EEL 4930/Stats Final Examination

December 12, 2017

The time for this test is two hours. The exam consists of two parts: a closed-book part, in which you may use up to two formula sheets; and an open book, open notes, open computer part. Note that some questions are worth more than other questions. You must sign the honor statement at the end of the test in order to receive any credit.

Binomial(N, p) PMF

$$P_X(k) = \begin{cases} \binom{N}{p} p^k (1-p)^{N-k}, & k = 0, 1, \ldots N \\ 0, & \text{otherwise} \end{cases}$$

$$E[X] = Np \qquad \text{Var}[X] = Np(1-p)$$

Geometric(p) PMF

$$P_X(k) = \begin{cases} (1-p)^{k-1} p & k = 1, 2, 3, \ldots \\ 0, & \text{otherwise} \end{cases}$$

$$E[X] = \frac{1}{p} \qquad \text{Var}[X] = \frac{1-p}{p^2}$$

Poisson(α) PMF

$$P_X(k) = \begin{cases} \frac{\alpha^k}{k!} e^{-\alpha}, & k = 0, 1, \ldots \\ 0, & \text{otherwise} \end{cases}$$

$$E[X] = \alpha \qquad \text{Var}[X] = \alpha$$

Q-Function

Definition

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2}\, dt$$

Good Approximation (good for programming in calculator)

$$Q(x) \approx \left[\frac{1}{(1-a)x + a\sqrt{x^2+b}} \right] \frac{1}{\sqrt{2\pi}} e^{-x^2/2},$$

where $a = 1/\pi$, $b = 2\pi$

Simple Upper Bound

$$Q(x) < \frac{1}{2} e^{-x^2/2}$$

Relation to Error Functions

$$Q(x) = \frac{1}{2} \operatorname{erfc}\left(\frac{x}{\sqrt{2}} \right), \qquad \operatorname{erfc}(x) = 2Q(x\sqrt{2})$$

Property

$$Q(-x) = 1 - Q(x)$$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	5.0000e-01	4.9601e-01	4.9202e-01	4.8803e-01	4.8405e-01	4.8006e-01	4.7608e-01	4.7210e-01	4.6812e-01	4.6414e-01
0.10	4.6017e-01	4.5620e-01	4.5224e-01	4.4828e-01	4.4433e-01	4.4038e-01	4.3644e-01	4.3251e-01	4.2858e-01	4.2465e-01
0.20	4.2074e-01	4.1683e-01	4.1294e-01	4.0905e-01	4.0517e-01	4.0129e-01	3.9743e-01	3.9358e-01	3.8974e-01	3.8591e-01
0.30	3.8209e-01	3.7828e-01	3.7448e-01	3.7070e-01	3.6693e-01	3.6317e-01	3.5942e-01	3.5569e-01	3.5197e-01	3.4827e-01
0.40	3.4458e-01	3.4090e-01	3.3724e-01	3.3360e-01	3.2997e-01	3.2636e-01	3.2276e-01	3.1918e-01	3.1561e-01	3.1207e-01
0.50	3.0854e-01	3.0503e-01	3.0153e-01	2.9806e-01	2.9460e-01	2.9116e-01	2.8774e-01	2.8434e-01	2.8096e-01	2.7760e-01
0.60	2.7425e-01	2.7093e-01	2.6763e-01	2.6435e-01	2.6109e-01	2.5785e-01	2.5463e-01	2.5143e-01	2.4825e-01	2.4510e-01
0.70	2.4196e-01	2.3885e-01	2.3576e-01	2.3270e-01	2.2965e-01	2.2663e-01	2.2363e-01	2.2065e-01	2.1770e-01	2.1476e-01
0.80	2.1186e-01	2.0897e-01	2.0611e-01	2.0327e-01	2.0045e-01	1.9766e-01	1.9489e-01	1.9215e-01	1.8943e-01	1.8673e-01
0.90	1.8406e-01	1.8141e-01	1.7879e-01	1.7619e-01	1.7361e-01	1.7106e-01	1.6853e-01	1.6602e-01	1.6354e-01	1.6109e-01
1.00	1.5866e-01	1.5625e-01	1.5386e-01	1.5151e-01	1.4917e-01	1.4686e-01	1.4457e-01	1.4231e-01	1.4007e-01	1.3786e-01
1.10	1.3567e-01	1.3350e-01	1.3136e-01	1.2924e-01	1.2714e-01	1.2507e-01	1.2302e-01	1.2100e-01	1.1900e-01	1.1702e-01
1.20	1.1507e-01	1.1314e-01	1.1123e-01	1.0935e-01	1.0749e-01	1.0565e-01	1.0383e-01	1.0204e-01	1.0027e-01	9.8525e-02
1.30	9.6800e-02	9.5098e-02	9.3418e-02	9.1759e-02	9.0123e-02	8.8508e-02	8.6915e-02	8.5343e-02	8.3793e-02	8.2264e-02
1.40	8.0757e-02	7.9270e-02	7.7804e-02	7.6359e-02	7.4934e-02	7.3529e-02	7.2145e-02	7.0781e-02	6.9437e-02	6.8112e-02
1.50	6.6807e-02	6.5522e-02	6.4255e-02	6.3008e-02	6.1780e-02	6.0571e-02	5.9380e-02	5.8208e-02	5.7053e-02	5.5917e-02
1.60	5.4799e-02	5.3699e-02	5.2616e-02	5.1551e-02	5.0503e-02	4.9471e-02	4.8457e-02	4.7460e-02	4.6479e-02	4.5514e-02
1.70	4.4565e-02	4.3633e-02	4.2716e-02	4.1815e-02	4.0930e-02	4.0059e-02	3.9204e-02	3.8364e-02	3.7538e-02	3.6727e-02
1.80	3.5930e-02	3.5148e-02	3.4380e-02	3.3625e-02	3.2884e-02	3.2157e-02	3.1443e-02	3.0742e-02	3.0054e-02	2.9379e-02
1.90	2.8717e-02	2.8067e-02	2.7429e-02	2.6803e-02	2.6190e-02	2.5588e-02	2.4998e-02	2.4419e-02	2.3852e-02	2.3295e-02

