Lab01-Copy1

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1 Bayesian Learning (732A73) Lab 1

Hoda Fakharzadehjahromy, Ravinder Alta

2 Assisgnment 1

2.0.1 (a)

True mean for $Beta(\alpha, \beta)$ is:

$$E[\theta] = \frac{\alpha}{\alpha + \beta}$$

True σ^2 for $Beta(\alpha, \beta)$ is:

$$\sigma^2(\theta) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

The prior function for this problem is:

$$p(\theta) \propto \theta^{\alpha_0 - 1}(1 - \theta^{\beta_0 - 1})$$

The likelihood function is :

$$p(y_1, ..., y_n|\theta) \propto \prod_{i=1}^{n} p(y_i|\theta) = \prod_{i=1}^{n} \theta^{y_i}(1-\theta)^{1-y_i} = \theta^s(1-\theta)^f$$

posterior is :

posterior

$$p(\theta|y) \propto Beta(\alpha_0 + s, \beta_0 + f) = Beta(11, 19)$$

[1]:
```
s = 8
n = 24
f = 16
alpha_0 = beta_0 = 3
```

[2]:
```
NDraws = 10000
res = matrix(,NDraws,3)
colnames(res) = c("iter","mean","sd")
```

```
alpha_n=alpha_0+s
beta_n = beta_0+f
TrueMean = (alpha_n)/(alpha_n+beta_n)
TrueVar = sqrt((alpha_n*beta_n)/((alpha_n+beta_n)^2*(alpha_n+beta_n+1)))
TrueMean
TrueVar
```

0.366666666666667

0.0865507910219387

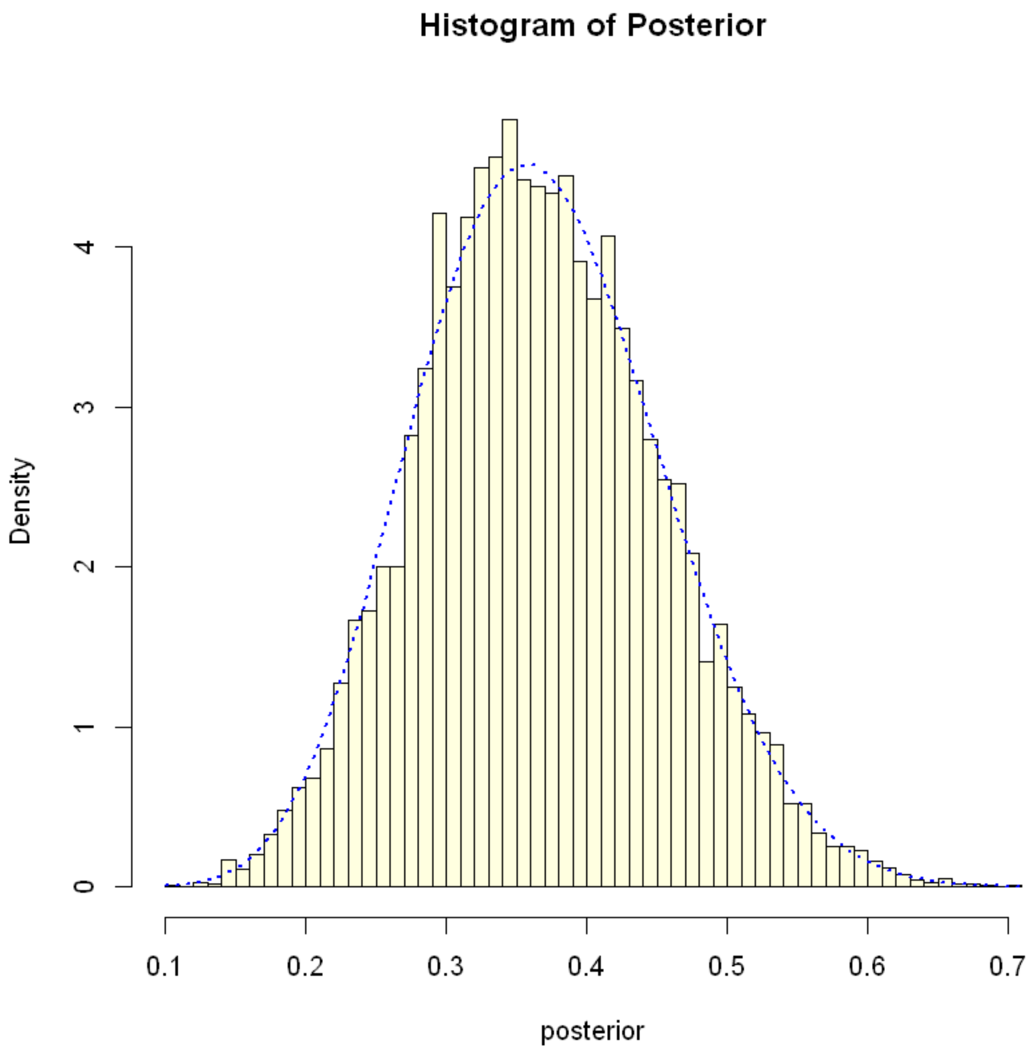
[3]:
```
PostB <- function(alpha= alpha_n,beta =beta_n ,n=NDraws){
    for(i in 1:n){
        postB = rbeta(i,alpha,beta)
        res[i,] = c(i,mean(postB),sd(postB))

    }

    return(list(post=postB,mean_sd=res))
}
Posterior = PostB()
```

