PhysiCell User Guide (Version 1.2.0)

PhysiCell Project (Lead: Paul Macklin)

Revision: July 30, 2017

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1 Introduction and citing PhysiCell

This user guide will teach you how to download and use PhysiCell [3], as well as document the key classes and functions. Wherever possible, it will demonstrate with specific examples. Please note that this guide will be periodically updated. Users should check PhysiCell.MathCancer.org for the latest version. The PhysiCell method paper is in revision for additional peer review at PLoS Computational Biology [3].

If you use PhysiCell, please cite it as:

We built our model using PhysiCell (Version 1.2.0) [1].

[1] A. Ghaffarizadeh, S.H. Friedman, S.M. Mumenthaler, and P. Macklin, PhysiCell: an Open Source Physics-Based Cell Simulator for 3-D Multicellular Systems, bioRxiv 088773, 2016. DOI: 10.1101/088773.

We will update this citation information as PhysiCell progresses through peer review.

If you prefer to cite a peer-reviewed journal publication while waiting for our main method paper, you you can also cite an earlier (2-D) version of the code by our 2012 paper in the Journal of Theoretical Biology:

P. Macklin, M.E. Edgerton, A.M. Thompson, and V. Cristini. Patient-calibrated agent-based modelling of ductal carcinoma in situ (DCIS): From microscopic measurements to macroscopic predictions of clinical progression. J. Theor. Biol. 301:122-40, 2012. DOI: 10.1016/j.jtbi.2012.02.002.

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2 Further resources for help

The PhysiCell project posts tips and tutorials at its blog:

http://www.mathcancer.org/blog/physicell-tutorials/

Users are encouraged to frequently visit the blog for these tips. This user manual may be updated more frequently than PhysiCell. Please check the PhysiCell project website for updates:

http://PhysiCell.MathCancer.org

Lastly, users can support help tickets at SourceForge:

https://sourceforge.net/p/physicell/tickets/

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3 Preparing your development environment

PhysiCell was designed to be cross-platform compatible, without need for a package manager system or any other complex installation. In principle, any C++11 compliant compiler with OpenMP support should work. In practice, we use g++ as our "gold standard." You'll also want to ensure that your build environment supports makefiles. Command-line zip/unzip is also helpful, but not required.

Please note that OSX (and its associated developer tools) ships with "g++" instead of g++: it uses LLVM/Clang and an alias to *pretend* to be g++. Unfortunately, the version of LLVM/Clang that Apple ships does not fully support OpenMP, and so compiling can fail on those platforms without further setup. For OSX, we recommend following our Homebrew-based tutorial to install real g++.

Full tutorials on installing a 64-bit g++/OpenMP environment (on Windows via mingw-w64 and on OSX using Homebrew) can be found at:

http://www.MathCancer.org/blog/physicell-tutorials/

Linux users should already have a 64-bit g++ environment installed by default.

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3.1 Virtual Machine option

Starting with the 1.2.x releases, we began distributing PhysiCell as zipped source (preferred), Docker containers, and virtual appliances for use in VirtualBox and other virtual machine software, allowing us to distribute a full PhysiCell development environment (including 64-bit g++, support for makefiles, zip/unzip, ImageMagick, mencoder, eog, a text editor, and the most up-to-date version of PhysiCell). This should make it simpler to start learning and using PhysiCell, even in cases where developers are not free to install or modify their own build environments, or have difficulty installing and configuring g++.

Please visit the PhysiCell blog for information on running VirtualDub and using the virtual appliance. (Section 2).

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3.2 Docker option

Starting with the 1.2.x releases, we began distributing PhysiCell as zipped source (preferred) and Docker containers of PhysiCell, allowing us to distribute a full PhysiCell development environment (including 64-bit g++, support for makefiles, zip/unzip, a text editor, and the most up-to-date version of PhysiCell). This should make it simpler to start learning and using PhysiCell, even in cases where developers are not free to install or modify their own build environments, or have difficulty installing and configuring g++.

Please visit the PhysiCell blog to check for blog posts on running Docker. (Section 2).

