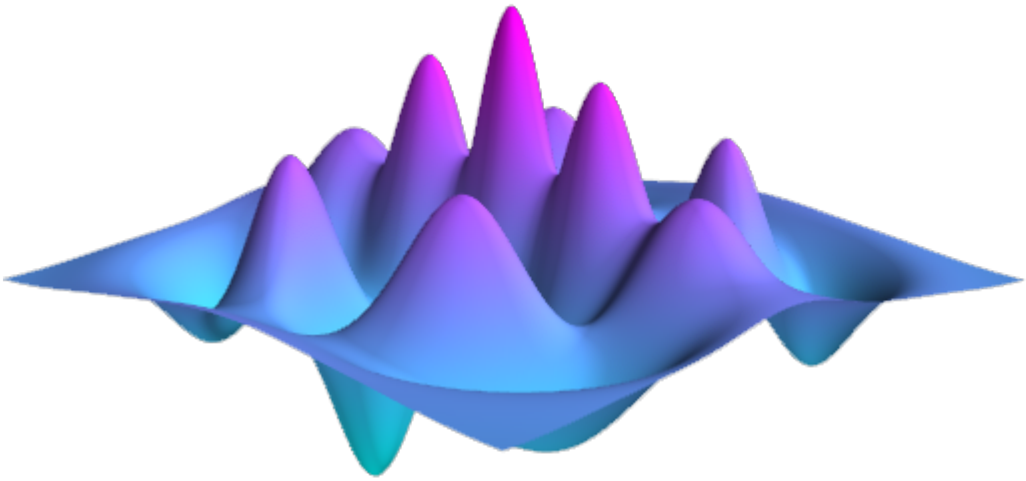
QuTiP



Supporting Organizations

QuTiP is currently supported by these organizations:

RIKEN | MOONSHOT RESEARCH & DEVELOPMENT PROGRAM | PRIFYSGOL ABERYSTWYTH UNIVERSITY | INSTITUT QUANTIQUE UNIVERSITÉ DE SHERBROOKE | PRESTO SAKIGAKE

QuTiP is proud to be affiliated to:

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The development of QuTiP was partially supported by the following organizations:

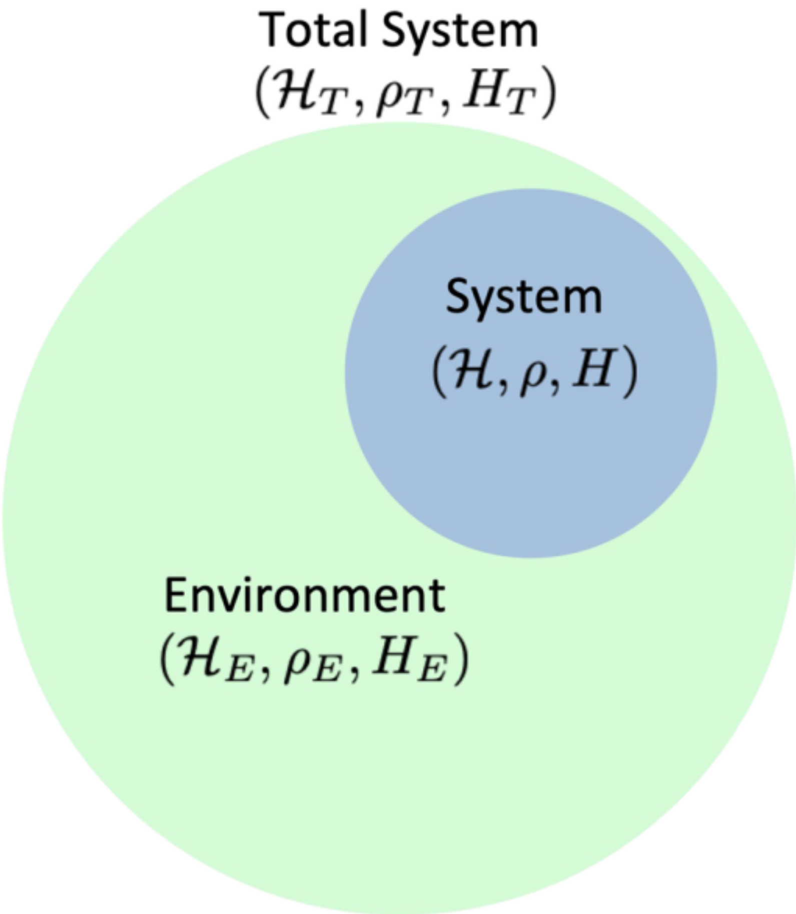
고려대학교 KOREA UNIVERSITY | 日本学術振興会 Japan Society for the Promotion of Science

🕑 : 0 minutes

QuTiP and open quantum systems

QuTiP is a Python package for simulating *open quantum systems*.