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.00	2.2750e-02	2.2216e-02	2.1692e-02	2.1178e-02	2.0675e-02	2.0182e-02	1.9699e-02	1.9226e-02	1.8763e-02	1.8309e-02
2.10	1.7864e-02	1.7429e-02	1.7003e-02	1.6586e-02	1.6177e-02	1.5778e-02	1.5386e-02	1.5003e-02	1.4629e-02	1.4262e-02
2.20	1.3903e-02	1.3553e-02	1.3209e-02	1.2874e-02	1.2545e-02	1.2224e-02	1.1911e-02	1.1604e-02	1.1304e-02	1.1011e-02
2.30	1.0724e-02	1.0444e-02	1.0170e-02	9.9031e-03	9.6419e-03	9.3867e-03	9.1375e-03	8.8940e-03	8.6563e-03	8.4242e-03
2.40	8.1975e-03	7.9763e-03	7.7603e-03	7.5494e-03	7.3436e-03	7.1428e-03	6.9469e-03	6.7557e-03	6.5691e-03	6.3872e-03
2.50	6.2097e-03	6.0366e-03	5.8677e-03	5.7031e-03	5.5426e-03	5.3861e-03	5.2336e-03	5.0849e-03	4.9400e-03	4.7988e-03
2.60	4.6612e-03	4.5271e-03	4.3965e-03	4.2692e-03	4.1453e-03	4.0246e-03	3.9070e-03	3.7926e-03	3.6811e-03	3.5726e-03
2.70	3.4670e-03	3.3642e-03	3.2641e-03	3.1667e-03	3.0720e-03	2.9798e-03	2.8901e-03	2.8028e-03	2.7179e-03	2.6354e-03
2.80	2.5551e-03	2.4771e-03	2.4012e-03	2.3274e-03	2.2557e-03	2.1860e-03	2.1182e-03	2.0524e-03	1.9884e-03	1.9262e-03
2.90	1.8658e-03	1.8071e-03	1.7502e-03	1.6948e-03	1.6411e-03	1.5889e-03	1.5382e-03	1.4890e-03	1.4412e-03	1.3949e-03
3.00	1.3499e-03	1.3062e-03	1.2639e-03	1.2228e-03	1.1829e-03	1.1442e-03	1.1067e-03	1.0703e-03	1.0350e-03	1.0008e-03
3.10	9.6760e-04	9.3544e-04	9.0426e-04	8.7403e-04	8.4474e-04	8.1635e-04	7.8885e-04	7.6219e-04	7.3638e-04	7.1136e-04
3.20	6.8714e-04	6.6367e-04	6.4095e-04	6.1895e-04	5.9765e-04	5.7703e-04	5.5706e-04	5.3774e-04	5.1904e-04	5.0094e-04
3.30	4.8342e-04	4.6648e-04	4.5009e-04	4.3423e-04	4.1889e-04	4.0406e-04	3.8971e-04	3.7584e-04	3.6243e-04	3.4946e-04
3.40	3.3693e-04	3.2481e-04	3.1311e-04	3.0179e-04	2.9086e-04	2.8029e-04	2.7009e-04	2.6023e-04	2.5071e-04	2.4151e-04
3.50	2.3263e-04	2.2405e-04	2.1577e-04	2.0778e-04	2.0006e-04	1.9262e-04	1.8543e-04	1.7849e-04	1.7180e-04	1.6534e-04
3.60	1.5911e-04	1.5310e-04	1.4730e-04	1.4171e-04	1.3632e-04	1.3112e-04	1.2611e-04	1.2128e-04	1.1662e-04	1.1213e-04
3.70	1.0780e-04	1.0363e-04	9.9611e-05	9.5740e-05	9.2010e-05	8.8417e-05	8.4957e-05	8.1624e-05	7.8414e-05	7.5324e-05
3.80	7.2348e-05	6.9483e-05	6.6726e-05	6.4072e-05	6.1517e-05	5.9059e-05	5.6694e-05	5.4418e-05	5.2228e-05	5.0122e-05
3.90	4.8096e-05	4.6148e-05	4.4274e-05	4.2473e-05	4.0741e-05	3.9076e-05	3.7475e-05	3.5936e-05	3.4458e-05	3.3037e-05
