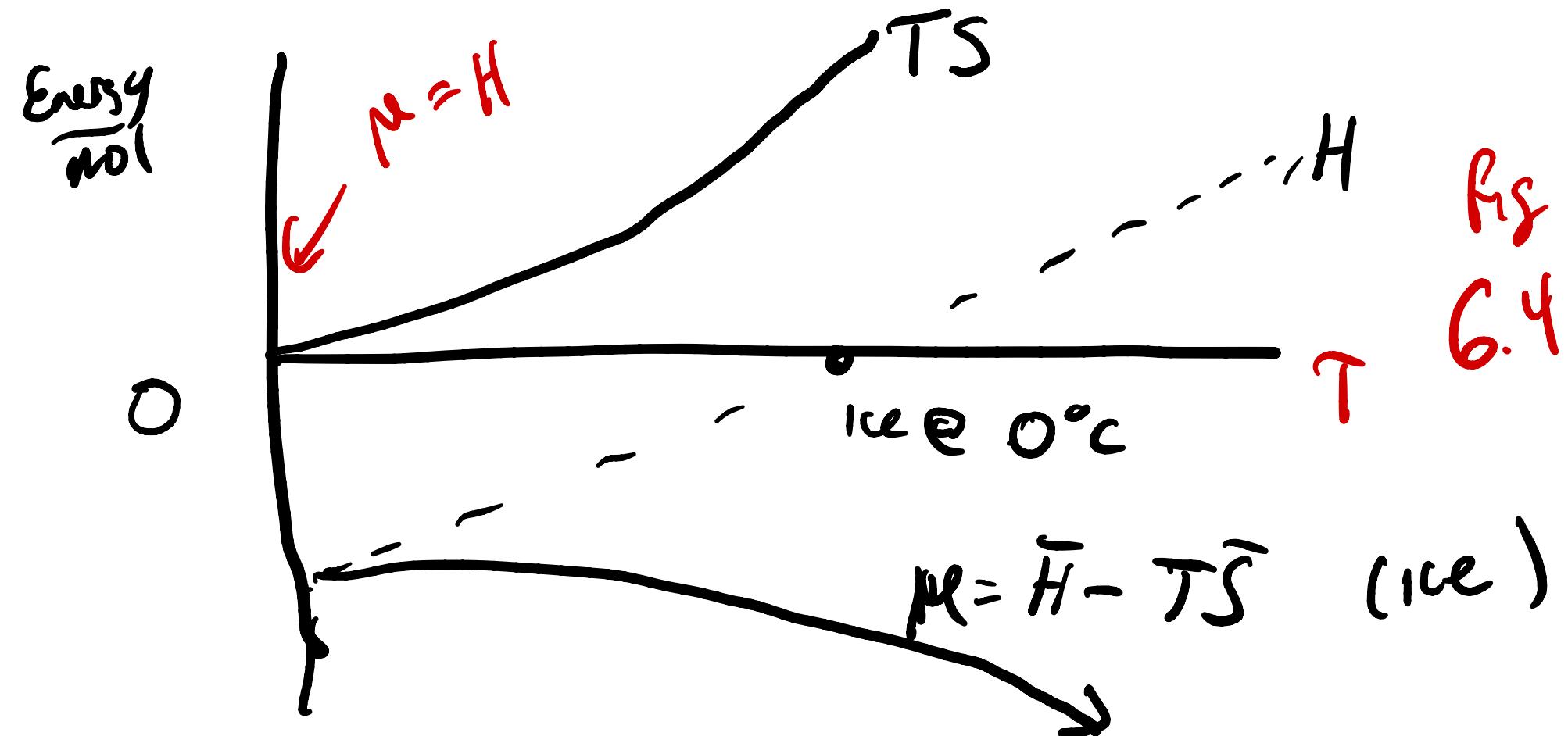
$$\Delta H^{\text{fus}} = g^{\text{fus}}$$



