EEL 4930 Stats – Lecture 24

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Central Limit Theorem

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CENTRAL LIMIT THEOREM

- Consider a sum of (independent) random variables:

- If $X_i, \quad i = 1, 2, \ldots$ is a sequence of independent random variables with the same distribution and finite variance, then the distribution function of

$$\lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} X_i$$

converges to a common distribution function

- This is the **Central Limit Theorem**

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- The limiting distribution is that of a **Gaussian random variable**
- The density of a Gaussian RV X is

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\},$$

which has two parameters: (mean) μ and (variance) $\sigma^2 \geq 0$

DISTRIBUTION FUNCTION

Distribution Function

- The CDF of a Gaussian RV is given by

$$\begin{aligned}
F_X(x) &= P(X \leq x) \\
&= \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(t-\mu)^2}{2\sigma^2}\right\} dt,
\end{aligned}$$

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which cannot be evaluated in closed form

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- This is called the Normal distribution, and its CDF is

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{t^2}{2}\right\} dt$$

- Mathematicians use the “error function” (erf) to define the CDF of the normal distribution:

$$\Phi(x) = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{x}{\sqrt{2}}\right)\right],$$

where

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2} dt$$

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- I will be supplying you with a Q-function table and a list of approximations to $Q(x)$

Q-Function

Definition

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2}\ dt$$

Good Approximation (good for programming in calculator)

$$Q(x) \approx \left[\frac{1}{(1-a)x + a\sqrt{x^2+b}}\right] \frac{1}{\sqrt{2\pi}} e^{-x^2/2},$$

where $a = 1/\pi$, $b = 2\pi$

Simple Upper Bound

$$Q(x) < \frac{1}{2} e^{-x^2/2}$$

Relation to Error Functions

$$Q(x) = \frac{1}{2}\operatorname{erfc}\left(\frac{x}{\sqrt{2}}\right), \qquad \operatorname{erfc}(x) = 2Q(x\sqrt{2})$$

Property

$$Q(-x) = 1 - Q(x)$$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	5.0000e-01	4.9601e-01	4.9202e-01	4.8803e-01	4.8405e-01	4.8006e-01	4.7608e-01	4.7210e-01	4.6812e-01	4.6414e-01
0.10	4.6017e-01	4.5620e-01	4.5224e-01	4.4828e-01	4.4433e-01	4.4038e-01	4.3644e-01	4.3251e-01	4.2858e-01	4.2465e-01
0.20	4.2074e-01	4.1683e-01	4.1294e-01	4.0905e-01	4.0517e-01	4.0129e-01	3.9743e-01	3.9358e-01	3.8974e-01	3.8591e-01
0.30	3.8209e-01	3.7828e-01	3.7448e-01	3.7070e-01	3.6693e-01	3.6317e-01	3.5942e-01	3.5569e-01	3.5197e-01	3.4827e-01
0.40	3.4458e-01	3.4090e-01	3.3724e-01	3.3360e-01	3.2997e-01	3.2636e-01	3.2276e-01	3.1918e-01	3.1561e-01	3.1207e-01
0.50	3.0854e-01	3.0503e-01	3.0153e-01	2.9806e-01	2.9460e-01	2.9116e-01	2.8774e-01	2.8434e-01	2.8096e-01	2.7760e-01
0.60	2.7425e-01	2.7093e-01	2.6763e-01	2.6435e-01	2.6109e-01	2.5785e-01	2.5463e-01	2.5143e-01	2.4825e-01	2.4510e-01
0.70	2.4196e-01	2.3885e-01	2.3576e-01	2.3270e-01	2.2965e-01	2.2663e-01	2.2363e-01	2.2065e-01	2.1770e-01	2.1476e-01
0.80	2.1186e-01	2.0897e-01	2.0611e-01	2.0327e-01	2.0045e-01	1.9766e-01	1.9489e-01	1.9215e-01	1.8943e-01	1.8673e-01
0.90	1.8406e-01	1.8141e-01	1.7879e-01	1.7619e-01	1.7361e-01	1.7106e-01	1.6853e-01	1.6602e-01	1.6354e-01	1.6109e-01
1.00	1.5866e-01	1.5625e-01	1.5386e-01	1.5151e-01	1.4917e-01	1.4686e-01	1.4457e-01	1.4231e-01	1.4007e-01	1.3786e-01
1.10	1.3567e-01	1.3350e-01	1.3136e-01	1.2924e-01	1.2714e-01	1.2507e-01	1.2302e-01	1.2100e-01	1.1900e-01	1.1702e-01
1.20	1.1507e-01	1.1314e-01	1.1123e-01	1.0935e-01	1.0749e-01	1.0565e-01	1.0383e-01	1.0204e-01	1.0027e-01	9.8525e-02
1.30	9.6800e-02	9.5098e-02	9.3418e-02	9.1759e-02	9.0123e-02	8.8508e-02	8.6915e-02	8.5343e-02	8.3793e-02	8.2264e-02
1.40	8.0757e-02	7.9270e-02	7.7804e-02	7.6359e-02	7.4934e-02	7.3529e-02	7.2145e-02	7.0781e-02	6.9437e-02	6.8112e-02
1.50	6.6807e-02	6.5522e-02	6.4255e-02	6.3008e-02	6.1780e-02	6.0571e-02	5.9380e-02	5.8208e-02	5.7053e-02	5.5917e-02
1.60	5.4799e-02	5.3699e-02	5.2616e-02	5.1551e-02	5.0503e-02	4.9471e-02	4.8457e-02	4.7460e-02	4.6479e-02	4.5514e-02
1.70	4.4565e-02	4.3633e-02	4.2716e-02	4.1815e-02	4.0930e-02	4.0059e-02	3.9204e-02	3.8364e-02	3.7538e-02	3.6727e-02
1.80	3.5930e-02	3.5148e-02	3.4380e-02	3.3625e-02	3.2884e-02	3.2157e-02	3.1443e-02	3.0742e-02	3.0054e-02	2.9379e-02
1.90	2.8717e-02	2.8067e-02	2.7429e-02	2.6803e-02	2.6190e-02	2.5588e-02	2.4998e-02	2.4419e-02	2.3852e-02	2.3295e-02

