Rush Hour: Optimization of the Boston T

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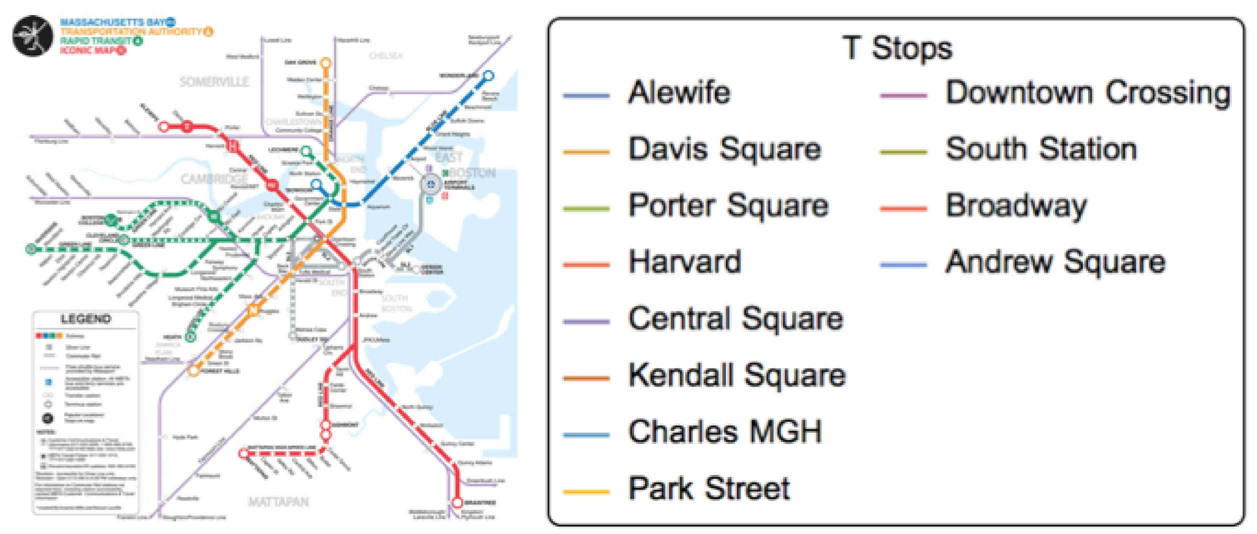
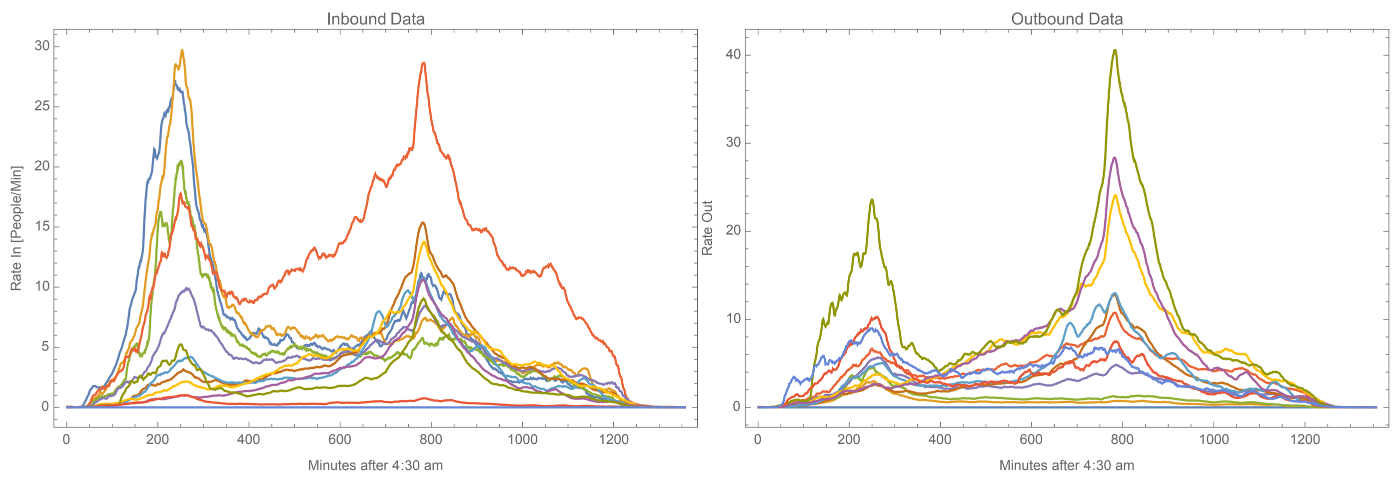
Objective

To find the optimal times for the arrival and departures of trains on a portion of the Boston MBTA Red Line during Rush Hour. We employ two models: one with instant loading and infinite train capacity and another with a finite capacity and a loading rate that decreases as the capacity of the train is reached.

