Red-Black Gauss–Seidel method

Рівняння Пуассона (2D)

$$\boxed{\nabla^2 u = f(x,y)}$$

$$\boxed{\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x,y)}$$

Рівняння Пуассона (2D)

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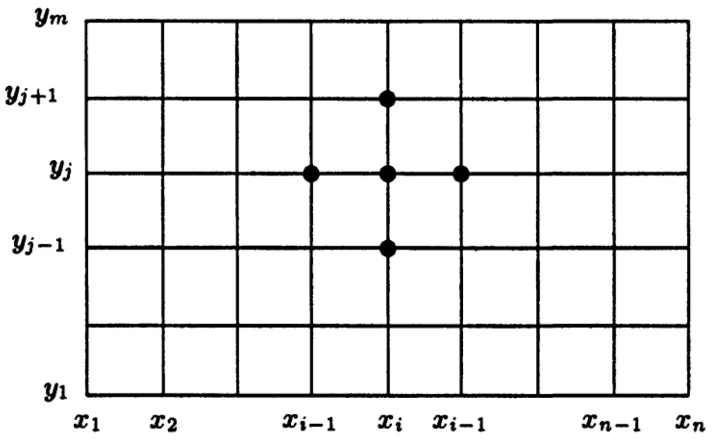
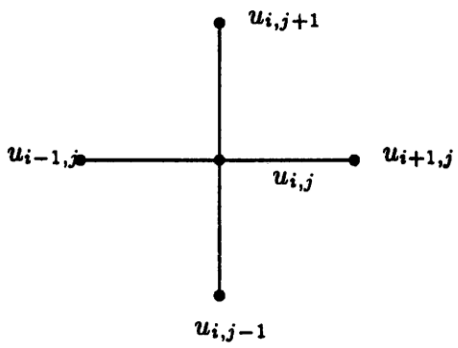
$$\boxed{\nabla^2 u = \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h_x^2} + \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h_y^2} = f(x, y)}$$

Рівняння Пуассона (2D)

$$\boxed{\frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h_x^2} + \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h_y^2} = f(x,y)}$$

$$\boxed{\frac{u_{i+1,j} + u_{i-1,j}}{h_x^2} + \frac{u_{i+1,j} + u_{i-1,j}}{h_y^2} - u_{i,j}\left(\frac{2}{h_x^2} + \frac{2}{h_y^2}\right) = f(x,y)}$$





Рівняння Пуассона (2D)

$$\boxed{u_{i,j} = (u_{i+1,j} + u_{i-1,j})b + (u_{i+1,j} + u_{i-1,j})c + \frac{f}{a}}$$

$$a = \frac{2}{h_x^2} + \frac{2}{h_y^2}, \qquad b = \frac{1}{h_x^2}, \qquad c = \frac{2}{h_y^2}$$

$$A = \begin{bmatrix}
a & b & \cdots & c & & & & & \\
b & a & b & \cdots & c & & & & \\
 & \ddots & \ddots & \ddots & & \ddots & & & \\
c & \cdots & b & a & b & \cdots & c & & \\
 & \ddots & & \ddots & \ddots & \ddots & & \ddots & \\
 & & c & \cdots & b & a & b & \cdots & c \\
 & & & \ddots & & \ddots & \ddots & \ddots & \\
 & & & & c & \cdots & b & a & b \\
 & & & & & c & \cdots & b & a
\end{bmatrix} \qquad x = \begin{bmatrix} u(0,0) \\ u(0,1) \\ \vdots \\ u(0,N_y) \\ \vdots \\ u(x,y) \\ \vdots \\ u(N_x,N_y) \end{bmatrix}$$

Рівняння Пуассона (2D)

$$\boxed{u_{i,j} = (u_{i+1,j} + u_{i-1,j})b + (u_{i+1,j} + u_{i-1,j})c + \frac{f}{a}}$$

$$a = \frac{2}{h_x^2} + \frac{2}{h_y^2}, \qquad b = \frac{1}{h_x^2}, \qquad c = \frac{2}{h_y^2}$$

$$A = \begin{bmatrix} a & b & \cdots & c & & & & & & \\ b & a & b & \cdots & c & & & & & \\ & \ddots & \ddots & \ddots & & \ddots & & & & \\ c & \cdots & b & a & b & \cdots & c & & & \\ & \ddots & & \ddots & \ddots & \ddots & & \ddots & & \\ & & c & \cdots & b & a & b & \cdots & c & \\ & & & \ddots & & \ddots & \ddots & \ddots & & \\ & & & & c & \cdots & b & a & b & \\ & & & & & c & \cdots & b & a & \end{bmatrix} \qquad x = \begin{bmatrix} u(0,0) \\ u(0,1) \\ \vdots \\ u(0,N_y) \\ \vdots \\ u(x,y) \\ \vdots \\ u(N_x,N_y) \end{bmatrix}$$

$$\begin{aligned} u(x_1, y_j) &= u_{1,j} && \text{для } 2 \le j \le m-1 \\ u(x_i, y_1) &= u_{i,1} && \text{для } 2 \le i \le n-1 \\ u(x_n, y_j) &= u_{n,j} && \text{для } 2 \le j \le m-1 \\ u(x_i, y_m) &= u_{i,m} && \text{для } 2 \le i \le n-1 \end{aligned}$$

