Wasserstoff

$R_W = 1fm \sqrt[3]{A}$

Lösungen: $\psi_{nlm} = R_{nl}(r)\, Y_{lm}(\vartheta,\varphi)$

E. Energien $E_n = -R_y \frac{Z^2}{n^2}$ $\quad R_y = \frac{\mu \cdot e^4}{8\varepsilon_0^2 h^2} = -13{,}6eV$

Radius $R_W = 1fm \sqrt[3]{A}$ $\quad r_n = \frac{(\hbar c)n^2}{mc^2 Z_{eff}} \hat{=} 1fm$

Bohrsche Radius $a_B = \frac{4\pi\varepsilon_0 \hbar^2}{\mu e^2}$

Feinstruktur Aufspaltung $E_{nj} = E_n \left[1 + \frac{(Z\alpha)^2}{n}\left(\frac{1}{j+\frac{1}{2}} - \frac{3}{4n}\right)\right]$

Lamb-shift: Quantenfluktuationen am Kern ⟹ je stärker desto näher Aufhebung Feinstruktur

Helium

Zentralfeldnäherung $\psi(\vec{r}_1,\vec{r}_2) = \psi_a(\vec{r}_1)\,\psi_b(\vec{r}_2)$

Bindungsenergie $E_{He} = -2^2 \cdot 13{,}6eV - 13{,}6eV = 68eV$ $\quad$ realität $= -79{,}93eV$

Pauli-Verbot Fermionen Antisym Bosonen sym

Spin-Wellenfkt $S=0$ Triplett $S=1$ Singulett

Termschema Helium $S=0$ Parahelium $S=1$ Orthohelium ↓ Höhere En.

Dipolstr. übergänge (Kein $J=0 \to 0$)

$\Delta L = \pm 1 \quad \Delta m_L = \pm 1 \quad \Delta S = 0 \quad \Delta J = 0, \pm 1$

Mehrelektronensysteme

(Leiche Atom)

LS-Kopplung $|L_1 - L_2| \le L \le |L_1 + L_2|$ $\quad J = L + S$

$S=0$ Antisym $\quad (-1)^L = 1$ Sym $\quad |S_1 - S_2| \le S \le |S_1 + S_2|$

${}^{2S+1}L_J$ $\quad$ Pauliverbot bei $n s n' d$

(Schwere Atome)

JJ Kopplung $|L_1 - S_1| \le j_1 \le |L_1 + S_1|$

$|j_1 - j_2| \le J \le |j_1 + j_2|$ $\quad |L_2 - S_2| \le j_2 \le |L_2 + S_2|$

Hundsche Regel $\quad 2n^2$ elektronen ?

S_1 $S_2 P_2$ $S_3 P_3 d_3$ $S_4 P_4 d_4 f_4$

1) Schale voll ⟹ $J = S = L = 0$

2) Schale offen ⟹ S_{max} ist am tiefsten

3) Zustand S_{max} mit L_{max} zsm

p: $+1 \quad 0 \quad -1$ $\quad L = +1 + 0 = 1 \quad S = \frac{1}{2} + \frac{1}{2}$

4) s, p, d, f Unterschalen < fast voll $J = |L - S|$; fast leer $J = L + S$ $\quad v = \frac{c}{\lambda}$

Ionisationsenergie $E_{ION} = R_y \frac{Z_{eff}^2}{n^2}$ $\quad E = h\nu$ (nicht $J=0 \to 0$)

Dipolübergänge $\Delta L = \pm 1 \quad \Delta M = 0, \pm 1 \quad \Delta J \quad \Delta S = 0$

Linienbreite $\Delta E \tau \approx \hbar$ $\quad \gamma = \frac{1}{\tau_i} + \frac{1}{\tau_k}$

Leistungsdichte $P(\omega) = P_0 \frac{\gamma / 2\pi}{(\omega - \omega_0)^2 + (\gamma/2\pi)^2}$

Röntgenstrahlung $\Delta E = -(Z - \sigma)^2 R_y \left(\frac{1}{n^2} - \frac{1}{m^2}\right)$

max λ Röntgen $\lambda_g = \frac{hc}{eU_B}$

Brag reflexion $2d \sin(2\theta) = n \cdot \lambda$

Absorbtion Röntgen $I(d) = I_0 e^{-\mu d}$

Moseley Gesetz $\sqrt{\frac{1}{\lambda_K}} \propto (Z - 1)$ $\quad E = h\nu$ $\quad \lambda \hat{=} K$-Kante λ

Fundamentale Materiebausteine und WW

Hadronen	3 Quarks	Fermionen	$\frac{1}{2}$ spin
Mesonen	2 Quarks	Bosonen	1 spin
Leptonen	e, ν	Austauschteilchen	photon, Boson

WW zwischen Teilchen	strong	em.	weak
Lepton.	–	✓	✓
neutrino	–	–	✓
Quarks	✓	✓	✓

Teilchen $\pi^+(u\bar{d})\ \pi^-(\bar{u}d)\ K^+(u\bar{s})\ K^-(\bar{u}s)$ $\quad p(uud)\ n(udd)\ \Lambda^0(uds)$