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3.3 Suggested tutorials and resources

Working in PhysiCell requires some knowledge of C++, Makefiles, and command-line routines in Unix-like systems. If you are not familiar with these skillsets, we recommend the following resources:

3.3.1 Linux and Makefile tutorials

- 1. A tutorial on learning the Linux shell: http://linuxcommand.org/lc3_learning_the_shell.php In particular, I recommend:
 - (a) What is "The Shell"?
 - (b) Navigation
 - (c) Looking Around
 - (d) Manipulating Files
- 2. UNIX Tutorial for Beginners: http://www.ee.surrey.ac.uk/Teaching/Unix/In particular, I suggest the following:
 - (a) Tutorial 1 (navigation, creating directories, etc.)
 - (b) Tutorial 2 (moving, copying files, etc.)
 - (c) Tutorial 4 (wildcards, filename conventions, etc.)
 - (d) Tutorial 7 (compiling and runnign software, etc.)
 - (e) Tutorial 8 (UNIX and environment variables, etc.) (Note: May not work in Windows/MinGW.)
- 3. The GNU Bash Reference Manual: http://www.gnu.org/software/bash/manual/bash.html
- 4. A good makefile tutorial: http://www.cprogramming.com/tutorial/makefiles.html
- 5. Another good makefile tutorial: http://mrbook.org/blog/tutorials/make/
- 6. One more makefile tutorial: http://makefiletutorial.com/
- 7. The GNU Make Reference: https://www.gnu.org/software/make/manual/make.html

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3.3.2 C++ references

The following websites are good references for C++:

- 1. **CPlusPlus.com:** http://www.cplusplus.com/ Excellent, detailed documentation on C++, as well as tutorials.
- 2. C++ Reference: http://en.cppreference.com/w/ Another good reference guide to C++.
- 3. **LearnCpp.com:** http://www.learncpp.com/ A series of tutorials on C++.

- 4. StackOverflow (C++ tag): https://stackoverflow.com/questions/tagged/c%2b%2b A sometimes overwhelming resource for problem solving in C++.
- 5. C++ section on CodeGuru: http://www.codeguru.com/cpp/cpp/

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3.3.3 Matlab tutorials

If you use Matlab for visualization and/or postprocessing, we recommend:

- 1. MATLAB tutorials at Mathworks: https://www.mathworks.com/academia/student_center/tutorials/mltutorial_launchpad.html
- 2. Introduction to MATLAB for Engineering Students: http://bit.ly/2cQBXDc
- 3. A Beginner's Guide to MATLAB: http://homen.vsb.cz/~lud0016/NM/matlab_guide.pdf (By Christos Xenophontos at Loyola College).
- 4. MATLAB Academy: https://matlabacademy.mathworks.com/?s_eid=ppc_23223967642
- 5. MATLAB Tutorial: https://www.tutorialspoint.com/matlab/
- 6. Octave: If you do not have access to MATLAB or prefer an open source alternative, have a look at this cross-platform package: https://www.gnu.org/software/octave/

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3.3.4 VirtualBox and related information (virtual appliances)

If you use the virtual appliance, we suggest the following tutorials and resources:

- 1. VirtualBox: http://virtualbox.org
 - This is an excellent cross-platform (Windows, Linux, OSX, etc.) virtual machine software that can import the virtual appliance version of PhysiCell.
- 2. Alpine Linux: https://alpinelinux.org/

Alpine Linux is a lean and secure version of linux we installed in the PhysiCell virtual appliance.

3. Alpine Linux Wiki: https://wiki.alpinelinux.org

A helpful site for using Alpine Linux.

- 4. VirtualBox shared folders in Alpine: https://wiki.alpinelinux.org/wiki/VirtualBox_shared_folders
- 5. Alpine Linux users forum: https://forum.alpinelinux.org/
- 6. XFCE4 project: https://xfce.org/

We use the XFCE4 desktop environment in our virtual appliance, so this may be helpful.

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3.3.5 Docker resources

If you use Docker, we suggest the following tutorials:

- 1. Docker user introduction: https://docs.docker.com/get-started/
- 2. Docker for beginners: https://prakhar.me/docker-curriculum/
- 3. Docker tutorial: https://www.tutorialspoint.com/docker/

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3.3.6 Recommended additional tools

Lastly, we find the following tools and resources very useful for postprocessing and visualization:

