EEL 4930 Stats – Lecture 21

PROPERTIES OF DISTRIBUTION FUNCTIONS

1. $0 \leq F_X(x) \leq 1$

   Pf: $F_X(x)$ is a prob. measure

2. $F_X(-\infty) = 0$ and $F_X(\infty) = 1$

   Proof is technical.

   Basically, $F_X(-\infty)$ and $F_X(\infty)$ are defined as limits, and the corresponding subsets of the samples space $\{s \in S : X \leq x\}$ are either shrinking to $\emptyset$ or S

3. $F_X(x)$ is monotonically nondecreasing,
   i.e., $F_X(a) \leq F_X(b)$ iff $a \leq b$

$$\text{Pf: } P\{X \in (-\infty, b]\} = P(X \in (-\infty, a]) + P(X \in (a, b])$$

$$\Rightarrow F_X(b) = F_X(a) + P(a < X \leq b) \tag{1}$$

4. $P(a < X \leq b) = F_X(b) - F_X(a)$

   Pf: rewriting equation (1)

5. $F_X(x)$ is continuous on the right,
   i.e., $F_X(b) = \lim_{h \to 0} F_X(b+h) = F_X(b)$

   (The value at a jump discontinuity is the value after the jump.)

   Proof is rather technical and will be omitted.

6. $P(X > x) = 1 - F_X(x)$

$$\text{Pf: } \{X > x\} = \{X \leq x\}^c$$
$$\Rightarrow P(X > x) = 1 - P(X \leq x) = 1 - F_X(x)$$