EEL 4930 Stats – Lecture 19

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RANDOM VARIABLES (RVS)

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- What is a random variable?

Numeric occurrence that is random

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- We define a random variable is defined on a probability space $(S, \mathscr{F}, P)$ as a function

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RANDOM VARIABLES (RVS)

- What is a random variable?

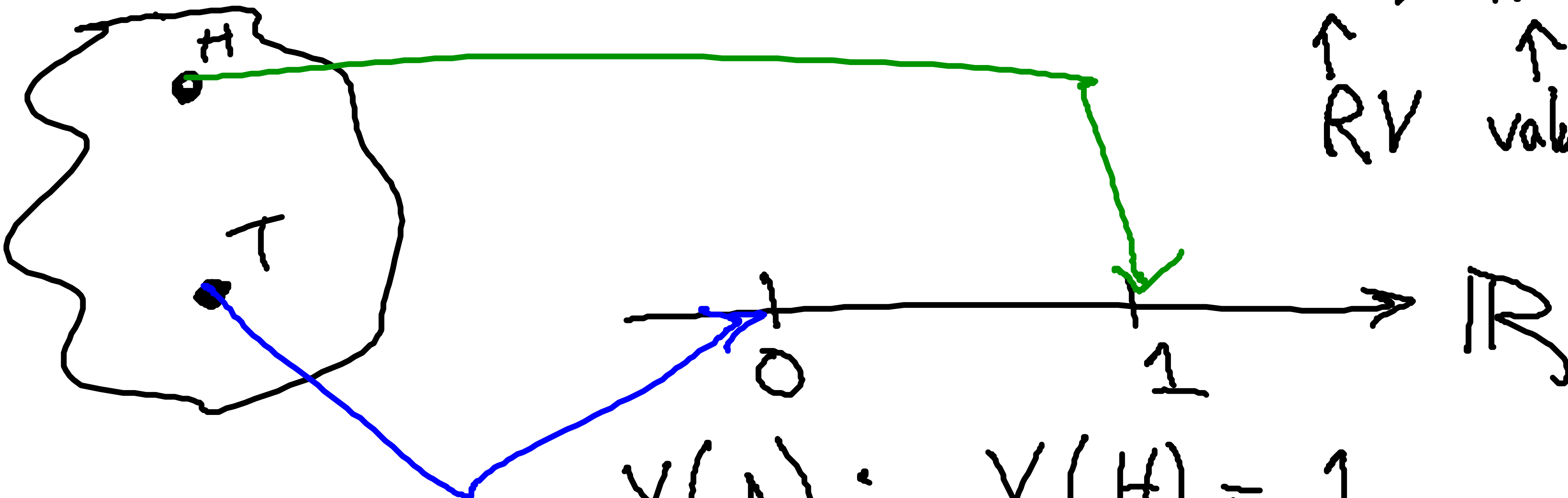
- We define a random variable is defined on a probability space $(S, \mathscr{F}, P)$ as a function from S to $\mathscr{R}$