- 1. **ImageMagick** is a cross-flatform image manipulation suite, which can (among other things) resize and crop images, change formats, and insert text labels. In PhysiCell, ImageMagick is especially useful for converting SVG files to PNG, JPG, or other raster-based formats for further animation.
 - URL: http://imagemagick.org
 - Typical use: magick mogrify -format jpg snapshot*.svg (Converts all files of the form snapshot*.svg to the png format.)
 - Typical use: magick snapshot*.jpg animation.gif (Combines all snapshot*.jpg files into a (huge!) animated gif file.)
- 2. **Mencoder** is a cross-platform, open source mpeg encoder, useful for creating compressed movies. In PhysiCell, we use mencoder to convert series of simulation snapshots to movies.
 - URL (Linux): Use your package manager to install mplayer (which includes mencoder).
 - URL (OSX): Use Homebrew or MacPorts to install mplayer (which includes mencoder).
 - URL (Windows): http://mplayerwin.sourceforge.net/
 - Typical use:

```
mencoder "mf://snapshot*.jpg" -ovc lavc -lavcopts vcodec=mpeg4:vbitrate=10000:mbd=2:trell -mf fps=24:type=jpg -nosound -o out.avi (Converts all the snapshot*.jpg files into an mpeg4-encoded movie named out.avi, with a 8kbps variable bit rate and 24 frames per second.)
```

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4 Overall codebase structure

PhysiCell was created by extending the Basic_Agent class in BioFVM [2] to include a fuller cell phenotype and force-based cell motion. In the overall software philosophy, we structure PhysiCell in several critical subdirectories:

- 1. **BioFVM** This includes a working copy of the BioFVM multi-substrate diffusion code [2]. Users should be able to fully replace these files with any later version of BioFVM. Note that BioFVM distributions also include pugixml (an efficient cross-platform XML parser) [6].
- 2. **config** While this directory is not currently used, PhysiCell will eventually parse XML files with configuration information in this directory. Any custom configuration files should be placed in this directory.
- 3. **core** The core library files for PhysiCell are kept in this directory. Users should not modify functions in the core library.
- 4. **custom_modules** This directory is the right place to put custom user code. Moreover, users should list their custom code files in **custom_modules/PhysiCell_custom.h**.
- 5. **modules** This is where we place non-core (but helpful) standard code modules for distribution with PhysiCell. Currently, we include pathology, MultiCellDS [1], and SVG (scalable vector graphic) functions. Future releases may include basic modules for extracellular matrix.

The following subdirectories are also included:

- 1. **archives** If you use the make zip or make archive rules, the compressed archives will be placed here.
- 2. documentation This directory includes user guides (like this one!).
- 3. matlab This includes basic Matlab files for handling PhysiCell outputs. (This and other postprocessing will be a major PhysiCell focus over the next few years.)
- 4. **examples** The examples from the PhysiCell method paper will be placed here. They may not necessarily be updated for compatibility with every PhysiCell release. (They are currently compatible up to version 1.1.1.)
- 5. **licenses** License files for BioFVM [2], pugixml [6], PhysiCell, and any other (BSD-compatible) dependencies are kept here.
- 6. **output** Some examples will put their outputs here.
- 7. **povray** This includes some basic utilities for visulization by povray (an open source raytracing package) [9].
- 8. **template_projects** This directory stores basic templates for 2D and 3D projects. A good start to a new project is to use the make template2D or make template3D makefile rules, which will copy the appropriate template to main.cpp. See Section 5.

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5 Using project templates

As of Version 1.1.0, PhysiCell includes templates for 2-D and 3-D projects. These template projects set up all the critical BioFVM (microenvironment) and PhysiCell data structures and give examples on seeding a few cells. When beginning a new project with PhysiCell, we strongly recommend starting with these templates.

To start a new 2-D project (based on the template), go to the command line in the root PhysiCell directory, and run:

make template2D

This will copy template_projects/template2D.cpp to main.cpp in the root directory. From here, modify main.cpp as you see fit, add new custom codes in the custom_modules directory, and edit the Makefile as needed to include these codes. Once you're ready, type

make

to build your new project. By default, the executable will be called project (project.exe on windows). To change the name of the executable, modify the PROJECT_NAME variable in the Makefile.

To start a new 3-D project based on a template, type make template3D and continue as before.

Now, run the code:

./project

(in Windows, omit the "./".) This will generate (among other things) a series of SVG images that visualize the simulation once per hour.

More examples will be introduced on the PhysiCell blog. See Section 2.