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.00	2.2750e-02	2.2216e-02	2.1692e-02	2.1178e-02	2.0675e-02	2.0182e-02	1.9699e-02	1.9226e-02	1.8763e-02	1.8309e-02
2.10	1.7864e-02	1.7429e-02	1.7003e-02	1.6586e-02	1.6177e-02	1.5778e-02	1.5386e-02	1.5003e-02	1.4629e-02	1.4262e-02
2.20	1.3903e-02	1.3553e-02	1.3209e-02	1.2874e-02	1.2545e-02	1.2224e-02	1.1911e-02	1.1604e-02	1.1304e-02	1.1011e-02
2.30	1.0724e-02	1.0444e-02	1.0170e-02	9.9031e-03	9.6419e-03	9.3867e-03	9.1375e-03	8.8940e-03	8.6563e-03	8.4242e-03
2.40	8.1975e-03	7.9763e-03	7.7603e-03	7.5494e-03	7.3436e-03	7.1428e-03	6.9469e-03	6.7557e-03	6.5691e-03	6.3872e-03
2.50	6.2097e-03	6.0366e-03	5.8677e-03	5.7031e-03	5.5426e-03	5.3861e-03	5.2336e-03	5.0849e-03	4.9400e-03	4.7988e-03
2.60	4.6612e-03	4.5271e-03	4.3965e-03	4.2692e-03	4.1453e-03	4.0246e-03	3.9070e-03	3.7926e-03	3.6811e-03	3.5726e-03
2.70	3.4670e-03	3.3642e-03	3.2641e-03	3.1667e-03	3.0720e-03	2.9798e-03	2.8901e-03	2.8028e-03	2.7179e-03	2.6354e-03
2.80	2.5551e-03	2.4771e-03	2.4012e-03	2.3274e-03	2.2557e-03	2.1860e-03	2.1182e-03	2.0524e-03	1.9884e-03	1.9262e-03
2.90	1.8658e-03	1.8071e-03	1.7502e-03	1.6948e-03	1.6411e-03	1.5889e-03	1.5382e-03	1.4890e-03	1.4412e-03	1.3949e-03
3.00	1.3499e-03	1.3062e-03	1.2639e-03	1.2228e-03	1.1829e-03	1.1442e-03	1.1067e-03	1.0703e-03	1.0350e-03	1.0008e-03
3.10	9.6760e-04	9.3544e-04	9.0426e-04	8.7403e-04	8.4474e-04	8.1635e-04	7.8885e-04	7.6219e-04	7.3638e-04	7.1136e-04
3.20	6.8714e-04	6.6367e-04	6.4095e-04	6.1895e-04	5.9765e-04	5.7703e-04	5.5706e-04	5.3774e-04	5.1904e-04	5.0094e-04
3.30	4.8342e-04	4.6648e-04	4.5009e-04	4.3423e-04	4.1889e-04	4.0406e-04	3.8971e-04	3.7584e-04	3.6243e-04	3.4946e-04
3.40	3.3693e-04	3.2481e-04	3.1311e-04	3.0179e-04	2.9086e-04	2.8029e-04	2.7009e-04	2.6023e-04	2.5071e-04	2.4151e-04
3.50	2.3263e-04	2.2405e-04	2.1577e-04	2.0778e-04	2.0006e-04	1.9262e-04	1.8543e-04	1.7849e-04	1.7180e-04	1.6534e-04
3.60	1.5911e-04	1.5310e-04	1.4730e-04	1.4171e-04	1.3632e-04	1.3112e-04	1.2611e-04	1.2128e-04	1.1662e-04	1.1213e-04
3.70	1.0780e-04	1.0363e-04	9.9611e-05	9.5740e-05	9.2010e-05	8.8417e-05	8.4957e-05	8.1624e-05	7.8414e-05	7.5324e-05
3.80	7.2348e-05	6.9483e-05	6.6726e-05	6.4072e-05	6.1517e-05	5.9059e-05	5.6694e-05	5.4418e-05	5.2228e-05	5.0122e-05
3.90	4.8096e-05	4.6148e-05	4.4274e-05	4.2473e-05	4.0741e-05	3.9076e-05	3.7475e-05	3.5936e-05	3.4458e-05	3.3037e-05
