Final Project: Context-Tree Weighting

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Algorithm, Application, Language Choice

- *Algorithm: Context-Tree Weighting (CTW)*
- *Application: File Compression*
- *Language: Python 3*

Where It Is Used

- *CTW solves the class of polynomial-time problems.*
- *It's primarily used in text compression.*
- *CTW can be used in applications where there is a data source and we want to predict the probability of the next symbol.*

Other applications

- *CTW can be used in route prediction applications. "Route prediction is a problem which deals with, given a sequence of road network graph edges already traveled by the user, predict the most probable edge of the network to be traveled (Tiwari 2008)."*
- *CTW can be used for content recognition purposes (*Z. Dawy 2004)

Alternative algorithms

- *Lempel-Ziv 78 (LZ78)*
- *Probabilistic Suffix Trees (PST)*
- *Prediction by Partial Match (PPM)*

Reason for choice

The choice of CTW for this project was motivated by the Hutter Prize. The CTW was recommended on the Hutter Prize FAQ page as a place to start with the challenge.

How Your Project Works

Constructor

I wrote a class called ctw. We create a new instance of the ctw class and pass a sequence s (with a binary alphabet {0,1}) and a depth d to the constructor. The constructor ‘constructs’ a d-bounded context-tree from the initial sequence.

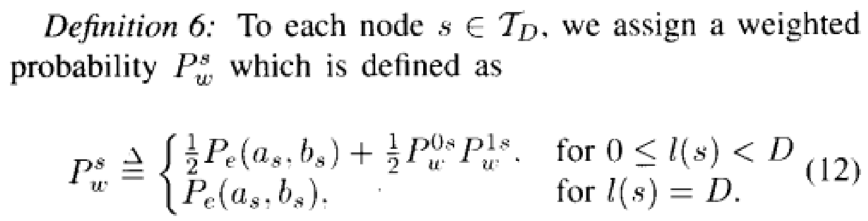
```python
# build the tree
def __init__(self,s,d):
    self.seq, self.depth, self.n, self.tree, self.Pw = s, d, len(s), {}, {}
    self.tree['root']=[0,0]
    for i in range(d,len(s)):
        for j in range(0,d+1):
            context= 'root' if j==0 else s[i-j:i]
            if context not in self.tree: self.tree[context]=[0,0]
            self.tree[context][int(s[i:i+1])]+=int(1)
```

addSymbol

addSymbol appends a new symbol b to the sequence. b must be in the alphabet {0,1}. The context-tree is updated by traversing the context path (from root to leaf) and incrementing zero-counts in nodes if symbol b is a zero or incrementing one-counts in nodes if symbol b is a one.

```python
# get next symbol and update context path
def addSymbol(self,b):
    self.tree['root'][int(b)]+=int(1)
    for i in range(1,self.depth+1):
        context=self.seq[self.n-i:]
        if context not in self.tree:
            self.tree[context]=[0,0]
        self.tree[context][int(b)]+=int(1)
    self.seq+=b
    self.n+=1
```

probs



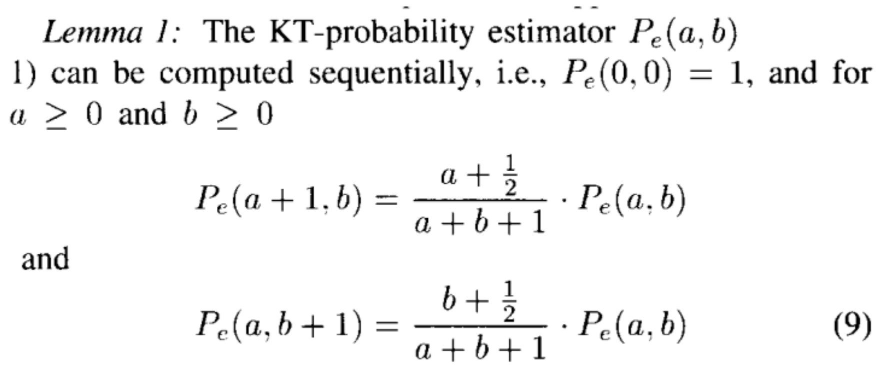
Definition 6: To each node $s \in \mathcal{T}_D$, we assign a weighted probability P_w^s which is defined as