[4]:
```
Posterior$mean_sd[1,3]=0
head(Posterior$mean_sd)
```

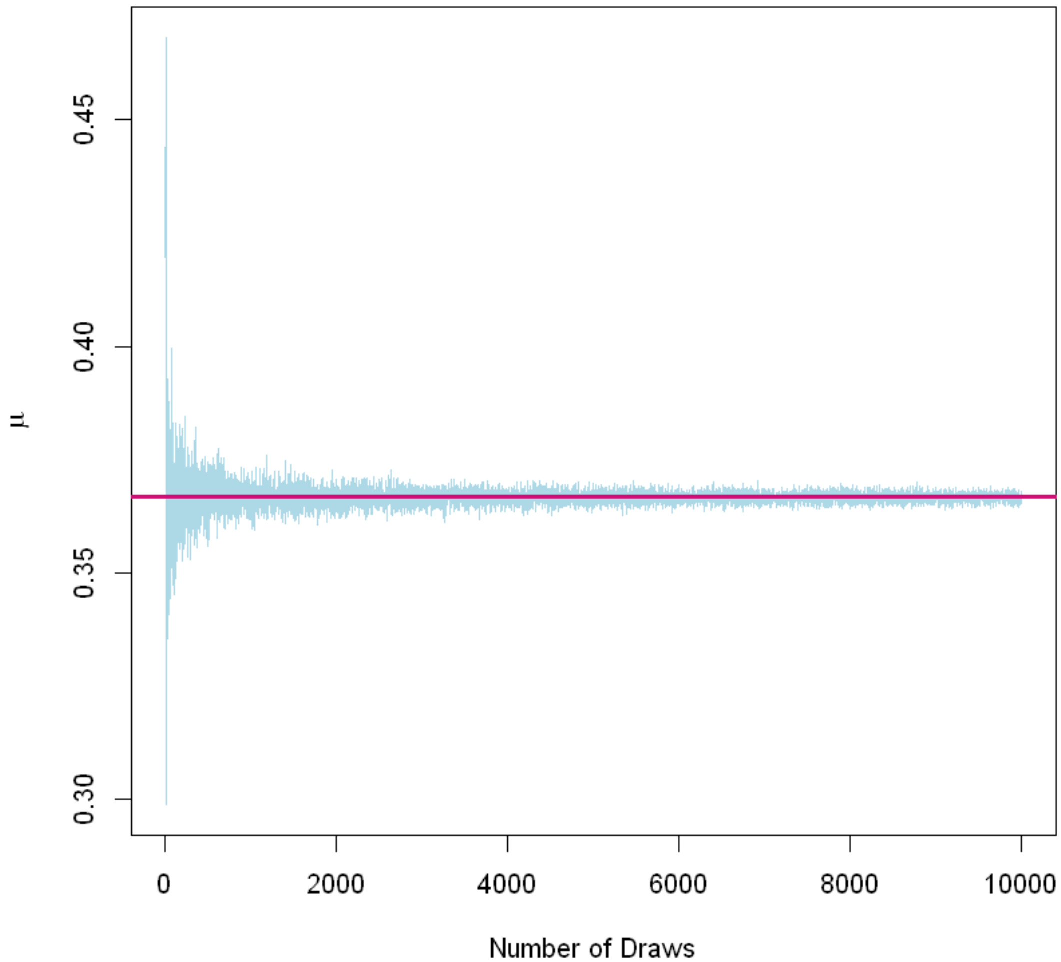
A matrix: 6 × 3 of type dbl

iter	mean	sd
1	0.4197914	0.00000000
2	0.4681419	0.02404432
3	0.2988524	0.02031929
4	0.3461667	0.08010881
5	0.3839380	0.05407235
6	0.3565289	0.07064133

