Let $p_{LNA} = P_{sub}\{$ chip is a low-noise amplifier $\}$

$$\frac{1}{\sqrt{2\pi(1)}} \exp\left\{-\frac{x^2}{2(1)}\right\} p_{LNA}$$

$$\underset{PA}{\overset{LNA}{\gtrless}} \frac{1}{\sqrt{2\pi 4}} \exp\left\{-\frac{x^2}{2(4)}\right\}\left[1-p_{LNA}\right]$$

$$\frac{2\,p_{LNA}}{1-p_{LNA}} \underset{PA}{\overset{LNA}{\gtrless}} \exp\left\{\frac{x^2}{2} - \frac{x^2}{8}\right\}$$

$$\ln\left(\frac{2\,p_{LNA}}{1-p_{LNA}}\right) \underset{PA}{\overset{LNA}{\gtrless}} \frac{4x^2 - x^2}{8}$$

$$\ln\left(\frac{2\,p_{LNA}}{1-p_{LNA}}\right) \underset{PA}{\overset{LNA}{\gtrless}} \frac{3x^2}{8}$$

$$x^2 \underset{PA}{\overset{LNA}{\lessgtr}} \frac{8}{3} \ln\left(\frac{2\,p_{LNA}}{1-p_{LNA}}\right)$$

$$x \underset{PA}{\overset{LNA}{\lessgtr}} \pm \sqrt{\frac{8}{3} \ln\left(\frac{2\,p_{LNA}}{1-p_{LNA}}\right)}$$

i) Note that

$$\ln\left(\frac{2\rho_{LNA}}{1-\rho_{LNA}}\right)$$ may be

negative if

$$\frac{2\rho_{LNA}}{1-\rho_{LNA}} < 1$$

$$\Leftrightarrow \quad 2\rho_{LNA} < 1-\rho_{LNA}$$

$$3\rho_{LNA} < 1$$

$$\rho_{LNA} < 1/3$$

So, the math breaks down b/c we can't take the sqrt of a negative value when

$$\rho_{LNA} < 1/3$$