$$P_w^s \triangleq \begin{cases} \frac{1}{2}P_e(a_s, b_s) + \frac{1}{2}P_w^{0s}P_w^{1s}, & \text{for } 0 \leq l(s) < D \\ P_e(a_s, b_s), & \text{for } l(s) = D. \end{cases} \quad (12)$$

```
# calculate probabilities for nodes in the tree
def probs(self,s):
    [a,b] = self.tree[s] if s in self.tree else [0,0]
    if s=='': [a,b]=self.tree['root']
    if(len(s)==self.depth):
        self.Pw[s]=self.Pe(a,b)
    else:
        self.probs('0'+s)
        self.probs('1'+s)
        self.Pw[s] = (self.Pe(a,b) + self.Pw['0'+s] * self.Pw['1'+s])/2
```

Pe (Krichevsky–Trofimov estimator)

Here is the definition of the KT-probability estimator that was used in the ‘probs’ function above. Note that the mathematical definition has been modified (a is substituted with (a-1)) for the purpose of writing a top-down recursive function (so that we can use P(0,0) as the base case).



Lemma 1: The KT-probability estimator $P_e(a, b)$

1) can be computed sequentially, i.e., $P_e(0, 0) = 1$, and for $a \geq 0$ and $b \geq 0$

$$P_e(a+1, b) = \frac{a + \frac{1}{2}}{a + b + 1} \cdot P_e(a, b)$$

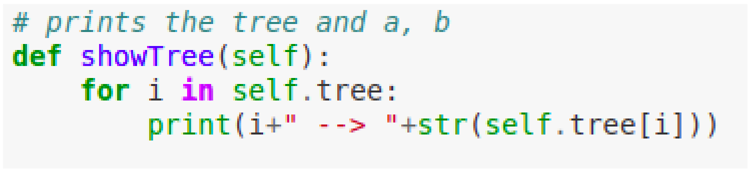
and

$$P_e(a, b+1) = \frac{b + \frac{1}{2}}{a + b + 1} \cdot P_e(a, b) \quad (9)$$

```
# Krichevsky–Trofimov estimator
def Pe(self,a,b):
    if a: return ((a-0.5)/(a+b))*self.Pe(a-1,b)
    elif b: return ((b-0.5)/(a+b))*self.Pe(a,b-1)
    else: return 1
```

showTree

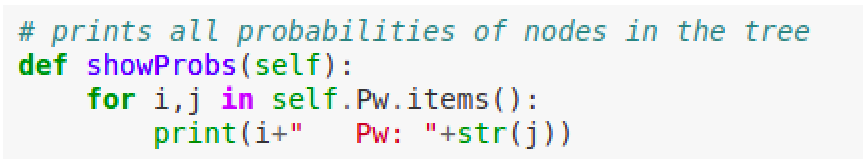
This function iterates through all nodes in the tree and prints each node's context string along with its corresponding pair: [a,b] which are the number of zeros and ones at that context



```
# prints the tree and a, b
def showTree(self):
    for i in self.tree:
        print(i+" --> "+str(self.tree[i]))
```

showProbs

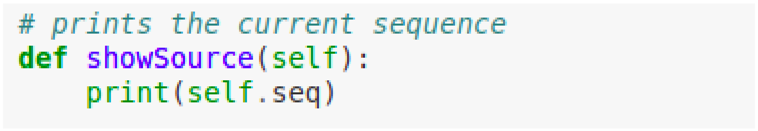
This function iterates through every node in the tree and prints the context string and the corresponding probability or weight for that context.



```
# prints all probabilities of nodes in the tree
def showProbs(self):
    for i,j in self.Pw.items():
        print(i+"   Pw: "+str(j))
```

showSource

Simply prints the sequence of symbols that have been seen so far.



```
# prints the current sequence
def showSource(self):
    print(self.seq)
```

Run time

The runtime of my context-tree constructor (__init__ function) is O(ND) where N is the length of the binary string sequence s and D is the depth of the context-tree.
The runtime of addSymbol is O(D) because the function traverses the context-path which has a length equal to the depth of the tree.
The runtime of probs is O(2^(D+1)) because there are 2^(D+1)-1 nodes in the tree.

Program usage or README

Run ctw.py on the command line with the following command:

```
$ python3 ctw.py
```

The ctw.py file contains a demo function which shows how to use the ctw class.

References

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http://prize.hutter1.net/