Relativitische Kinematik

Energie-Impuls $E^2 = \vec{p}^2c^2 + m^2c^4$

4er-Vektor $(E, c\vec{p})$ $\quad a \cdot b = a_0 b_0 - \vec{a}\vec{b}$

Ruhesystem $p^2 = m^2c^4$

E_{cms}^2 Schwerpunktsenerg. $s = (p_1 + p_2)^2$

Hochrel. näherung $E_i \approx c|p_i|$

Hochrel teilchen auf Target $s = 2Mc^2E_1 + M^2c^4$

Rel Kinematik $\beta = \frac{v}{c}$ $\quad \gamma = \frac{1}{\sqrt{1-\beta^2}}$ $\quad \beta\gamma = \frac{|p|}{mc}$

$E = \gamma mc^2 \quad |p| = \gamma m \beta c \quad \beta = \frac{c|p|}{E} \quad \gamma = \frac{E}{mc^2}$

Zerfall $A \to 1 + 2$ $\quad m_A = \sqrt{(q_1 + q_2)^2} / c^2$

Streukinematik $q^2 = -4c^2 |p_f|^2 \sin^2\frac{\theta}{2}$

Skalarprod. $\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cos\theta$

Beschreibung von Streuprozessen

Hamilton-Operator $\hat{H} = \hat{H}_0 + \hat{H}_{WW}$

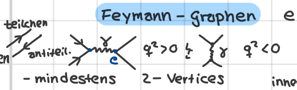
übergangsmatrixelement

$A(i \to f) = A_{fi} = \langle f | \hat{H}_{WW} | i \rangle$

$\Gamma = \frac{A_{fi}}{t}$

Feynman-Graphen

elm



$q^2 > 0$ $\quad q^2 < 0$

- mindestens 2-Vertices
- Vertex gilt 4er-Impuls

innere Linien ≙ propagator

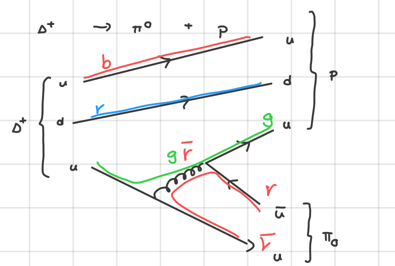
Tree-Level $A_{fi} \sim \frac{e^2}{q^2} \sim \frac{\alpha}{q^2}$

Loop-Level $A \sim e^4 \sim \alpha^2$ $\quad$ 2 photonen

2loop level $\quad$ 3-photonen

Laufende Kopplungskonstante $\alpha(q^2)$

Feymann Diagram starke WW



Wikungsquerschnitt

N_i Rate; A oberfläche; V_i geschw.; n_i dichte st.

Teilchenfluss $\Phi = \frac{\dot{N}_i}{A} = n_i v_i$

diff Wirkungsquer. $\left(\frac{d\sigma}{d\Omega}\right) = \frac{d\dot{N}_S(\theta,\varphi)}{\Phi N_t\, d\Omega}$ $\quad$ # targets

tot. Wirkungsquer $\sigma = \frac{\dot{N}_S}{\Phi N_t} = \int_{4\pi} \left(\frac{d\sigma}{d\Omega}\right) d\Omega$

barn $1b = 10^{-28} m^2$

Gesamtrate $N_S = \Phi N_t A_t$ $\quad N_S = \mathcal{L}_{int}\, \sigma$ $\quad A_t = \pi R_t^2$

dünnes Target $\sigma = \frac{\dot{N}_S}{\Phi N_t} = A_t = \pi R_t^2$

p-p WW $b_{max} = 2R_p \approx 1{,}6 fm$ $\quad$ Avoga dro

#Streuzentren $\boxed{N_t = n_t A d}$ $\quad n_t = \frac{\mathcal{S} N_A}{M_{mol}}$

WW Streurate Teilchenstrahl $\boxed{\dot{N}_S = \dot{N}_i n_t \cdot d \cdot \sigma}$

Dickes Target $dW = \frac{\dot{N}_S}{\dot{N}_i} = n_t\, dx\, \sigma$

freie Weglänge $\boxed{\lambda = \frac{1}{n_t \sigma} = \frac{M_{mol}}{\mathcal{S} N_A \sigma}}$

Luminosität $\boxed{L = \Phi N_t}$ $\quad \mathcal{L}_{int} = \int_T \mathcal{L}\, dt$

Kollision 2 Strahlen $\dot{N}_S = ((v_1 + v_2) n_1 n_2 V)\, \sigma$

Speicherring $\mathcal{L} = f_n \frac{N_A N_B}{A}$ $\quad N_S = \frac{\mathcal{S} N_A}{M_{mol}}$

Lebensdauer und Zerfallsbreite

$N(t) = N_0 e^{-\frac{t}{\tau}}$ $\quad T_{1/2} = \tau \ln 2$

W'heit dichte $P(E) = \frac{1}{2\pi} \frac{\Gamma}{(E - mc^2) + \Gamma/4}$

Linienbreite $\boxed{\Gamma\tau = \hbar}$

Zerfallsbreite $\Gamma_i = B_i \Gamma$

Fermi's Goldene Regel

Übergangsrate $\frac{1}{\tau} = \frac{\Gamma_{fi}}{\hbar} = W_{fi}$

$\sigma = \frac{V}{v} W_{fi}$

Fermi's g. Regel $W_{fi} = \frac{2\pi}{\hbar} |A_{fi}|^2 \mathcal{S}(E_f)$

Zustandsdichte $\mathcal{S}(E_f) = \frac{dN}{dE}(E_f)$

Phasenraumzelle $dN = \frac{Ldp_x}{2\pi\hbar}$ $\quad dN = \frac{Vd^3p}{h^3}$ $\quad N = \frac{V \int d^3p}{(2\pi\hbar)^3}$

