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ERRORS AND PERFORMANCE TRADEOFFS IN HYPOTHESIS TESTING

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Errors and Performance Tradeoffs in Hypothesis Testing

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1. **False Alarm** (Type I Error, also called False Positive)
   - occurs if we accept a hypothesis when it is not true
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$$P_{fa} = P(\text{false alarm})$$

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- the tradeoff is controlled by choosing the significance level, a, to which the p-value is compared
  - the value a is the probability that we will reject the null hypothesis, H_0 when it is in fact true
  - equivalently, it is the probability of accepting the alternative hypothesis, H_A, when H_A is false

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  - thus, we will consider the implications with respect to H_A when labeling errors
  - so for the case that we accept H_A when it is false, we call that a **false alarm**/Type I error
  - then $P_{fa} = a$

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  - i.e., we increase the **Probability of Miss/Type II error**, P_m
- the converse is also true

BINARY DECISIONS FROM CONTINUOUS DATA

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- We have many situations where we have a continuous measurement that depends on some underlying binary phenomena
- For example, we may wish to determine the presence of a disease based on the measurement of some chemical
  - Then the distribution of the data depends on whether the disease is present or not

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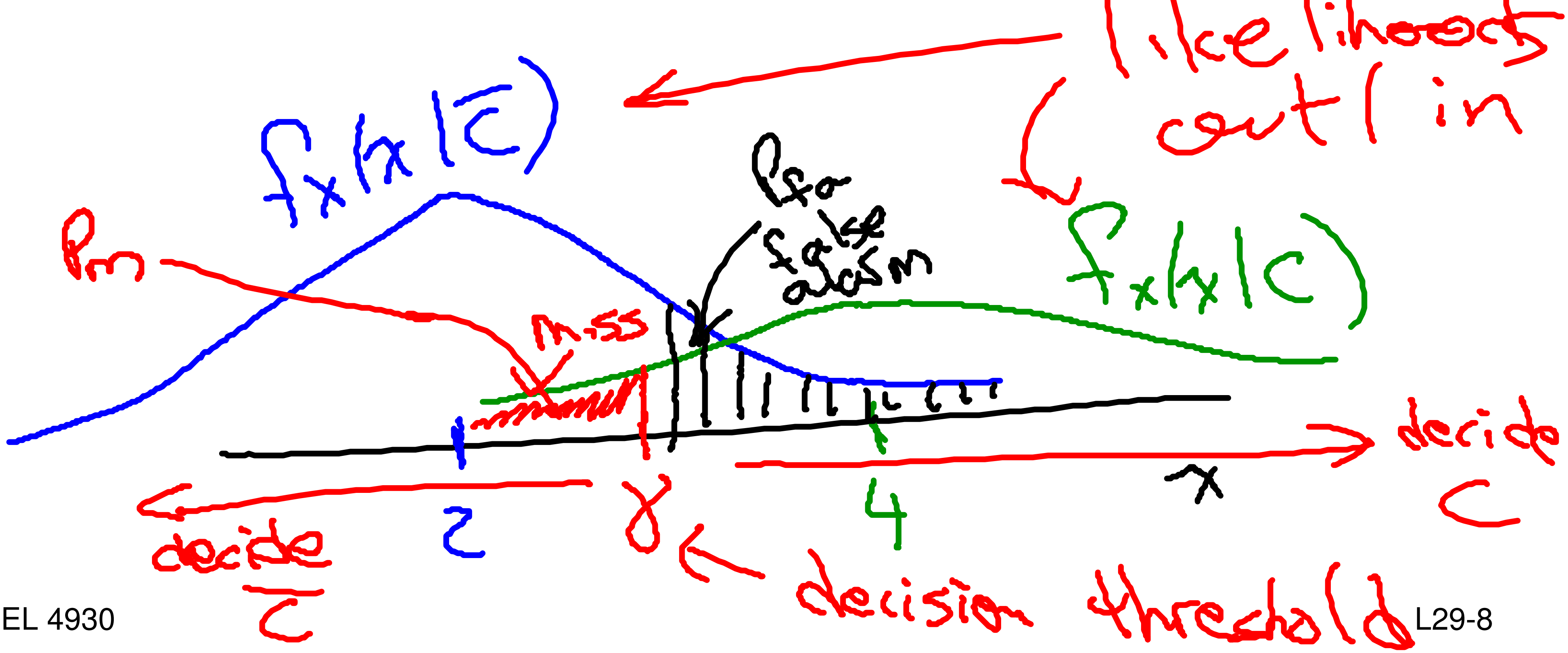
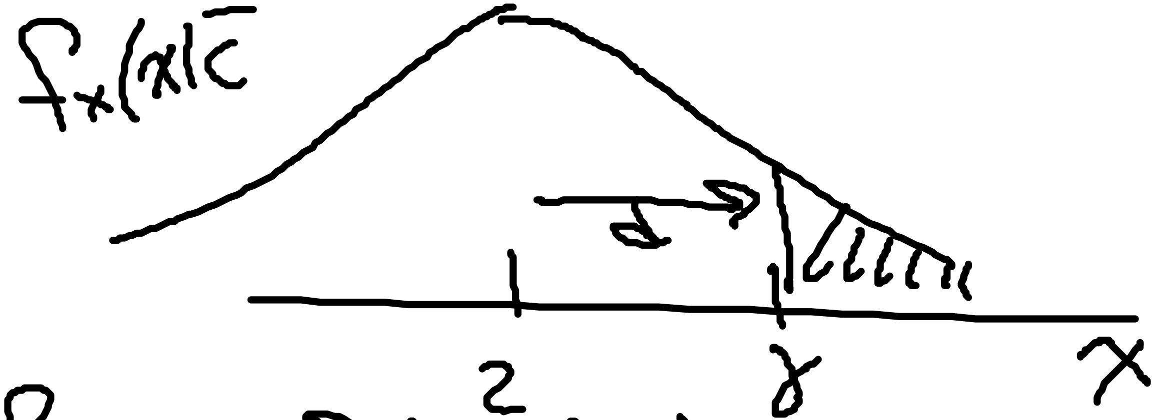
- In many cases, the decision regions are determined by a single threshold γ, like $R_0 = x < \gamma$ and $R_1 = x \geq \gamma$

