EEL 4930 Stats – Lecture 25

EXPECTED VALUE

- Consider again set of observations $x_1, x_2, \ldots, x_n$
- Then the *average* of the data is

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

- We would like to define a similar notion for a random variable X, but take the average over the *ensemble* of potential values of X
- This value is the *expected value*, *ensemble mean*, or simply *mean* of X
- We can use *relative frequency* to connect the two:

The *expected value* or *mean* of a random variable X is[1]

$$\mu_X = E[X] \quad =$$

if X is a discrete random variable, and is

$$\mu_X = E[X] \quad =$$

if X is a continuous random variable.

WHY DO WE CARE ABOUT THE MEAN?

- In a repeated experiment, the limit of the average value is the mean
  - In fact, we will show that we can determine a limit on the number of times the experiment must be repeated to ensure that the average is within a range around the mean with a specified probability (Chebyshev's inequality, covered later)
- If we wish to use a constant value to estimate a random variable, then the mean is the value that minimizes the mean-square error
- Note that $E[X]$ may be infinite

EX Rolling a fair 6-sided die

EX Bernoulli RV

Notes

[1] In some special cases, we would not define the expected value because it is of the form $-\infty+\infty$. We won't cover those in this class.