EEL 4930 Lecture 8

INTRODUCTION TO CONDITIONAL PROBABILITY

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(See the Jupyter notebook)

Another example

Another example

EX

EX Defective computers in a lab

Another example

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EX Defective computers in a lab

A computer lab contains

Another example

EX **EX Defective computers in a lab**

A computer lab contains

- two computer from manufacturer A, one of which is defective

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A computer lab contains

- two computer from manufacturer A, one of which is defective
- three computers from manufacturer B, two of which are defective

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EX Defective computers in a lab

A computer lab contains

- two computer from manufacturer A, one of which is defective
- three computers from manufacturer B, two of which are defective

A user sits down at a computer at random.

Let the properties of the computer he sits down at be denoted by a two letter code, where the first letter is the manufacturer and the second letter is D for a defective computer and N for a non-defective computer. (We add a subscript to differentiate computers with the same two-letter code.)

$$S = \{AD, AN, BD_1, BD_2, BN\}$$

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Let

- E_A be the event that the selected computer is from manufacturer A

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Let

- E_A be the event that the selected computer is from manufacturer A
- E_B be the event that the selected computer is from manufacturer B

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Let

- E_A be the event that the selected computer is from manufacturer A $= \{AD, AN\}$ $|E_A| = 2$
- E_B be the event that the selected computer is from manufacturer B $= \{BD_1, BD_2, BN\}$ $|E_B| = 3$
- E_D be the event that the selected computer is defective $= \{AD, BD_1, BD_2\}$

$|E_D| = 3$

$|S| = 5$

$E_B = \overline{E_A}$

Find

$P(E_A) =$ 2/5	$P(E_B) =$ 3/5	$P(E_D) =$ 3/5

$$S = \{AD, AN, BD_1, BD_2, BN\}$$

Find

$P(E_A) =$ _______	$P(E_B) =$ _______	$P(E_D) =$ _______

$$S = \{AD, AN, BD_1, BD_2, BN\}$$

- Now, suppose that I select a computer and tell you its manufacturer. Does that influence the probability that the computer is defective?

$$S = \{AD, AN, BD_1, BD_2, BN\}$$

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- Ex: Suppose I tell you the computer is from manufacturer A. Then what is the prob. that it is defective?

$1/2$

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We denote this prob. as $P(E_D|E_A)$

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- Ex: Suppose I tell you the computer is from manufacturer A. Then what is the prob. that it is defective?

$$\frac{1}{2}$$

We denote this prob. as $P(E_D|E_A)$
(means: *the conditional probability of event E_D given that event E_A occured*)