1T-Zustandsdichte $\mathcal{S} = \frac{V}{(2\pi\hbar)^3} \frac{pE}{c^2} \int d\Omega$

2T.-Zustandsdichte $\mathcal{S} = \frac{V}{(2\pi\hbar)^3} \frac{P_1}{c^2} \frac{E_1 E_2}{E_1 + E_2} \int d\Omega$

$a + b \to 1 + 2$

Wirkungsquerschnitt 2T.

$\sigma = \frac{1}{2c} \frac{2\pi}{\hbar} \frac{1}{(2\pi\hbar)^3} \frac{|P_1|}{c^2} \frac{E_1 E_2}{E_1 + E_2} \int |A_{fi}|^2 d\Omega_1$

falls $m_1 = m_2$ $\quad \frac{d\sigma}{d\Omega} \sim (E_1 + E_2)^2 |A_{fi}|^2$

2-Teilchenzerfallsbreite $\quad$ bei $m_1 = m_2$ $\quad \frac{d\Gamma}{d\Omega} \sim m_A |p_1| |A|$

$\Gamma = 2\pi \frac{1}{(2\pi\hbar)^3} \frac{|P_1|}{c^2} \frac{E_1 E_2}{m_A c^2} \int |A_{fi}|^2 d\Omega$

Raumwinkel $\Omega = \frac{A}{r^2}$

W'keit WW $P = \frac{N_S}{N_i}$

WW von Teilchen mit Materie

Bethe-Bloch-Formel $\quad$ e-dichte $n_e = \left(\mathcal{S} N_A \frac{Z}{A}\right)$

$\left\langle \frac{dE}{dx} \right\rangle_{ion} = n_e 4\pi r_e^2 m_e c^2 z^2 \frac{1}{\beta^2} \left(\ln \frac{2m_e c^2 \gamma^2 \beta^2}{I} - \beta^2\right)$

$\frac{1}{\mathcal{S}} \left\langle \frac{dE}{dx} \right\rangle = 0{,}307 \frac{MeV}{g} cm^2 z^2 \frac{Z}{A} \frac{1}{\beta^2} \left(\ln \frac{2m_e c^2 \gamma^2 \beta^2}{I} - \beta^2\right)$

$-\frac{1}{\mathcal{S}} \left\langle \frac{dE}{dx} \right\rangle = 2 \frac{MeV\, cm^2}{g}$

Energieverlust Elektronen $\left\langle \frac{dE}{dx} \right\rangle = \left\langle \frac{dE}{dx} \right\rangle_{ion} + \left\langle \frac{dE}{dx} \right\rangle_{Brems}$

$\left\langle \frac{dE}{dx} \right\rangle_{Brems} = -\frac{1}{X_0} E$ ⟹ $E(x) = E_0 e^{-\frac{x}{X_0}}$

$\frac{1}{X_0} = \mathcal{S} N_A \frac{Z^2}{A} \cdot 4\alpha \cdot r_e^2 \ln\left(\frac{183}{Z^{\frac{1}{3}}}\right)$

Cherenkov-Strahlung $\quad \cos\theta = \frac{1}{\beta n}$ $\quad$ Str. $v > \frac{c}{n}$

WW Photon mit Materie

$I(x) = I_0 e^{-\mu x}$ $\quad \mu = \mu_{ph} + \mu_{comb} + \mu_{paar}$

Photoeffekt $\sim \frac{1}{E_\gamma}$

Compton-Effekt $\Delta\lambda = \frac{h}{cme}(1 - \cos\theta)$

$E'_\gamma = E_\gamma \frac{1}{1 + \frac{E_\gamma}{mec^2}(1 - \cos\theta)}$

$E'_{\gamma\, min} = \frac{E_\gamma}{1 + 2\varepsilon}$ $\quad E_{kin,max} = E_\gamma - E'_\gamma = E_\gamma \frac{2\varepsilon}{1 + 2\varepsilon}$

Paarbildung $> 2{,}04 MeV$

$\sigma_{paar} = \frac{7}{9} \cdot 4\alpha r_e^2 Z \ln \frac{183}{Z^{\frac{1}{3}}} = \frac{7}{9} \frac{A}{\mathcal{S} N_A} \frac{1}{X_0}$

Inverser comp. eff. $\quad E'_\gamma > E_\gamma$

Paarbildung $E_\gamma > 1{,}02 MeV$ an Kernen; $E_\gamma > 2{,}04 MeV$ für Rückstoß an Elektron

Paar. prod. $\sigma_{paar} = \frac{7}{9} \frac{A}{\mathcal{S} N_A} \frac{1}{X_0}$

mittlere Weglänge bis zur Paarbildung $\lambda_{paar} = \frac{1}{n\sigma} = \frac{9}{7} X_0$