Motivation

Many commuters rely on the Boston MBTA everyday as an important means of transportation. Currently, the train system is setup so that trains depart on regular intervals throughout the day. We wonder what schedule of departure times would minimize the average wait time and how drastically different it is from the equal spacing case.





Model Basics

- We model the Red Line of the Boston T from Alewife to Andrew Square.
- We vary the total number of trains between 10 and 50, evenly split between the inbound and outbound direction.
- Model 1: Every train has an infinite capacity and loading is instantaneous.
- Model 2: Every train has a finite capacity that scales inversely with the number of trains. The loading rate is given by: $\dot{T} = \kappa(C - T)/C$ where C is the train capacity and T is the number of people on the train.
- We want to **minimize the number of man-minutes.**
- We perform the optimization using the Nelder-Mead algorithm.

Results

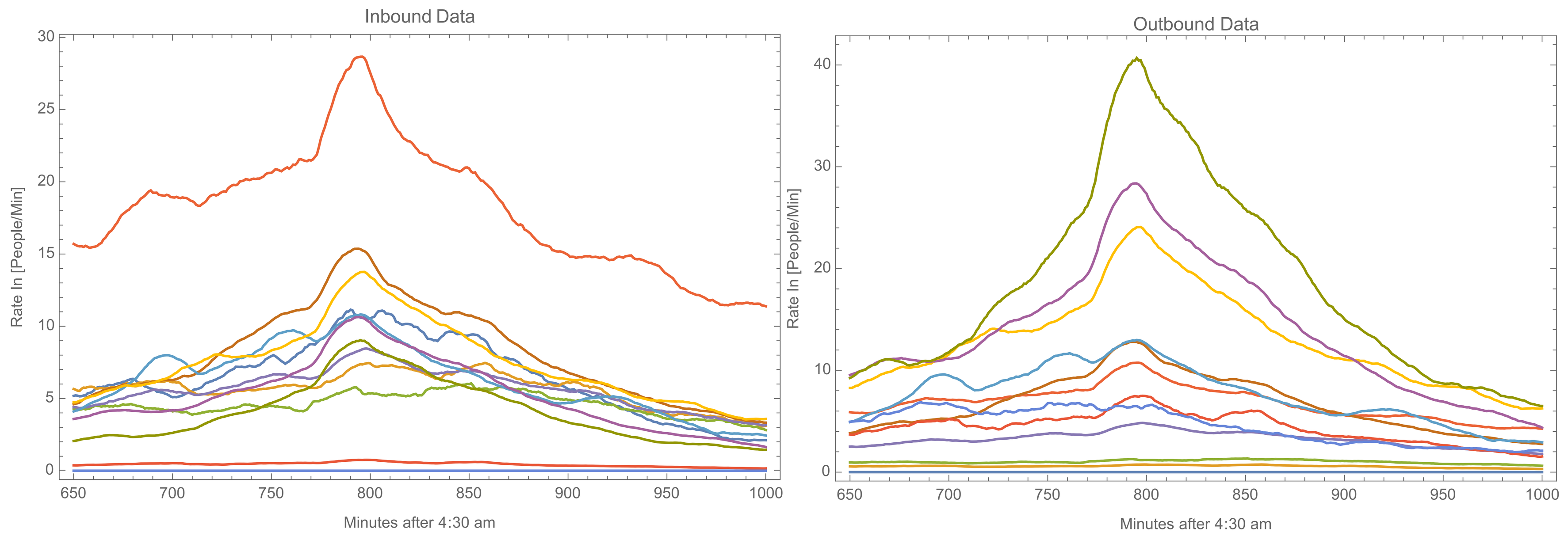
- **Instant Loading, Infinite Capacity**

No.	Even	Optimal	% Imp.
10	1.795×10^6	1.749×10^6	2.61
20	980136.	951872.	2.88
30	670365.	653114.	2.57
40	509399.	500182.	1.81
50	409248	404338.	1.12