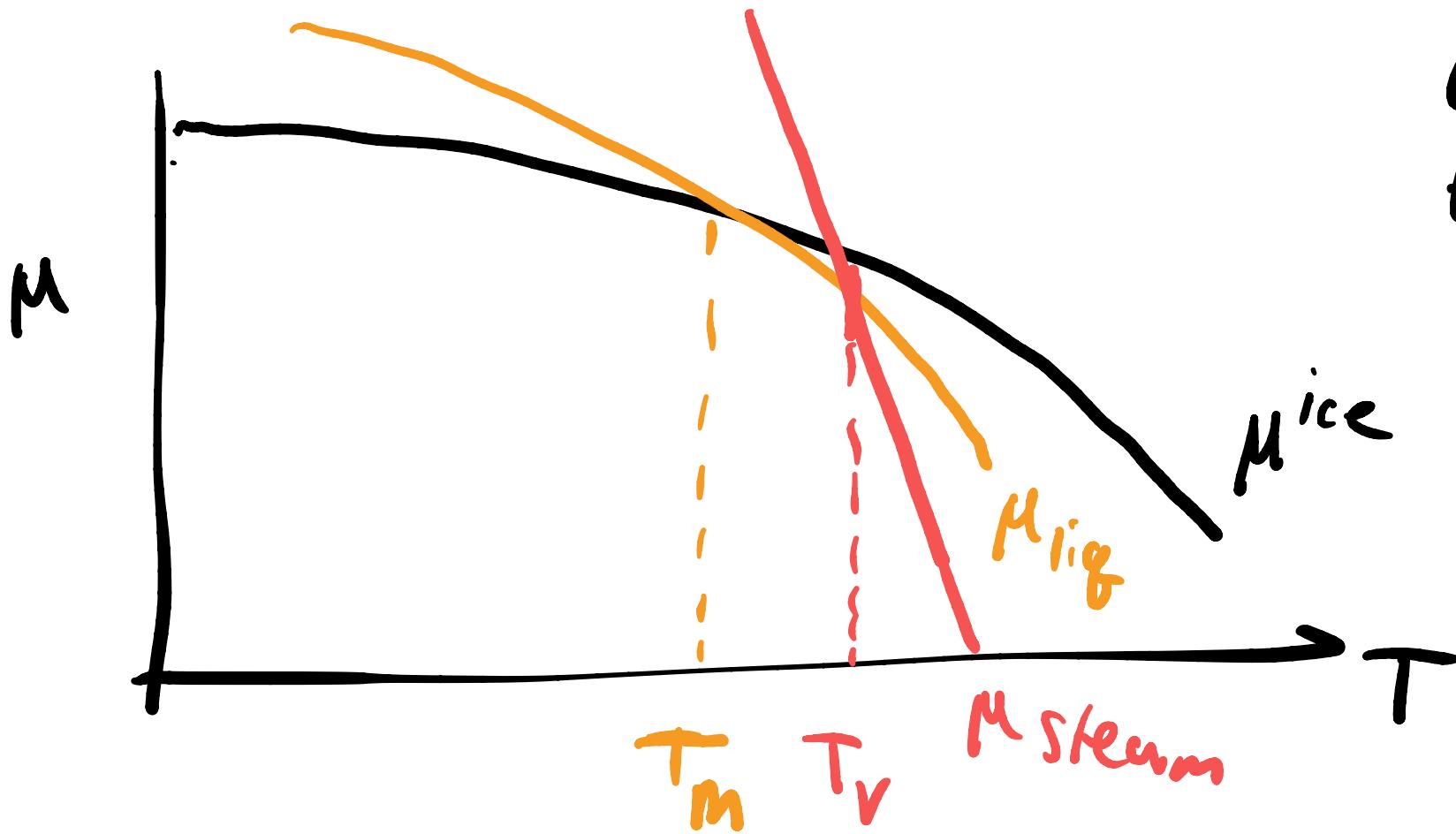
Boo! details of integrals

$\bar{H}(T)$ for H_2O

$\bar{S}(T)$ for H_2O *real section*



6.5
fig