4.00	3.1671e-05	3.0359e-05	2.9099e-05	2.7888e-05	2.6726e-05	2.5609e-05	2.4536e-05	2.3507e-05	2.2518e-05	2.1569e-05
4.10	2.0658e-05	1.9783e-05	1.8944e-05	1.8138e-05	1.7365e-05	1.6624e-05	1.5912e-05	1.5230e-05	1.4575e-05	1.3948e-05
4.20	1.3346e-05	1.2769e-05	1.2215e-05	1.1685e-05	1.1176e-05	1.0689e-05	1.0221e-05	9.7736e-06	9.3447e-06	8.9337e-06
4.30	8.5399e-06	8.1627e-06	7.8015e-06	7.4555e-06	7.1241e-06	6.8069e-06	6.5031e-06	6.2123e-06	5.9340e-06	5.6675e-06
4.40	5.4125e-06	5.1685e-06	4.9350e-06	4.7117e-06	4.4979e-06	4.2935e-06	4.0980e-06	3.9110e-06	3.7322e-06	3.5612e-06
4.50	3.3977e-06	3.2414e-06	3.0920e-06	2.9492e-06	2.8127e-06	2.6823e-06	2.5577e-06	2.4386e-06	2.3249e-06	2.2162e-06
4.60	2.1125e-06	2.0133e-06	1.9187e-06	1.8283e-06	1.7420e-06	1.6597e-06	1.5810e-06	1.5060e-06	1.4344e-06	1.3660e-06
4.70	1.3008e-06	1.2386e-06	1.1792e-06	1.1226e-06	1.0686e-06	1.0171e-06	9.6796e-07	9.2113e-07	8.7648e-07	8.3391e-07
4.80	7.9333e-07	7.5465e-07	7.1779e-07	6.8267e-07	6.4920e-07	6.1731e-07	5.8693e-07	5.5799e-07	5.3043e-07	5.0418e-07
4.90	4.7918e-07	4.5538e-07	4.3272e-07	4.1115e-07	3.9061e-07	3.7107e-07	3.5247e-07	3.3476e-07	3.1792e-07	3.0190e-07
5.00	2.8665e-07	2.7215e-07	2.5836e-07	2.4524e-07	2.3277e-07	2.2091e-07	2.0963e-07	1.9891e-07	1.8872e-07	1.7903e-07
5.10	1.6983e-07	1.6108e-07	1.5277e-07	1.4487e-07	1.3737e-07	1.3024e-07	1.2347e-07	1.1705e-07	1.1094e-07	1.0515e-07
5.20	9.9644e-08	9.4420e-08	8.9462e-08	8.4755e-08	8.0288e-08	7.6050e-08	7.2028e-08	6.8212e-08	6.4592e-08	6.1158e-08
5.30	5.7901e-08	5.4813e-08	5.1884e-08	4.9106e-08	4.6473e-08	4.3977e-08	4.1611e-08	3.9368e-08	3.7243e-08	3.5229e-08
5.40	3.3320e-08	3.1512e-08	2.9800e-08	2.8177e-08	2.6640e-08	2.5185e-08	2.3807e-08	2.2502e-08	2.1266e-08	2.0097e-08
5.50	1.8990e-08	1.7942e-08	1.6950e-08	1.6012e-08	1.5124e-08	1.4283e-08	1.3489e-08	1.2737e-08	1.2026e-08	1.1353e-08
5.60	1.0718e-08	1.0116e-08	9.5479e-09	9.0105e-09	8.5025e-09	8.0224e-09	7.5686e-09	7.1399e-09	6.7347e-09	6.3520e-09
5.70	5.9904e-09	5.6488e-09	5.3262e-09	5.0215e-09	4.7338e-09	4.4622e-09	4.2057e-09	3.9636e-09	3.7350e-09	3.5193e-09
5.80	3.3157e-09	3.1236e-09	2.9424e-09	2.7714e-09	2.6100e-09	2.4579e-09	2.3143e-09	2.1790e-09	2.0513e-09	1.9310e-09
5.90	1.8175e-09	1.7105e-09	1.6097e-09	1.5147e-09	1.4251e-09	1.3407e-09	1.2612e-09	1.1863e-09	1.1157e-09	1.0492e-09