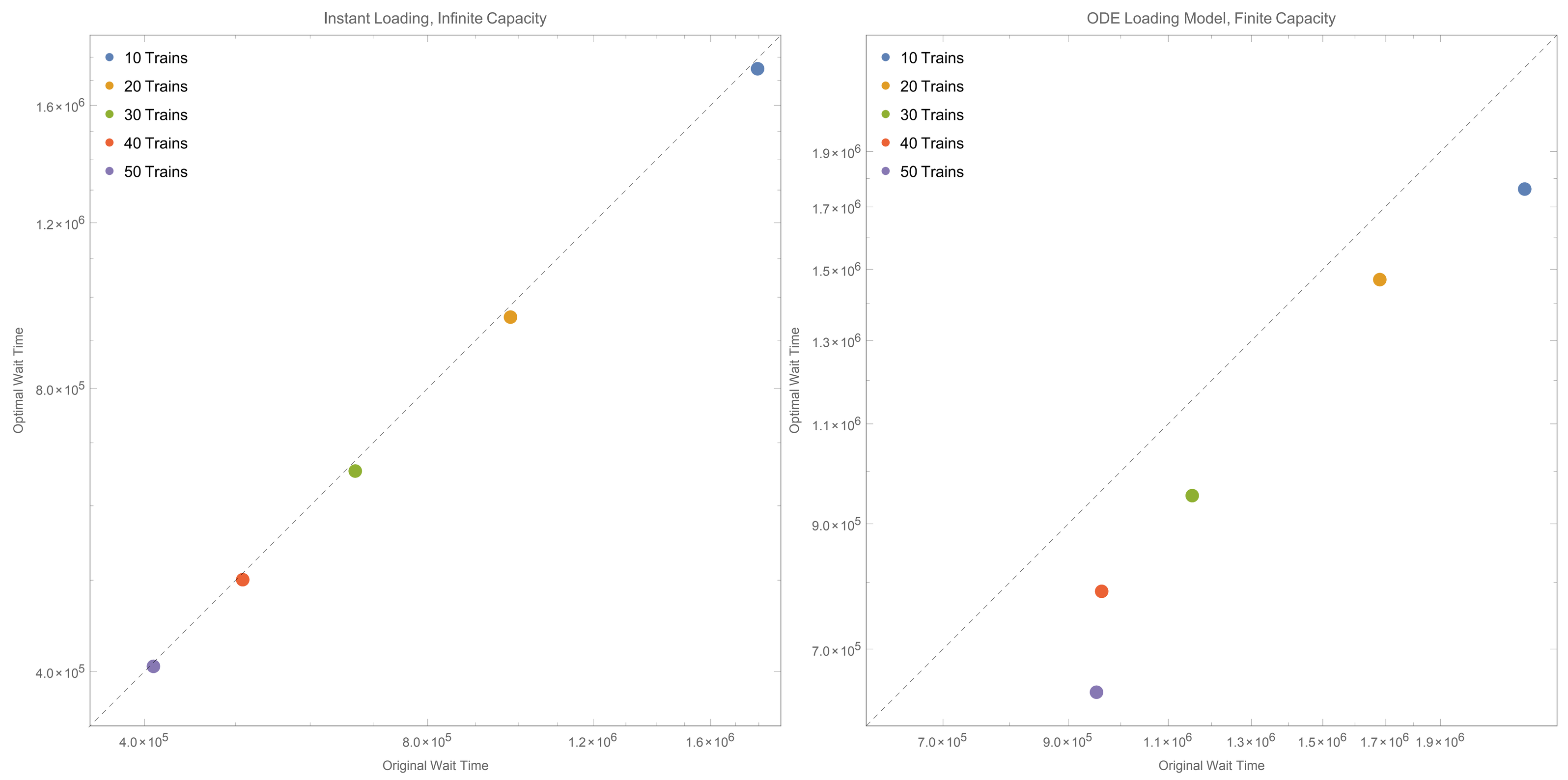
ODE Loading, Finite Capacity

No.	Even	Optimal	% Imp.
10	2.251×10^6	1.761×10^6	21.78
20	1.681×10^6	1.467×10^6	12.72
30	1.152×10^6	951905.	17.43
40	961942.	785325.	18.36
50	953316.	642123.	32.64

