EEL 4930 Stats – Lecture 19

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RANDOM VARIABLES (RVS)

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- What is a random variable?

Numeric occurrence that is random

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- What is a random variable?

- We define a random variable is defined on a probability space $(S, \mathscr{F}, P)$ as a function

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RANDOM VARIABLES (RVS)

- What is a random variable?

- We define a random variable is defined on a probability space $(S, \mathscr{F}, P)$ as a function from S to $\mathscr{R}$

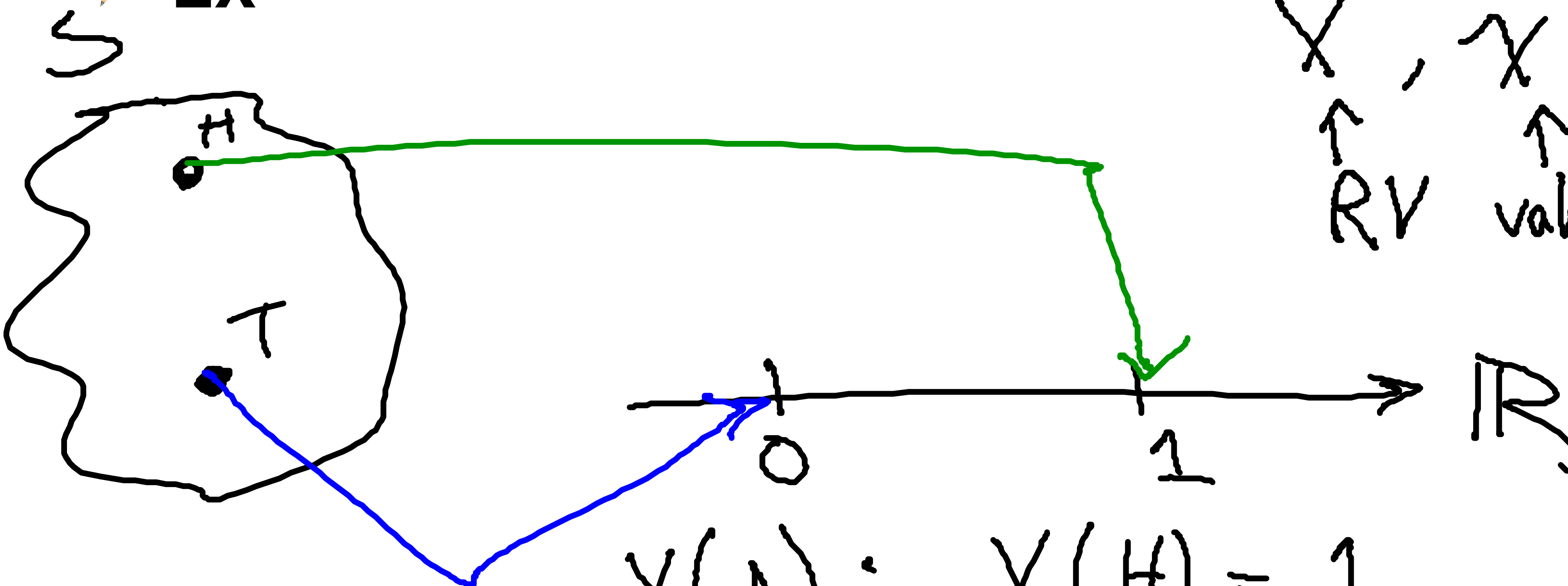
ζ S

EX

Create a binary RV from tossing a fair coin

X , x

RV value

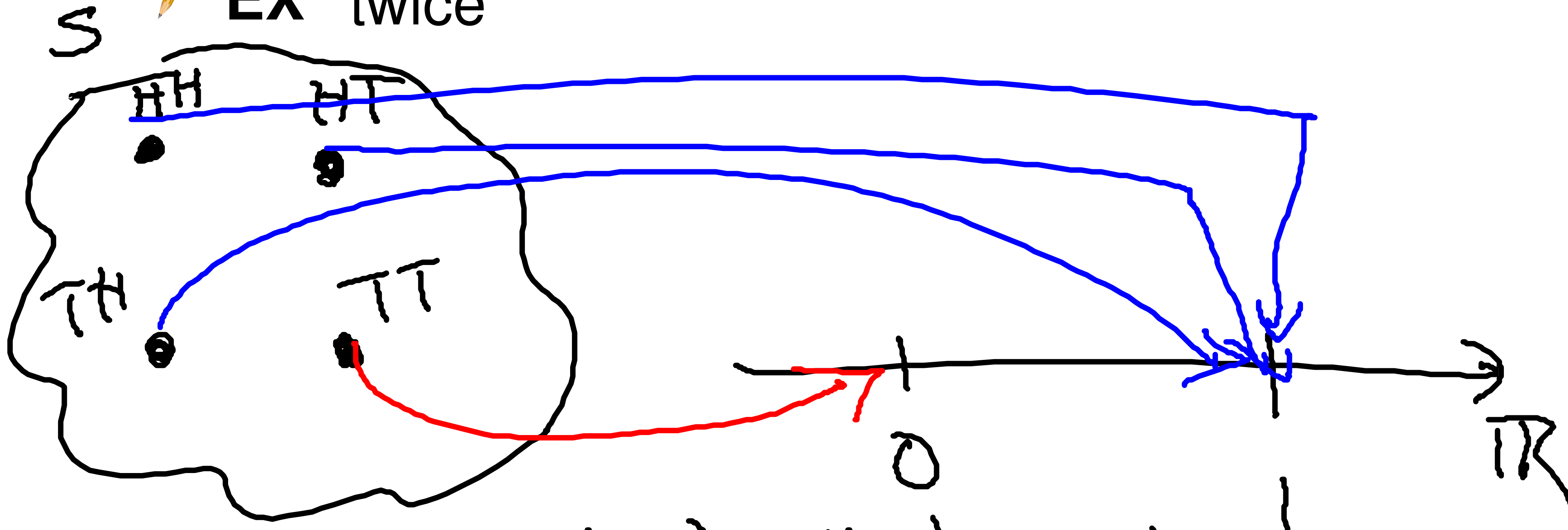


$$X(\zeta):\quad X(H) = 1$$
$$X(T) = 0$$

$$P[X(\zeta)=1] = P[\{\zeta \mid X(\zeta)=1\}]$$
$$= P[\{H\}] = 1/2$$
$$P[X(\zeta)=0] = P[\{T\}] = 1/2$$

$$P[X(\zeta)=x] = 0, \quad x \neq 0, \ x \neq 1$$

EX Create a binary RV from tossing a fair coin twice



$X(s)$: $X(HH) = X(HT) = X(TH) = 1$

$X(TT) = 0$

such that

$P[X(s)=1] = P[\{s \mid X(s)=1\}]$ not given!

$= P[\{HH, HT, TH\}] = 3/4$

$P[X=0] = P[\{TT\}] = 1/4$

EX Create another RV from tossing a fair coin ~~twice~~ same $(S, \mathcal{F}, P)$

$Y(s)$: $Y(HH) = 3$ $Y(HT) = 2$
$Y(TH) = 1$ $Y(TT) = 0$