$$S = \{AD, AN, BD_1, BD_2, BN\}$$

$$S = \{\boxed{AD}, AN, BD_1, BD_2, BN\}$$

Find

- $P(E_D|E_B) =$ ___2/3___
- $P(E_A|E_D) =$ ___1/3___
- $P(E_B|E_D) =$ ___2/3___

$$S = \{AD, AN, BD_1, BD_2, BN\}$$

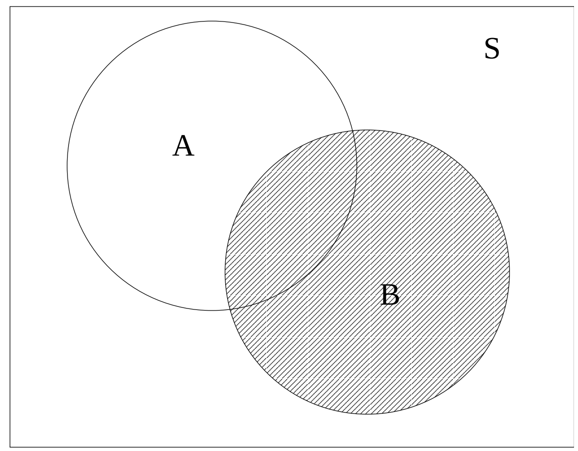
Find

- $P(E_D|E_B) =$ __________
- $P(E_A|E_D) =$ __________
- $P(E_B|E_D) =$ __________

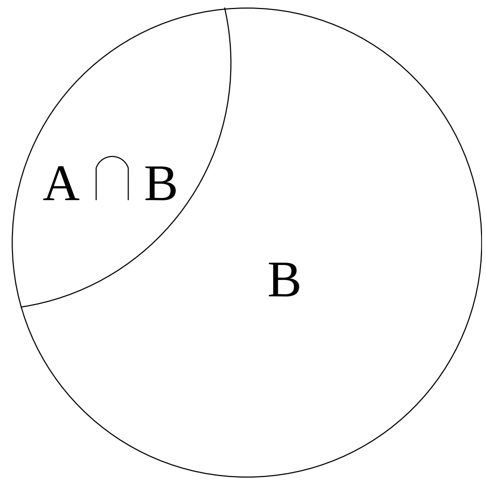
We need a systematic way of determining probabilities given additional information about the experiment outcome.

Formally Defining Conditional Probabilty

Consider the Venn diagram:



If we condition on B having occurred, then we can form the new Venn diagram:



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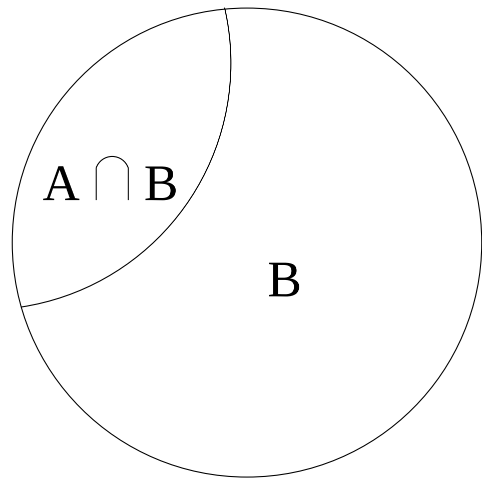

This diagram suggests that if $A \cap B = \emptyset$ then if B occurs, A could not have occurred.

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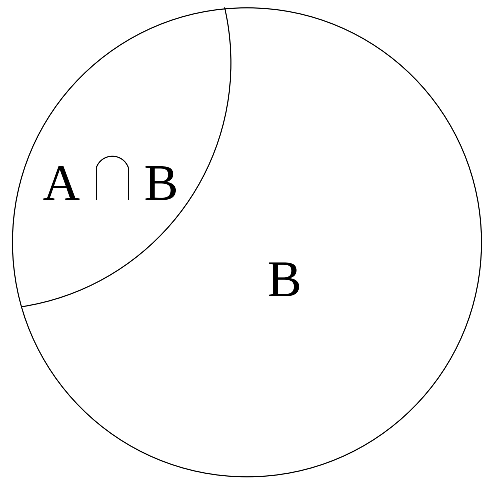

This diagram suggests that if $A \cap B = \emptyset$ then if B occurs, A could not have occurred.

Similarly if $B \subset A$, then if B occurs, the diagram suggests that A must have occurred.

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For $A \in \mathscr{F}$, $B \in \mathscr{F}$, the *conditional probability* of event A *given* that event B occurred is

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For $A \in \mathscr{F}$, $B \in \mathscr{F}$, the *conditional probability* of event A *given* that event B occurred is

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \text{ for } P(B) > 0.$$

Claim: If $P(B) > 0$, the conditional probability $P(|B)$ satisfies the axioms on the original sample space

$(S, \mathscr{F}, P(\cdot|B))$

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Check the axioms:

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$(S, \mathscr{F}, P(\cdot|B))$

Check the axioms:

1.

$$P(S|B) =$$

Claim: If $P(B) > 0$, the conditional probability $P(|B)$ satisfies the axioms on the original sample space

$(S, \mathscr{F}, P(\cdot|B))$

Check the axioms:

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$$P(S|B) = \frac{P(S \cap B)}{P(B)}$$

Claim: If $P(B) > 0$, the conditional probability $P(|B)$ satisfies the axioms on the original sample space
$(S, \mathscr{F}, P(\cdot|B))$

Check the axioms:

1.