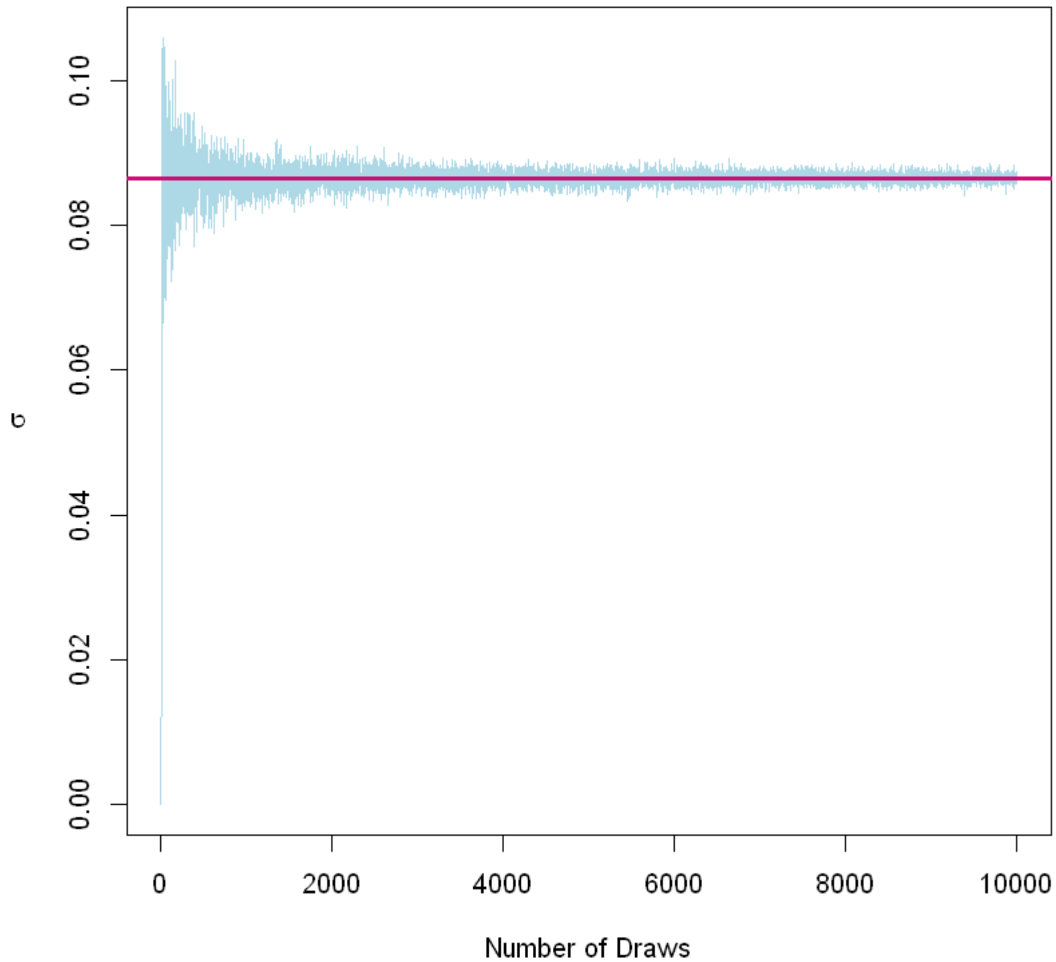
[5]:
```
x <-seq( from =-5, to =10 , by =0.001)
hist(Posterior$post,col='lightyellow',freq =␣
↪FALSE,breaks=50,xlab='posterior',main='Histogram of Posterior')
curve(dbeta(x,alpha_n,beta_n),add=TRUE,col='blue', lwd =2,lty=3)
```



[6]:
```
plot(x=1:NDraws,y=Posterior$mean_sd[,2],col='lightblue',lwd␣
↪=1,type='l',xlab="Number of Draws",ylab=expression(mu))
abline(h=TrueMean,col='deeppink3',lwd=3)
```



[7]:
```
plot(x=1:NDraws,y=Posterior$mean_sd[,3],col='lightblue',lwd␣
 ↪=1,type='l',xlab="Number of Draws",ylab=expression(sigma))
abline(h=TrueVar,col='deeppink3',lwd=3)
```



As we can see from the two plots above,for while the number of draws increases, both μ and σ converge to the true values.

2.0.2 (b)

we can see that our estimation of $Pr(\theta > 0.4|y)$ is very close to true theoratical value.

[8]:
```
exact_prb =1- pbeta(0.4,alpha_n,beta_n)
pos_prob = length(Posterior$post[Posterior$post > 0.4])/length(Posterior$post)
prb = data.frame(exact_prb,pos_prob)
colnames(prb) = c('excact_prob',"simulated_value")
prb
```