4.00	3.1671e-05	3.0359e-05	2.9099e-05	2.7888e-05	2.6726e-05	2.5609e-05	2.4536e-05	2.3507e-05	2.2518e-05	2.1569e-05
4.10	2.0658e-05	1.9783e-05	1.8944e-05	1.8138e-05	1.7365e-05	1.6624e-05	1.5912e-05	1.5230e-05	1.4575e-05	1.3948e-05
4.20	1.3346e-05	1.2769e-05	1.2215e-05	1.1685e-05	1.1176e-05	1.0689e-05	1.0221e-05	9.7736e-06	9.3447e-06	8.9337e-06
4.30	8.5399e-06	8.1627e-06	7.8015e-06	7.4555e-06	7.1241e-06	6.8069e-06	6.5031e-06	6.2123e-06	5.9340e-06	5.6675e-06
4.40	5.4125e-06	5.1685e-06	4.9350e-06	4.7117e-06	4.4979e-06	4.2935e-06	4.0980e-06	3.9110e-06	3.7322e-06	3.5612e-06
4.50	3.3977e-06	3.2414e-06	3.0920e-06	2.9492e-06	2.8127e-06	2.6823e-06	2.5577e-06	2.4386e-06	2.3249e-06	2.2162e-06
4.60	2.1125e-06	2.0133e-06	1.9187e-06	1.8283e-06	1.7420e-06	1.6597e-06	1.5810e-06	1.5060e-06	1.4344e-06	1.3660e-06
4.70	1.3008e-06	1.2386e-06	1.1792e-06	1.1226e-06	1.0686e-06	1.0171e-06	9.6796e-07	9.2113e-07	8.7648e-07	8.3391e-07
4.80	7.9333e-07	7.5465e-07	7.1779e-07	6.8267e-07	6.4920e-07	6.1731e-07	5.8693e-07	5.5799e-07	5.3043e-07	5.0418e-07
4.90	4.7918e-07	4.5538e-07	4.3272e-07	4.1115e-07	3.9061e-07	3.7107e-07	3.5247e-07	3.3476e-07	3.1792e-07	3.0190e-07
5.00	2.8665e-07	2.7215e-07	2.5836e-07	2.4524e-07	2.3277e-07	2.2091e-07	2.0963e-07	1.9891e-07	1.8872e-07	1.7903e-07
5.10	1.6983e-07	1.6108e-07	1.5277e-07	1.4487e-07	1.3737e-07	1.3024e-07	1.2347e-07	1.1705e-07	1.1094e-07	1.0515e-07
5.20	9.9644e-08	9.4420e-08	8.9462e-08	8.4755e-08	8.0288e-08	7.6050e-08	7.2028e-08	6.8212e-08	6.4592e-08	6.1158e-08
5.30	5.7901e-08	5.4813e-08	5.1884e-08	4.9106e-08	4.6473e-08	4.3977e-08	4.1611e-08	3.9368e-08	3.7243e-08	3.5229e-08
5.40	3.3320e-08	3.1512e-08	2.9800e-08	2.8177e-08	2.6640e-08	2.5185e-08	2.3807e-08	2.2502e-08	2.1266e-08	2.0097e-08
5.50	1.8990e-08	1.7942e-08	1.6950e-08	1.6012e-08	1.5124e-08	1.4283e-08	1.3489e-08	1.2737e-08	1.2026e-08	1.1353e-08
5.60	1.0718e-08	1.0116e-08	9.5479e-09	9.0105e-09	8.5025e-09	8.0224e-09	7.5686e-09	7.1399e-09	6.7347e-09	6.3520e-09
5.70	5.9904e-09	5.6488e-09	5.3262e-09	5.0215e-09	4.7338e-09	4.4622e-09	4.2057e-09	3.9636e-09	3.7350e-09	3.5193e-09
5.80	3.3157e-09	3.1236e-09	2.9424e-09	2.7714e-09	2.6100e-09	2.4579e-09	2.3143e-09	2.1790e-09	2.0513e-09	1.9310e-09
5.90	1.8175e-09	1.7105e-09	1.6097e-09	1.5147e-09	1.4251e-09	1.3407e-09	1.2612e-09	1.1863e-09	1.1157e-09	1.0492e-09

