Sistem Persamaan Linear

Part-A

Topics

- SPL dan ekistensi solusi
- SPL sederhana – segitiga
- Transformasi Gauss
- Eliminasi Gauss
- Faktorisasi LU
- Pivoting

SPL dan Ekistensi Solusi

Secara umum SPL : $Ax = b$.

① A bujur sangkar → square sys. ⇒ ada 1 solusi jika A nonsingular

② # baris > # kolom → over determined sys. ⇒ tdk ada solusi.

③ # baris < # kolom. → Under determined ⇒ tak hingga banyak solusi.

PA bagi ini akan dibahas type ①

Asumsi : A nonsingular

Solusi SPL: $Ax=b$

- Jika A = matrix diagonal $\Rightarrow$ misal $Dx=b$.
  (matrix D adl diagonal jika $D_{ij}=0$ jika $i \neq j$).
  $\Rightarrow x_1 = b_1 / D_{11}$, $x_i = b_i / D_{ii}$.
- Jika A adl matrix segitiga bawah (i.e. $A_{ij}=0$ if $i<j$)

$\Delta 0$: $$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \Rightarrow \begin{aligned} x &= 2 \\ y &= 1 - 2x = -3 \\ z &= x + y - 1 = -2 \end{aligned}$$

Secara umum SPL: $Ux = b$, dg U matrix segitiga bawah:

akan memiliki solusi:

$\rightarrow x_1 = b_1 / u_{11}$

loop

$$x_2 = (b_2 - u_{21} \cdot x_1) / u_{22}$$

$$x_3 = (b_3 - u_{31} x_1 - u_{32} x_2) / u_{33}$$

$$\vdots$$

$$x_i = \Big(b_i - \sum_{j=1}^{i-1} u_{ij} \cdot x_j\Big) / u_{ii}$$

$$\begin{bmatrix} u_{11} & 0 & \cdots & 0 \\ u_{21} & u_{22} & 0 \cdots & 0 \\ \vdots & & & \\ u_{n1} & \cdots & & u_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

<u>Alg</u>: $Lx = b$.

```
x1 = b1 / L11
for i = 2:n
    s = 0
    for j = 1:i-1
        s = s + Lij * xj
    end
    xi = (bi - s) / Lii
end
```

$$Ux = b$$

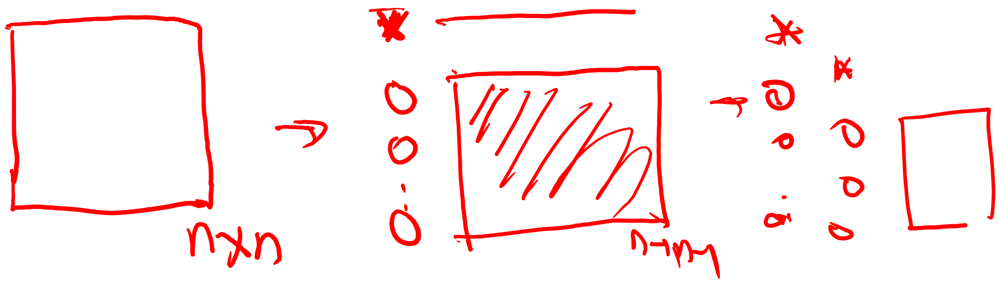
$$x(n) = b(n) / U(n,n)$$

$$x(n-1) = [\, b(n-1) - U(n-1,n)\, x(n) \,] / U(n-1,n-1)$$

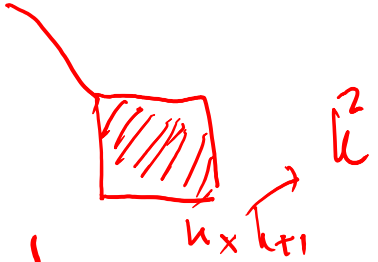
$$x(i) = \left[b(i) - \sum_{j=i+1}^{n} U_{ij} x_j \right] / U_{ii}$$

Alg. GE: $Ax=b$. $A \in \mathbb{R}^{n\times n}$

$B = [A|b] \to [U|\tilde{b}]$;




```
for k = 1 : n-1
    for i = k+1 : n
        m = B(i,k)/B(k,k).
        for j = k : n+1
            B(i,j) = B(i,j) - m*B(k,j).
        end
    end
end
```



Total cost

$$\sum_{1}^{n-1} k^2 = \frac{n(n-1)(2n-1)}{6} \approx \frac{n^3}{3}$$

GE as linear transformation process

Matrix Gauss

$$M_k = \begin{array}{c} \begin{array}{ccccc} & & i & & \end{array} \\ \begin{bmatrix} 1 & & & & 0 \\ & 1 & & & \\ & & 1 & & \\ 0 & & -m_1 & & \\ & & -m_2 & & 1 \end{bmatrix} \end{array}$$

→ call matrix transfor. as 1st model zeroing

$A_1 \to M_1 A$

Goal: $\begin{pmatrix}1\\2\\3\end{pmatrix} \to \begin{pmatrix}1\\0\\0\end{pmatrix} \to M = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}\begin{bmatrix}1\\2\\3\end{bmatrix} = \begin{bmatrix}1\\0\\0\end{bmatrix}$

$M_2 A_1 \to A_2$

$\to M_{n-1} \cdot M_{n-2} \cdots M_1 A = U$

Faktorisasi LU

Recall: $M_{n-1} \cdot M_{n-2} \cdots M_1 A = U$

$$\Leftrightarrow \quad A = [M_{n-1} M_{n-2} \cdots M_1]^{-1} \cdot U$$

$$= \underline{(M_1^{-1} \cdot M_2^{-1} \cdots M_{n-1}^{-1})} U$$

Secara umum ~~matrix~~ inverse dr M diperoleh hanya dg mengganti ~~elem~~ tanda elem yg di bwh diagonal.

es. $M_1 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} \Rightarrow M_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$

∘ Perkalian matrix Gauss = digabung

Cth:
$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & -2 & 1 \end{bmatrix}$$

∘ Masing² matrix Gauss adl segitiga bawah.

∘∘ $M_1^{-1} \cdot M_2^{-1} \cdots M_{n-1}^{-1}$ = segitiga bawah L,
yg isinya adalah matriks m //

ê $TA = U$

$Tb = \tilde{b}$

Tq: $A \xrightarrow{GE} LU$

$\boxed{A = LU}$

$AX = B$

$(LU)X = B$

U → tgiác trên

L → tgiác dưới

Vd: $Ax = b \Leftrightarrow L\underbrace{Ux}_{} = b$

① $Ly = b$

② $Ux = y$