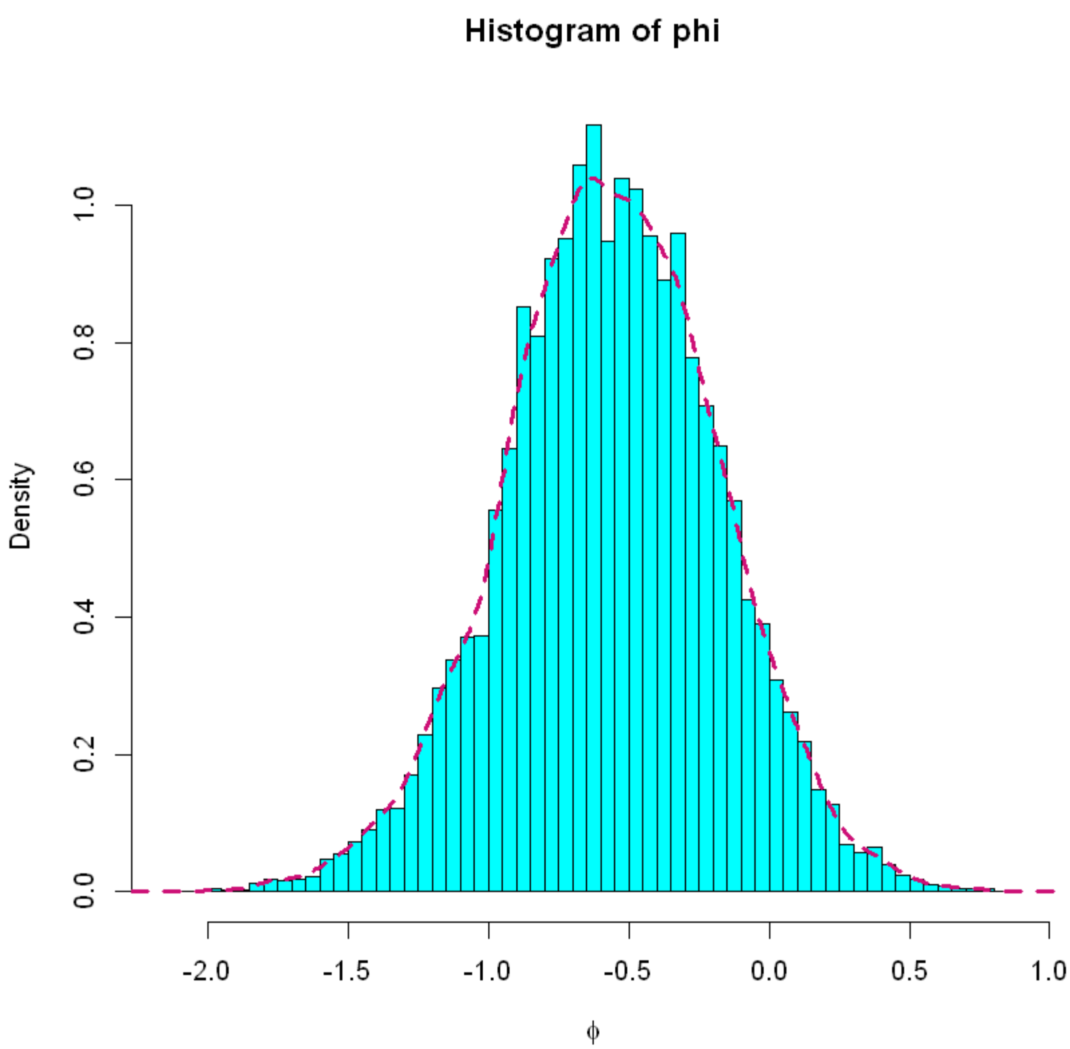
A data.frame: 1 × 2

excact_prob	simulated_value
<dbl>	<dbl>
0.3426654	0.3425

2.0.3 (C)

Histogram and kernel density of of the data simulated from the posterior distribution of the $\phi = log(\theta\ (1-\theta))$ with 10000 draws.

[9]:
```
phi = log(Posterior$post/(1- Posterior$post))
hist(phi,breaks=100,probability = TRUE,col='cyan',xlab=expression(phi))
lines(density(phi),col='deeppink3',lty=2,lwd=3)
```



3 Assignment 2

1- Simulate 10, 000 draws from the posterior of 2 (assuming = 3.8) and com- pare it with the theoretical Inv − 2(n, 2) posterior distribution.

[10]:
```
m = 10000 #sample size
mu = 3.8
observ = c(38,20, 49, 58, 31, 70, 18, 56, 25,78)
n = length(observ)
```

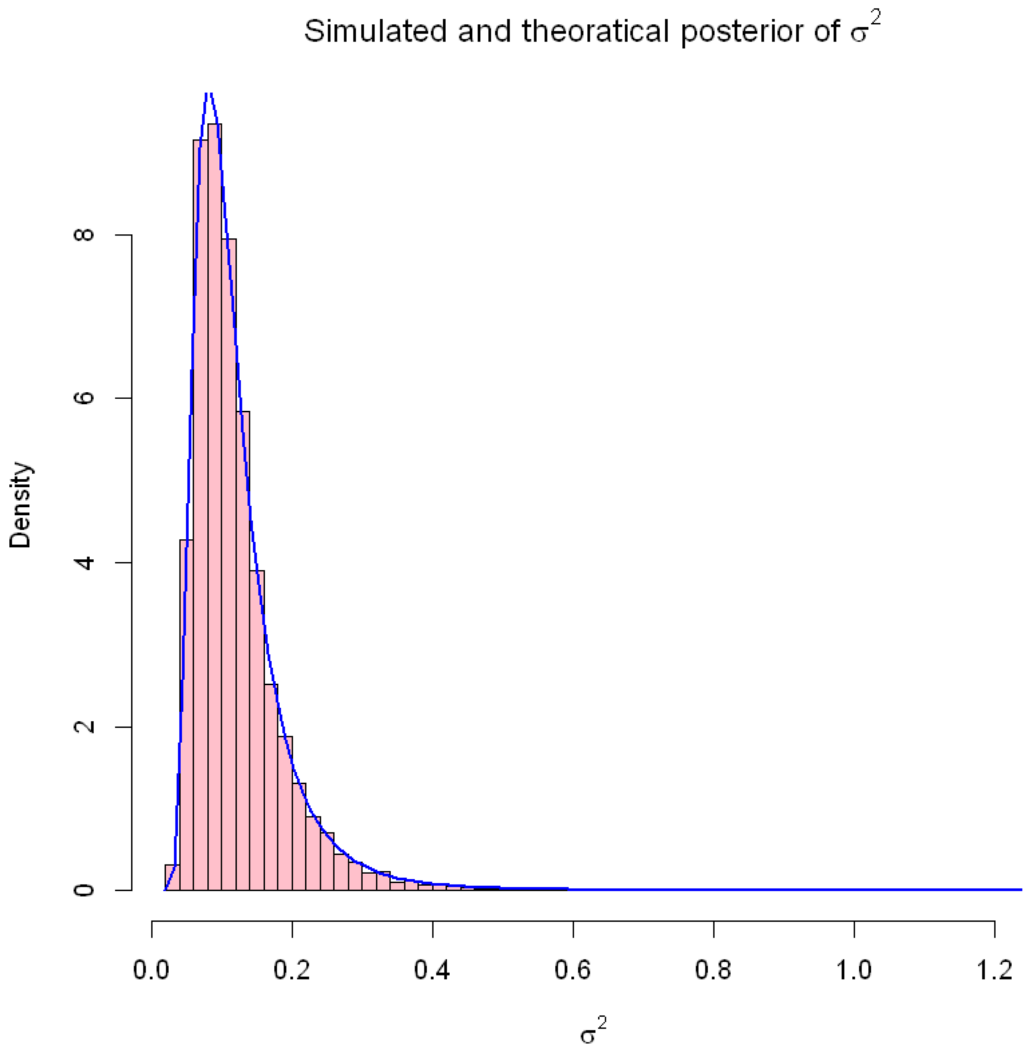
[11]:
```
tau2 = function(data,mu){
    sum((log(data) -mu)^2 )/n
}
```

