EEL 4930 Stats – Lecture 20

IMPORTANT RANDOM VARIABLES

Discrete RVs

1. Bernoulli RV

   - An event $A \in \mathcal{F}$ is considered a "success"
   - A Bernoulli RV X is defined by

$$X = \begin{cases} 1, & s \in A \\ 0, & s \notin A \end{cases}$$

   - The pmf for a Bernoulli RV X can be found formally as

$$\begin{aligned} P(X=1) &= P\Big(X(s)=1\Big) \\ &= P\Big(\{s|s \in A\}\Big) = P(A) \triangleq p \end{aligned}$$

   So,

$$P(X=x) = \begin{cases} p & x=1 \\ 1-p & x=0 \\ 0 & x \neq 0,1 \end{cases}$$

   - **Engineering examples:** Whether a bit is 0 or 1, whether a bit is in error, whether a component has failed, whether something has been detected

   - **Examples using Jupyter Notebook**

2. Binomial RV

   - A Binomial RV represents the number of sucesses on n independent Bernoulli trials
   - Thus, a Binomial RV can also be defined as the sum of n independent Bernoullis RVs
   - Let X= # of successes
     Then the pmf of X is given by

$$P[X=k] = \begin{cases} \binom{n}{k} p^k (1-p)^{n-k}, & k = 0, 1, \ldots, n \\ 0, & \text{o.w.} \end{cases}$$

   - **Engineering examples:** The number of bits in error in a packet, the number of defective items in a manufacturing run

- **Examples using Jupyter Notebook**

3. Geometric RV

- A Geometric RV occurs when independent Bernoulli trials are conducted until the first success
- X = # number of trials required
  Then the pmf or X is given by

$$P[X=k] = \begin{cases} (1-p)^{k-1}p, & k=1,2,\ldots,n \\ 0, & \text{o.w.} \end{cases}$$

- **Engineering examples:** The number of retransmissions required for a packet, number of white dots between black dots in the scan of a black and white document
- **Examples using Jupyter Notebook**

4. Poisson RV

- Models events that occur randomly in space or time
- Let λ= the # of events/(unit of space or time)
- Consider observing some period of time or space of length t and let $\alpha = \lambda t$
- Let N= the # events in time (or space) t
- The pmf of the Poission random variable is

$$P_N(n) = \begin{cases} \frac{\alpha^n}{n!}e^{-\alpha}, & n=0,1,\ldots \\ 0, & \text{o.w.} \end{cases}$$

- For large α, the Poisson pmf has a bell shape. For example, see the pmf when $\alpha = 20$
- **Engineering examples:**
  - calls coming in to a switching center
  - packets arriving at a queue in a network
  - processes being submitted to a scheduler
    The following examples are adopted from A First Course in Probability by Sheldon Ross:
  - # of misprints on a group of pages in a book
  - # of people in a community that live to be 100 years old
  - # of wrong telephone numbers that are dialed in a day
  - # of α-particles discharged in a fixed period of time from some radioactive material
  - # of earthquakes per year
  - # of computer crashes in a lab in a week
- **Examples using Jupyter Notebook**