Elektromagnetischer Schauer

$x_{max} = \frac{\ln E / E_c}{\ln 2} X_0$

Hadronischer Schauer

$\lambda_{ww} = \frac{1}{n\sigma_{ww}} = \frac{A}{\mathcal{S} N_A} \frac{1}{\sigma_{ww}}$

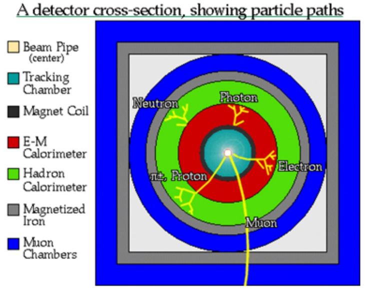
$n, p, \pi^\pm, K^\pm, K^0, A \longrightarrow$ sek. Hadronen

Detektoren

Spurdetektor $\quad$ Elektromag. $\quad$ Kalorimeter

Spektrometer $\quad$ Myon detekto

A detector cross-section, showing particle paths



Symmetrie und Erhaltungssätze

Noether $\cdot$ Kont Symm → add. QZ; $\cdot$ DISK Symm → mult. QZ $\quad 0 = [\hat{H}, \hat{Q}] = \frac{dQ}{dt}$

S, U, em Ladungserhaltung Q_{el}: π^- -1; p 1; π^0 0; π^+ $+1$; π^- -1; n 0

S, em Leptonenzahlerhalt. $L_\mu = const$, $L_\tau = const$, $L_e = const$, $L = L_e + L_\mu + L_\tau = const$

$L_e(e^-) = +1 \quad L_e(e^+) = -1 \quad L_e(\nu_e) = +1 \quad L_e(\bar{\nu}_e) = -1$

S, U, em Baryonenzahl $q = \frac{1}{3} \quad \bar{q} = \frac{1}{3}$ $\quad$ Mesonen $= 0$; Baryonen $= 1$; Antibaryone $= -1$

S. WW Isospinn $\quad p = |\frac{1}{2}, +\frac{1}{2}\rangle \quad n = |\frac{1}{2}, -\frac{1}{2}\rangle$ $\quad \pi_+ |1,1\rangle \quad \pi_0 |1,0\rangle \quad \pi_- |1,-1\rangle$ $\quad$ u Quark $\frac{1}{2} + \frac{1}{2}$; d $\frac{1}{2} - \frac{1}{2}$

Verhältnis $\frac{\sigma_1}{\sigma_2} = \frac{|\langle I = 1, I_3 = 0 | \hat{A}_{ww} | pn \rangle|^2}{|\langle I = 1, I_3 = 1 | \hat{A}_{ww} | pp \rangle|^2} = \frac{\langle I, I_3 | p \rangle}{\langle I, I_3 | pn \rangle}$ $\quad$ Clebsch Gordo

Wellenfkt $\psi = \Phi_{Raum}\, \chi_{spin}\, \xi_{isospin}$

S WW Strangeness $\quad$ Quarks $S = 1 \quad \bar{S} = -1$

Teilchen	K^0	$\bar{K}^0$	K^+	K^-	Λ	$\bar{\Lambda}$
S	+1	-1	+1	-1	-1	+1

Parität $\hat{P}(p = n = \Lambda = \Lambda_c = \Lambda_b =) + 1$

rel Drehimpuls $\eta_P(T) = \eta_P(1) \cdot \eta_P(2) \cdot (-1)^L$ $\quad$ rel Bahndreh

$J^P(\pi^+) = 0^- \quad J^P(\pi^0) = 0^- \quad J^P(p) = \frac{1}{2}^+ \quad J^P(n) = \frac{1}{2}^+$

Paritätsverletzung (weak WW)

Zerfall $\quad$ Teilchen LH; antiteilchen RH

Ladungskonjugation $\quad \hat{C}|\text{teilchen}\rangle = |\text{Antiteilchen}\rangle$ $\quad \eta_C(\gamma) = -1$

$\pi^0 \to \gamma + \gamma$ $\quad C\ (+1) \quad (-1) \cdot (-1)$

Dipolmoment $\mu_{el} = e\vec{d}$

	$\vec{x}$	$\vec{p}$	$\vec{L}\ \vec{s}$	$h = \vec{L} \cdot \hat{p}$	Q_{el}	Lep.	Bar.	Farb	I_3	E	B
$\hat{C}$	+	+	+	+	–	–	–	–	–	–	–
P	–	–	+	–	+	+	+	+	+	–	+
$\hat{T}$	+	–	–	+	+	+	+	+	+	+	–

Symmetrie	starke WW	e.m WW.	schwache WW
P	✓	✓	–
C	✓	✓	–
T	✓	✓	–
CP	✓	✓	–
CPT	✓	✓	✓

Erhaltungsgröße	Stark	Em	Schwach
Energie Impuls	✓	✓	✓
Ladung	✓	✓	✓
Baryonenzahl	✓	✓	✓
Leptonenzahl	✓	✓	✓
Isospin	✓	✗	✗
Strangeness	✓	✓	✗
Charm	✓	✓	✗
Parität	✓	✓	✗
Ladungskonjugation	✓	✓	✗
CP	✓	✓	✗
CPT	✓	✓	✓