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(This is a fairly recent result that is in very few textbooks. This form has a finite range of integration that is often easier to work with.)

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$$
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&= 1 - Q\left(\frac{x-\mu}{\sigma}\right)
\end{aligned}
$$

Note that the denominator above is σ, not σ^2. Many students use the wrong value when solving problems!

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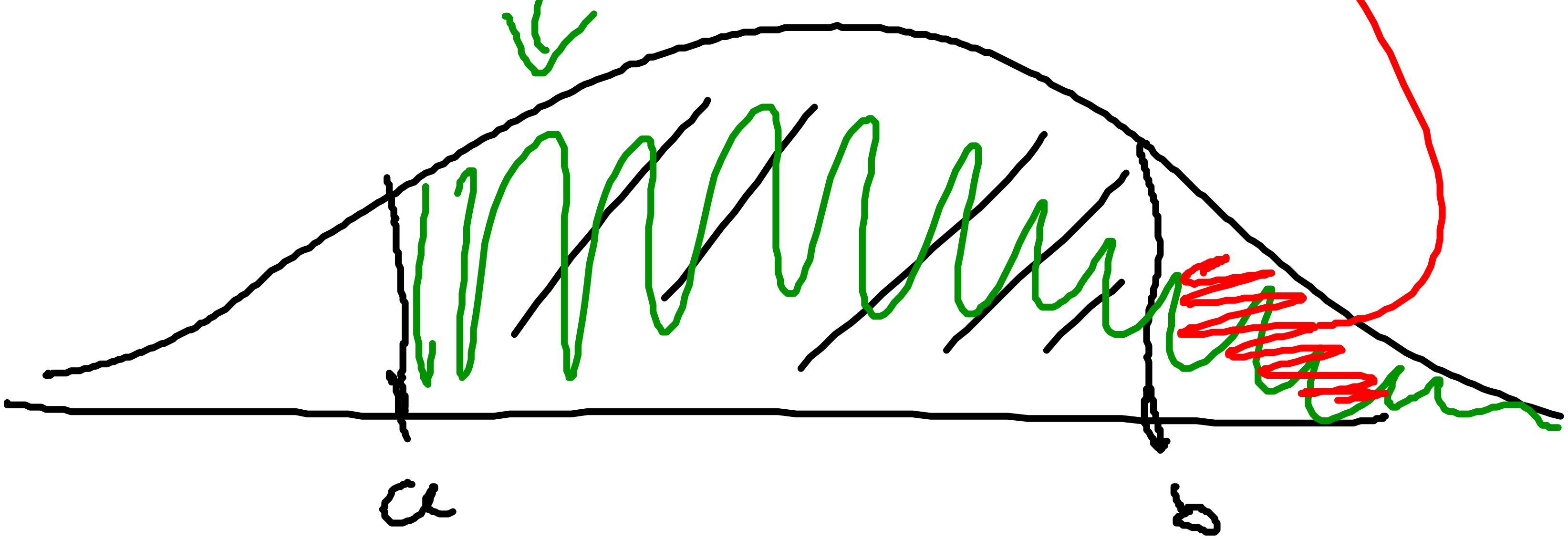

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- **Engineering examples:** Noise sample in an electrical device, complex Gaussian models combined amplitude and phase of wireless signal received in multipath environment, sum of accumulated errors

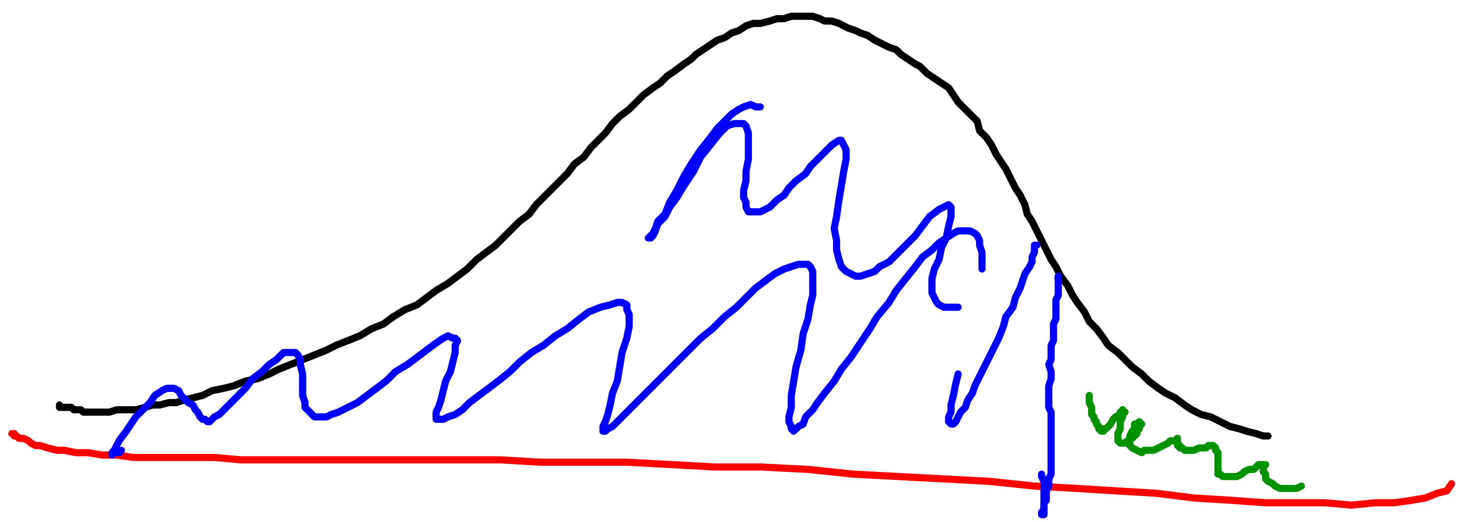
More on Computing Gaussian Tail Probs

- Any Gaussian probabilities can be decomposed in terms of Gaussian tail probabilities