An open quantum system is a quantum *system interacting* within an *environment*, like so:



Reference: doi:10.1063/1.5115323

The system evolves according to a Schrödinger equation (1926):

$$i\hbar \frac{d\psi}{dt} = H(t)\psi$$

where $\Psi \in \mathcal{H}$.

Or if the system is represented by a density matrix (i.e. an essemble) then by the von Neumann equation (1927?):

$$\frac{\partial \rho}{\partial t} = \frac{1}{i\hbar}[H(t), \rho]$$

where ρ is an operator $\rho : \mathcal{H} \to \mathcal{H}$ where

$$\rho = \sum_j p_j |\psi_j\rangle\langle\psi_j|$$

and the p_j are real, positive (or zero) and sum to 1.

Practicalities:

- The system Hilbert space is tractably small (e.g. a few tens of qubits).
- The environment Hilbert space is intractably large (e.g. a small antenna and its surrounding electromagnetic field)
- The environment is coupled to the system via an interaction term, H_I in the total system Hamiltonian, H_T.
- The total system evolves unitarily according to the Schrödinger or Von Neuman equation with $H_T = H_S + H_I + H_E$.
- The environment and its effect on the system must be approximated in some way.

Approximating the effects of the environment:

- Typically the system plus environment is modelled by a new Hilbert space, $\mathcal{H}_A$, and a new evolution equation for it.
- Both $\mathcal{H}_A$ and the new evolution equation will dependent on the environment.
- In the simplest examples, $\mathcal{H}_A$, will sometimes *look* exactly like $\mathcal{H}$. Do not be fooled.
- The new evolution equation will be non-unitary.

Why do we care about open quantum systems?

The only quantum systems we care about are ones we can interact with -- i.e. the open ones!

Actually this is not entirely true. Often a quantum system will be well approximated by a classical model before we interact with it (e.g. a soccer ball) so in many cases a classical model will suffice.

And sometimes we will want a model for a quantum system by itself.

But when we want to interact with a quantum system -- i.e. to incorporate non-classical effects into our classical world -- we need models which are open quantum systems.

In particular, *controlling* a quantum system -- i.e. making it do stuff -- requires interacting with it, so controlled quantum systems are typically open ones.

Open quantum systems cover a LOT of interesting cases:

- Quantum computers!
- Measurement!
- Quantum thermodynamics!
- Classical computers!
- Biology! (e.g. photosynthesis)
- Quantum statistical mechanics
- Quantum sensors
- Quantum cosmology
- Gravitational wave detectors!

One can easily keep going. A lot of the basic theory of open quantum systems is established. There are some really hard problems still to tackle (measurement, non-Markovian interactions, large-scale and precision controllable devices) and applications everywhere.

A Fabry-Pérot interferometer in a thermal bath

i.e. the very "simplest" example.

- The system (i.e. the interferometer) is represented by quantum harmonic oscillator with frequency ω_c and Hamiltonian $H = \hbar\omega(N + \frac{1}{2})$ and $N = a^\dagger a$ (the number of the excitation).
- The bath may emit photons into the interferometer cavity, or absorb photons from it. This raises or lowers the number of excitations in cavity.