Kern und Nukleon Struktur

Rutherford

$$\left(\frac{d\sigma}{d\Omega}\right)_{Ruther} = \frac{z^2 Z^2 \alpha^2}{4E^2 \sin^4(\frac{\vartheta}{2})}(\hbar c)^2$$

$$\left(\frac{d\sigma}{d\Omega}\right)^{\text{nicht rel.}}_{Ruther} = \frac{z^2 Z^2 \alpha^2}{16 E_{KIN}^2 \sin^4(\frac{\vartheta}{2})}(\hbar c)^2$$

Mott

Streuung Spin $\frac{1}{2}$ an schweren Ta. ohne Spin

$$\left(\frac{d\sigma}{d\Omega}\right)_{mott} = \left(\frac{d\sigma}{d\Omega}\right)_{Ruther} \cos^2\left(\frac{\vartheta}{2}\right)$$

$$\left(\frac{d\sigma}{d\Omega}\right)^{\text{nicht rel}}_{mott} = \left(\frac{d\sigma}{d\Omega}\right)_{Ruther}\left(1-\beta^2 \sin^2\left(\frac{\vartheta}{2}\right)\right)$$

Formfaktor

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{mott} \cdot \underbrace{|F(q^2)|^2}_{\leq 1}$$

Ladungsverteilung	$f(r)$	Formfaktor $F(q^2)$
Punkt	$\delta(r)/4\pi$	1
exp.	$(a^3/8\pi)\exp(-ar)$	$(1+q^2/a^2/\hbar^2)^{-2}$
gauß	$(a^2/2\pi)^{\frac{3}{2}}\exp(-a^2r^2/2)$	$\exp(-q^2/2a^2\hbar^2)$
hom. Kugel	$\frac{3}{4}\pi R^3$ $r \leq R$; 0 $r > R$	$3\alpha^{-3}(\sin\alpha - \alpha\cos\alpha)$, $\alpha = \lvert q \rvert R/\hbar$

$$\rho(r) = \frac{\rho_0}{1+e^{(r-c)/a}}$$

Nucleonspin und mag. Momente

$$\vec{\mu}_{p,n} = g_{p,n}\,\mu_N \frac{\vec{S}}{\hbar} \qquad \mu_N = \frac{e\hbar}{2M_N} = 3{,}15 \cdot 10^{-14}\ \frac{\text{MeV}}{}$$

Elektron an Proton Streuung

$$\left(\frac{d\sigma}{d\Omega}\right)_{Dirac} = \left(\frac{d\sigma}{d\Omega}\right)_{Ruther} \cdot \left(\cos^2\left(\frac{\vartheta}{2}\right) + \frac{Q^2}{2c^4 M_p^2}\sin^2\frac{\vartheta}{2}\right)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{Dirac} = \left(\frac{d\sigma}{d\Omega}\right)_{mott}\left(1+2\tau\tan^2\frac{\vartheta}{2}\right) \qquad \tau = \frac{Q^2}{4c^4M_p^2}$$

Rückstoß

$$\left(\frac{d\sigma}{d\Omega}\right)^{\text{Rück.}}_{Mott} = \left(\frac{E_f}{E_i}\right)\left(\frac{d\sigma}{d\Omega}\right)_{mott}$$

$$\left(\frac{d\sigma}{d\Omega}\right)^{\text{Rück}}_{Dirac} = \left(\frac{E_f}{E_i}\right)\left(\frac{d\sigma}{d\Omega}\right)_{Dirac}$$

Rosenbluth-Formel

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{mott} \cdot \left(\frac{G_E^2(Q^2)+\tau G_m^2(Q^2)}{1+\tau} + 2\tau G_m^2 \tan^2\frac{\vartheta}{2}\right)$$

$$G_E(0) = 1 \qquad G_m(0) = 2{,}79 \qquad \Rightarrow G_E = \frac{G_M(Q^2)}{2{,}79}$$

Dipol-Formel

$$G_E(Q^2) = \left(1+\frac{Q^2}{0.71\ \text{GeV}^2}\right)^{-2}$$

Proton: $\langle r^2 \rangle = \frac{12}{a^2} = 0{,}658\ \text{fm}^2$

Formfaktor $q = 2|p_f|\sin^2\left(\frac{\vartheta}{2}\right)$ — Nullstelle Ablesen

Nullstelle $|q|R \hat{=} 4{,}5$

starke WW

Farbladung

Quarks r, g, b

antiquarks $\bar{r}, \bar{g}, \bar{b}$

Gluonen $r\bar{g}$ $r\bar{b}$ $g\bar{r}$ $b\bar{g}$ $b\bar{r}$ $g\bar{b}$

$\frac{1}{\sqrt{2}}(r\bar{r}-g\bar{g})$, $\frac{1}{\sqrt{6}}(r\bar{r}+g\bar{g}+2b\bar{b})$

