EEL 4930 Stats – Lecture 26

EXPECTED VALUE

The *expected value* or *mean* of a random variable X is[1]

$$\mu_X = E[X] \quad =$$

if X is a discrete random variable, and is

$$\mu_X = E[X] \quad =$$

if X is a continuous random variable.

- **Important Property:** Expected value is a linear operator. If X and Y are random variables, and a and b are abitrary constants, then

$$E[aX + bY] = aE[X] + bE[Y]$$

Note that this does not require that X and Y be independent.

EX *Example: Expected Value of Binomial RV*

Let $B_i, \quad i = 1, 2, \ldots, N$ be a sequence of independent Bernoulli random variables with common parameter p. Then

$$X = \sum_{i=1}^{N} B_i$$

is a Binomial (N, p) random variable.

Using the linear property,

$$\begin{aligned} E[X] &= E\left[\sum_{i=1}^{N} B_i\right] \\ &= \sum_{i=1}^{N} E\left[B_i\right] \\ &= \sum_{i=1}^{N} p \\ &= Np \end{aligned}$$

We can derive the same result from the PMF, but it is *way* more complicated – I will post the math to the web site.

EX A continuous, nonuniform density

<u>EXPECTED VALUE OF A FUNCTION OF A RV</u>

If $Y = g(X)$, it is not necessary to compute the pdf or cdf of Y to find its expected value:

$$E[Y] =$$

- This is sometimes known as the

2

- Expected value of a constant, $E[c] = c$

- Note that $E[f(X)] \neq f(E[X])$

- **In-class assignment**

  Recall that if x_i are samples drawn from a random variable X, then

  $$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} x_i = E[X].$$

  Create a Uniform random variable object using scipy.stats. Draw 10,000 sample values from it, and use the sample values to estimate $(E[U])^2$ and $E[U^2]$.

  Analytical Solution:

- Find the value c that minimizes the expected mean-square error to a random variable X, $E[(X-c)^2]$

MOMENTS

- Moments of a random variable are expected values of the random variable raised to some power
- For a central moment, the mean is subtracted from the random variable before it is raised to a power
- Because different powers spread the values of the random variable in different ways, moments can provide additional information about a random variable than the mean
  - Variance is the second central moment and provides a measure of how much the probability density or mass of random variable is spread away from the mean
- Some common moments (expected values):
  - nth moment of X:

$$E[X^n] \quad =$$

  - nth central moment of X:

$$E[(X-\mu_X)^n] \quad =$$

    where $\mu_X = E[X]$.

  - Variance of X is 2nd central moment:

$$\begin{aligned}\mathrm{Var}[X] &= E[(X-\mu_X)^2] \\ &= \\ &= \\ &= \end{aligned}$$

    (this latter formula is usually a more convenient way to find the variance.)

- The variance of a Gaussian random variable is the parameter σ^2 (you can get it through integration by parts or some clever manipulation)

<u>PROPERTIES OF VARIANCE:</u>

1. $\mathrm{Var}[c] =$

2. $\mathrm{Var}[X + c] =$

3. $\mathrm{Var}[cX] =$