$$\mu = \frac{G}{n}$$

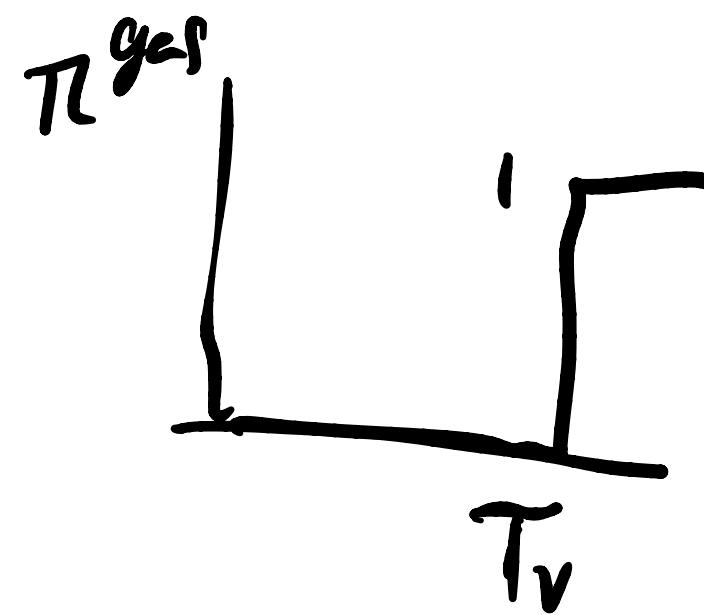
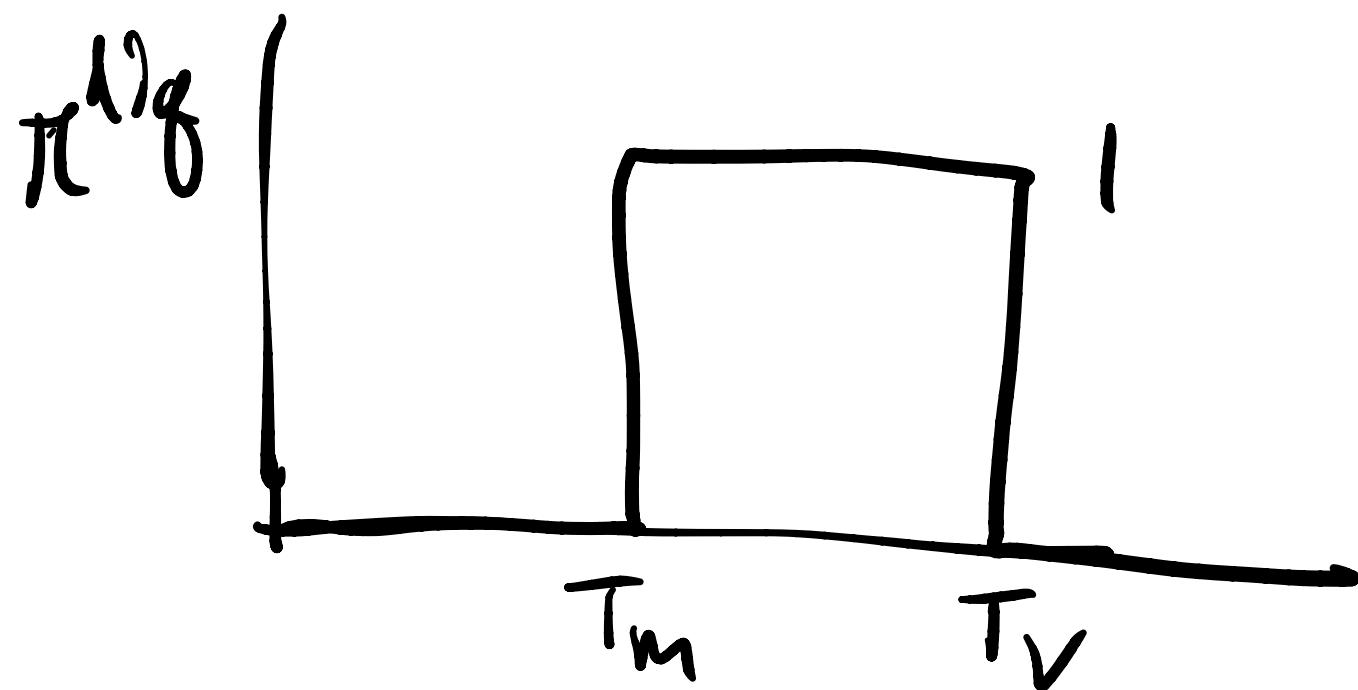
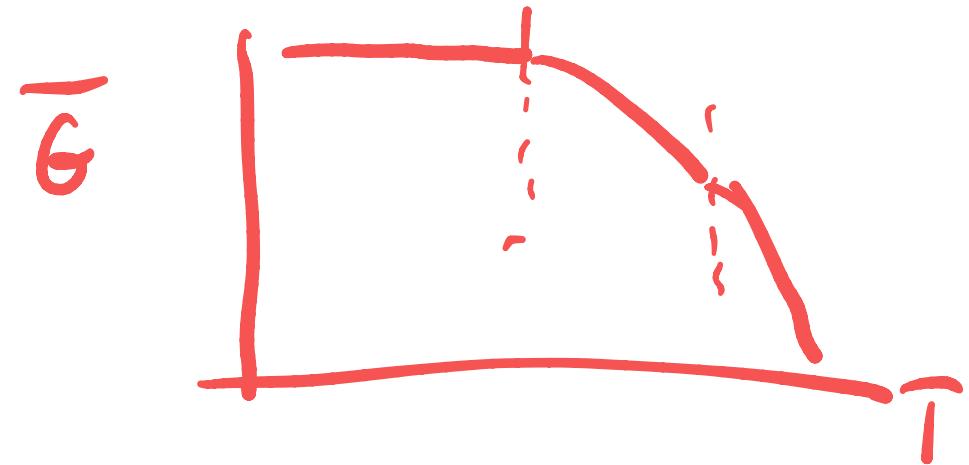