[12]:
```
#install.packages('invgamma')
library(invgamma)
tau2(observ,mu )
```

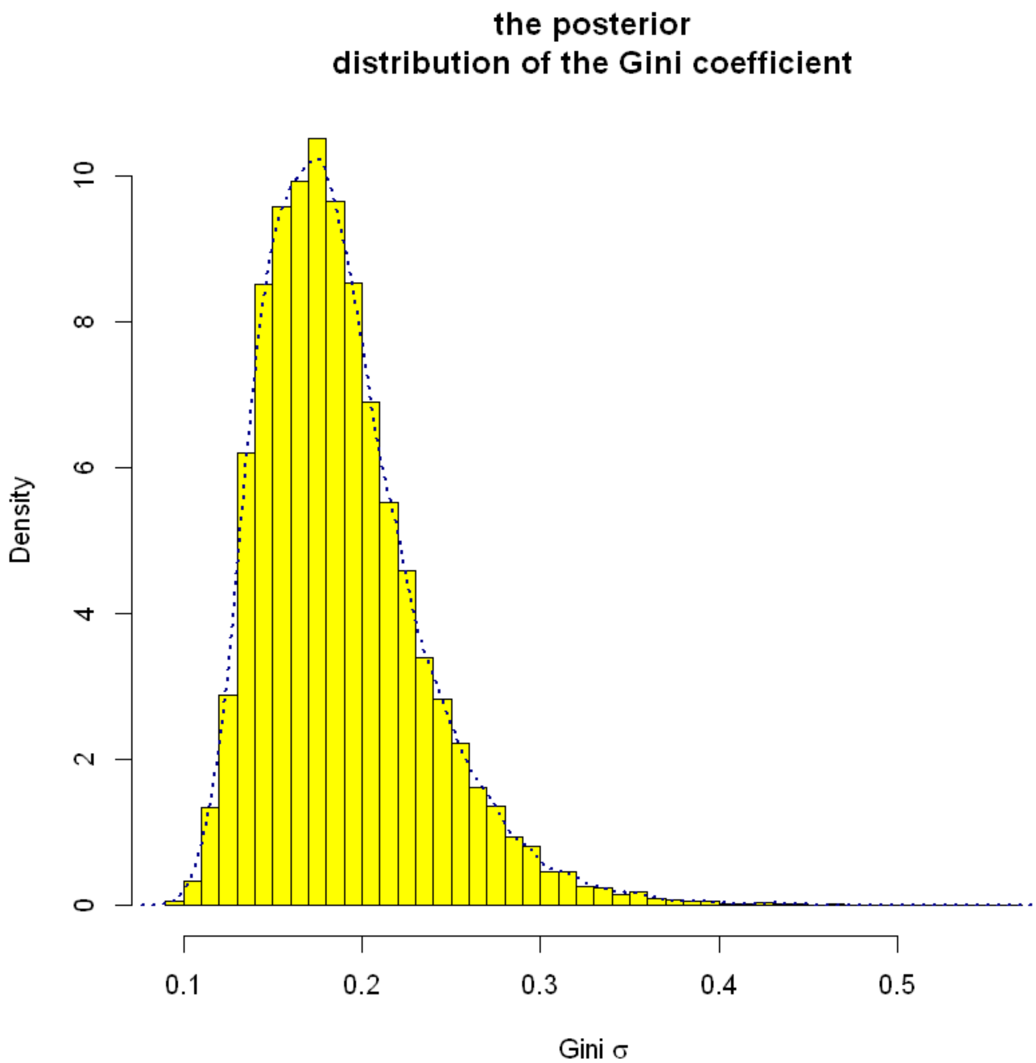
0.261043665681099

[13]:
```
post_sigma2 <- function(m){
    set.seed(12345)
    rinvchisq(n = m,df = n,ncp = tau2(observ,mu))
}
```

[14]:
```
x <-seq( from =0, to =10 , by =0.001)
hist(post_sigma2(m),probability = TRUE,col='pink',breaks =
↪50,main=expression(paste('Simulated and theoratical posterior of ', sigma^2)
↪),
    ,xlab = expression(paste(sigma ^2)))
curve(dinvchisq(x ,df = n,ncp = tau2(observ,mu)),add=TRUE,col='blue',lwd=2)
```



[15]:
```
sigma2 = post_sigma2(m)
g_sigma = sqrt(sigma2)/sqrt(2)
gpdf =2* pnorm(q = g_sigma,mean = 0,sd = 1)-1
hist(gpdf,probability = TRUE,col='yellow',breaks=50,main='the posterior
distribution of the Gini coefficient',xlab=expression(paste('Gini ', sigma)))
lines(density(gpdf),col='darkblue',lty=3,lwd=2)
```



[16]:
```
alpha_conf = 0.1
q_lower = quantile(gpdf,alpha_conf/2)
q_upper = quantile(gpdf,1-alpha_conf/2)
c(q_lower, q_upper)
true_mean = mean(gpdf)
```

5\% 0.130774883443954 **95\%** 0.271906220898121

[17]:
```
true_mean
```

0.188106773563226

HPD: shortest possible interval that under the posterior has the siginificance probability (ie. 0.9)

[18]:

```
KerDenG=density(gpdf)
xx = KerDenG$x
yy = sort(KerDenG$y,decreasing = TRUE,index.return=TRUE)
```

we will use the cumulative sum of kernel density values to find the shortest interval in data that contains 0.9 of the data. the reason we can use cumulative to find HPD interval is because our data is unimodal.

[19]:

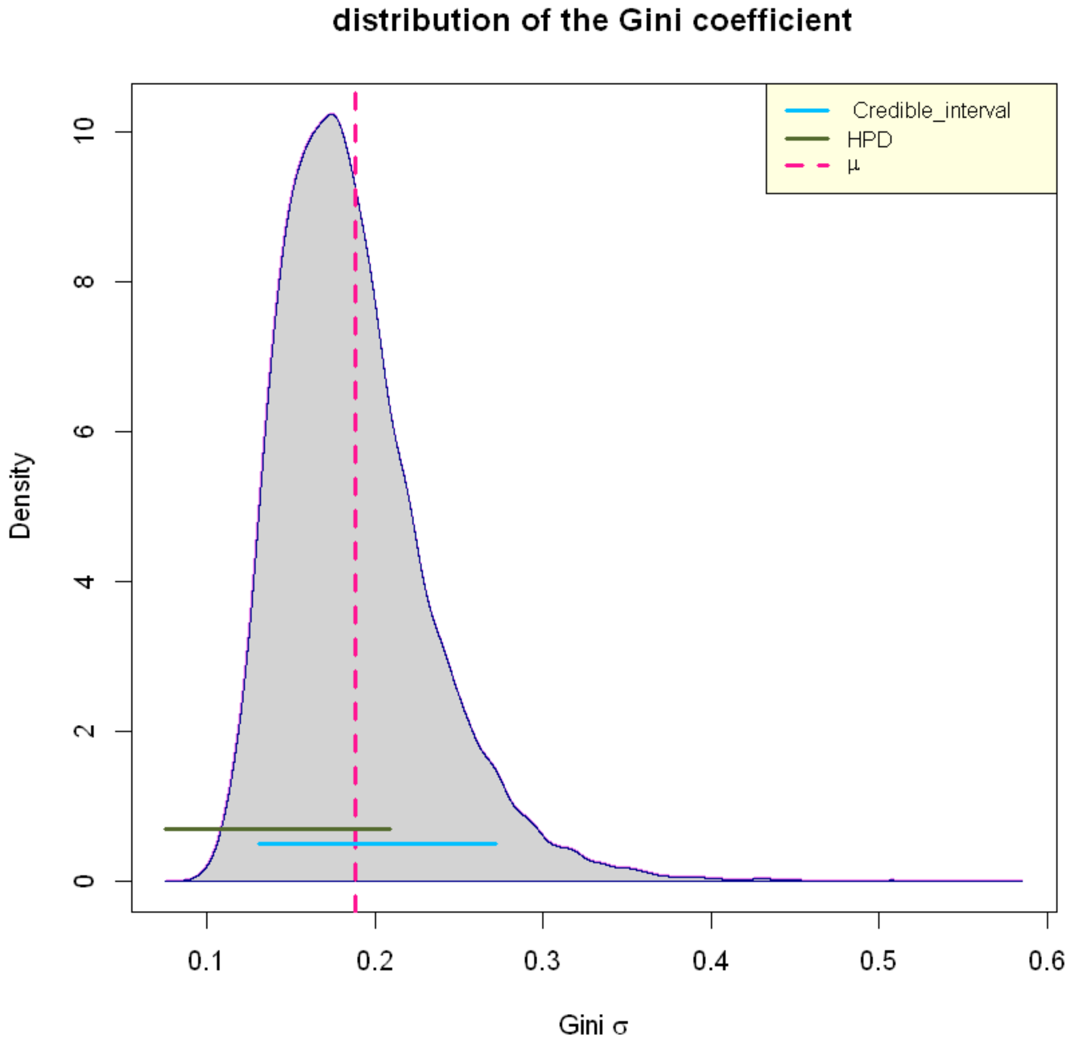
```
CumSum = cumsum(yy$x)
HPD_D = CumSum[length(CumSum)]*0.9
temp = which(CumSum<HPD_D)
HPD_interval = range(xx[which(CumSum<HPD_D)])
```

[20]:

```
df = data.frame('HPD_interval'=HPD_interval,'credible_interval'=c(q_lower,␣
↪q_upper))
```

[21]:

```
y_temp=density(gpdf,n=10000)
plot(x=y_temp$x,y=y_temp$y,type = 'l', lwd = 2, col = 'violet',main=␣
↪'distribution of the Gini coefficient'
     ,xlab=expression(paste('Gini ', sigma)),ylab='Density')
polygon(x=y_temp$x,y=y_temp$y, col = 'lightgrey',border = 'darkblue')
abline(v = true_mean, col="deeppink", lwd=3, lty=2)
#abline(v=q_upper)
#abline(v=q_lower)
#abline(v=HPD_interval[1],col='blue')
#abline(v=HPD_interval[2],col='blue')

segments(x0 =q_lower,y0 =0.5,x1 =q_upper,y1 =  0.5,col='deepskyblue1' ,lwd=3)
segments(x0=HPD_interval[1],y0=0.7,x1=HPD_interval[2],y=0.
↪7,col='darkolivegreen',lwd=3)

legend("topright",
  legend = c(" Credible_interval", "HPD",expression(mu)),
  col = c('deepskyblue1','darkolivegreen','deeppink'), lty=c(1,1,2), cex=0.
↪8,lwd=3,bg='lightyellow')
```



4 Assignment 3

4.0.1 (a) Posterior :

$$p(\kappa \mid y_1, y_2, ..., y_n) \propto p(y_1, y_2, ..., y_n \mid \kappa) \cdot p(\kappa)$$

$$p(\kappa \mid y_1, y_2, ..., y_n) \propto \left[\frac{1}{I_0(\kappa)}\right]^n \cdot \exp\left[\sum_{i=1}^{n} \kappa \cdot cos(y_i - \mu) - \lambda\kappa\right]$$

$$p(\kappa \mid y_1, y_2, ..., y_n) \propto \left[\frac{1}{I_0(\kappa)}\right]^n \cdot \exp\left[\sum_{i=1}^{n} \kappa \cdot cos(y_i - 2.39) - \kappa\right]$$

[22]:
```
y_data =c(-2.44, 2.14, 2.54, 1.83, 2.02, 2.33, -2.79, 2.23, 2.07, 2.02)
```

[23]:
```
n =length(y_data)
mu = 2.39
n
```

10

[24]:
```
# a function to compute prior for k (exponential with lambda = 1)
prior_k <- function(k){
    dexp(x = k,rate = 1)
}
```

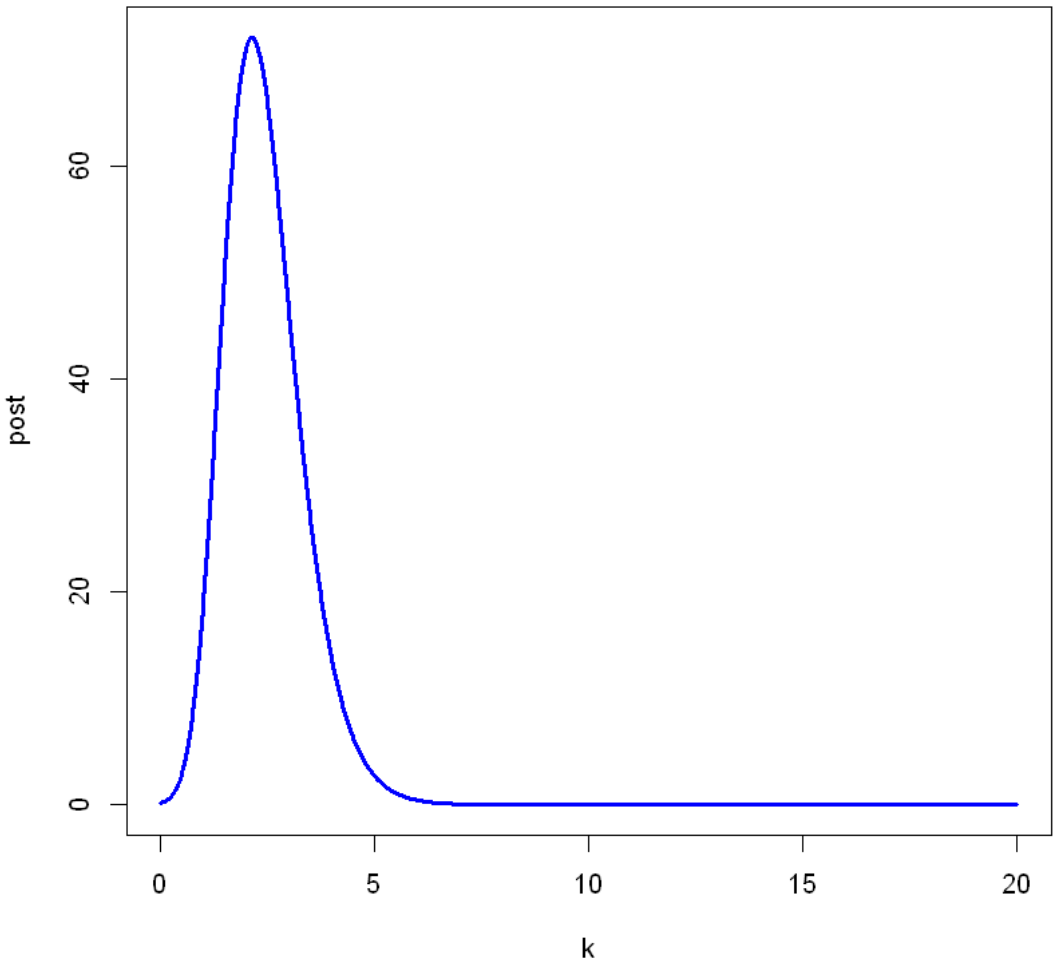
[25]:
```
likelikood_k <- function(y,mu ,n,k){

    exp(k*sum(cos(y-mu)))/(2*pi*besselI(k,nu = 0)^n)
}
```

[26]:
```
k= seq(from = 0,to = 20,by = 0.01)
```

[27]:
```
#
posterior_k <- function(y=y_data,mu=mu ,n=n,k=k){
    likelikood_k(y,mu ,n,k )*prior_k(k)
}
```

[28]:
```
post=posterior_k(y=y_data,mu=mu ,n=n,k=k)
```

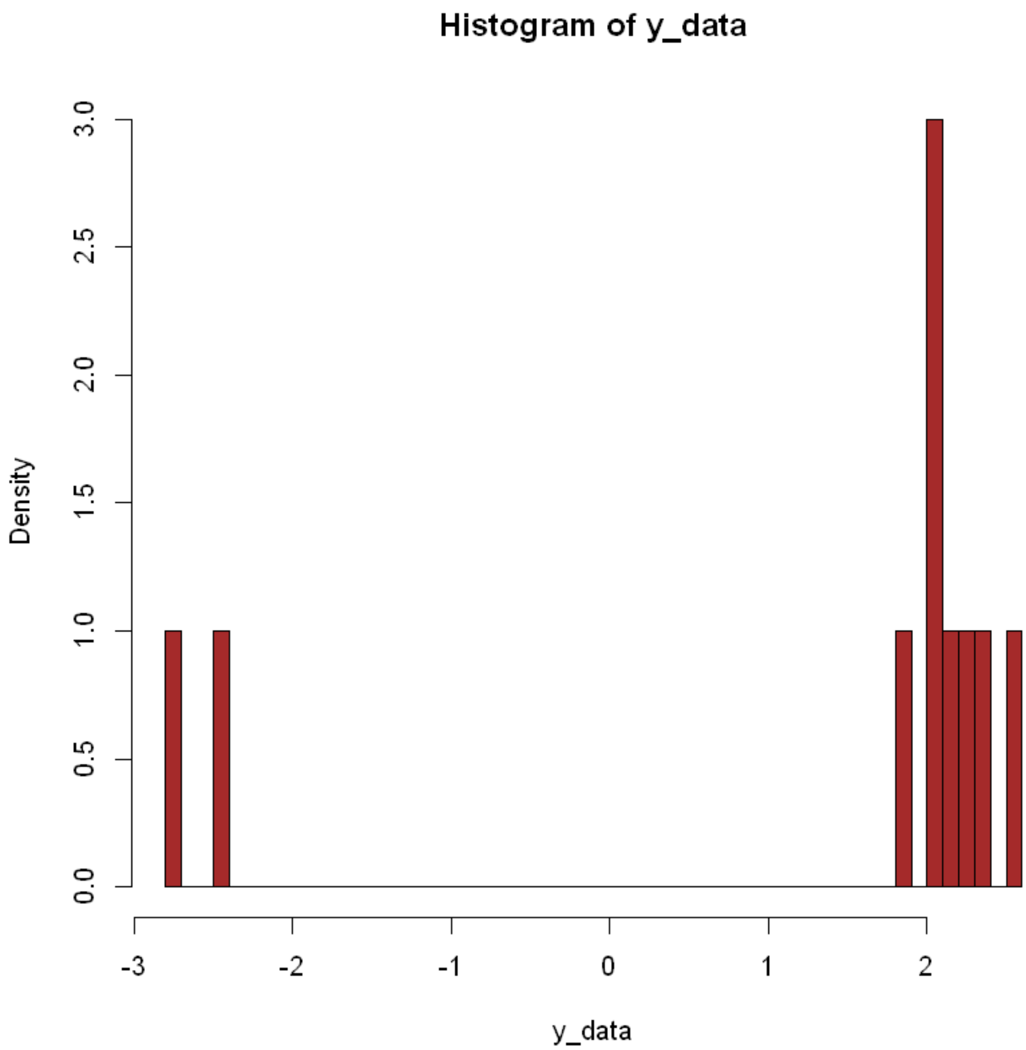
[29]:
```
plot(y=post,x=k,col='blue',type='l',lwd=3)
```



[30]:
```
k[which.max(post)]
```

2.12

[31]:
```
hist(y_data,probability = TRUE,breaks = 50,col='brown')
```



[32]:
```
y_data[which(y_data<0)]
```

1. -2.44 2. -2.79