As the approximate Hilbert space $\mathcal{H}_A$ we will use $\mathcal{H}$ itself, but the evolution will be non-unitary and will be government by a Lindblad equation:

$$\frac{\partial \rho}{\partial t} = \frac{i}{\hbar}[H, \rho] + \sum_{i=1}^{2} \gamma_i \left(L_i^\dagger \rho L_i - \frac{1}{2}\left\{ L_i^\dagger L_i, \rho \right\} \right)$$

where:

$$\begin{aligned} L_1 &= a, \qquad \gamma_1 = \frac{\gamma}{2}(n+1) \qquad (1) \\ L_2 &= a^\dagger, \qquad \gamma_2 = \frac{\gamma}{2} n \qquad (2) \end{aligned}$$

L_1 represents the possibility of the system emitting a photon into the bath, and L_2 the possibility of the system absorbing a photon from the bath.

γ is the strength of the coupling between the bath and the system.

n is the average occupancy of the bath.

The evolution equation is an example of a Lindblad equation (1976). These describe the evolution of systems coupled to Markovian baths (i.e. where the effect of the bath on the system depends only on the current state of the approximated system and not on the complete evolution history).

The operators L are called Lindblad or jump operators.

The terms $L_i^\dagger \rho L_i - \frac{1}{2}\left\{ L_i^\dagger L_i, \rho \right\}$ may be represented by a superoperator called the Liouvillian.

The Lindblad equation and Liouvillians form the basis of many more complex approximations which capture richer interactions between a system and an environment, so they are well worth studying in detail.

QuTiP includes direct support for solving Lindblad master equations via the master equation solver, `mesolve` .

🕑 : 15 minutes

Simple two-level system (i.e. a qubit)

In [1]:
```
import qutip
```

In [2]:
```
import numpy as np
import matplotlib.pyplot as plt
```

In [3]:
```
q = qutip.ket("0")
q
```

Out[3]: Quantum object: dims = [[2], [1]], shape = (2, 1), type = ket

$$\begin{pmatrix}1.0\\0.0\end{pmatrix}$$

In [4]:
```
H =  1 / np.sqrt(2) * qutip.Qobj([
    [1, 1],
    [1, -1],
])
H
```

Out[4]: Quantum object: dims = [[2], [2]], shape = (2, 2), type = oper, isherm = True

$$\begin{pmatrix}0.707 & 0.707\\0.707 & -0.707\end{pmatrix}$$

See https://en.wikipedia.org/wiki/Quantum_logic_gate for the matrices for more quantum gates.

In [5]:
```
H * q
```

Out[5]: Quantum object: dims = [[2], [1]], shape = (2, 1), type = ket

$$\begin{pmatrix}0.707\\0.707\end{pmatrix}$$

Composing systems

In [6]:
```
qq = qutip.ket("00")
qq
```

Out[6]: Quantum object: dims = [[2, 2], [1, 1]], shape = (4, 1), type = ket

$$\begin{pmatrix}1.0\\0.0\\0.0\\0.0\end{pmatrix}$$

In [7]:
```
H_gates = qutip.tensor(H, H)
H_gates
```

Out[7]: Quantum object: dims = [[2, 2], [2, 2]], shape = (4, 4), type = oper, isherm = True

$$\begin{pmatrix}0.500 & 0.500 & 0.500 & 0.500\\0.500 & -0.500 & 0.500 & -0.500\\0.500 & 0.500 & -0.500 & -0.500\\0.500 & -0.500 & -0.500 & 0.500\end{pmatrix}$$

In [8]:
```
H_gates * qq
```

Out[8]: Quantum object: dims = [[2, 2], [1, 1]], shape = (4, 1), type = ket

$$\begin{pmatrix}0.500\\0.500\\0.500\\0.500\end{pmatrix}$$

Evolving a single qubit system over time

If the Hamiltonian is time-independent, it always represents a rotation around some axis. A simple example is:

$$H = \frac{1}{2}\delta\sigma_x + \frac{1}{2}\epsilon\sigma_z$$

where ϵ is the energy difference between the two states and δ is the strenght of the tunneling between them.