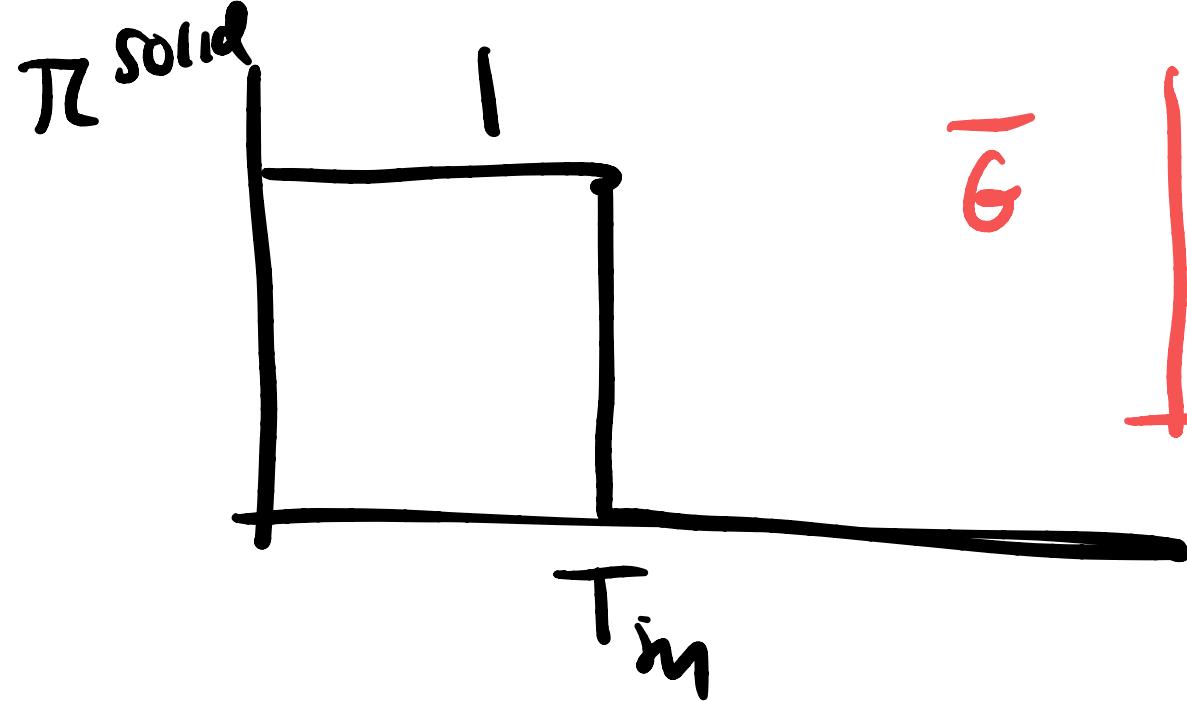
$$\begin{aligned} G &= H - TS \\ \mu &= \frac{H}{n} - T \frac{S}{n} \\ &= \bar{H} - \bar{S} \end{aligned}$$

