EEL 4930 Stats – Lecture 22

CONTINUOUS RANDOM VARIABLES

A random variable is __________________ [1] if its distribution function is continuous function.

- Continuous random variables do not have probability at any discrete points.I.e., $P(X = x) = 0 \ \ \forall x \in \mathbb{R}$

A random variable is _____________ [2] if its distribution function is a staircase function.

- Discrete random variables have all of the probability concentrated at a discrete set points.
- It is possible to have a random variable for which some of the probability is concentrated at individual points and some of the probability is distributed over continous ranges
- These are called __________ [3] random variables and will not be covered in this class
- For discrete RVs, we can see how the probability is distributed across the values by using the PMF, $P_X(x) = P(X = x)$
- For continous RVs, $P_X(x) = 0$ for all x
- So how to see how the probability is distributed across the values?
- Solution: see how the **probability density** is distributed instead of the probability:

PROBABILITY DENSITY FUNCTION

The ______________________________________ (______), $f_X(x)$, of a random variable X is the derivative (which may not exist at some places) of ____:

- Properties:
  1. $f_X(x) \geq 0, -\infty < x < \infty$
  2. $F_X(x) = \int_{-\infty}^{x} f_X(t)dt$
  3. $\int_{-\infty}^{+\infty} f_X(x)dx = 1$
  4. $P(a < X \leq b) = \int_a^b f_X(x)dx, a \leq b$
  5. If $g(x)$ is a nonnegative piecewise continuous function with finite integral

     $$\int_{-\infty}^{+\infty} g(x)dx = c, -\infty < c < +\infty,$$

     then $f_X(x) = \frac{g(x)}{c}$ is a valid pdf.

     Pf omitted.

- Note that if $f(x)$ exists at x, then $F_X(x)$ is continuous at x and thus $P[X = x] = F(x) - F(x^-) = 0$
- Recall that this does not mean that x never occurs, but that the occurrence is extremely unlikely