In [9]:
```
def H_q(epsilon, delta):
    """ Return a single qubit Hamiltonian with energy difference epsilon
    return 0.5 * delta * qutip.sigmax() + 0.5 * epsilon * qutip.sigmaz()
```

In [10]:
```
H_q(1, 0) * qutip.ket("0")
```

Out[10]: Quantum object: dims = [[2], [1]], shape = (2, 1), type = ket

$$\begin{pmatrix}0.500\\0.0\end{pmatrix}$$

In [11]:
```
H_q(1, 0) * qutip.ket("1")
```

Out[11]: Quantum object: dims = [[2], [1]], shape = (2, 1), type = ket

$$\begin{pmatrix} 0.0 \\ -0.500 \end{pmatrix}$$

In [12]:
```
H_q(1, 0).eigenenergies()
```

Out[12]:
```
array([-0.5,  0.5])
```

In [13]:
```
H_q(2, 0).eigenenergies()
```

Out[13]:
```
array([-1.,  1.])
```

In [14]:
```
evals, evecs = H_q(1, 0).eigenstates()
for energy, state in zip(evals, evecs):
    print("Energy:", energy)
    print("State:", state)
    print()
```

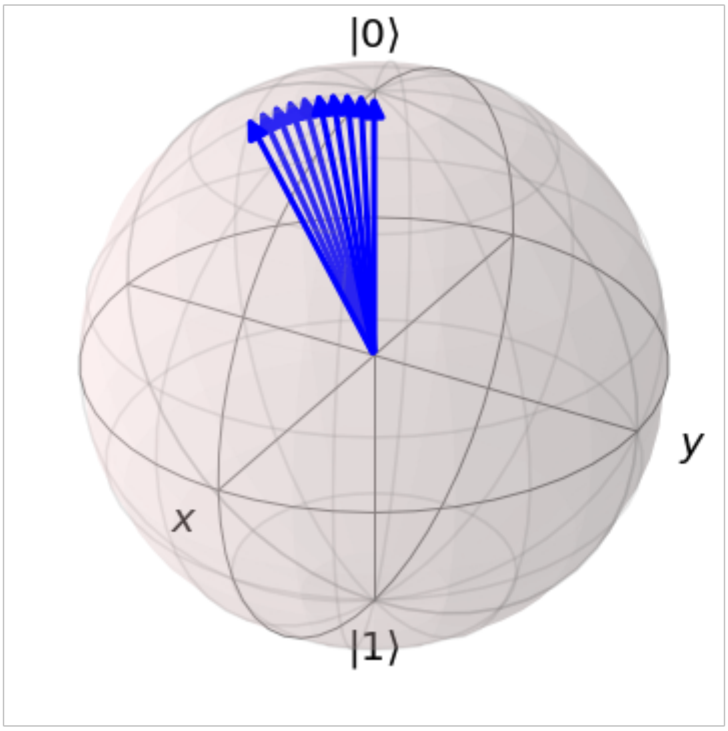
```
Energy: -0.5
State: Quantum object: dims = [[2], [1]], shape = (2, 1), type = ket
Qobj data =
[[ 0.]
 [-1.]]

Energy: 0.5
State: Quantum object: dims = [[2], [1]], shape = (2, 1), type = ket
Qobj data =
[[-1.]
 [ 0.]]
```

In [15]:
```
evals, evecs = H_q(0, 0.5).eigenstates()
for energy, state in zip(evals, evecs):
    print("Energy:", energy)
    print("State:", state)
    print()
```

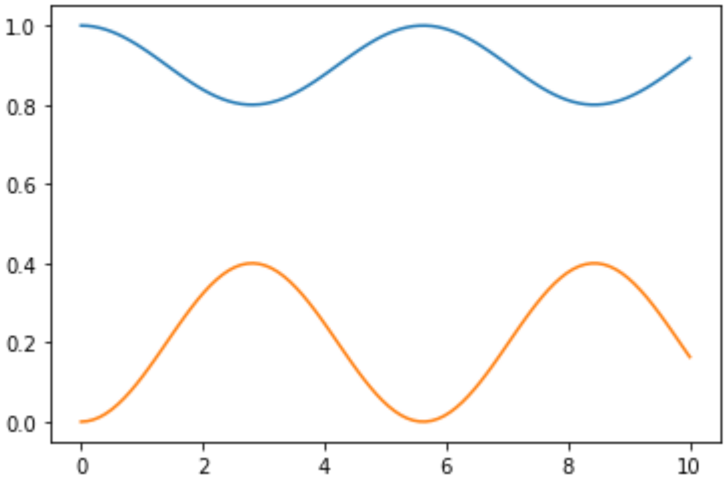
```
Energy: -0.25
State: Quantum object: dims = [[2], [1]], shape = (2, 1), type = ket
Qobj data =
[[-0.70710678]
 [ 0.70710678]]

Energy: 0.25
State: Quantum object: dims = [[2], [1]], shape = (2, 1), type = ket
Qobj data =
[[0.70710678]
 [0.70710678]]
```