Mixtures

$$\chi^\pi = \frac{n^\pi}{n_{\text{total}}}$$

or $\chi^{\text{gas}} = \frac{n^{\text{gas}}}{n_{\text{solid}} + n_{\text{liq}} + n_{\text{gas}}}$

$$\bar{G} = \sum_\pi \chi^\pi \mu^\pi$$



That was all constant pressure

What is effect of
being @ different fixed pressures

Biggest effect on gas

(What is the compressibility)

$$d\mu^\pi = \underbrace{-S^\pi dT}_{@dT \approx 0} + \bar{V}^\pi dP$$

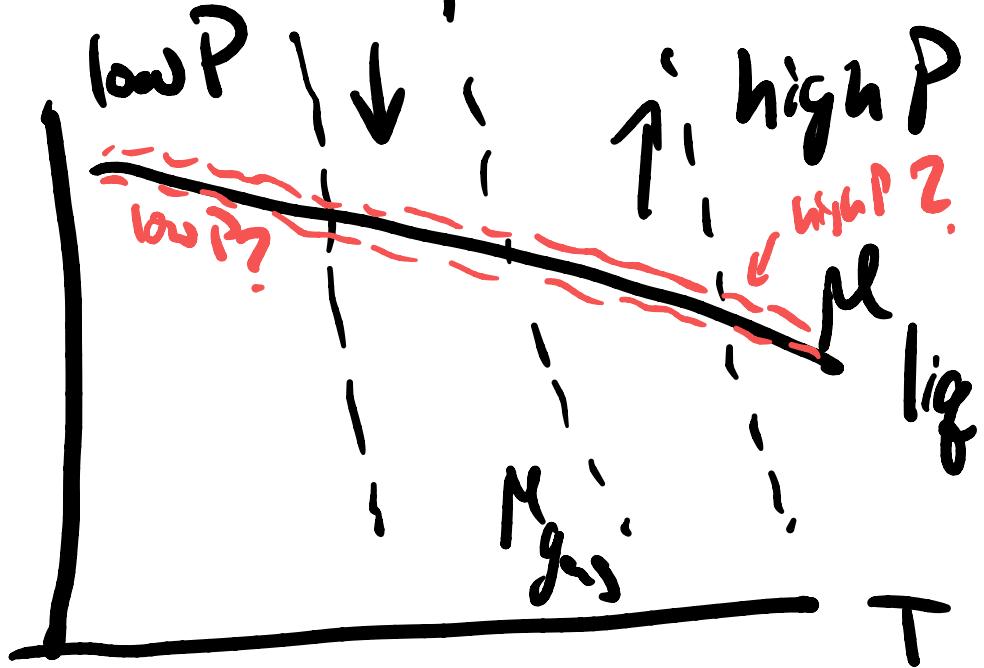
$$PV=nRT$$
$$J=V/n$$

$$d\mu_{\text{gas}} = \bar{V}_{\text{gas}} dP = \frac{RT}{P} dP$$

If ideal gas

$$\Delta \mu = \int_{1\text{ atm}}^P d\mu = \int_{1\text{ atm}}^P \frac{RT}{\tilde{P}} d\tilde{P}$$