$$P(S|B) = \frac{P(S \cap B)}{P(B)} = \frac{P(B)}{P(B)}$$

Claim: If $P(B) > 0$, the conditional probability $P(|B)$ satisfies the axioms on the original sample space

$(S, \mathscr{F}, P(\cdot|B))$

Check the axioms:

1.

$$P(S|B) = \frac{P(S \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$$

2. Given $A \in \mathscr{F}$,

$$P(A|B) = \frac{P(A \cap B)}{P(B)},$$

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and $P(A \cap B) \geq 0$, $P(B) \geq 0$

$$\Rightarrow P(A|B) \geq 0$$

3. If $A \subset \mathscr{F}$, $C \subset \mathscr{F}$, and $A \cap C = \emptyset$,

$$P(A \cup C | B) \ =$$

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\begin{aligned}
P(A \cup C | B) &= \frac{P\left[(A \cup C) \cap B\right]}{P[B]} \\
&= \frac{P\left[(A \cap B) \cup (C \cap B)\right]}{P[B]}.
\end{aligned}
$$

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P(A \cup C | B) = \frac{P\left[A \cap B\right]}{P\left[B\right]} + \frac{P\left[C \cap B\right]}{P[B]}
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3. If $A \subset \mathscr{F}$, $C \subset \mathscr{F}$, and $A \cap C = \emptyset$,

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$$
\begin{aligned}
P(A \cup C|B) &= \frac{P\left[A \cap B\right]}{P\left[B\right]} + \frac{P\left[C \cap B\right]}{P[B]} \\
&= P(A|B) + P(C|B)
\end{aligned}
$$

EX **Check prev. example:** $S = \{AD, AN, BD, BD, BN\}$

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Relating Conditional and Unconditional Probs

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(a) $P(A|B) \geq P(A)$

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(a) $P(A|B) \geq P(A)$

(b) $P(A|B) \leq P(A)$

Relating Conditional and Unconditional Probs

Which of the following statements are true?

(a) $P(A|B) \geq P(A)$

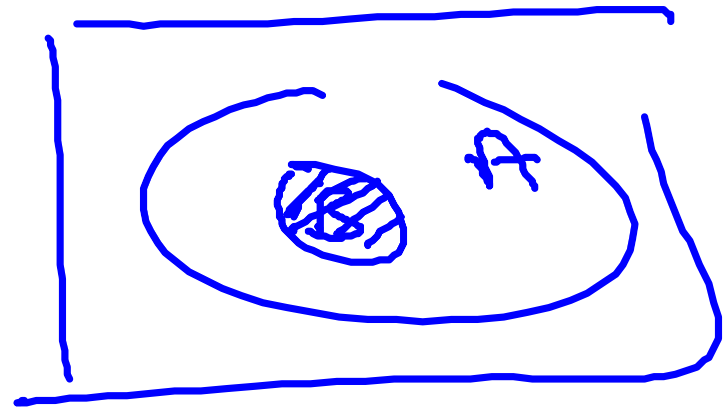
(b) $P(A|B) \leq P(A)$

(c) Not necessarily (a) or (b)

$$A \cap B = \emptyset$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{P(B)} = 0$$

If $B \subset A$



$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(B)}{P(B)} = 1$$

Conditional Probability for Discrete Sample Spaces with Equal Probabilities

Conditional Probability for Discrete Sample Spaces with Equal Probabilities

Conditional probability, independence, and mutually exclusive events for discrete sample spaces with equal probabilities:

EX **Take 2:** XOR of Two Independent Binary Values

Flip a fair coin with sides labeled '0' and '1' two times. Let E_i denote a '1' on the top face on flip i. Let F denote the event that the XOR of the values observed on the top faces on the two flips is '1'.

