EEL 4930 Midterm Examination

October 30, 2017

The time for this test is 1 hour. The test has two parts, each of which is worth 50 points.

The first part is a closed book test, but you are allowed one formula sheet. The formula sheet cannot contain any examples. You should **write your name on the formula sheet** and turn it in with your exam. You may use a calculator on this test. You must show your work to receive credit for a problem. Note that some problems are worth more points than other problems, and the problems are not necessarily sorted in order of difficulty or point value.

You need to use your computer to answer the questions on the second part of the test. Instructions on accessing the second part of the test are on the last page.

1. (10 Points) What grade do you believe that you have EARNED so far in this class? Why?

2. (20 Points) A group of students has 2 Apple laptops, 7 Windows laptops, 6 Apple desktops, and 9 Windows desktops.

   A computer is selected at random. Let A, W, L, and D denote the events that the computer is an Apple computer, a Windows computer, a laptop computer, or a desktop computer, respectively.

   (a) Find $P(A)$

$$|S| = 2+7+6+9 = 24$$

$$|A| = 2+6 = 8$$

$$P(A) = \frac{|A|}{|S|} = \frac{8}{24} = \frac{1}{3}$$

(b) Find $P(D)$

$$|D| = 6+9 = 15$$

$$P(D) = \frac{|D|}{|S|} = \frac{15}{24} = \frac{5}{8}$$

(c) Find $P(A \cup D)$

$$P(A \cup D) = P(A) + P(D) - P(A \cap D)$$

$$|A \cap D| = 6$$

$$P(A \cup D) = \frac{8 + 15 - 6}{24}$$

$$= \frac{17}{24}$$

(d) Are A and D statistically independent? Write YES or NO. Justify your answer.

$$P(A)\cdot P(D) \stackrel{?}{=} P(A\cap D)$$

$$\left(\frac{1}{3}\right)\left(\frac{5}{8}\right) \stackrel{?}{=} \frac{|A\cap D|}{|S|}$$

$$\frac{5}{24} \stackrel{?}{=} \frac{6}{24} \quad X$$

NO.

(e) Are A and D mutually exclusive? Write YES or NO. Justify your answer.

$$P(A\cap D) = \frac{6}{24} > 0 \qquad \text{NO.}$$

3. (20 points) The medical school has developed a new test for prostate cancer in men. Clinical trials show:

   - the test comes back positive with probability 0.99 in men with prostate cancer
   - the test comes back positive with probability 0.1 in men with a non-cancerous condition called BPH
   - the test comes back positive with probability 10^{-4} in people who have neither prostate cancer nor BPH

   The test will be given to men over 50 who have not been previously diagnosed with prostate cancer or BPH. Men in this group have prostate cancer with probability 10^{-3}, and they have BPH with probability 5×10^{-2}. (BPH and prostate cancer should be considered mutually exclusive conditions.)

   Evaluate this test by answering the following questions:

   (a) What is the probability that the test comes back positive?

Let C = man has cancer
B = man has BPH
H = man healthy (neither C nor H)
T = test positive

$$P(T) = P(T|C)P(C) + P(T|B)P(B) + P(T|H)P(H)$$

Law of Total Prob.

$$P(T) = (0.99)(10^{-3}) + (0.1)(5\times10^{-2}) + (10^{-4})(1 - 10^{-3} - 5\times10^{-2})$$

$$\approx 6.08 \times 10^{-3}$$

(b) What is the probability of someone having prostate cancer when the test comes back negative?

$$P(C|\bar{T}) = \frac{P(\bar{T}|C)P(C)}{P(\bar{T})}$$

$$\approx \frac{(1-0.99)(10^{-3})}{1-6.08\times10^{-3}}$$

$$\approx 1.0\times10^{-5}$$

(c) What is the probability of someone having prostate cancer when the test comes back positive?

$$P(C|T) = \frac{P(T|C)P(C)}{P(T)}$$

$$\approx \frac{(0.99)(10^{-3})}{6.08 \times 10^{-3}}$$

$$\approx 0.163$$