Statistical Independence

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- In this case, we say that A is *statistically independent (s.i.)* of B, since the probabilities of A are not affected by knowledge of A having occurred

- By the chain rule, $P(A \cap B) = P(A|B)P(B)$. So if $P(A) > 0$, $P(B > 0)$, and $P(A|B) = P(A)$, then

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&= \frac{P(A|B)P(B)}{P(A)} \\
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Events A and B are *statistically independent (s.i.)* if and only if (iff)

$$P(A \cap B) = P(A)P(B).$$

- Events that arise from completely separate random phenomena are statistically independent.

EX **Take 2:** XOR of Two Independent Binary Values

Flip a fair coin with sides labeled '0' and '1' two times. Let E_i denote a '1' on the top face on flip i. Let F denote the event that the XOR of the values observed on the top faces on the two flips is '1'.

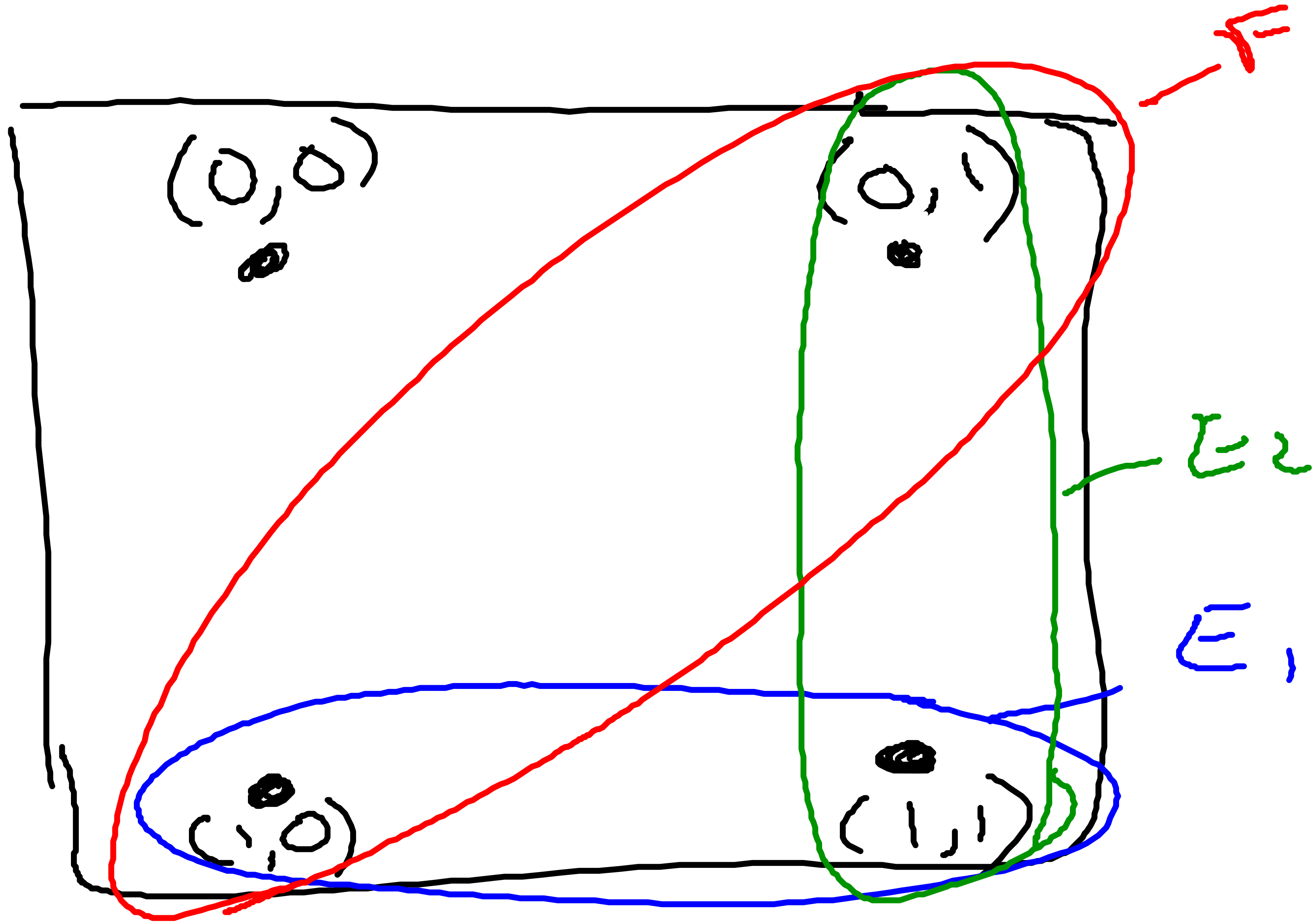
Flip 1	Flip 2	XOR	Prob
0	0	0	1/4
1	0	1	1/4
0	1	1	1/4
1	1	0	1/4

$P(E_1)$

$1/2$

E_2

F



$$P(E_1) = P(E_2) = P(F) = 1/2$$

$P(E_1|E_2) = 1/2$ $\quad$ $P(E_2|E_1) = 1/2$ $\quad$ $P(F|E_1) = \frac{1}{2}$

$P(F|E_1 \cap E_2) = 0$, F and $E_1 \cap E_2$ are m.e.

- Events that arise from completely separate random phenomena are statistically independent.

EX **Take 3:** A fair six-sided die is rolled twice. What is the probability of observing a 1 or a 2 on the top face on either roll of the die?

$$P(E_1 \cup E_2)$$

$$= P(E_1) + P(E_2) - P(\underbrace{E_1 \cap E_2}_{\text{s.i.}})$$

$$= 1/3 + 1/3 - \underbrace{P(E_1)}_{(1/3)}\underbrace{P(E_2)}_{(1/3)}$$

$$= 5/9$$

~~Take~~

$$P(E_1 \cup E_2) = 1 - P\left(\overline{E_1 \cup E_2}\right)$$

$$= 1 - P(\underbrace{\overline{E_1} \cap \overline{E_2}}_{s.i.})$$

$$= 1 - P(\overline{E_1})P(\overline{E_2})$$

$$= 1 - (2/3)(2/3)$$

$$= 1 - [1 - P(E_1)][1 - P(E_2)]$$

$$= 1 - [1 - p]^2$$

$$p = P(E_1) = P(E_2)$$

EEL 4930 Lecture 9:

One-dimensional Statistics

Populations and Sampling

POPULATIONS AND SAMPLING

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When the population is too large to directly measure the parameters of interest, then we try to draw inferences from a subset of the population

A sample from a population is a subset of the population that can be used to draw inferences about the parameters of interest.

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- A sample is usually drawn randomly from the population
- We usually require that each member of the sample is chosen independently from other members
- Often, but not always, each member in the population is equally likely to be included in the sample

A statistic is a measurement of a quality or property on a sample that is used to assess a parameter of the whole population.

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- For example, consider the problem of determining whether a coin is fair or two-sided. The result of flipping a coin one time provides no useful information for determining that

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There are generally two cases that we will encounter

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2. Sometimes the experiment has already been carried out or is not under the control of the statistician. For instance, the statistician wants to assess something based on an existing survey or compare effects of a change in laws on a set of states. In this case, the ~~population~~ sample size is fixed

We will be using Python to compute statistics on samples and determine whether and how well these statistics represent the parameters of the populations being studied