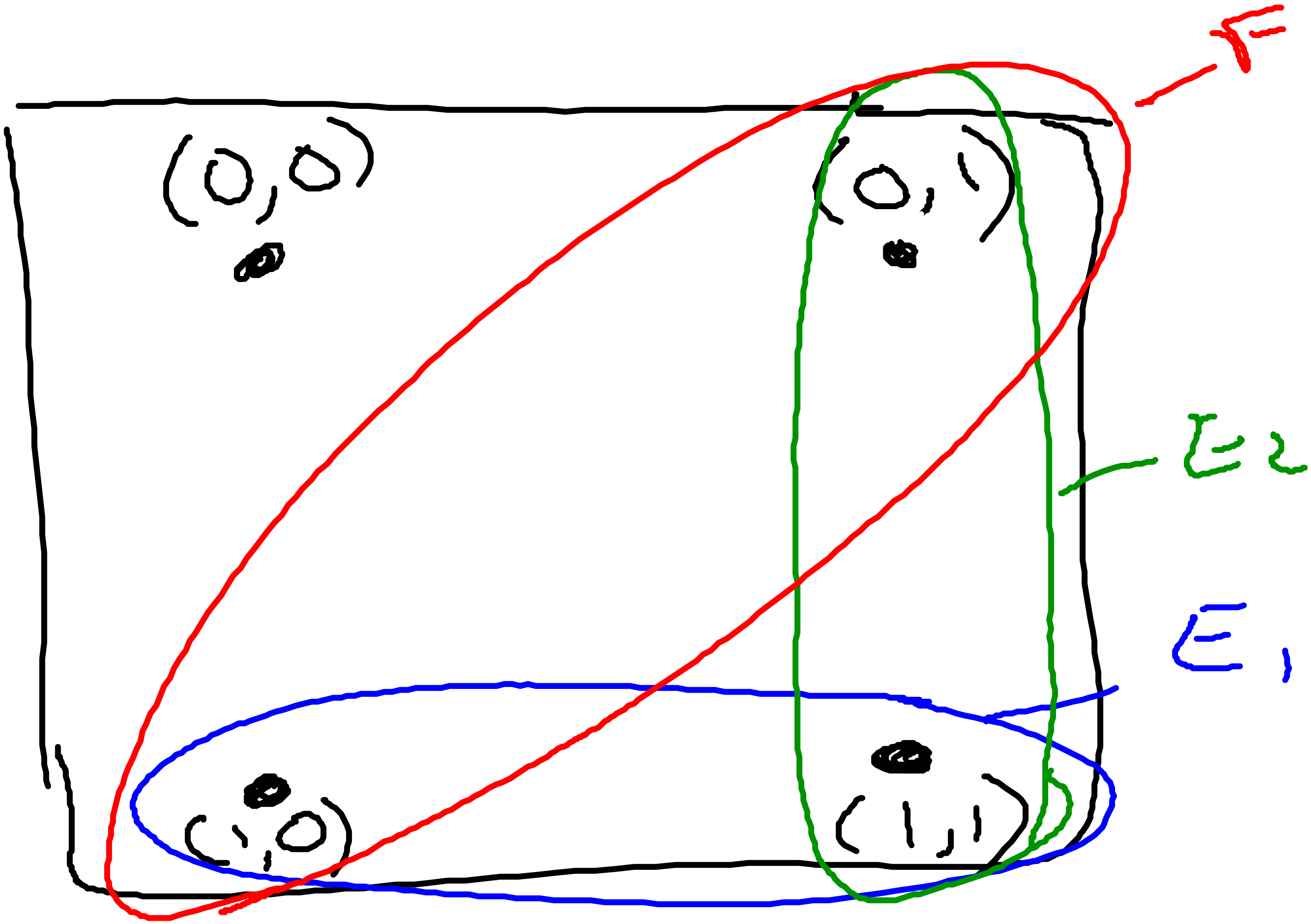
Flip 1	Flip 2	XOR	Prob
0	0	0	1/4
1	0	1	1/4
0	1	1	1/4
1	1	0	1/4

F

P(E1)

1/2

E2



$P(E_1) = P(E_2) = P(F) = 1/2$

$P(E_1 | E_2) = 1/2$ $\quad P(E_2 | E_1) = 1/2$ $\quad P(F | E_1) = \frac{1}{2}$

$P(F | E_1 \cap E_2) = 0$, F and $E_1 \cap E_2$ are m.e.

