Notes on this assignment

- **This is an analytical assignment. You are expected to calculate all answers using probability theory.**
- Do not just provide a numerical answer; show how you got that answer. For this particular assignment, most of the problems have very few steps, but you should still show the formula that resulted in the numerical answer.
- Submit your answers to this assignment as a PDF. It is recommended that you handwrite your answers and then scan them. I recommend the Scannable by Evernot app for scanning to PDF; it is free.

Problems

I. Simulations of fair experiments

1. Consider rolling a fair die three times. Let E_i be the event that the total value rolled is equal to i. Use combinatorics to find the relative frequencies of the events $E_3, E_4,$ and E_5

2. No response required. This is included to maintain numbering consistency with Homework 1.

Hypothesis Testing

Use your knowledge of combinatorics and statistical independence to answer these questions

3. If a 6-sided die is rolled 12 times and all the values are <=4, should we feel confident that the dice is not fair (i.e., reject the null hypothesis with p=0.01)?

4. If a pair of dice is rolled 4 times (i.e., each die is rolled 6 times) and the sum of the dice is always less than or equal to 6, should we feel confident that the dice are not fair?

5. Repeat problem 4 if the pairs of dice are rolled 6 times instead of 4 times.

Simulation of Systems with Hidden State and Conditional Probabilities

Total Probability: Simplest Form

Let A and B be any two events. (I.e., in general, you can apply this form to any two events. It is often useful when the probability of A has some natural dependence on B.)

Then

$$\begin{aligned} A &= A \cap S \\ &= A \cap (B \cup \overline{B}) \\ &= (A \cap B) \cup (A \cap \overline{B}) \end{aligned}$$

Note that $(A \cap B) \cap (A \cap \overline{B}) = A \cap (B \cap \overline{B}) = A \cap \emptyset = \emptyset$, so $A \cap B$ and $A \cap \overline{B}$ are mutually exclusive. (If you don't see this immediately, draw a Venn diagram!)

Thus,

$$P(A) = P(A \cap B) + P(A \cap \overline{B})$$

Now, we can apply the chain rule to each of these terms:

$$P(A) = P(A|B)P(B) + P(A|\overline{B})P(\overline{B})$$

6. A magician has in her pocket **two** fair coins and a two-headed coin. She chooses one at random and flips it. What is the probability that the result is heads?

Hint: You can use counting methods to solve this, but it is much more interesting and less tedious to use Total Probability. When using Total Probability in this type of scenario, we usually need to condition on the hidden state. (I.e., it plays the role of B in the Total Probability form given above.)

7. A magician has in her pocket two fair coins and a two-headed coin. The magician withdraws a coin and flips it. If it comes up Heads, what is the probability that a second flip of that same coin will also be Heads?

Hint 1: Use the definition of conditional probability along with Total Probability.

Hint 2: If we know which type of coin is flipped, then the outcomes become statistically independent. Show that they are not independent if we do not know which type of coin is flipped.

8. A magician has in her pocket two fair coins and a two-headed coin. The magician withdraws a coin and flips it twice. If it comes up Heads both times, what is the probability that a third flip of that same coin will also be Heads?

Hint: Again, this is easiest if you apply knowledge of Total Probability and statistical independence.