EX The PSA values for men in their 60s without cancer are approximately Gaussian(2,$\sigma^2 = 1$). The PSA values are values for men in their 60s with cancer are approximately Gaussian(4, $\sigma^2 = 2$).

C = cancer

1. For $P_{fa} = 10\%$, find P_m.

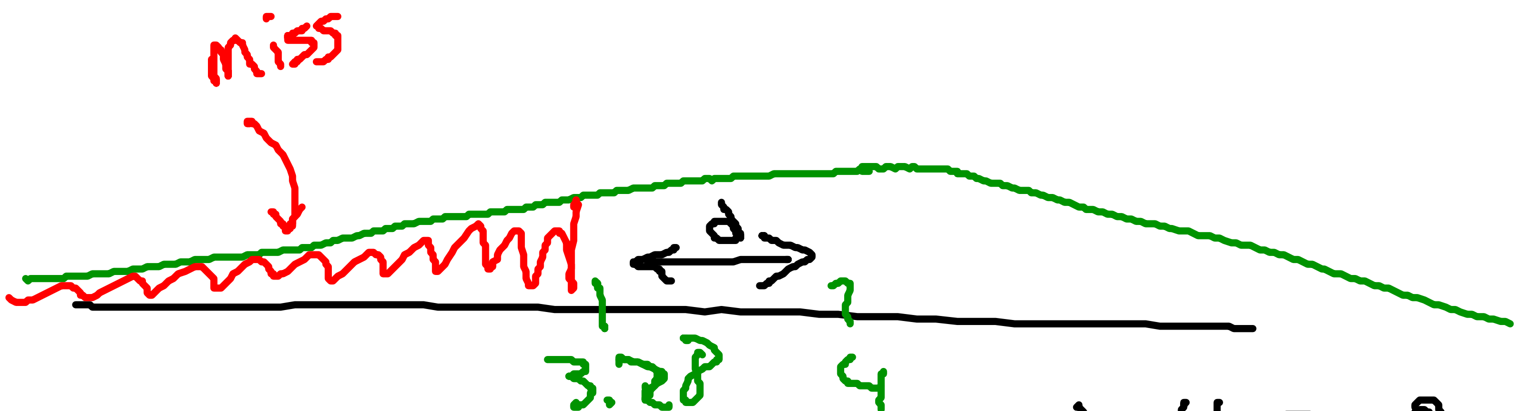




$$P_{fa} = Q\left(\frac{\gamma-2}{\cancel{\sigma}\,1}\right) = 0.1$$

$$\gamma - 2 = Q^{-1}(0.1) = 1.28$$

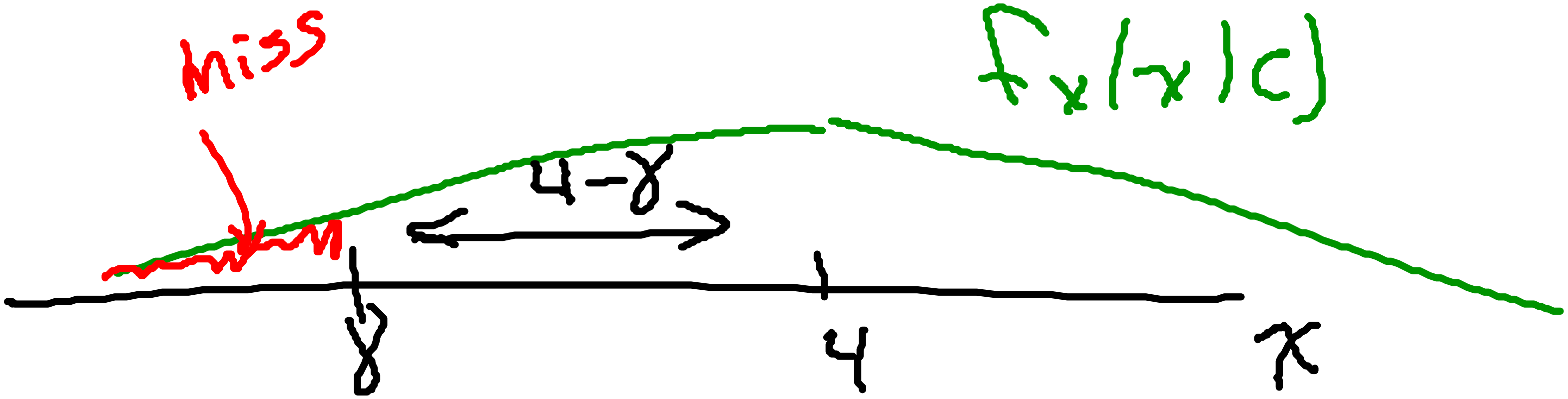
$$\gamma = 3.28$$



$$d = 4 - 3.28 = 0.72$$

$$P_m = Q\left(\frac{d}{\sigma}\right) = Q\left(\frac{0.72}{\sqrt{2}}\right) \approx 0.305$$

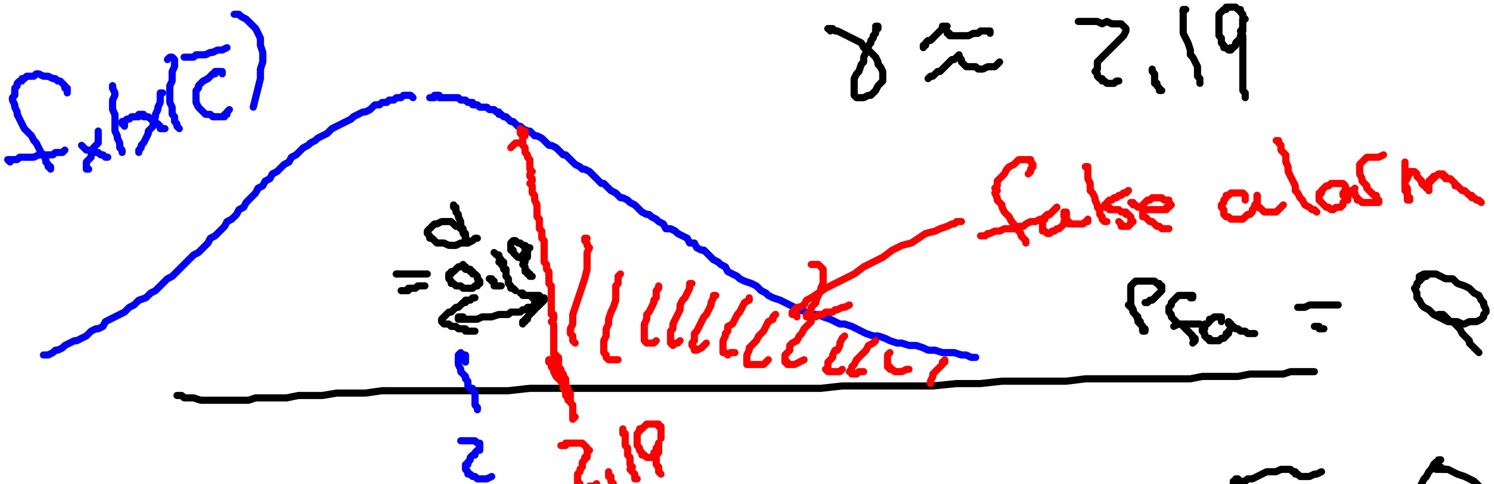
2. For $P_m = 10\%$, find P_{fa}.



$$P_m = Q\left(\frac{4-\gamma}{\sqrt{2}}\right) = 0.1$$

$$4-\gamma/\sqrt{2} = 1.28$$

$$\gamma \approx 2.19$$



$$P_{fa} = Q\left(\frac{0.19}{1}\right)$$

$$\approx 0.425$$

Visualizing Tradeoffs in Hypothesis Testing: ROC Curves

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  - ROC= receiver operating characteristic
  - ROC curves were developed for RADAR systems but are widely used in fields that do statistical tests, such as biomedicine

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- Instead:
  - the x-axis is FPR= false positive rate = P_{fa}, and
  - the y-axis is TPR= true positive rate = $1 - P_m$