In [16]:
```
results = qutip.sesolve(
    H=H_q(1, 0.5),
    psi0=qutip.ket("0"),
    tlist=np.linspace(0, 10, 100),
    e_ops=[
        qutip.bra("0"),
        qutip.bra("1"),
    ],
    options=qutip.Options(store_states=True),
)
```

In [17]:
```
bloch = qutip.Bloch()
bloch.vector_color = ["b"]
bloch.add_states(results.states[:10])
bloch.show()
```



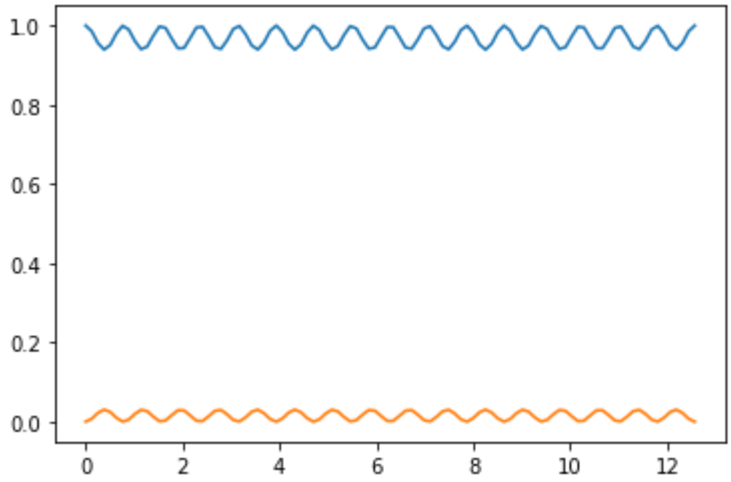
In [18]:
```
fig = plt.figure()
plt.plot(results.times, np.real(results.expect[0]))
plt.plot(results.times, np.real(results.expect[1]));
```



With a small modification we can make the system Hamiltonian time-dependent

```
In [19]: results = qutip.sesolve(
             H=[[H_q(1, 2), "cos(4 * t)"]],
             psi0=qutip.ket("0"),
             tlist=np.linspace(0, 2 * 2 * np.pi, 100),
             e_ops=[
                 qutip.bra("0"),
                 qutip.bra("1"),
             ],
             options=qutip.Options(store_states=True),
         )
```

```
In [20]: fig = plt.figure()
         plt.plot(results.times, np.real(results.expect[0]))
         plt.plot(results.times, np.real(results.expect[1]));
```



🕑 : 30 minutes

Now for a completely different part of qutip -- i.e. qutip-qip

QuTiP's QIP (quantum information processing) package allows one to describe quantum circuits. Many software packages allow one to describe quantum circuits these days. What is special about QuTiP's version is that one can then run these circuits on simulated open quantum systems -- i.e. with physical hamiltonians, coupled to models of baths, with quantum noise and with quantum control.

Let's implement a very simple quantum algorithm ...

The Deutsch–Jozsa algorithm

The Deutsch–Jozsa algorithm is the simplest quantum algorithm that offers an exponential speed-up compared to the classical one. It assumes that we have a function $f : \{0,1\}^n \rightarrow \{0,1\}$ which is either balanced or constant.

Constant means that $f(x)$ is either 1 or 0 for all inputs.

Balanced means that $f(x)$ is 1 for half of the input domain and 0 for the other half.

A more rigorous definition can be found at https://en.wikipedia.org/wiki/Deutsch-Jozsa_algorithm.

The implementation of the Deutsch–Jozsa algorithm includes n input qubits and 1 ancilla initialised in state $ \lvert 1 \rangle.

At the end of the algorithm, the first n qubits are measured on the computational basis.

If the function is constant, the result will be 0 for all n qubits.