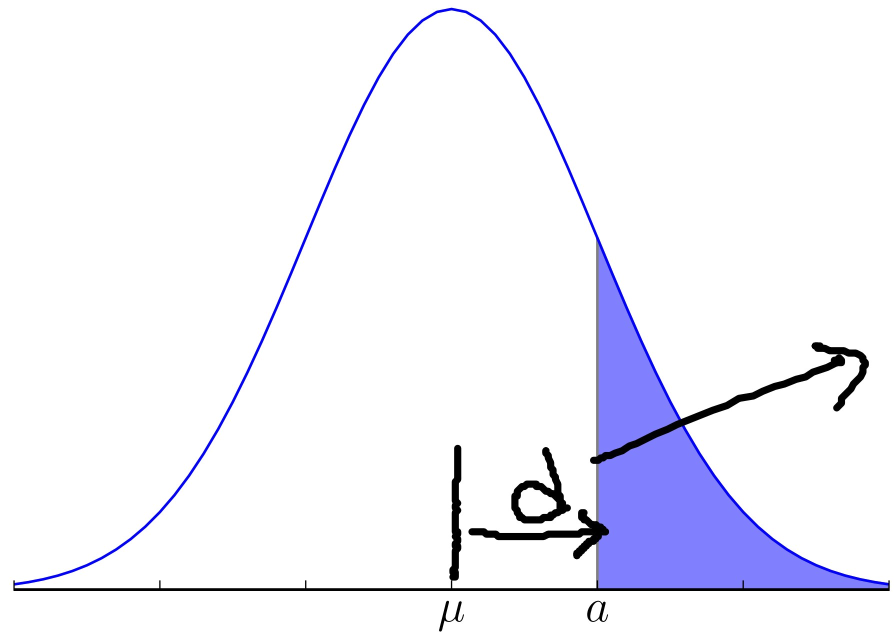
MORE ON COMPUTING GAUSSIAN TAIL PROBS

- Any Gaussian probabilities can be decomposed in terms of Gaussian tail probabilities

- There are 2 cases of the tail probabilities



- **Case 1:** $P(X \geq a)$, where $a > \mu$

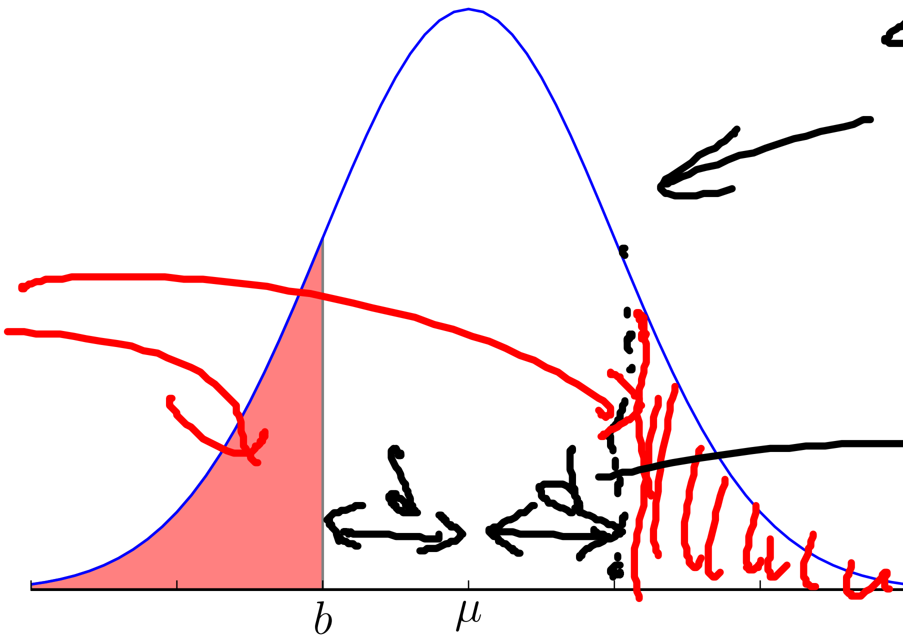


$a - \mu$

$$P(X \geq a) = Q\left(\frac{a-\mu}{\sigma}\right) = Q\left(\frac{d}{\sigma}\right)$$

- **Case 2:** $P(X \leq b)$, where $b < \mu$

Equal by symmetry



symmetric around μ

$d = \mu - b$

$\mu - b$

$2\mu - b$

$$P(X \geq 2\mu - b) = Q\left(\frac{2\mu - b - \mu}{\sigma}\right) = Q\left(\frac{\mu - b}{\sigma}\right) = Q\left(\frac{d}{\sigma}\right)$$

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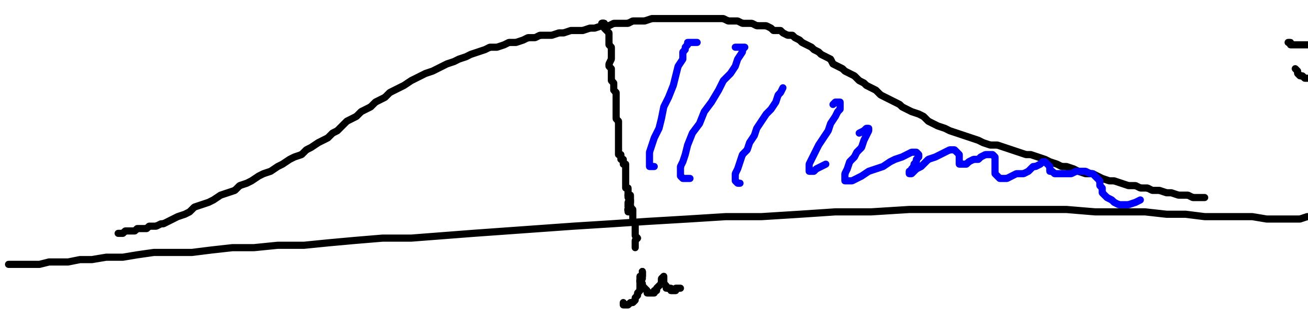
Let X represent a randomly chosen student's grade.

