# Homework 1: Introduction to Algorithmic Analysis and Recurrence

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CSC-372 Analysis of Algorithms

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Section 1 DUE: Thursday, Aug 27th, at 7AM
Section 2 DUE: Thursday, Sept 3 th, at 7AM

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## Section 2: Recursion Analysis

**1) (26 pt) Determine the run time (bit-O) for the following recurrence formula using the tree or substitution method. You may use the master method only to check your answer.**

$$T(n) = \begin{cases} 1 & n = 1 \\ 3T(\frac{n}{4}) + n^2 & n > 1 \end{cases}$$

Substitution Method:

1. We guess that the form of the solution is $T(n) = O(n^2 log(n))$.

2. We want to show by mathematical induction that $T(n) \leq d \cdot n^2 log(n)$ for some constant $d > 0$.

$$T(n) = 3T\left(\frac{n}{4}\right) + n^2$$

Now substitute for the $T\left(\frac{n}{4}\right)$ term in the above equation:

$$\begin{aligned} T(n) &= 3 \cdot d\left(\frac{n}{4}\right)^2 \cdot log\left(\frac{n}{4}\right) + n^2 \\ &= \frac{3}{16} d \cdot n^2 \cdot log\left(\frac{n}{4}\right) + n^2 \\ &\leq d \cdot n^2 \cdot log(n) + n^2 \end{aligned}$$

When d is sufficiently large we drop the $n^2$ term:

$$d \cdot n^2 \cdot log(n)$$

**2) (26 pt) Determine the run time (big-O) for the following recurrence formula using the tree or substitution method. You may use the master method only to check your answer.**

$$T(n) = \begin{cases} 1 & n = 1 \\ 2T(\frac{n}{3}) + n^3 & n > 1 \end{cases}$$

Substitution Method:

1. We guess that the form of the solution is $T(n) = O(n^3 log(n))$.

2. We want to show by mathematical induction that $T(n) \leq d \cdot n^3 log(n)$ for some constant $d > 0$.

$$T(n) = 2T\left(\frac{n}{3}\right) + n^3$$

Now substitute for the $T\left(\frac{n}{3}\right)$ term in the above equation:

$$\begin{aligned} T(n) &= 2 \cdot d\left(\frac{n}{3}\right)^3 \cdot log\left(\frac{n}{3}\right) + n^3 \\ &= \frac{1}{3} d \cdot n^3 \cdot log\left(\frac{n}{3}\right) + n^3 \\ &\leq d \cdot n^3 \cdot log(n) + n^3 \end{aligned}$$

When d is sufficiently large we drop the $n^3$ term:

$$d \cdot n^3 \cdot log(n)$$

3) (12 pt) Determine which case of the Master Theorem applies for the following recurrences. Include the values of a, b, and k (and ideally $b^k$) as proof of your selection. Also, include the final big-theta formula. You also have teh option of a recurrence relation that cannot use the master method as described in class, in which case, state it "fails".

a. $T(n) = 2T(\frac{n}{2}) + \sqrt{n}$