If it is balanced, $|00...0\rangle$ will never be measured.

The following example is implemented for the balanced two-qubit function $f(\psi)$ defined by:

$$
\begin{aligned}
f(00) &= 0 \quad (3)\\
f(01) &= 1 \quad (4)\\
f(10) &= 1 \quad (5)\\
f(11) &= 0 \quad (6)
\end{aligned}
$$

This function is balanced, so the probability of measuring state $|00\rangle$ should be 0.

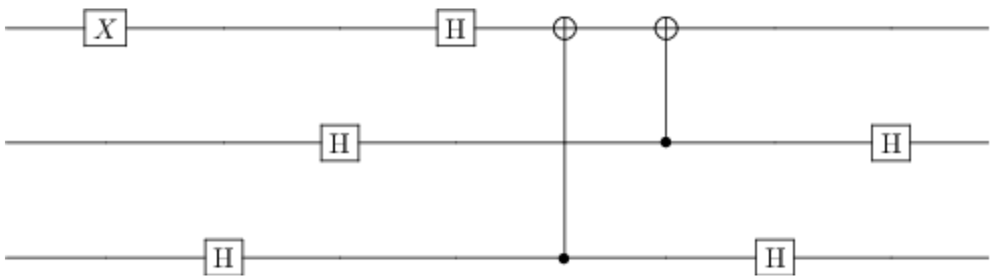
In [21]:
```
from qutip.qip.circuit import QubitCircuit
```

Note that QuTiP uses the name `SNOT` for the Hadamard gate that is more commonly named `H`.

In [22]:
```
qc = QubitCircuit(N=3)
qc.add_gate("X", targets=2)
qc.add_gate("SNOT", targets=0)
qc.add_gate("SNOT", targets=1)
qc.add_gate("SNOT", targets=2)

# function f(x)
qc.add_gate("CNOT", controls=0, targets=2)
qc.add_gate("CNOT", controls=1, targets=2)

qc.add_gate("SNOT", targets=0)
qc.add_gate("SNOT", targets=1)
```

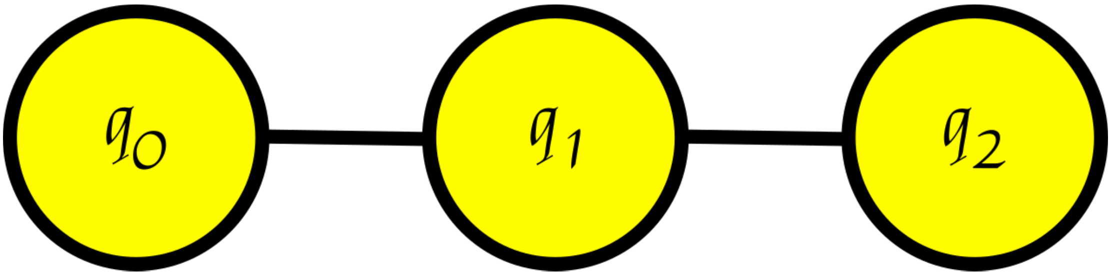
In [23]:
```
qc
```

Out[23]:



: 45 minutes

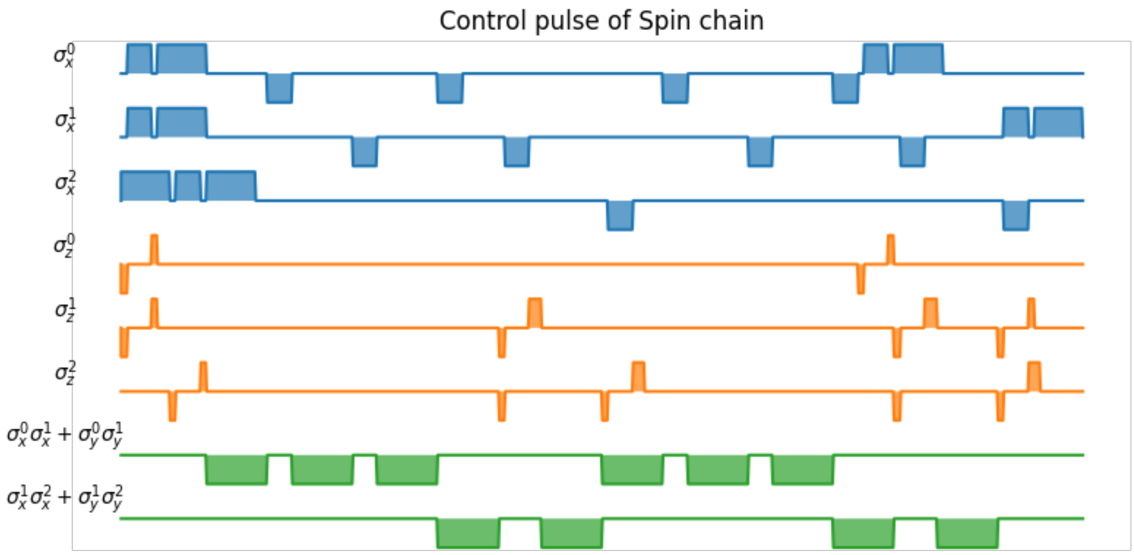
Simulating the circuit with the linear spin chain model