$$P[Y(s) = y] = \begin{cases} 1/4, & y \in \{0,1,2,3\} \\ 0, & \text{o.w.} \end{cases}$$

Are X & Y independent?

NO.

If $X = 0$, then $s = TT \Rightarrow Y(s) = 0$

$P[Y(s) = 0] = 1$

DISCRETE RANDOM VARIABLES

A discrete random variable

DISCRETE RANDOM VARIABLES

A discrete random variable has nonzero probability at a countable number of values.

Probability Mass Function

PROBABILITY MASS FUNCTION

For a discrete RV, the *probability mass function*

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For a discrete RV, the *probability mass function* (pmf)

PROBABILITY MASS FUNCTION

For a discrete RV, the *probability mass function* (pmf) is

$$P(X = x) = P[X \leq x] - P[X < x]$$

$$P[X(\zeta) = x]$$

Probability Mass Function

For a discrete RV, the *probability mass function* (pmf) is

$$P(X = x) \;=\; P[X \leq x] - P[X < x]$$

EX: Roll a fair 6-sided die

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X = # on top face

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$$P(X = x) =$$

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X= # on top face

$$P(X = x) = \begin{cases} 1/6, & x = 1, 2, \ldots, 6 \\ \end{cases}$$

EX: Roll a fair 6-sided die

X = # on top face

$$p_X(x) = P(X = x) = \begin{cases} 1/6, & x = 1, 2, \ldots, 6 \\ 0, & \text{o.w.} \end{cases}$$

specify value for all x!