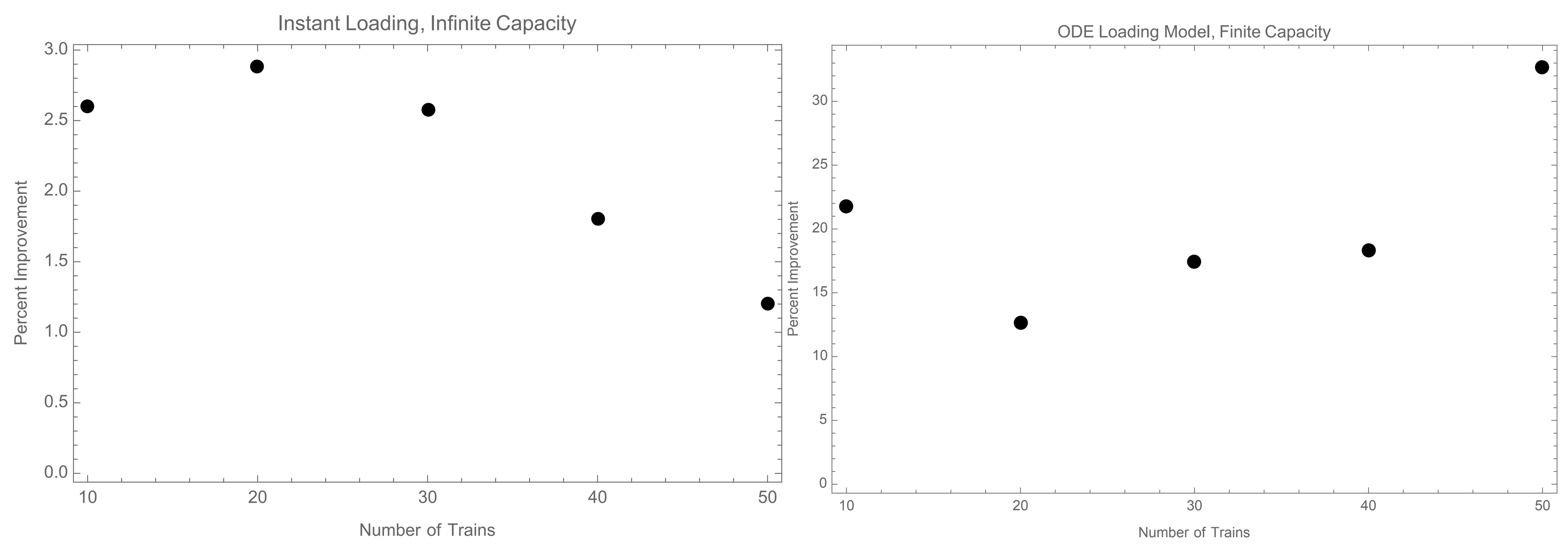
- **Below:** The inbound and outbound rates for the times we are considering.



- **Below:** Comparison of the number of minutes waited by commuters with an even distribution of trains and our optimal distribution. **Left:** Instant loading. **Right:** Finite train capacity and the ODE loading model.