$\frac{1}{\sqrt{3}}(r\bar{r}+g\bar{g}+b\bar{b})$

Wellenfkt $\psi = \psi_{Raum}\,\Phi_{flavor}\,\chi_{spin}\,\xi_{farbe}$ (symm.; ξ antisym)

$$\xi = \frac{1}{\sqrt{6}}\sum_{r,g,b}\varepsilon_{ijk}\,u_i u_j u_k$$

Feymann Vertices

Wirkungsquerschnitt

$$\sigma(c\bar{c}) = \underset{\#\text{farbe}}{N} \cdot \sum_q Q_{q_i}^2 \qquad Q_c = \frac{2}{3} \qquad Q_b = \frac{1}{3} \quad Q = u$$

$$\sigma_{Had} = \sigma_{\mu\mu}\cdot N_c \sum_q Q_{q_i}^2$$

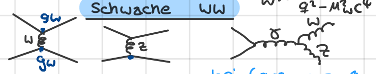
$$R_{Had} = \frac{\sigma_{Had}}{\sigma_{\mu\mu}} = N_c \sum_{q_i} Q_{q_i}^2$$

$\sigma \sim \frac{1}{s}$ bei Energie änderung

Energiebereich	Q.	R_{had}
$E < 3$ GeV	u, d, s	$3\cdot\left(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}\right) = 2$
$2m_cc^2 < E < 2m_bc^2 \approx 10$ GeV	u, d, s, c	$\frac{10}{3}$
$2m_bc^2 < E <$ 50 GeV	u, d, s, c, b	$\frac{11}{3}$

Schwache WW

$$W \sim \frac{1}{q^2 - M_W^2 c^4}$$



bei farg ma gw·3!

Schwacher Isospin $|T = \frac{1}{2}, T_3 = \pm\frac{1}{2}\rangle$

$$\begin{matrix}+\frac{1}{2}\\-\frac{1}{2}\end{matrix} \begin{pmatrix}\nu_e\\e^-\end{pmatrix}_{LH} \begin{pmatrix}\nu_\mu\\\mu^-\end{pmatrix}_{LH} \begin{pmatrix}\nu_\tau\\\tau^-\end{pmatrix}_{LH} \downarrow W^+ \uparrow W^-$$

Quarks

$$\begin{matrix}+\frac{1}{2}\\-\frac{1}{2}\end{matrix} \begin{pmatrix}u\\d\end{pmatrix}_{LH} \begin{pmatrix}c\\s\end{pmatrix}_{LH} \begin{pmatrix}t\\b\end{pmatrix}_{LH} \downarrow W^+ \uparrow W^-$$

β Zerfall – α-Zerfall

Neutronenzerfall $n \rightarrow p + e^- + \bar{\nu}_e$

β^--Kernzerfall ${}^A_ZX \rightarrow {}^{A}_{Z+1}Y + e^- + \bar{\nu}_e$

β^+-Kernzerfall ${}^A_ZX \rightarrow {}^A_{Z-1}Y + e^+ + \nu_e$

e^--Einfang (EC) ${}^A_ZX + e^- \rightarrow {}^A_{Z-1}Y + \nu_e$

${}^A_ZX \rightarrow {}^{A-4}_{Z-2}X + He^{2+}$

3 Teilchen phasenraum

$$\left(\frac{dw_{fi}}{p_e^2\,dp_e}\right)^{\frac{1}{2}} = \text{const}\left(|A_{fi}|^2\right)^{\frac{1}{2}}(E_0 - E_e)$$

Lebensdauer

$$\Gamma = E_0^5 \qquad \tau \sim \frac{1}{E_0^5} \qquad \tau = \frac{1}{\Gamma}$$

$$|A_{fi}|^2 = \frac{2\pi^3}{f\cdot\tau}\,\frac{\hbar^7}{m_e^5c^4}$$

Teilchen	Quark	Ladung	m_0c^2[MeV]	Spin
d	d	$-\frac{1}{3}$	4.67	1/2
u	u	$+\frac{2}{3}$	2.16	1/2
s	s	$-\frac{1}{3}$	93.4	1/2
c	c	$+\frac{2}{3}$	$1{,}27\cdot10^3$	1/2
b	b	$-\frac{1}{3}$	$4{,}18\cdot10^3$	1/2
t	t	$+\frac{2}{3}$	$0{,}17\cdot10^6$	1/2
e^+/e^-	–	+1/-1	0.511	1/2
μ^+/μ^-	–	+1/-1	105.66	1/2
τ^+/τ^-	–	+1/-1	$1{,}777\cdot10^3$	1/2
π^+/π^-	$u\bar{d}$ / $d\bar{u}$	+1/-1	139,6	0
π^0	$\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})$	0	135,0	0
K^+/K^-	$u\bar{s}$ / $s\bar{u}$	+1/-1	493,7	0
K^0	$d\bar{s}$	0	497,6	0
ρ^0	$\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})$	0	775,6	1
$\omega(1420)$	$\frac{1}{\sqrt{2}}(d\bar{d}+u\bar{u})$	0	$1{,}019\cdot10^3$	1
Φ	$s\bar{s}$	0	936.6	1
n	udd	0	928,3	1/2
p	uud	+1	$1{,}67\cdot10^3$	1/2
Ω^-	sss	-1	$1{,}672\cdot10^3$	3/2
Δ^0	udd	0	$1{,}232\cdot10^3$	3/2
Δ^+	uud	+1	$1{,}232\cdot10^3$	3/2
Δ^{++}	uuu	+2	$1{,}232\cdot10^3$	3/2
Δ^-	ddd	-1	$1{,}232\cdot10^3$	3/2
Λ	uds	0	$1{,}115\cdot10^3$	1/2
Σ^+	uus	+1	—	1/2
Σ^0	uds	0	—	1/2
Ξ^-	dds	-1		1/2
γ	–	0		1
W^+/W^-	–	+1/-1	$80{,}3\cdot10^3$	1
Z	–	0	$91{,}1\cdot10^3$	1
g	–	0	0	1
Higgs	–	0	$125{,}3\cdot10^3$	0