EX: Flip a fair coin until heads occurs

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X = # of flips

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$$P(X = x) =$$

EX: Flip a fair coin until heads occurs

X = # of flips

$$P(X = x) = \begin{cases} \left(\frac{1}{2}\right)^x, & x = 1, 2, \ldots \\ 0, & \text{o.w.} \end{cases}$$

infinite support

$$\sum_{x=1}^{\infty} \left(\frac{1}{2}\right)^x = 0.11111\ldots$$
$$= 1$$

EX: Flip a fair coin until heads occurs

X = # of flips

$$P(X = x) = \begin{cases} \left(\frac{1}{2}\right)^x, & x = 1, 2, \ldots \\ 0, & \text{o.w.} \end{cases}$$

Cumulative Distribution Function

If $(S, \mathscr{F}, P)$ is a prob. space with $X(\omega)$ a real RV on S, the **cumulative distribution function**

CUMULATIVE DISTRIBUTION FUNCTION

If $(S, \mathscr{F}, P)$ is a prob. space with $X(\omega)$ a real RV on S, the **cumulative distribution function** (cdf)

Cumulative Distribution Function

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If $(S, \mathscr{F}, P)$ is a prob. space with $X(\omega)$ a real RV on S, the **cumulative distribution function** (cdf), denoted $F_X(x)$ is

$$F_X(x) = P\Big[\{\omega | X(\omega) \in (-\infty, x]\} \text{ and } \omega \in S\Big]$$

x is included in interval!

Cumulative Distribution Function

If $(S, \mathscr{F}, P)$ is a prob. space with $X(\omega)$ a real RV on S, the **cumulative distribution function** (cdf), denoted $F_X(x)$ is

$$\begin{aligned} F_X(x) &= P\Big[\{\omega | X(\omega) \in (-\infty, x]\} \\ &\qquad\qquad \text{and } \omega \in S\Big] \\ &= P(X \le x). \end{aligned}$$

Cumulative Distribution Function

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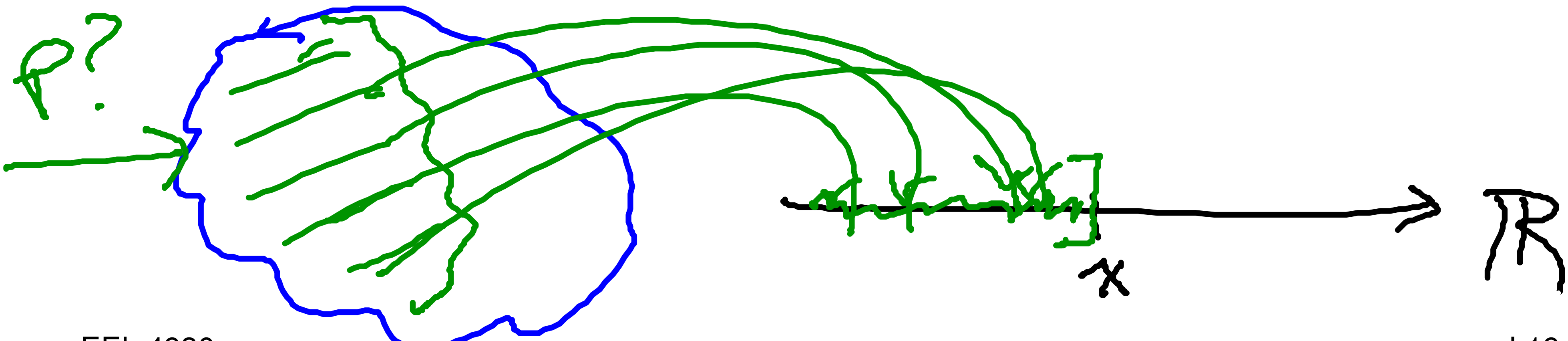
$$



- $F_X(x)$ is also sometimes called the *probability distribution function (PDF)*, but I will avoid this terminology to avoid confusion with another function we will use, called the probability density function (pdf)

- $F_X(x)$ is a prob. measure

- $F_X(x)$ is a prob. measure
  - Thus $F_X(x)$ inherits all the properties of a probability measure (axioms and corollaries still apply)