(a) What is the probability that the student's grade is above μ?

$$P(X > \mu) = Q\left(\frac{\mu - \mu}{\sigma}\right) = Q\left(\frac{0}{\sigma}\right) = 1/2$$



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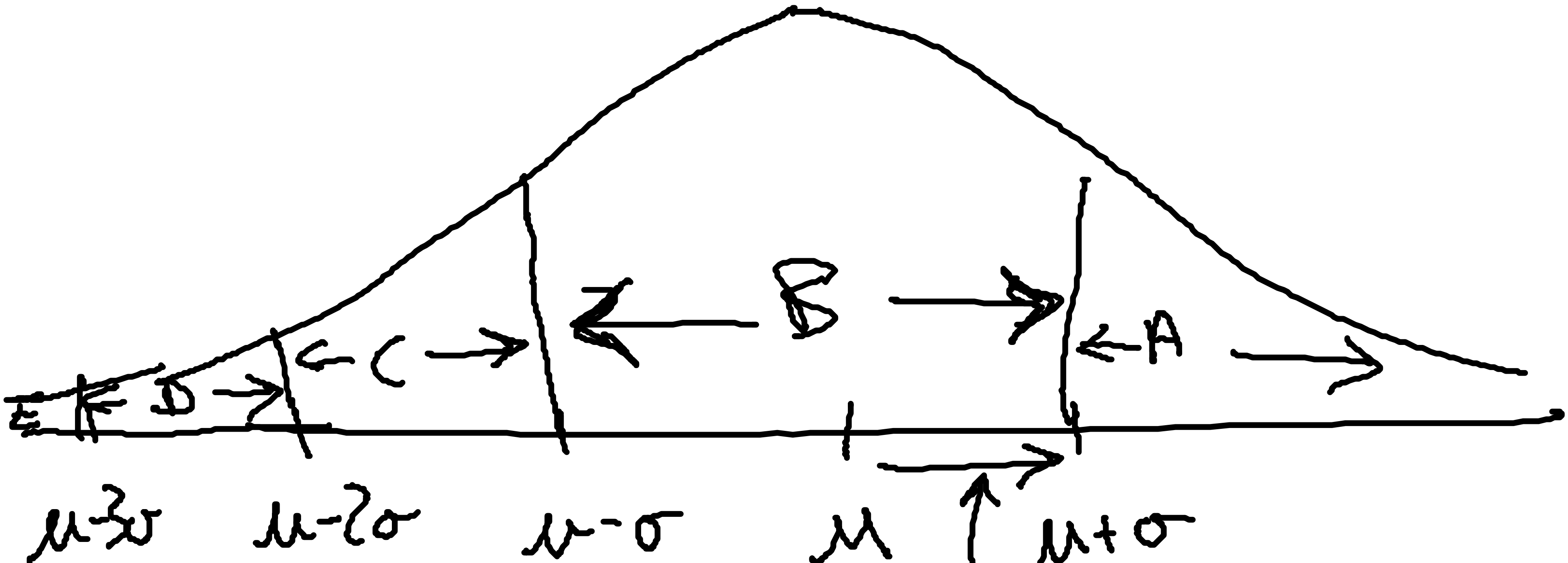
- If the grades are more than 3σ below the mean, assign E