Your Name:

1. (24 points) Let $f_X(x)$ be the density for a random variable X, where

$$f_X(x) = \begin{cases} a\sqrt{x}, & 0 \le x < 1 \\ 0, & \text{otherwise,} \end{cases}$$

and a is a constant.

   (a) Find the numeric value of a for $f_X(x)$ to be a valid density function.

(b) Find the expected value (mean) of X.

(c) Find the variance of X.

(d) Find $E[2\sqrt{x}+1]$

2. (20 points)

Part I. A professor makes an average of 2 errors in the problem descriptions of his final exams.

(a) What is the probability that a given final exam has no errors?

(b) What is the probability that the exam will have 3 or more errors?

Part II. Over many years, it is observed that a professor makes an average of 100 mistakes during lectures per semester.

(c) Give an **approximation** for the probability that the professor makes more than 125 mistakes during a semester.

Your Name:

Open Book, Open Computer Part

3. (16 Points) Let X_i be Rice random variables with shape parameter $b = 0.5$. You can create a scipy.stats object with this shape parameter using

```
import scipy.stats as stats
myrice=stats.rice(0.5)
```

Let Y be a sum of X_i's,

$$Y = \sum_{i=0}^{N-1} X_i$$

How large should N be for Y to be approximately Gaussian? Justify your answer using techniques demonstrated in class.

Upload your responses as a Jupyter notebook to the E-Learning Assignment Final Problem 3.

4. (32 points) A phone manufacturer is considering whether it should switch from Gorilla Glass 4 (GG4) to Gorilla (GG5). They distribute phones with both types of glass to employees. The phones have a drop sensor that blocks usage of the phone and alerts an employee to return the phone if it is dropped. The dropped phones are disassembled, and the damage is classified as: no damage, only glass damaged, only internals damaged, both glass and internals damaged. The results for the first 100 dropped phones are as follows:

	No damage	Only glass	Only internals	Both damaged
GG4	18	13	12	15
GG5	24	6	6	6

Conduct a chi-squared contingency test to determine if the manufacturer should feel confident that GG5 offers different performance than GG4.

(a) Show by hand how to analytically calculate the expected value for the "GG4/No damage" entry in the table.

Submit your answers to the following problems in a Jupyter notebook that is uploaded to the E-learning assignment Final Problem 4. You may use whatever Python libraries you like in answering these questions (i.e., you do not have to reproduce all the work in a step-by-step fashion.)

(b) Give the expected values for the contingency table.

(c) Give the errors table.

(d) Give the value of the chi-squared statistic (without the continuity correction).

(e) Specify the number of degrees of freedom for this contingency table.

(f) Determine the p-value for this contingency table.

(g) Is the data sufficient to conclude that GG5 offers different performance than GG4 ($p < 0.05$)?

5. (8 points) What grade have you **earned** in this class? Briefly justify your answer.

HONOR STATEMENT

I understand that I am bound to uphold the honor code of the University of Florida. I have neither given nor received assistance on this examination. In addition, I did not use any outside materials on the this exam other than the two formula sheets that were allowed.

Sign Your Name: _____

Print Your Name: _____

Write the Date: _____

Turn in your formula sheets with your exam!!!