Using Conditional Probability to Decompose Events: Chain Rules, Partitions, and Total Probability

Chain Rules

Chain rule for expanding intersections

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Note that $P(A|B) \;=\; \dfrac{P(A \cap B)}{P(B)}$

Chain rule for expanding intersections

$$\text{Note that } P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A \cap B) = P(A|B)P(B)$$

Chain rule for expanding intersections

$$\begin{aligned}
\text{Note that } P(A|B) &= \frac{P(A \cap B)}{P(B)} \\
\Rightarrow P(A \cap B) &= P(A|B)P(B) \qquad (1)
\end{aligned}$$

$$\text{and } P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Chain rule for expanding intersections

$$\begin{aligned} \text{Note that } P(A|B) &= \frac{P(A \cap B)}{P(B)} \\ \Rightarrow P(A \cap B) &= P(A|B)P(B) \end{aligned} \tag{1}$$

$$\begin{aligned} \text{and } P(B|A) &= \frac{P(A \cap B)}{P(A)} \\ \Rightarrow P(A \cap B) &= P(B|A)P(A) \end{aligned}$$

Chain rule for expanding intersections

$$\text{Note that } P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A \cap B) = P(A|B)P(B) \quad (1)$$

$$\text{and } P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(A \cap B) = P(B|A)P(A) \quad (2)$$

- Eqns. (1) and (2) are chain rules for expanding the probability of the intersection of two events

- The chain rule can be easily generalized to more than two events

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Ex: Intersection of 3 events

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$$P(A \cap B \cap C) \ = \ P(A \cap B \cap C)$$

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Ex: Intersection of 3 events

$$P(A \cap B \cap C) \;=\; \frac{P(A \cap B \cap C)}{P(B \cap C)} \cdot P(B \cap C)$$

- The chain rule can be easily generalized to more than two events

Ex: Intersection of 3 events

$$P(A \cap B \cap C) \;=\; \frac{P(A \cap B \cap C)}{P(B \cap C)} \cdot \frac{P(B \cap C)}{P(C)} \cdot P(C)$$

- The chain rule can be easily generalized to more than two events

Ex: Intersection of 3 events

$$\begin{aligned} P(A \cap B \cap C) &= \frac{P(A \cap B \cap C)}{P(B \cap C)} \cdot \frac{P(B \cap C)}{P(C)} \cdot P(C) \\ &= P(A | B \cap C) \end{aligned}$$

- The chain rule can be easily generalized to more than two events

Ex: Intersection of 3 events

$$
\begin{aligned}
P(A \cap B \cap C) &= \frac{P(A \cap B \cap C)}{P(B \cap C)} \cdot \frac{P(B \cap C)}{P(C)} \cdot P(C) \\
&= P(A|B \cap C)P(B|C)
\end{aligned}
$$

- The chain rule can be easily generalized to more than two events

Ex: Intersection of 3 events

$$\begin{aligned} P(A \cap B \cap C) &= \frac{P(A \cap B \cap C)}{P(B \cap C)} \cdot \frac{P(B \cap C)}{P(C)} \cdot P(C) \\ &= P(A|B \cap C)P(B|C)P(C) \end{aligned}$$

STATISTICAL INDEPENDENCE

- In the last example, we had several probabilities of the form $P(A|B) = P(A)$

STATISTICAL INDEPENDENCE

- In the last example, we had several probabilities of the form $P(A|B) = P(A)$
- In this case, we say that A is *statistically independent (s.i.)* of B, since the probabilities of A are not affected by knowledge of A having occurred

- By the chain rule, $P(A \cap B) = P(A|B)P(B)$. So if $P(A) > 0$, $P(B > 0)$, and $P(A|B) = P(A)$, then

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

- By the chain rule, $P(A \cap B) = P(A|B)P(B)$. So if $P(A) > 0$, $P(B > 0)$, and $P(A|B) = P(A)$, then

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Events A and B are *statistically independent (s.i.)* if and only if (iff)

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EX **Take 3:** A fair six-sided die is rolled twice. What is the probability of observing a 1 or a 2 on the top face on either roll of the die?