In [24]:
```
from qutip.qip.device import LinearSpinChain
```

In [25]:
```
processor = LinearSpinChain(3)
processor.load_circuit(qc);
```

In [26]:
```
processor.plot_pulses(title="Control pulse of Spin chain", figsize=(8, 4))
```



- Because for the spin chain model interaction only exists between neighbouring qubits, SWAP gates are added between and after the first CNOT gate, swapping the first two qubits.
- The SWAP gate is decomposed into three iSWAP gates, while the CNOT is decomposed into two iSWAP gates plus additional single-qubit corrections.
- Both the Hadamard gate and the two-qubit gates need to be decomposed to native gates (iSWAP and rotation on the x and z axes).
- The compiled coefficients are square pulses and the control coefficients on σ_z and σ_x are also different, resulting in different gate times.

In [27]:
```
# Without decoherence

basis00 = qutip.basis([2,2], [0,0])
psi0 = qutip.basis([2,2,2], [0,0,0])
result = processor.run_state(init_state=psi0)
print("Probability of measuring state 00:")
print(np.real((basis00.dag() * qutip.ptrace(result.states[-1], [0,1]) * b
```

```
Probability of measuring state 00:
1.543502734240989e-08
```

In [28]:
```
# With decoherence

processor.t1 = 100
processor.t2 = 30
psi0 = qutip.basis([2,2,2], [0,0,0])
result = processor.run_state(init_state=psi0)
print("Probability of measuring state 00:")
print(np.real((basis00.dag() * qutip.ptrace(result.states[-1], [0,1]) * b
```

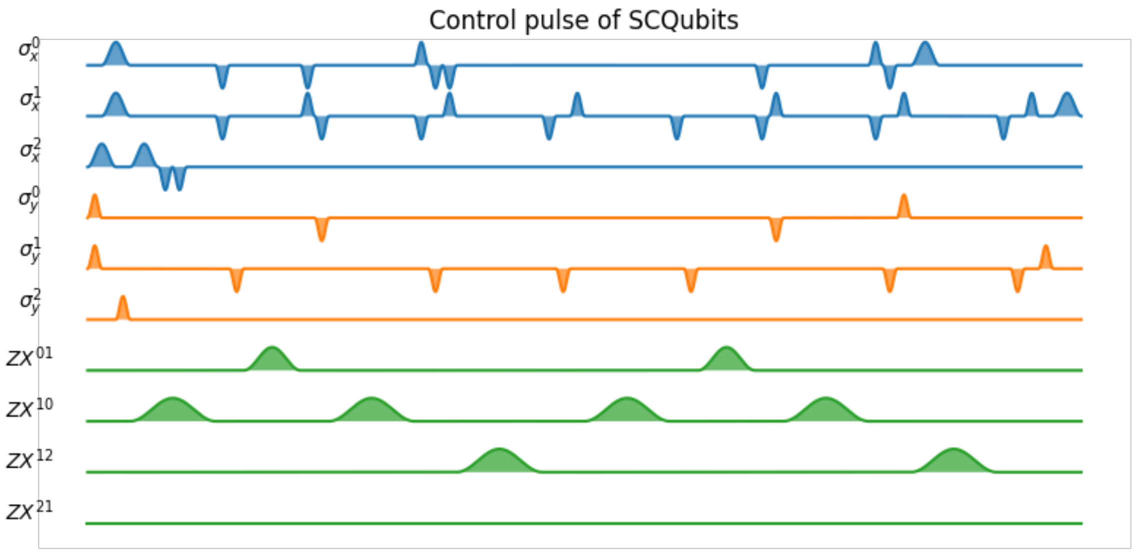
```
Probability of measuring state 00:
0.13730658185148123
```