$$= RT \ln(P / 1\text{ atm})$$



higher pressures

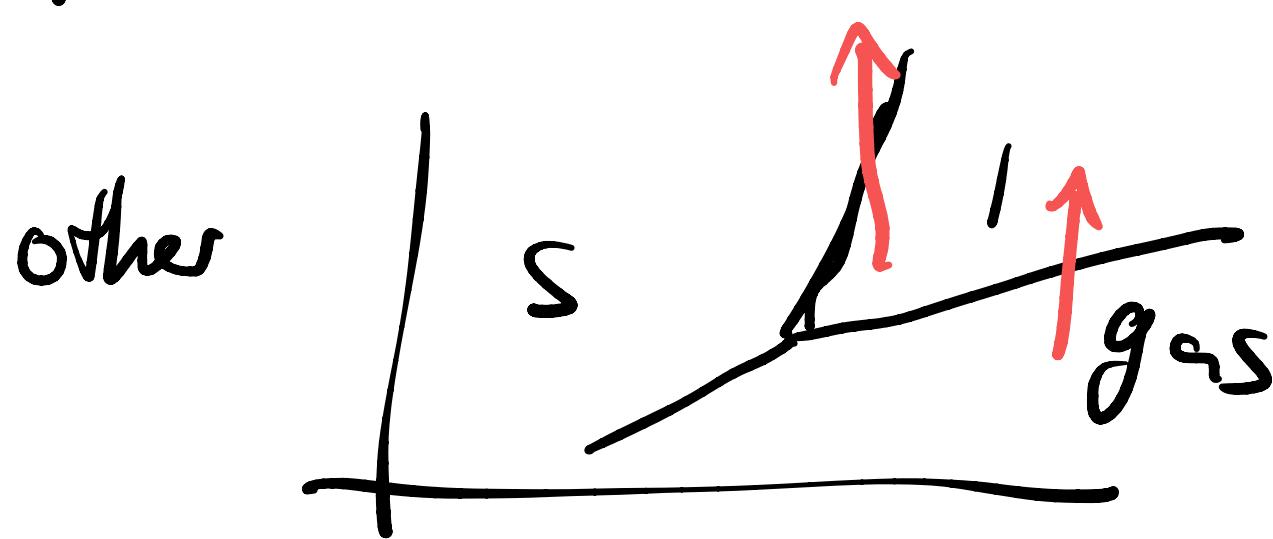
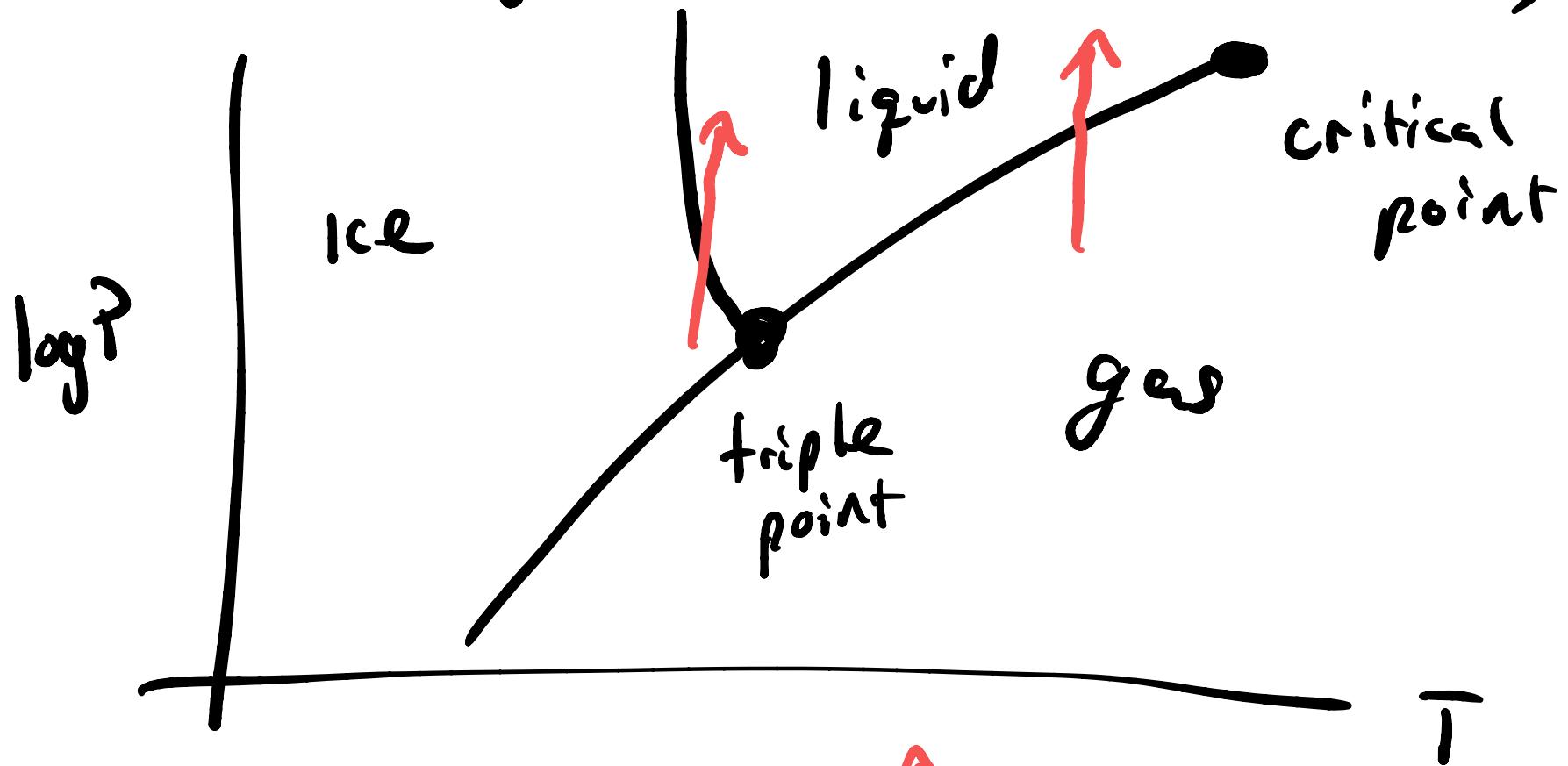
favor phases that are denser

