EEL 4930 Stats – Lecture 21

EEL 4930 Stats – Lecture 20

IMPORTANT RANDOM VARIABLES

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Discrete RVs

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$$X = \begin{cases} 1, & s \in A \\ & \end{cases}$$

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- **Examples using Jupyter Notebook**

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$$P[X=k]=\begin{cases}\binom{n}{k}p^k(1-p)^{n-k}, & k=0,1,\ldots,n\\ 0, & \text{o.w.}\end{cases}$$

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$$P[X=k] = \begin{cases} (1-p)^{k-1}p, & k = 1, 2, \ldots \\ 0, & \text{o.w.} \end{cases}$$

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- The pmf of the Poission random variable is

$$P_N(n) = \begin{cases} \frac{\alpha^n}{n!} e^{-\alpha}, & n = 0, 1, \ldots \\ 0, & \text{o.w.} \end{cases}$$

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  - packets arriving at a queue in a network
  - processes being submitted to a scheduler

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PROPERTIES OF DISTRIBUTION FUNCTIONS

1. $0 \leq F_X(x) \leq 1$

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Properties of Distribution Functions

1. $0 \leq F_X(x) \leq 1$

   Pf: $F_X(x)$ is a prob. measure

2. $F_X(-\infty) = 0$ and $F_X(\infty) = 1$

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Basically, $F_X(-\infty)$ and $F_X(\infty)$ are defined as limits, and the corresponding subsets of the samples space $\{s \in S : X \leq x\}$ are either shrinking to $\emptyset$ or S

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   Pf: rewriting equation (1)

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Proof is rather technical and will be omitted.

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