Рівняння Пуассона (2D)

$$\boxed{u_{i,j} = \left(u_{i+1,j} + u_{i-1,j}\right)b + \left(u_{i+1,j} + u_{i-1,j}\right)c + \frac{f}{a}}$$

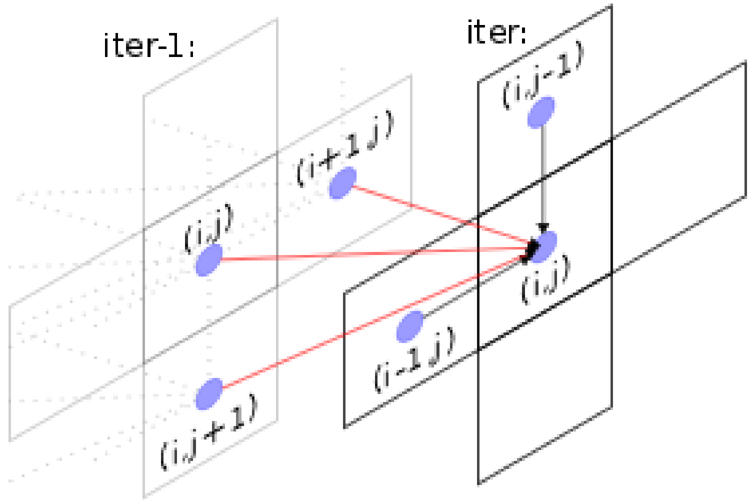
$$a = \frac{2}{h_x^2} + \frac{2}{h_y^2}, \qquad b = \frac{1}{h_x^2}, \qquad c = \frac{2}{h_y^2}$$

```
for(unsigned iter=0; iter < max_iterations; ++iter)
{
    for(x = 1; x < N_x-1; ++x)
    {
        for(y = 1; y < N_y-1; ++y)
        {
            u(x, y) = ((u(x-1,y) + u(x+1, y)) * b
                + (u(x, y-1) + u(x, y+1)) * c
                + f(x,y))/a;
        }
    }
}
```

Рівняння Пуассона (2D)

$$\boxed{u_{i,j} = (u_{i+1,j} + u_{i-1,j})b + (u_{i+1,j} + u_{i-1,j})c + \frac{f}{a}}$$

$$a = \frac{2}{h_x^2} + \frac{2}{h_y^2}, \qquad b = \frac{1}{h_x^2}, \qquad c = \frac{2}{h_y^2}$$



Рівняння Пуассона (2D)

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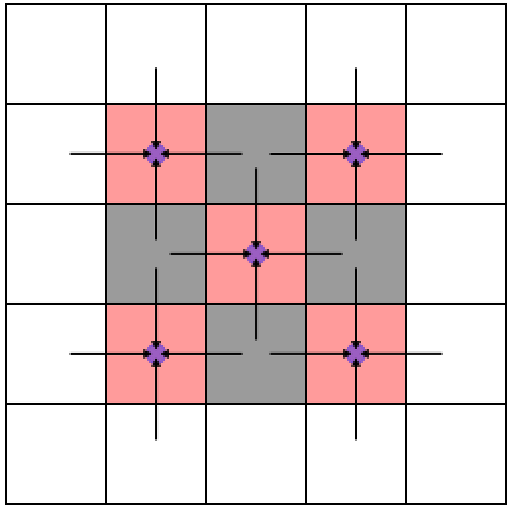

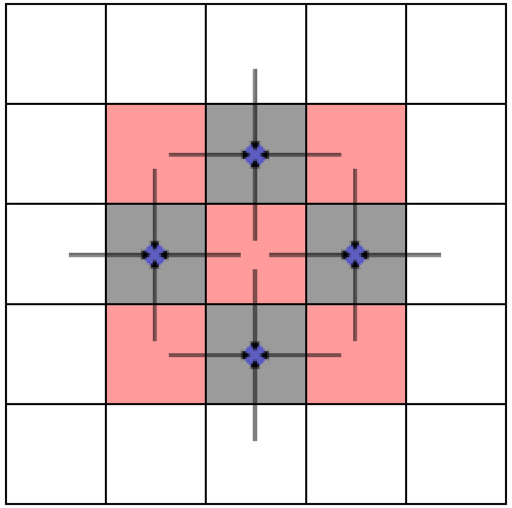

Рівняння Пуассона (2D)

$$u_{i,j} = \left(u_{i+1,j} + u_{i-1,j}\right)b + \left(u_{i+1,j} + u_{i-1,j}\right)c + \frac{f}{a}$$

```
for(iter=0; iter < max_iterations; ++iter)
{
#pragma omp parallel for
    for(x = 1; x < N_x-1; ++x)
    {
        for(y = (x%2) +1; y < N_y-1; y+=2)
        {
            u(x, y) = ((u(x-1,y) + u(x+1, y)) * b
                + (u(x, y-1) + u(x, y+1)) * c
                + f(x,y))/a;
        }
    }
#pragma omp parallel for
    for(x = 1; x < N_x-1; ++x)
    {
        for(y = ((x+1)%2) +1; y < N_y-1; y+=2)
        {
            u(x, y) = ((u(x-1,y) + u(x+1, y)) * b
                + (u(x, y-1) + u(x, y+1)) * c
                + f(x,y))/a;
        }
    }
}
```