Simulating the circuit with superconducting qubits

In [29]:
```
from qutip_qip.device import SCQubits
```

In [30]:
```
processor = SCQubits(num_qubits=3)
processor.load_circuit(qc);
```

In [31]:
```
processor.plot_pulses(title="Control pulse of SCQubits", figsize=(8, 4),
```



- For the superconducting-qubit processor, the compiled pulses have a Gaussian shape. This is crucial for superconducting qubits because the second excited level is only slightly detuned from the qubit transition energy. A smooth pulse usually prevents leakage to the non-computational subspace.
- Similar to the spin chain, SWAP gates are added to switch the zeroth and first qubit and one SWAP gate is compiled to three CNOT gates.
- The control ZX21 is not used because there is no CNOT gate that is controlled by the second qubit and acts on the first one.

In [32]:
```
# Without decoherence

basis00 = qutip.basis([3, 3], [0, 0])
psi0 = qutip.basis([3, 3, 3], [0, 0, 0])
result = processor.run_state(init_state=psi0)
print("Probability of measuring state 00:")
print(np.real((basis00.dag() * qutip.ptrace(result.states[-1], [0,1]) * b
```

```
Probability of measuring state 00:
0.0003445928447621024
```

In [33]:
```
# With decoherence

processor.t1 = 50.e3
processor.t2 = 20.e3
psi0 = qutip.basis([3, 3, 3], [0, 0, 0])
result = processor.run_state(init_state=psi0)
print("Probability of measuring state 00:")
print(np.real((basis00.dag() * qutip.ptrace(result.states[-1], [0,1]) * b
```

```
Probability of measuring state 00:
0.0604189948419380 65
```

See the "Quantum information processing" section on the QuTiP tutorials page.

Further reading

QuTiP and documentation

- Website: https://qutip.org/
- Documentation: https://qutip.org/docs/latest/index.html
- Tutorials: https://qutip.org/tutorials.html

QuTiP QIP and documentation

- Documentation: https://qutip-qip.readthedocs.io/
- Tutorials: https://qutip.org/tutorials.html

Community

- Discord (Unitary Fund): https://discord.com/invite/JqVGmpkP96
- GitHub: https://github.com/qutip
- Mailing list: https://groups.google.com/g/qutip/

End

In [34]:
```
qutip.about()
```

```
QuTiP: Quantum Toolbox in Python
================================
Copyright (c) QuTiP team 2011 and later.
Current admin team: Alexander Pitchford, Nathan Shammah, Shahnawaz Ahmed,
Neill Lambert, Eric Giguère, Boxi Li, Jake Lishman and Simon Cross.
Board members: Daniel Burgarth, Robert Johansson, Anton F. Kockum, Franco
Nori and Will Zeng.
Original developers: R. J. Johansson & P. D. Nation.
Previous lead developers: Chris Granade & A. Grimsmo.
Currently developed through wide collaboration. See https://github.com/qu
tip for details.

QuTiP Version:      4.7.0
Numpy Version:      1.21.0
Scipy Version:      1.8.0
Cython Version:     0.29.28
Matplotlib Version: 3.5.1
Python Version:     3.9.12
Number of CPUs:     8
BLAS Info:          OPENBLAS
OPENMP Installed:   False
INTEL MKL Ext:      False
Platform Info:      Linux (x86_64)
Installation path:  /home/simon/venvs/py3/qutip-venezuela-2022/lib/python
3.9/site-packages/qutip
==========================================================================
=======
Please cite QuTiP in your publication.
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=======
For your convenience a bibtex reference can be easily generated using `qu
tip.cite()`
```

In [ ]: