

## Lab 3 Part 1

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### 1.1 - Frequency Response of the Four-Point Averager

1.

Question 1: Using the Euler Formulas, show by hand that the frequency response for the 4-point running average operator is equivalent to the definition in the lab document.

```
img = imread("1.1.1.jpg");
image(img);
```

Handwritten derivation of the frequency response of a 4-point running average filter:

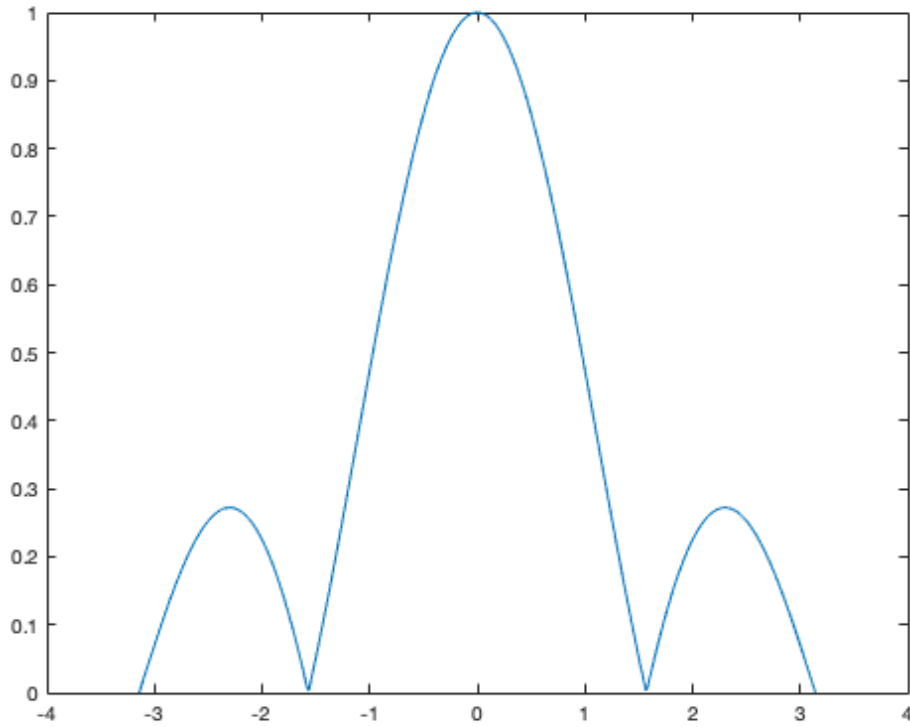
$$\begin{aligned}
 1.) \quad y[n] &= \sum_{k=0}^3 \frac{1}{4} x[n-k] \\
 &= \frac{1}{4} x[n] + \frac{1}{4} x[n-1] + \frac{1}{4} x[n-2] + \frac{1}{4} x[n-3] \\
 h[n] &= \frac{1}{4} \delta[n] + \frac{1}{4} \delta[n-1] + \frac{1}{4} \delta[n-2] + \frac{1}{4} \delta[n-3] \\
 H(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} h[n] e^{-j\omega n} \\
 H(e^{j\omega}) &= \frac{1}{4} [1 + e^{-j\omega} + e^{-2j\omega} + e^{-3j\omega}] \\
 H(e^{j\omega}) &= \frac{1}{4} e^{-j1.5\omega} [e^{j1.5\omega} + e^{j0.5\omega} + e^{-j0.5\omega} + e^{-j1.5\omega}] \\
 H(e^{j\omega}) &= \frac{1}{4} e^{-j1.5\omega} [2\cos(1.5\omega) + 2\cos(0.5\omega)] \\
 H(e^{j\omega}) &= \frac{2\cos(1.5\omega) + 2\cos(0.5\omega)}{4} e^{-j1.5\omega}
 \end{aligned}$$

2. Implement this equation directly in code and plot the magnitude and phase response of this filter.

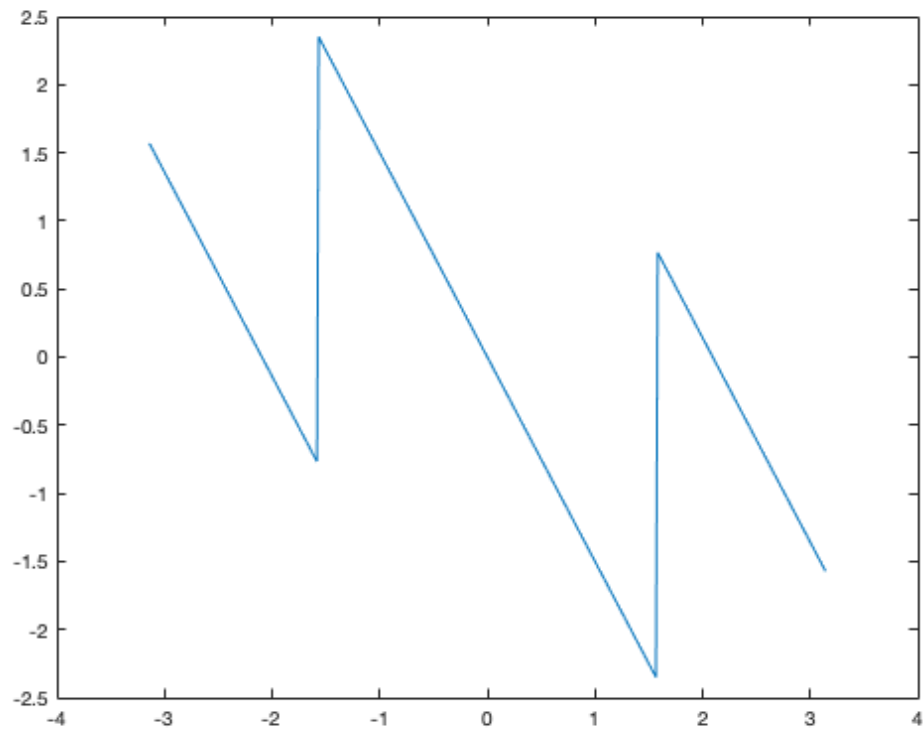
```
ww = -pi:(2*pi/399):pi;
H = 1/4*(2*cos(0.5*ww) + 2*cos(1.5*ww)).*exp(-j*1.5*ww);
mag = abs(H);
```

```
ang = angle(H);
```

```
plot(ww,mag);
```

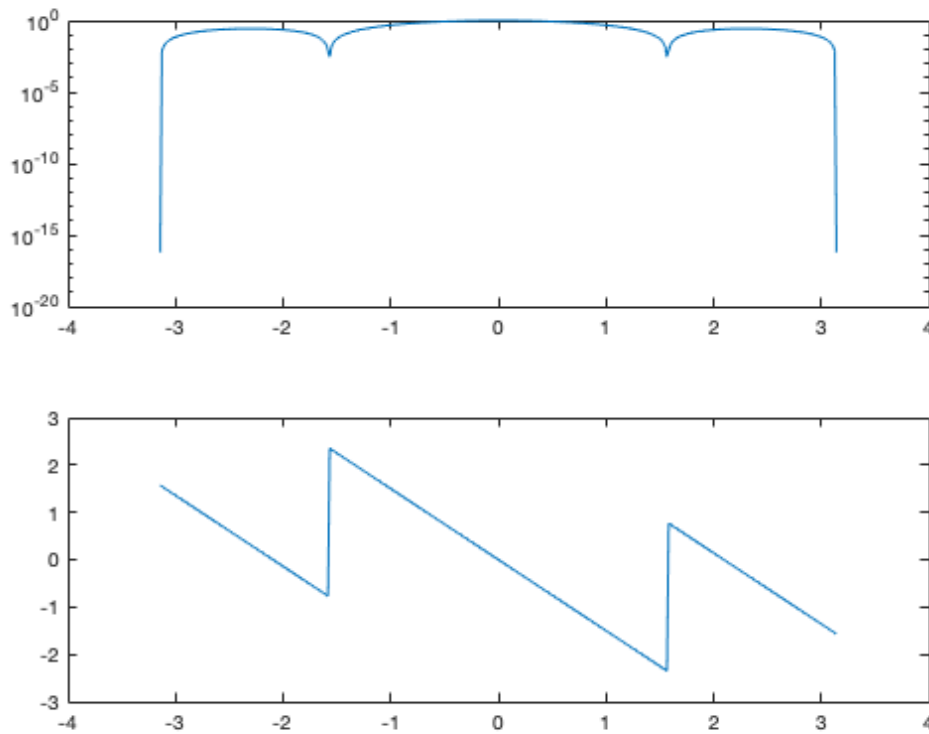


```
plot(ww,ang);
```



3.

```
bb = ones(1,4) / 4;  
aa = 1;  
N = 400;  
  
[HH,ww] = freekz(bb,aa,ww,'whole');  
subplot(2,1,1), semilogy(ww,abs(HH));    %-- magnitude  
subplot(2,1,2), plot(ww,angle(HH));
```



## 1.2 MATLAB find() function

```
b = ones(1,4) / 4;
w = -pi:pi/500:pi;
H = freekz(b,1,w);
index = find(abs(H) < 0.001)
```

index =

1            251            751            1001

Question 2: Does this match the frequency response that you plotted for the 4-point average?

Yes, it does, as you can see that the indexes occur at approximately 0,  $\pi/2$ ,  $3\pi/2$ , and  $2\pi$ . This matches the zeroes on the above plot.

```
index * pi/500
```

ans =

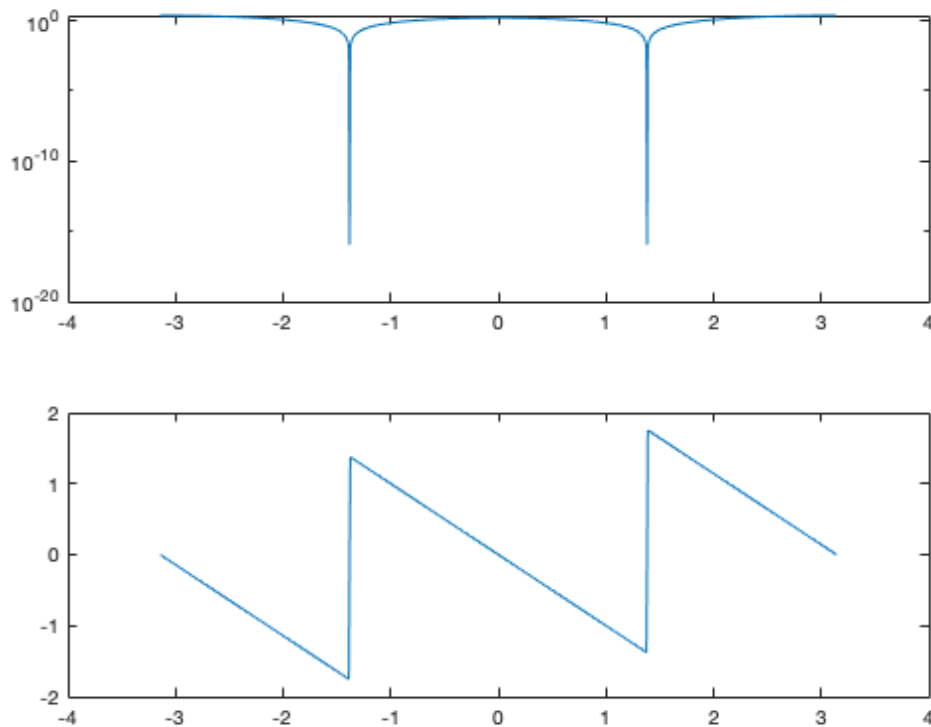
0.0063    1.5771    4.7187    6.2895

### 1.3 Nulling Filter

1. Design a filtering system that consists of the cascade of two FIR nulling filters that will eliminate the following input frequencies:  $\omega = 0.44\pi$ , and  $\omega = 0.7\pi$ . For this part, derive the filter coefficients of both nulling filters. Submit necessary code, and plots of the magnitude and phase responses of both filters, along with the cascaded system.

Filter 1.

```
wn = 0.44*pi;  
b0 = 1;  
b1 = -2*cos(wn);  
b2 = 1;  
  
filt1 = [b0,b1,b2];  
ww = -pi:pi/500:pi;  
filt1H = freekz(filt1,1,ww);  
  
subplot(2,1,1), semilogy(ww,abs(filt1H)); %-- magnitude  
subplot(2,1,2), plot(ww,angle(filt1H));
```



Filter 2.

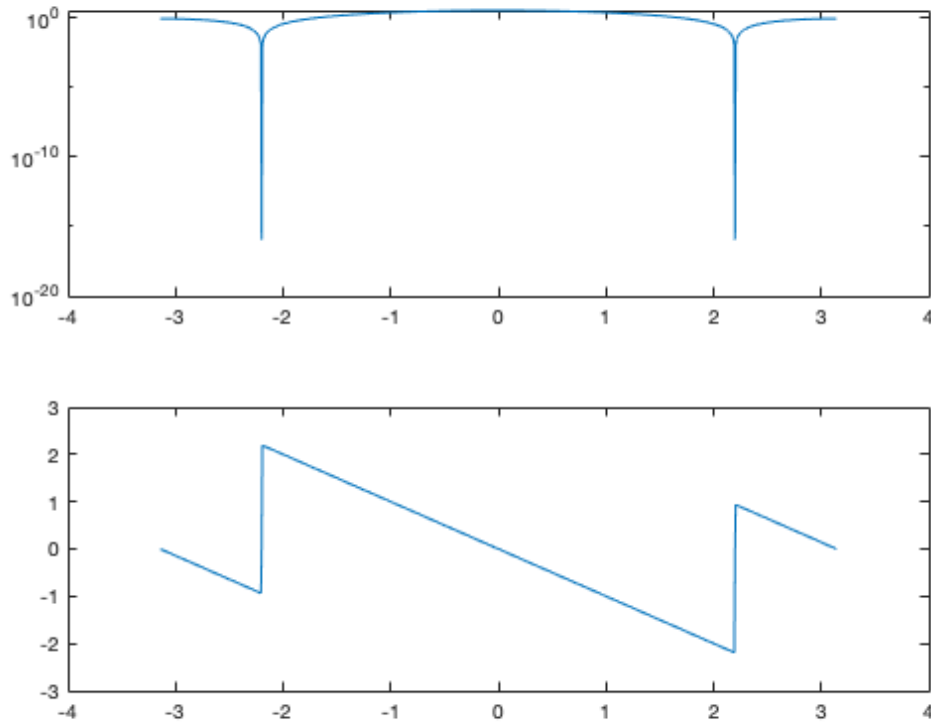
```
wn = 0.7*pi;  
b0 = 1;  
b1 = -2*cos(wn);  
b2 = 1;
```

```

filt2 = [b0,b1,b2];
ww = -pi:pi/500:pi;
filt2H = freekz(filt2,1,ww);

subplot(2,1,1), semilogy(ww,abs(filt2H)); %-- magnitude
subplot(2,1,2), plot(ww,angle(filt2H));

```



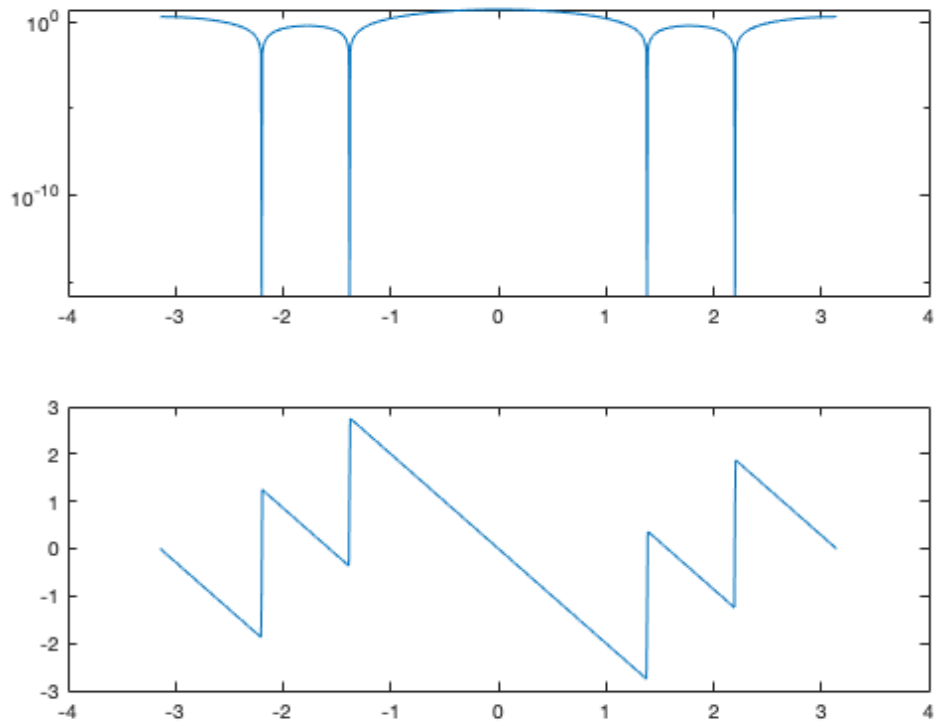
Cacaded filter.

```

ww = -pi:pi/500:pi;
casc = conv(filt1,filt2);
cascH = filt1H .* filt2H;

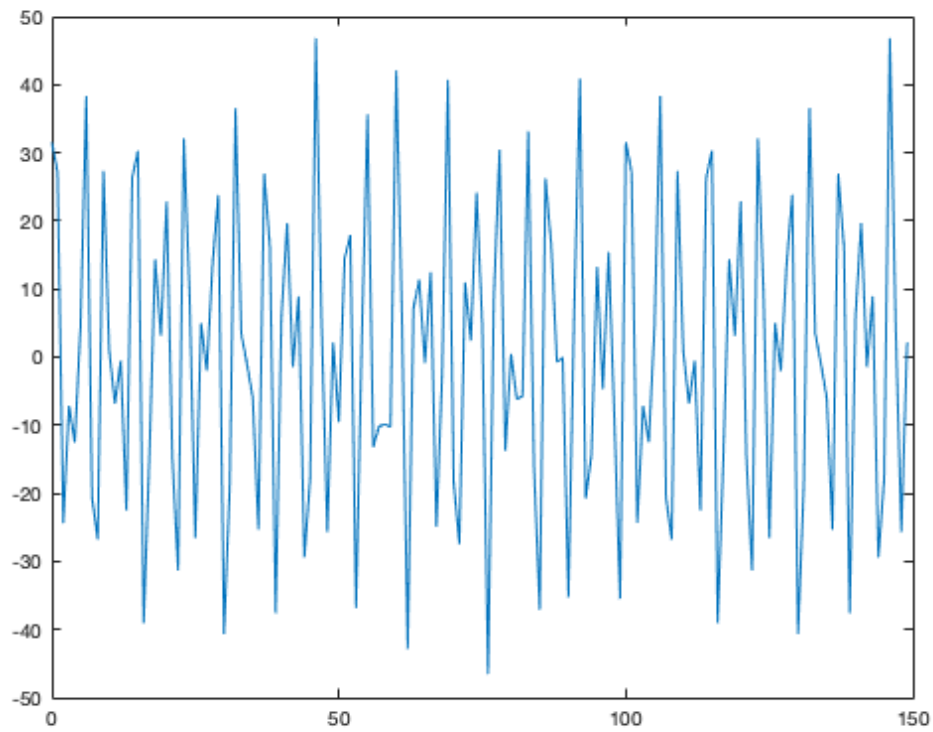
subplot(2,1,1), semilogy(ww,abs(cascH)); %-- magnitude
subplot(2,1,2), plot(ww,angle(cascH));

```



2. Generate an input signal that is the sum of three sinusoids

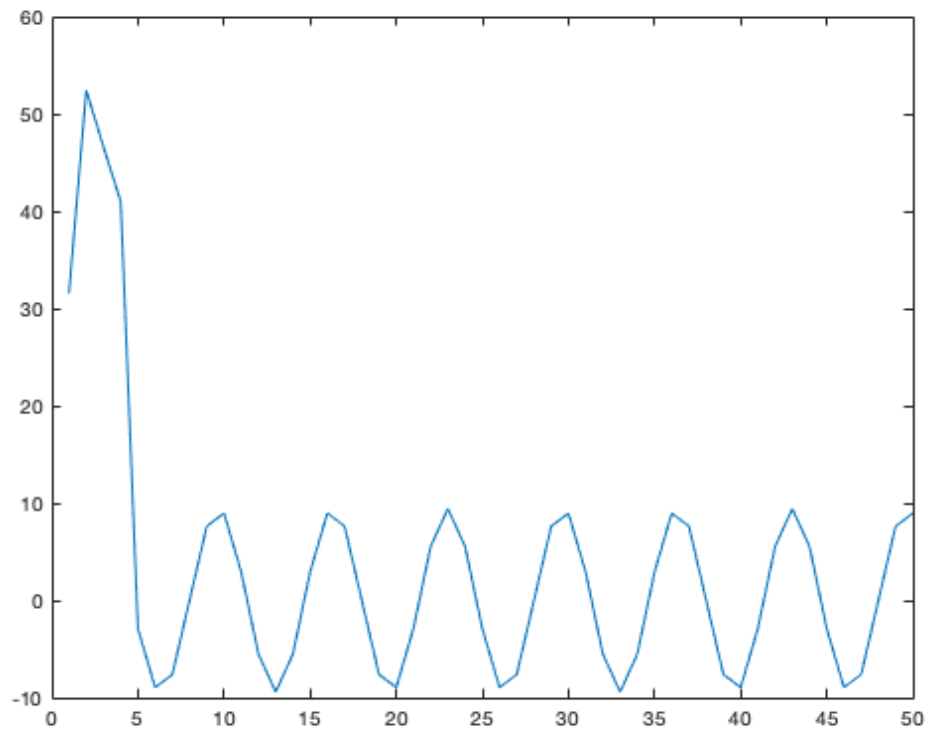
```
n = 0:1:149;  
xx = 5*cos(0.3*pi*n) + 22*cos(0.44*pi*n - pi/3) + 22*cos(0.7*pi*n - pi/4);  
clf  
plot(n,xx)
```



4.

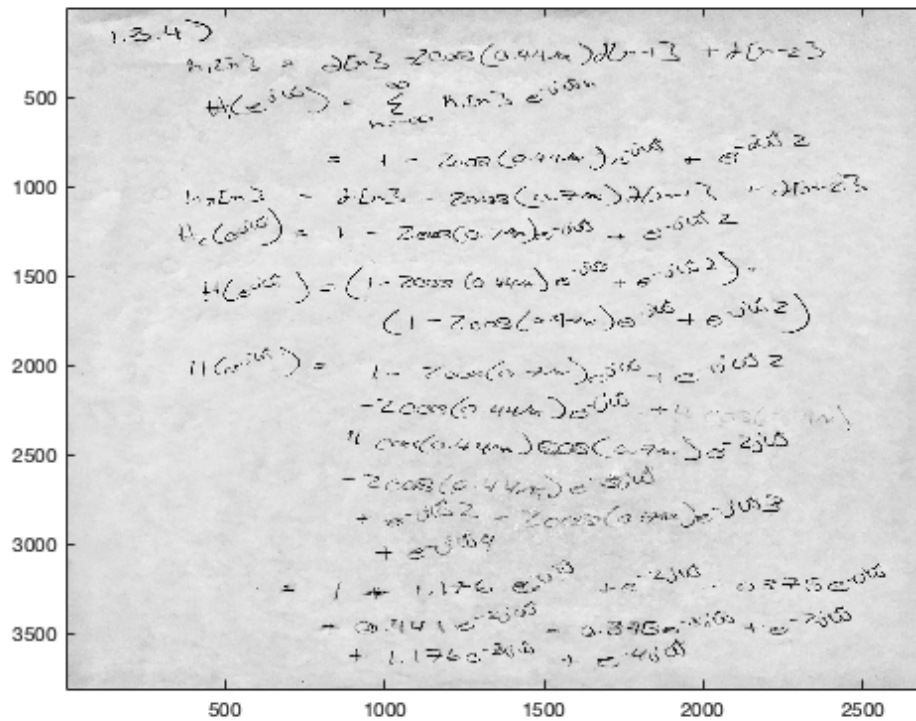
```
clf
out = conv(xx,casc);
plot(out(1:50));
```