for water:

liq water is more dense
than solid ice

ice: $P \uparrow$ $T_m \downarrow$

These diagram of a System (H_2O)

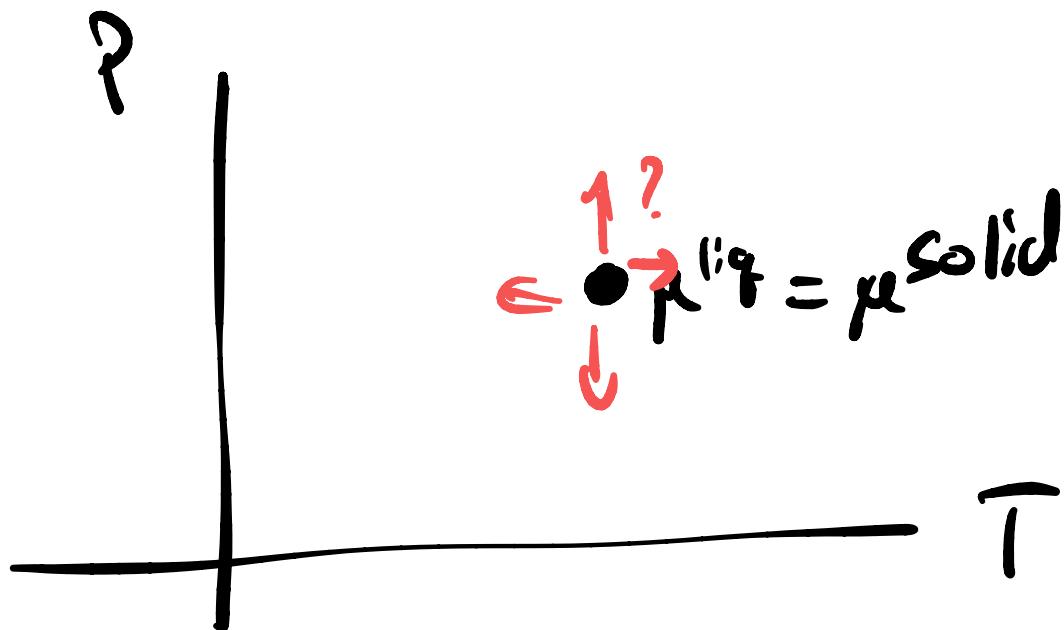


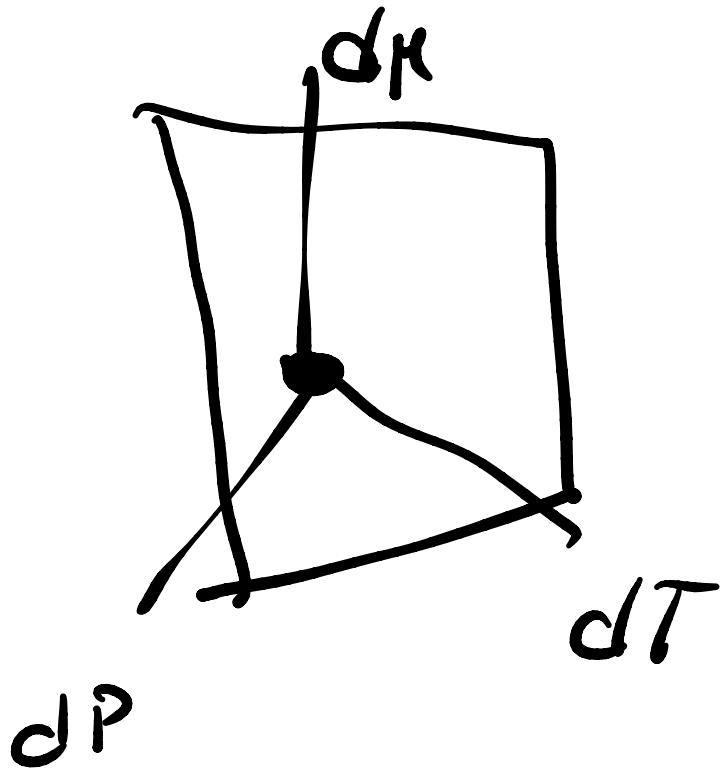
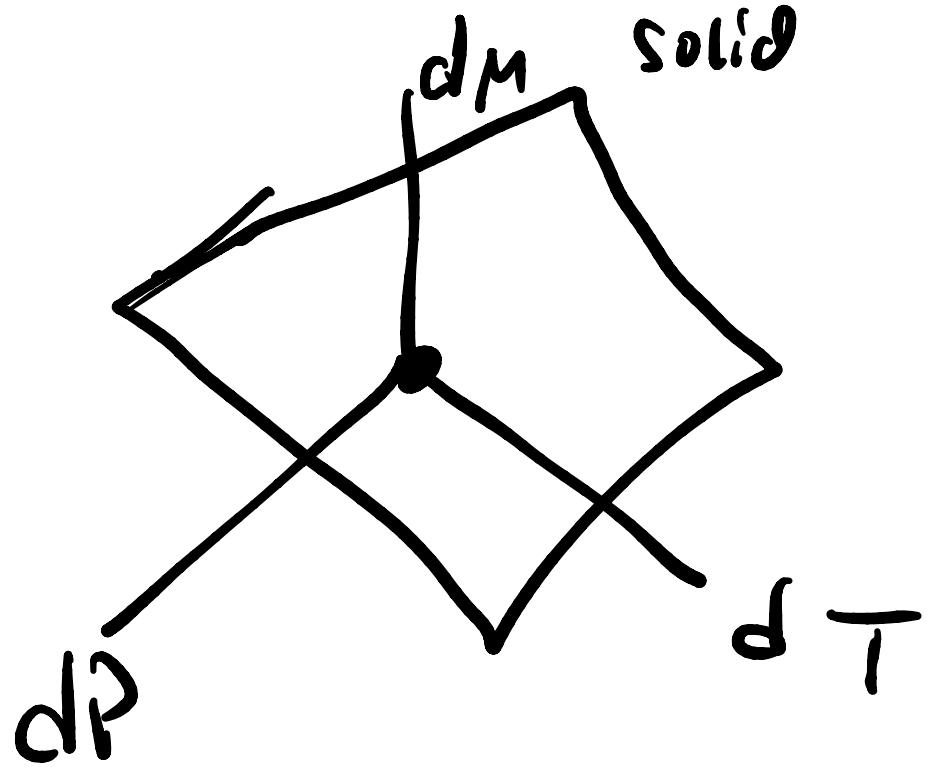
Restrictions on phase diagram

$$d\mu^{\pi} = \bar{V} dP - \bar{S} dT$$

One equation for each phase

When can they be equal





Intersect along a line



one line
coexistence

Only 1 point where 3 phases
can intersect

this line has a slope

$$\left(\frac{dP}{dT}\right) = \frac{\Delta \bar{S}^{1 \rightarrow 2}}{\Delta \bar{V}^{1 \rightarrow 2}}$$

←
next
time

Claussius-Claperyon Equation

Gibbs - Phase Rule

$$\# \text{ components} - \# \text{ coexisting} + 2 = \text{Degrees of freedom}$$

Things you can change & maintain eg