Determing the probability that a randomly chosen student gets each grade



$d_A = \sigma$

$$P(A) = P(X \geq \mu + \sigma) = Q\left(\frac{d_A}{\sigma}\right)$$

$$= Q\left(\frac{\sigma}{\sigma}\right) = Q(1)$$

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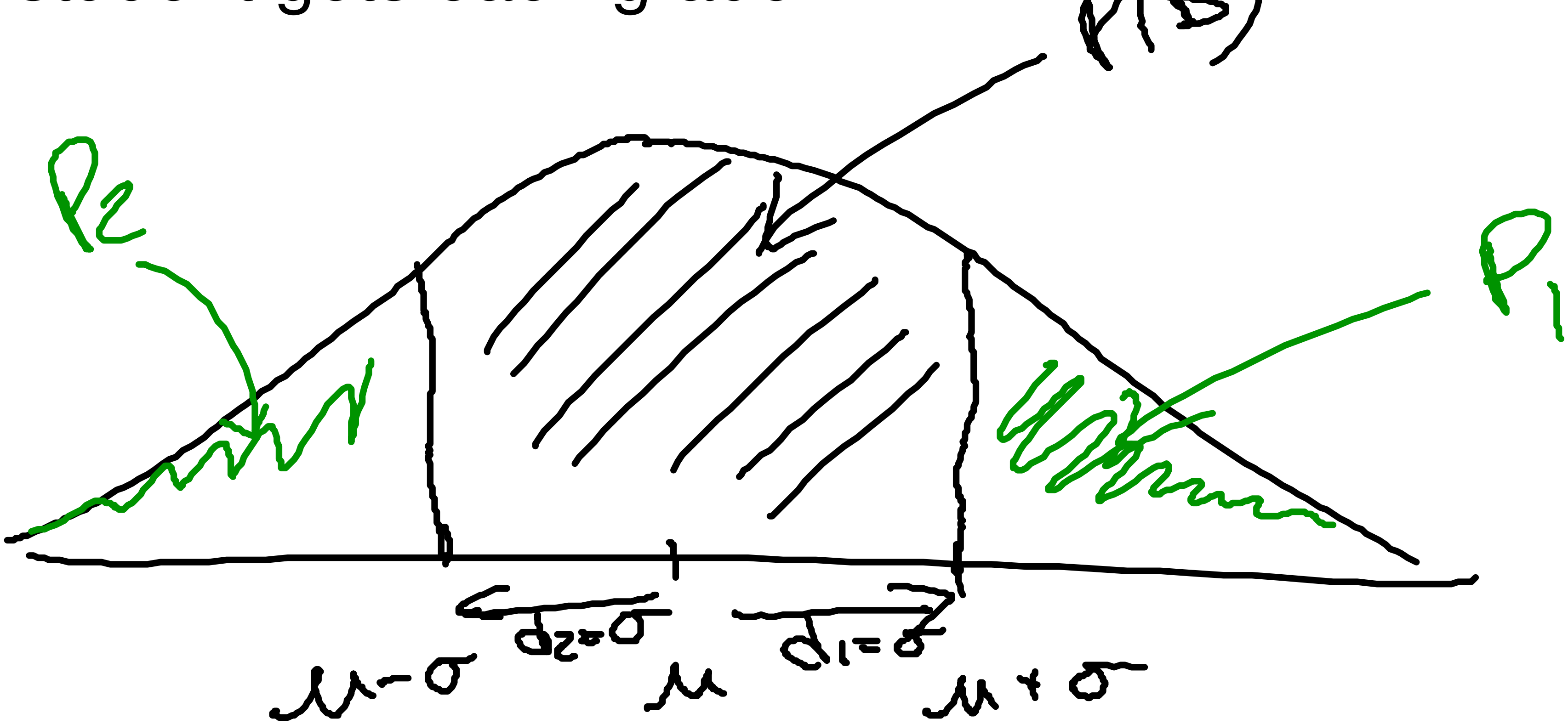
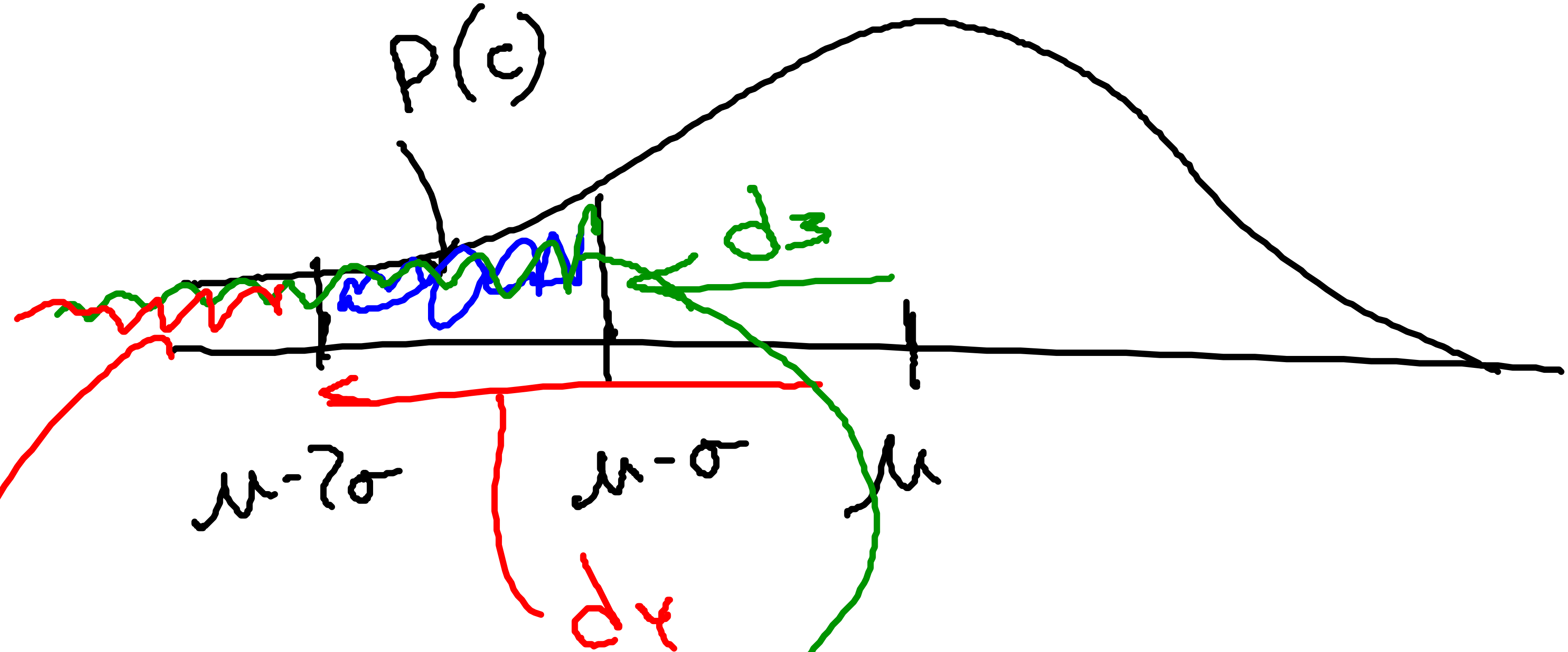


$$P(B) = 1 - P_1 - P_2$$
$$= 1 - Q\left(\frac{d_1}{\sigma}\right) - Q\left(\frac{d_2}{\sigma}\right)$$
$$= 1 - Q\left(\frac{\sigma}{\sigma}\right) - Q\left(\frac{\sigma}{\sigma}\right) = 1 - 2Q(1)$$

C:

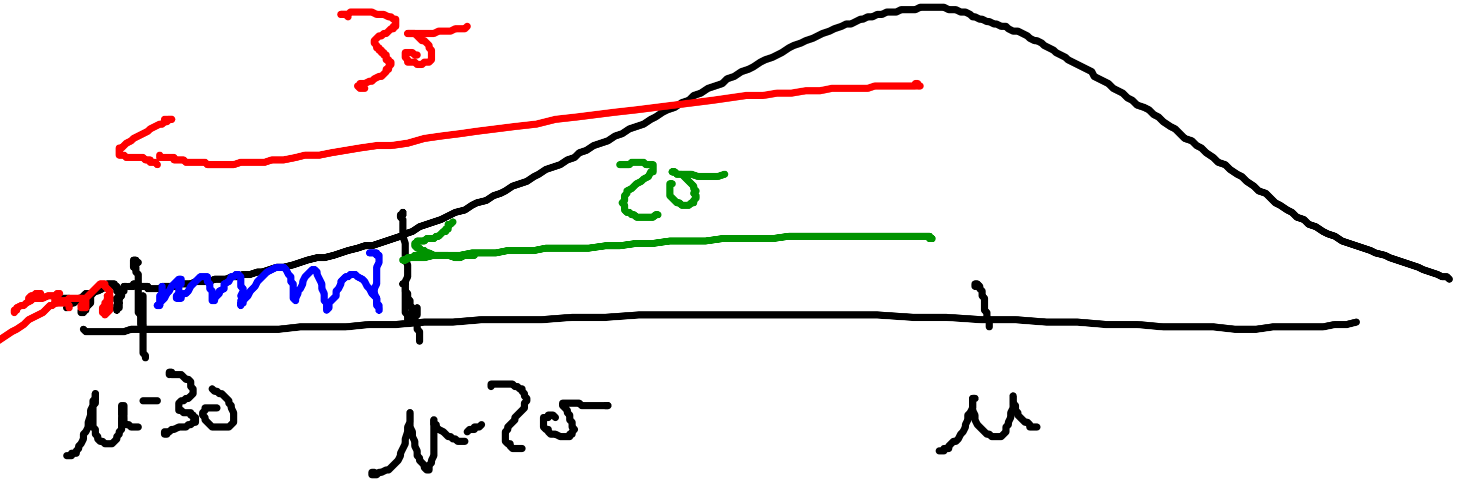


$$P(C) = Q\left(\frac{d_3}{\sigma}\right) - Q\left(\frac{d_4}{\sigma}\right)$$

$$= Q\left(\frac{\sigma}{\sigma}\right) - Q\left(\frac{2\sigma}{\sigma}\right)$$

$$= Q(1) - Q(2)$$

D.

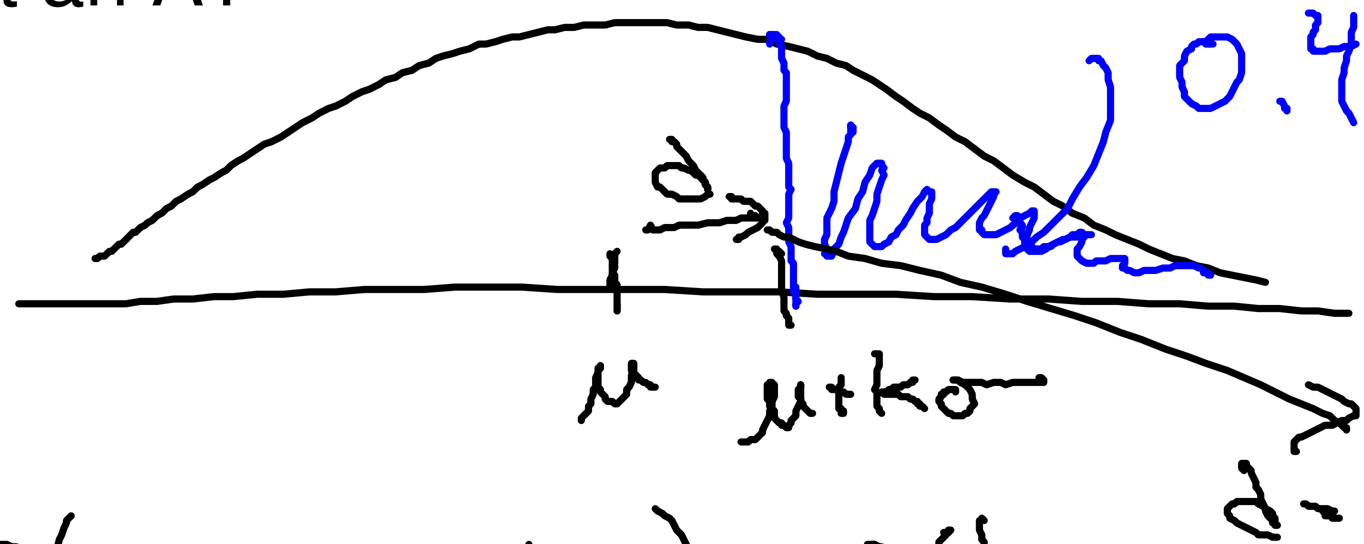


$$P(D) = Q\left(\frac{2\sigma}{\sigma}\right) - Q\left(\frac{3\sigma}{\sigma}\right)$$

$$= Q(2) - Q(3)$$

E: $P(X \le \mu - 3\sigma) = Q\left(\frac{3\sigma}{\sigma}\right) = Q(3)$

(c) Suppose the threshold to get an A is $k\sigma$ above the mean, what value of k is needed for 40% of the class to get an A?



$$\underline{P(X \geq \mu + k\sigma)} = 0.4$$

$$Q\left(\frac{d}{\sigma}\right) = Q\left(\frac{k\sigma}{\sigma}\right) = 0.4$$

$$Q(k) = 0.4$$

$$k = Q^{-1}(0.4)$$