$\varsigma \quad S$

EX Create a binary RV from tossing a fair coin

$X \; , \; x$

$\uparrow$ RV $\quad \uparrow$ value



$X(\varsigma): \quad X(H) = 1$

$\qquad\qquad X(T) = 0$

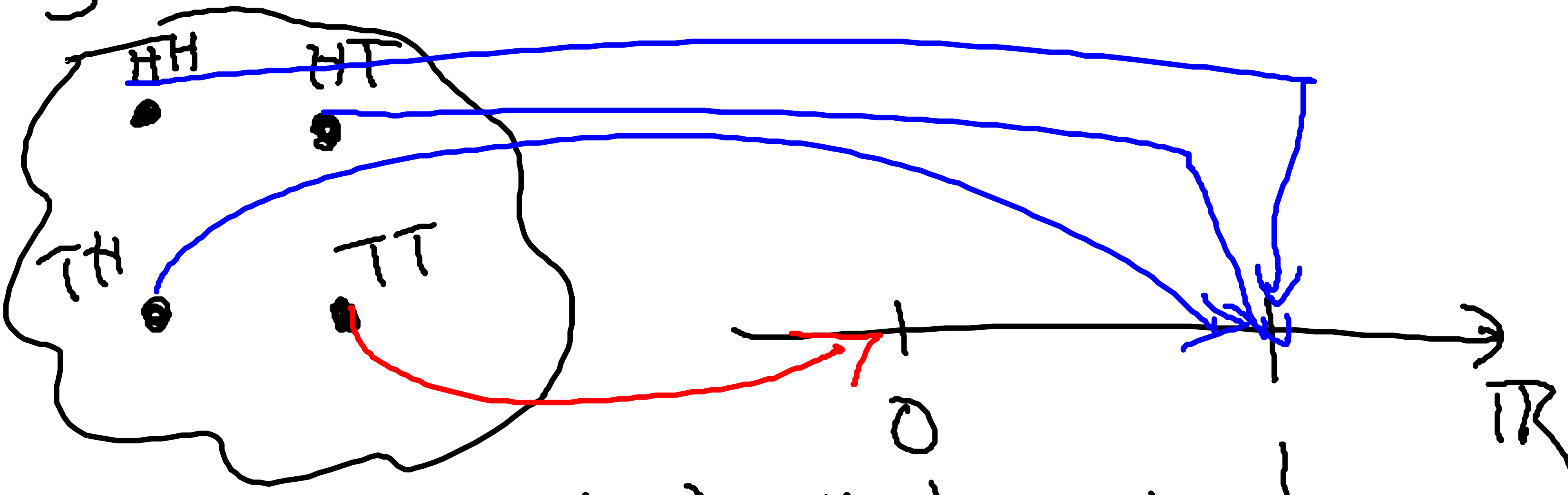
$P[X(\varsigma) = 1] = P[\{\varsigma \mid X(\varsigma) = 1\}]$

$\qquad = P[\{H\}] = 1/2$

$P[X(\varsigma) = 0] = P[\{T\}] = 1/2$

$P[X(\varsigma) = x] = 0, \; x \neq 0, \; x \neq 1$

EX Create a binary RV from tossing a fair coin twice



$$X(s): \quad X(HH) = X(HT) = X(TH) = 1$$
$$X(TT) = 0$$

$$P[X(s)=1] = P[\{s \mid X(s)=1\}]$$

such that

not given!

$$= P[\{HH, HT, TH\}] = 3/4$$

$$P[X=0] = P[\{TT\}] = 1/4$$

EX Create another RV from tossing a fair coin twice

Discrete Random Variables

A discrete random variable

DISCRETE RANDOM VARIABLES

A discrete random variable has nonzero probability at a countable number of values.

Probability Mass Function

PROBABILITY MASS FUNCTION

For a discrete RV, the *probability mass function*

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For a discrete RV, the *probability mass function* (pmf)

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EX: Roll a fair 6-sided die

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EX: Flip a fair coin until heads occurs

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CUMULATIVE DISTRIBUTION FUNCTION

If $(S, \mathscr{F}, P)$ is a prob. space with $X(\omega)$ a real RV on S, the **cumulative distribution function**

Cumulative Distribution Function

If $(S, \mathscr{F}, P)$ is a prob. space with $X(\omega)$ a real RV on S, the **cumulative distribution function** (cdf)

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&\qquad\qquad \text{and } \omega \in S\Big] \\
&= P(X \leq x).
\end{aligned}
$$

- $F_X(x)$ is also sometimes called the *probability distribution function (PDF)*, but I will avoid this terminology to avoid confusion with another function we will use, called the probability density function (pdf)

- $F_X(x)$ is a prob. measure

- $F_X(x)$ is a prob. measure
  - Thus $F_X(x)$ inherits all the properties of a probability measure (axioms and corollaries still apply)

EX Find and plot the cdfs for the previous two examples