$m_0(\mu^-) = 105{,}6$ MeV

Schale		K	L		M			N
Z	Element	1s	2s	2p	3s	3p	3d	4s
1	H Wasserstoff	1						
2	He Helium	2						
3	Li Lithium	2	1					
4	Be Beryllium	2	2					
5	B Bor	2	2	1				
6	C Kohlenstoff	2	2	2				
7	N Stickstoff	2	2	3				
8	O Sauerstoff	2	2	4				
9	F Fluor	2	2	5				
10	Ne Neon	2	2	6				
11	Na Natrium	2	2	6	1			
12	Mg Magnesium	2	2	6	2			
13	Al Aluminium	2	2	6	2	1		
14	Si Silizium	2	2	6	2	2		
15	P Phosphor	2	2	6	2	3		
16	S Schwefel	2	2	6	2	4		
17	Cl Chlor	2	2	6	2	5		
18	Ar Argon	2	2	6	2	6		
19	K Kalium	2	2	6	2	6		1
20	Ca Calcium	2	2	6	2	6		2
21	Sc Scandium	2	2	6	2	6	1	2
20	Ti Titan	2	2	6	2	6	2	2
23	V Vanadium	2	2	6	2	6	3	2
24	Cr Chrom	2	2	6	2	6	5	1
25	Mn Mangan	2	2	6	2	6	5	2
26	Fe Eisen	2	2	6	2	6	6	2
27	Co Kobalt	2	2	6	2	6	7	2

Schale $2E^2$ –		K	L		M			N			O	
Z	Element	1s	2s	2p	3s	3p	3d	4s	4p	4d	5s	5p
28	Ni Nickel	2	2	6	2	6	8	2				
29	Cu Kupfer	2	2	6	2	6	10	1				
30	Zn Zink	2	2	6	2	6	10	2				
31	Ga Gallium	2	2	6	2	6	10	2	1			
32	Ge Germanium	2	2	6	2	6	10	2	2			
33	As Arsen	2	2	6	2	6	10	2	3			
34	Se Selen	2	2	6	2	6	10	2	4			
35	Br Brom	2	2	6	2	6	10	2	5			
36	Kr Krypton	2	2	6	2	6	10	2	6			
37	Rb Rubidium	2	2	6	2	6	10	2	6		1	
38	Sr Strontium	2	2	6	2	6	10	2	6		2	
39	Y Yttrium	2	2	6	2	6	10	2	6	1	2	
40	Zr Zirkonium	2	2	6	2	6	10	2	6	2	2	
41	Nb Niob	2	2	6	2	6	10	2	6	4	1	
42	Mo Molybdän	2	2	6	2	6	10	2	6	5	1	
43	Tc Technetium	2	2	6	2	6	10	2	6	6	1	
44	Ru Ruthenium	2	2	6	2	6	10	2	6	7	1	
45	Rh Rhodium	2	2	6	2	6	10	2	6	8	1	
46	Pd Palladium	2	2	6	2	6	10	2	6	10		
47	Ag Silber	2	2	6	2	6	10	2	6	10	1	
48	Cd Cadmium	2	2	6	2	6	10	2	6	10	2	
49	In Indium	2	2	6	2	6	10	2	6	10	2	1
50	Sn Zinn	2	2	6	2	6	10	2	6	10	2	2
51	Sb Antimon	2	2	6	2	6	10	2	6	10	2	3
52	Te Tellur	2	2	6	2	6	10	2	6	10	2	4
53	I Iod	2	2	6	2	6	10	2	6	10	2	5
54	Xe Xenon	2	2	6	2	6	10	2	6	10	2	6

Klausur aufgabe

Verhältnis Zerfalls Kanäle

$$R = \frac{\Gamma(\Delta^0 \rightarrow p + \pi^-)}{\Gamma(\Delta^0 \rightarrow n + \pi^0)} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

Streuung

$$\sigma = \int_{4\pi}\frac{d\sigma}{d\Omega}d\Omega = \frac{\dot{N}_S}{\Phi\, N_t} \qquad [\sigma] = 1\text{b}$$

$$N_t = n_t \cdot \underbrace{A\cdot d}_{V} = \frac{\rho N_A}{M_{mol}}\cdot A\cdot d$$

$$\underset{\text{streurate}}{\dot{N}_S} = \underset{\text{T. Intensität}}{\dot{N}_i} \cdot n_t \cdot d \cdot \sigma$$

Raum winkel $\Delta\Omega = \frac{A}{r^2}$

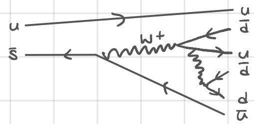
Neutrino WW

$$\lambda = \frac{1}{n_t\sigma} = \frac{M_{mol}}{\rho N_A\,\sigma} \qquad n_t = \frac{10 N_A}{18}$$

$$\Phi = \Phi_0\exp\left(-\frac{x}{\lambda}\right) \qquad \Phi_0 = 2\cdot10^6\ \frac{1}{\text{cm}^2}$$

