EEL 4930 Stats – Lecture 22

EEL 4930 Stats – Lecture 21

Properties of Distribution Functions

1. $0 \leq F_X(x) \leq 1$

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PROPERTIES OF DISTRIBUTION FUNCTIONS

1. $0 \le F_X(x) \le 1$ $= P(X \le x)$
   $= P(\{s \mid X(s) \le x\})$

   Pf: $F_X(x)$ is a prob. measure

2. $F_X(-\infty) = 0$ and $F_X(\infty) = 1$

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   Basically, $F_X(-\infty)$ and $F_X(\infty)$ are defined as limits, and the corresponding subsets of the samples space $\{s \in S : X \leq x\}$ are either shrinking to $\emptyset$ or S

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Pf: $P\{X \in (-\infty, b]\} \;=\; P(X \in (-\infty, a])$

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$$\Rightarrow F_X(b) = F_X(a) + \underbrace{P(a < X \leq b)}_{\geq 0}$$

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   Pf: rewriting equation (1)

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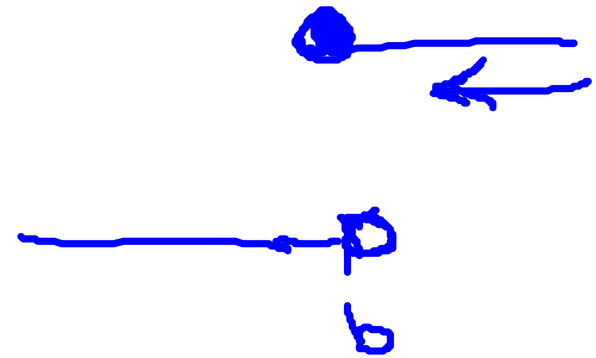

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Proof is rather technical and will be omitted.

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