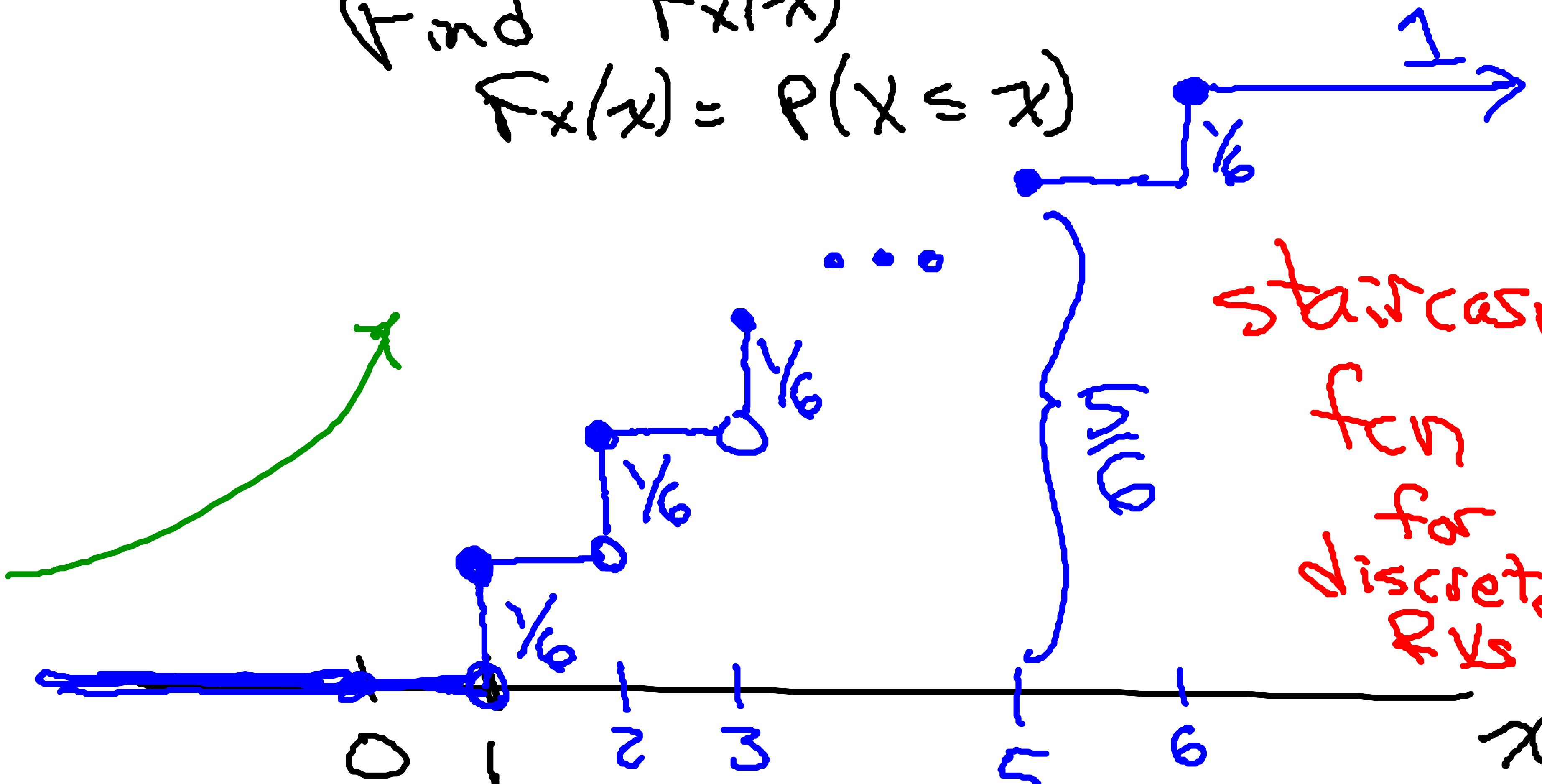
EX Find and plot the cdfs for the previous two examples

Ex) Roll a fair 6-sided die

X = # on top face

Find $F_X(x)$

$F_X(x) = P(X \le x)$



staircase fcn for discrete RVs

$F_X(x) = \begin{cases} 0 & x < 1 \\ 1/6, & 1 \le x < 2 \end{cases}$

$F_X(x)$

monotonically nondecreasing

$F_X(-\infty) = P(X \le -\infty) = 0$

$F_X(\infty) = P(X \le +\infty) = 1$

$P_X(x) = P(X = x) = P(X \le x) - P(X < x)$ limit from left

$= F_X(x) - F_X(x^-)$

$P(X=x)=$ "height" of jump in $F_X(x)$ at x