Schwacher Zerfall

$$K^+ \rightarrow \pi^+ + \pi^+ + \pi^+$$



Energieverlust

$$\frac{1}{\rho}\left\langle\frac{dE}{dx}\right\rangle = 2\ \text{MeV}\ \frac{\text{cm}^2}{\text{g}}$$

Zerfall Teilchen

$\gamma \cdot \tau = t$

$$M_0^2c^4 = 4M_\pi^2c^4 + 2c^2|p|^2(1-\cos\vartheta)$$

mindest energie

$$\begin{pmatrix}e^+ & e^-\\ p_+ & p_-\end{pmatrix}^2 = \begin{pmatrix}w^+ & & w^+\\ 0 & + & 0\end{pmatrix}^2$$

Kerne, Kernmodelle Kernzerfall

Massenspektrometer $\frac{m}{q} = R\cdot\frac{B_1B_2}{E}$ $\qquad \omega_c = \frac{q}{m}\cdot B$

Bindungsenergie $B(A,Z) = (Z\cdot m_p + (A-Z)m_n - m(A,Z))\cdot c^2$

$m_p = 938$ MeV $\quad m_n = 939$ MeV

Bethe-Weizsäcker

$$B(A,Z) = a_V A - a_0 A^{\frac{2}{3}} - a_c\frac{Z^2}{A^{\frac{1}{3}}} - a_S\frac{(N-Z)^2}{A} + \frac{\delta}{A^{\frac{1}{2}}}$$

stark WW

Volumenbeitrag $a_V = 15{,}67$ MeV

s. WW

Oberflächenbeitrag $a_0 = 17{,}23$ MeV

e.m

Coulomb-Abstoßung $a_c = 0{,}71$ MeV

Symmetrieterm $a_S = 23{,}3$ MeV

$$\text{Paarungsterm} = \begin{cases} +11.2\ \text{MeV} & Z \text{ und } N \text{ gerade}\\ 0\ \text{MeV} & Z \text{ oder } N \text{ gerade}\\ -11{,}2\ \text{MeV} & Z \text{ und } N \text{ ungerade}\end{cases}$$

Massenformel $M(A,Z) = Z\cdot m_p + (A-Z)m_n - B(A,Z)/c^2$

β-Zerfall $M(A,Z)$ als Fkt von Z

$$M(A,Z) = \alpha\cdot A - \beta\cdot Z + \gamma Z^2 + \frac{\delta}{A^{\frac{1}{2}}}$$

Fall unterschied: A ungerad / gerade

A ungerade ⇒ ug-Kern ⇒ $\delta = 0$ — Ru^{44} stabil, $< \beta^-\ \beta^+$ zerfall

2 parabeln $11{,}2/A^{\frac{1}{2}}$ MeV

A gerade ⇒ uu, gg Kern ⇒ β zerfall uu ⟷ gg Kern

minimum $\left.\frac{dm}{dZ}\right|_{Z=Z_0} = 0 \qquad Z_0 = \frac{A}{1{,}98+0{,}015A^{\frac{2}{3}}} < \frac{A}{2}$

α-Zerfall

α Teilchen — 7 MeV Bindungsenergie; 5 MeV Energiegewinn

Coulomb. $V_c(r) = 2(Z-2)\frac{\alpha\hbar c}{r}$

Tans. W'keit $T = e^{-2\frac{1}{\hbar}\sqrt{2m_\alpha c^2(V_0 - E_\alpha)}\,d}$

$$T = e^{-2G} \qquad G = \frac{1}{\hbar}\int_R^{r_1}\sqrt{2m_\alpha c^2|E_\alpha - V(r)|}\,dr \sim \frac{1}{\sqrt{E_\alpha}}$$

Lebensdauer $\frac{1}{\tau} = w(\alpha)\,v\,e^{-2G} \sim e^{-2/\sqrt{E_\alpha}}$

Geiger-Nutall-Regel $\log T_{\frac{1}{2}} = \frac{1}{\sqrt{E_\alpha}}$

Zerfallsreihen $A = 4n;\ 4n+1,\ 4n+2,\ 4n+3$

Kernspaltung (Sym. in) $Q = a_0A^{\frac{2}{3}}(1-2^{\frac{1}{3}}) + a_cZ^2A^{-\frac{1}{3}}(1-2^{-\frac{2}{3}})$

bei S, $A = 90$

$$= a_0' A^{\frac{2}{3}} + a_c' Z^2 A^{-\frac{1}{3}}$$

Abschätzung $\frac{Z^2}{A} \geq \frac{a_0'}{c - a_c'}$

Spalt. parameter $x = \frac{Z^2}{51\cdot A}$ $\qquad \frac{Z^2}{A} > 51$

Reaktorregelung $N(t) = N_0\exp\left(\frac{k_{eff}-1}{T}\cdot t\right)$

Fermigas $p_F^p = p_F^n = \frac{\hbar}{R_0}\left(\frac{9\pi}{8}\right)^{\frac{1}{3}} = 250$ MeV

Energie höchster besetz. $E_F^{p,n} \approx 33$ MeV $\qquad V_0 \approx 40$ MeV

Bed. Kernzerfall $B(A,Z) < B(A_1,Z_1) + (B(A_2,Z_2)$