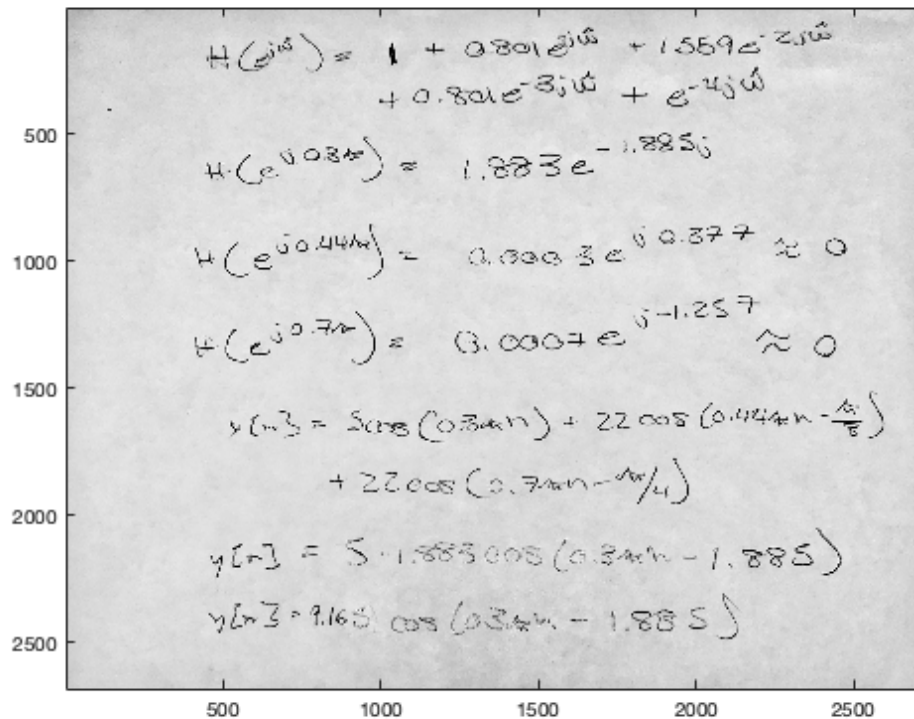


Determine (by hand) the exact mathematical formula (magnitude, phase and frequency) for the output signal for  $n \geq 5$ .

```
pg1 = imread("1.3.4 pg1.jpg");  
image(pg1);
```



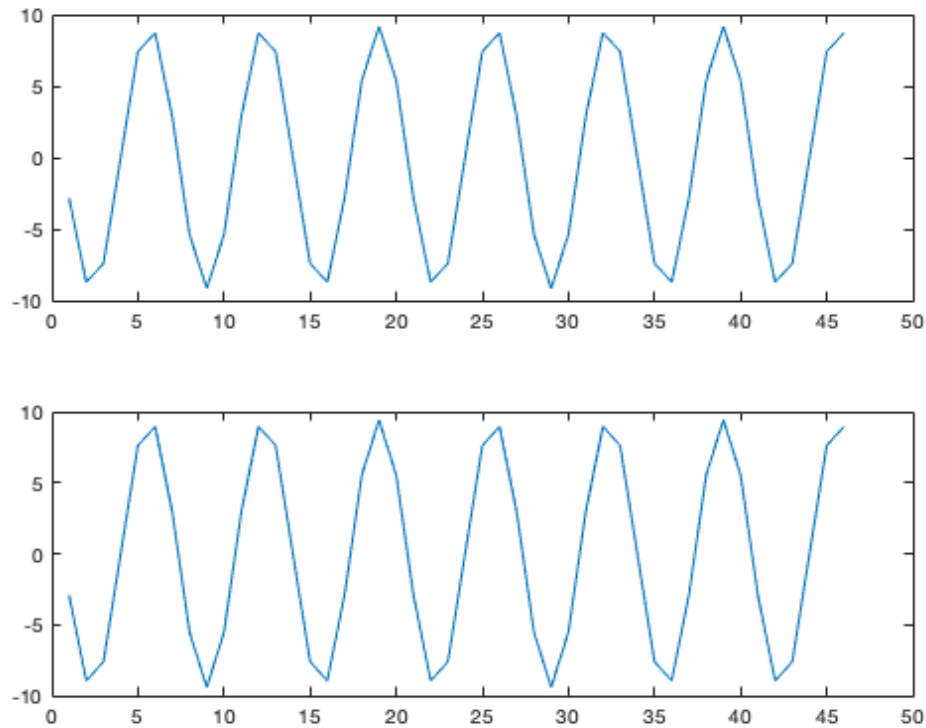
```
pg2 = imread("1.3.4 pg2.jpg");
image(pg2);
```



Plot the expected output of the cascaded filter and compare it to the plot obtained in 1.3.4 to show that it matches the filter output over the range  $5 \leq n \leq 50$ .

The above plot is the expected output of the cascaded filter in the range  $5 \leq n \leq 50$ . The below plot is the actual output of the cascaded filter in the range  $5 \leq n \leq 50$ . As you can see, in this sample range, they are literally identical.

```
yy = 9.165 * cos(0.3*pi*n - 1.885);
clf
subplot(2,1,1), plot(yy(5:50))
subplot(2,1,2), plot(out(5:50))
```



6.

Question 3: Explain why the output signal is different for the first few points. How many "start-up" points are found? How is this number related to the lengths of the filters designed previously?

The reason that the output signal is different for the first few points is because in reality, there is also a transient response from an FIR filter and not just the theoretical steady-state response that was obtained from my derivation. This transient response period is known as the settling period. For a length  $N$  filter we expect to find  $N-1$  samples in the transient response. Since our cascaded filter is of length 5 ( $3 + 3 - 1 = 5$ ), then we expect the first 4 samples to be the transient response. Thus, when we look from 5 samples onwards, they are the same.

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