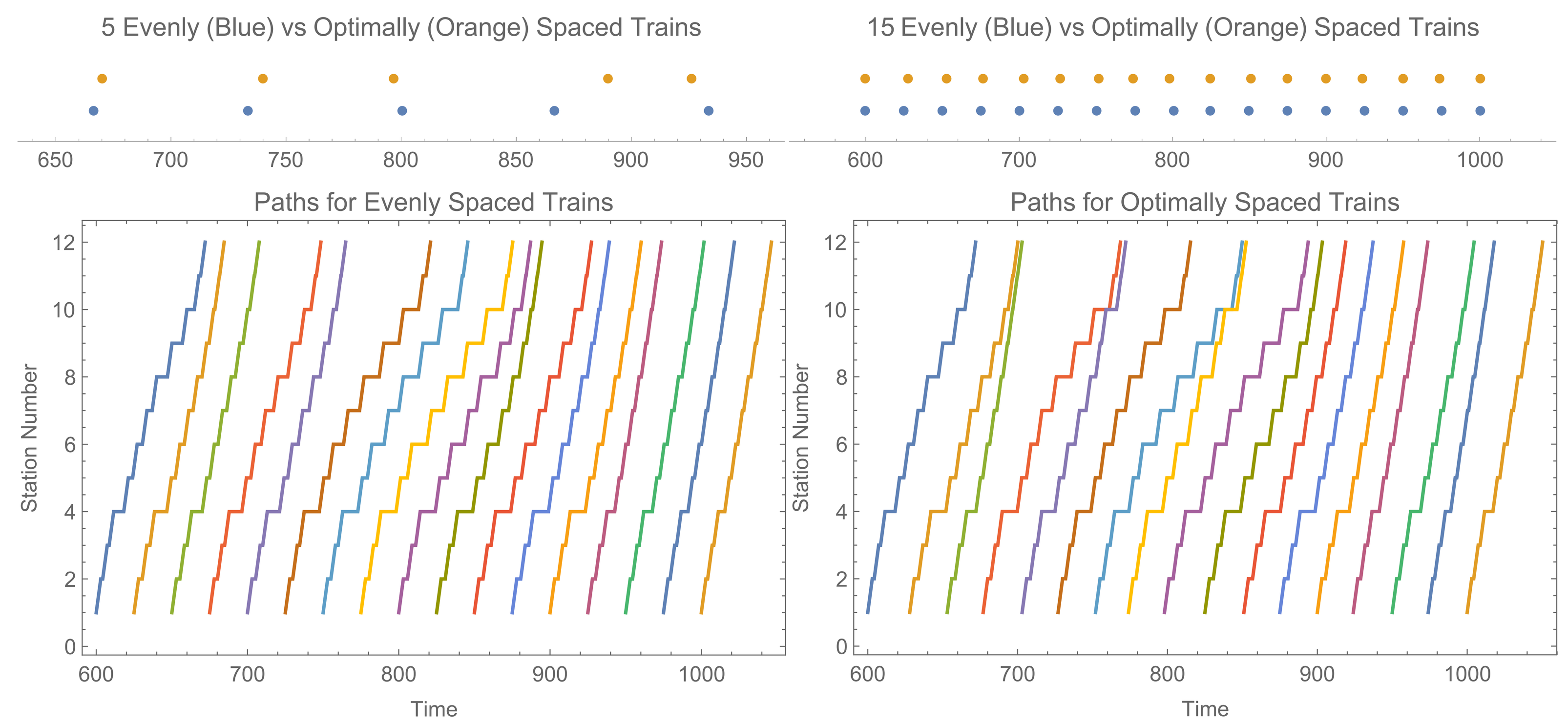


- **Below:** We plot the percent improvement over the evenly spaced time distribution that we are able to recover using our model.



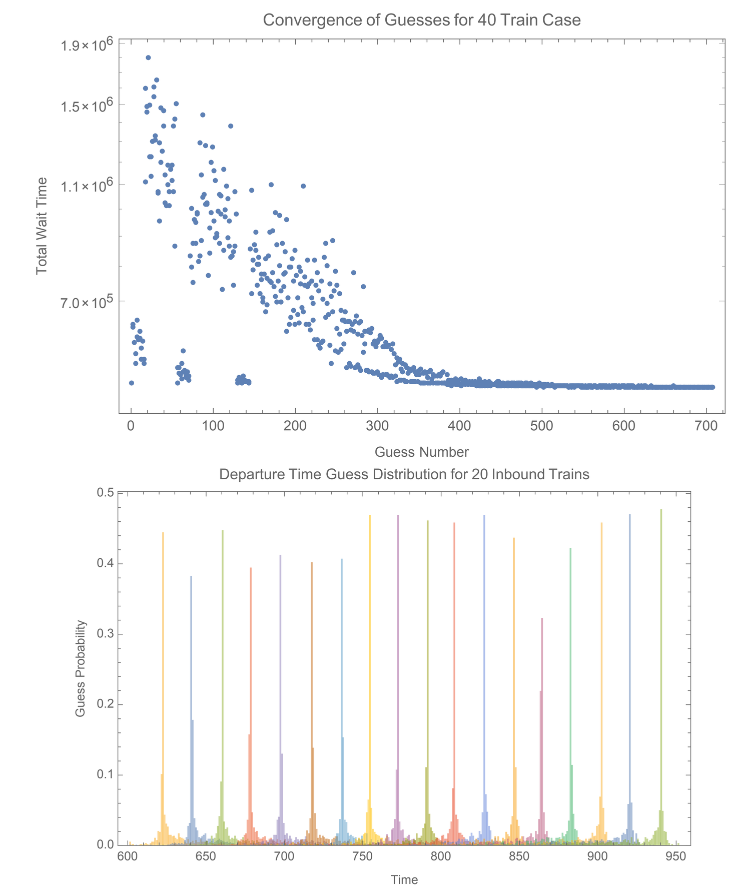
Conclusions

- Assuming instant loading, we find even spacing to be a quite efficient solution.
- Without instant loading, we find that we can do substantially better than even spacing- reducing waiting time by up to 30%.
- While in the case of instant loading the percent improvement tends to decrease with the number of trains, this is not the case with a loading rate inversely proportional to how full the train is.
- We find that the primary inefficiencies arise from trains unoptimally jamming.
- We compare the inbound dispatch times of the even and ideal case and find that there is a large change in the packing of trains in the optimal model.
- Relatively small differences in start time can result in a large change in train position down the track.



Future Work

- We would like to extend this work to the entire MBTA T network over a full day.
- We would need to model junctions and stations where more than one line meet.
- **Right:** we show the convergence (upper) and guess distribution (lower) of the Nelder-Mead minimization. Ideally we would run this many times to ensure we do not get trapped in a local minima.



Acknowledgments and References

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Data from: `github.com/mbtaviz`, MBTA Subway Image from: `www.mbta.com`

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