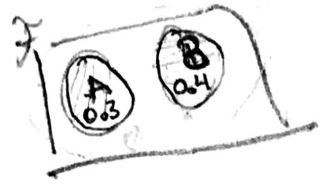
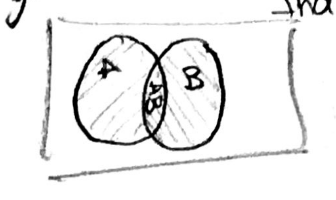
ME $\mathcal{F}$ Ind.





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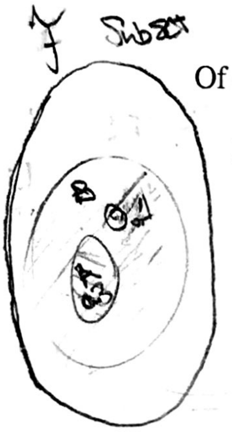
Homework



Provide **analytical** solutions (i.e., using mathematics, not simulations) to the following problems. Of course, you are welcome to use simulations to verify your answers.

1. Let $(S, \mathcal{F}, P)$ be a probability space with $A \in \mathcal{F}$ and $B \in \mathcal{F}$ such that $P(A) = 0.3$ and $P(B) = 0.4$. Find the following probabilities under the specified conditions. Note that I don't expect you to have to show much work in answering this question.

   (a) either A or B occurs if A and B are mutually exclusive $P = P(A) + P(B) = 0.3 + 0.4 = 0.7$
   (b) either A or B occurs if A and B are statistically independent $P = P(A) + P(B) - P(A)P(B)$ $= 0.3 + 0.4 - 0.12 = 0.58$
   (c) either A or B occurs if A is a subset of B $P = P(B) = 0.4$
   (d) A occurs but B does not occur if A and B are mutually exclusive $P = P(A) = 0.3$
   (e) A occurs but B does not occur if A and B are statistically independent $P = P(A) - P(A)P(B)$ $= 0.3 - 0.3*0.4$ $= 0.18$
   (f) A occurs but B does not occur if A is a subset of B $P = 0$
   (g) both A and B occur if A and B are mutually exclusive $P = 0$
   (h) both A and B occur if A and B are statistically independent $P = P(A)*P(B) = 0.3*0.4 = 0.12$
   (i) both A and B occur if A is a subset of B $P = P(A) = 0.3$
   (j) B occurs but A does not occur if A is a subset of B $P = P(B) - P(A) = 0.1$

2. A system has m controllers and n actuators. The system has redundancy built in so that it will be operational if at least one controller and two actuators have not failed. All controllers and actuators fail independently.

   If the probability that a controller fails is $p_c = 0.25$, and the probability that an actuator fails is $p_a = 0.1$, find the probability that the system will be operational for:

   (a) 1 controller, 2 actuators $P = 0.75 * 0.9 * 0.9 = .6075$
   (b) 2 controllers, 2 actuators $P = 0.75 * 0.75 * 0.9 * 0.9 + 2*0.75*0.25*0.9*0.9$ $= 0.759$
   (c) 2 controllers, 3 actuators On Scratch

3. Problem 61 from the 2nd ed. of *Probability, Statistics, and Random Processes For Electrical Engineering* by Leon-Garcia:

   A computer manufacturer uses chips from three sources. Chips from sources A, B, and C are defective with probabilities 0.001, 0.005, and 0.01, respectively. If a randomly selected chip is found to be defective, what is the probability that the manufacturer was A; that the manufacturer was C. *My clarification:* Assume that the computer manufacturer uses equal numbers of chips from each source. On Scratch

4. Prostate cancer is the most common type of cancer found in males. As an indicator of whether a male has prostate cancer, doctors often perform a test that measures the level of the PSA (prostate specific antigen) protein that is produced only by the prostate gland. Although PSA levels are indicative of cancer, the test is notoriously unreliable. Indeed, the

2) $P = (0.75)^2 (0.9)^3 + \left[\binom{2}{1}\left((0.75)(0.25)\right)\right]\left[\binom{3}{2}\right]$ *

c.) $(0.9)(0.9)(0.1)\Big] + (0.75)^2 * \binom{3}{2} * 0.9^2 * 0.1$

$+ \binom{2}{1}(0.75 * 0.25 ...)(0.9)^3 = 0.91125$

- Everything works
- ✓ Both controllers work, 2 actuators work
- ✓ 1 controller works, All actuators work
- ✓ 1 controller works, 2 actuators work
- ✓ 1

3) D → chip is defective

A → chip from source A

B → " B

C → " C

$P(A|D) = \dfrac{P(D|A) * P(A)}{P(D)}$

$= \dfrac{(0.001) \times (1/3)}{0.005\overline{3}}$

$= 0.0625$

$P(C|D) = \dfrac{P(D|C) * P(C)}{P(D)}$

$= \dfrac{(0.01) * (1/3)}{0.005\overline{3}}$

$= 0.625$

$P(D) = P(D|A) * P(A) + P(D|B) * P(B) + P(D|C) * P(C)$

$= (0.001)(1/3) + (0.005) * (1/3) + (0.01) * (1/3)$

$= 0.005\overline{3}$

4.) E → elevated PSA   NE → not elevated PSA

C cancerous man

NC noncancerous man

A.)

$P(C) = 0.7$ $P(E|C) = 0.268$ $P(NE|C) = 0.732$

$P(NC) = 0.3$ $P(E|NC) = 0.135$ $P(NE|NC) = 0.865$

a.) $P(C|E) = \dfrac{P(E|C) \times P(C)}{P(E)}$

$= \dfrac{0.268 * 0.7}{0.2281} = 0.822$

$P(E) = P(E|C) * P(C) + P(E|NC) * P(NC)$

$= 0.268 * 0.7 + 0.135 * 0.3 = 0.2281$

$P(NE) = 1 - P(E) = 1 - 0.2281 = 0.7719$

b.) $P(C|NE) = \dfrac{P(NE|C) \times P(C)}{P(NE)}$

$\dfrac{0.732 * 0.7}{0.7719} = 0.664$

B.)

$P(C) = 0.3$

$P(NC) = 0.7$

$P(E|NC) = 0.135$ $P(NE|C) = 0.732$

$P(E|C) = 0.268$ $P(NE|NC) = 0.865$

a.) $P(C|E) = \dfrac{P(E|C) * P(C)}{P(E)} = \dfrac{0.268 * 0.3}{0.1749} = 0.460$

$P(E) = P(E|C) * P(C) + P(E|NC) * P(NC)$ $P(NE) = 1 - P(E)$

$= 0.268 * 0.3 + 0.135 * 0.7 = 0.1749$ $= 0.8251$

b.) $P(C|NE) = \dfrac{P(NE|C) * P(C)}{P(NE)} = \dfrac{0.732 * 0.3}{0.8251}$

$= 0.266$

probability that a noncancerous man will have an elevated PSA level is approximately 0.135, with this probability increasing to approximately 0.268 if the man does have cancer. If, based on other factors, a physician is 70 percent certain that a male has prostate cancer, what is the conditional probability that he has the cancer given that

A

(a) the test indicated an elevated PSA level?

(b) the test did not indicate an elevated PSA level?

On Scratch

B

Repeat the preceding, this time assuming that the physician initially believes that there is a 30 percent chance that the man has prostate cancer.

3. From *Random Signal Analysis in Engineering Systems* by John J. Komo (slightly modified)

For the digital communication system shown in Figure 1, where $P(A0) = 0.6$ and $P(A1) = 0.4$, completely specify the MAP decision rule and calculate the overall probability of error under that rule.

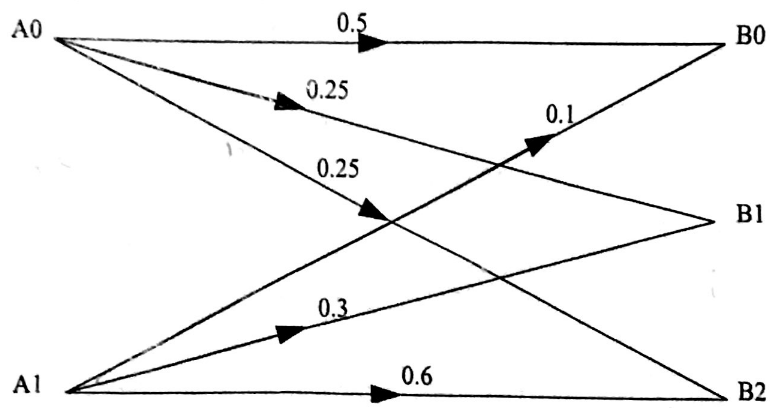


Figure 1: Two-to-three communication system channel diagram.

MAP RULE

given B0, choose A0

$P(A1|B0) = 0.4 * 0.1 = 0.04$

$P(A0|B0) = 0.6 * 0.5 = 0.30$

given B1, choose A0

$P(A1|B1) = 0.4 * 0.3 = 0.12$

$P(A0|B0) = 0.6 * 0.5 = 0.30$

given B2, choose A1

$P(A1|B2) = 0.4 * 0.6 = 0.24$

$P(A0|B2) = 0.6 * 0.25 = 0.15$

Prob Error

$P(Err) = P(A1|B0) + P(A1|B1) + P(A0|B2)$

$= 0.04 + 0.12 + 0.15$

$= 0.31$