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6 The BioFVM microenvironment

PhysiCell is built upon BioFVM [2], and so it uses the (bio)chemical microenvironment from BioFVM, including diffusion, decay, cell-based secretions/uptake, and bulk supply/uptake functions. All PhysiCell projects already include bioFVM once you have included PhysiCell:

```
#include "./core/PhysiCell.h"
```

We also suggest using the BioFVM and PhysiCell namespaces:

```
using namespace BioFVM;
using namespace PhysiCell;
```

6.1 Setting up and using the microenvironment

Within your main program loop (int main(int argc, char* argv[])), you need to declare and set up a Microenvironment. (Here, we also include some useful tips for setting space and time units.) We make use of BioFVM 1.1.5 improvements that include simplified setup functions:

```
/* Microenvironment setup */
default_microenvironment_options.name = "liver";
default_microenvironment_options.time_units = "min";
default_microenvironment_options.spatial_units = "micron";
default_microenvironment_options.dx = 20;
default_microenvironment_options.dy = 20;
default_microenvironment_options.dz = 20;
// set a Dirichlet outer boundary condition
default_microenvironment_options.outer_Dirichlet_conditions = true;
std::vector<double> bc_vector( 1 , 38.0 ); // 5% o2
default_microenvironment_options.Dirichlet_condition_vector = bc_vector;
// stick with a 3-D simulation
default_microenvironment_options.simulate_2D = false;
// set the domain size
default_microenvironment_options.X_range = {-500, 500};
default_microenvironment_options.Y_range = {-400, 400};
default_microenvironment_options.Z_range = {-100, 100};
// turn off gradient calculations
default_microenvironment_options.bool calculate_gradients = false;
// initialize BioFVM with these options
initialize_microenvironment();
BioFVM defaults to a 1 mm<sup>3</sup> domain centered at (0,0,0) with \Delta x = \Delta y = \Delta z = 20 \,\mu\text{m}, simulating 3D
with no Dirichlet conditions and no gradient calculations, and minutes time units and micron space units.
If you call initialize_microenvironment() without setting units, these defaults will be used.
The code also automatically chooses the correct 2D/3D diffusion solver, and sets the single diffusing field
to oxygen with diffusion coefficient 10^5 \mu \text{m}^2/\text{min} and decay coefficient 0.11/\mu \text{m} (for a 1 mm diffusion length
scale in the absence of cells).
By the end of these commands, the default Microenvironment is set to microenvironment. You can get
this address at any time using the BioFVM command:
Microenvironment* get_default_microenvironment( void )
You'll also need to set up PhysiCell's mechanics data structures (for cell-cell interaction testing) and match
them to BioFVM:
/* PhysiCell setup */
// prepare PhysiCell mechanics data structures
```

// mechanics voxel size

```
double mechanics_voxel_size = 30;
Cell_Container* cell_container = create_cell_container_for_microenvironment(
    microenvironment, mechanics_voxel_size);
```

Within your main program loop, you'll want to make sure that BioFVM is being called to update the biochemical environment:

```
while( t < t_max )
{
    // main loop contents ...

    // update the microenvironment

    microenvironment.simulate_diffusion_decay( diffusion_dt );
    if( default_microenvironment_options.calculate_gradients )
        { microenvironment.compute_all_gradient_vectors(); }

    // physicell functions
    t += diffusion_dt;
}</pre>
```

As of Version 1.2.0, PhysiCell has automated cell secretions/uptake, so there is no need to explicitly call the BioFVM cell-based functions in the main program loop.

Note that the PhysiCell templates already include these critical setup commands. See Section 5.

```
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```

6.2 Sampling the microenvironment

To make it easier to write functions involving microenvironment substrates at any cell's location, BioFVM 1.1.5 added the following function:

int Microenvironment::find_density_index(std::string name) This allows you to find the index of the substrate named name. Note that this function is case sensitive. Here's an example of this function in action:

```
bool bad_conditions( Microenvironment& M , int voxel_index )
{
    // find the correct indices the first time you run this function
    static int oxygen_index = M.find_density_index( "oxygen" );
    static int glucose_index = M.find_density_index( "glucose" );

static double oxygen_threshold = 2.5; // mmHg
    static double glucose_threshold = 0.05; // dimensionless
```

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6.3 Dirichlet conditions

BioFVM also allows you to set constant conditions at any voxel. (These are called Dirichlet nodes.) The relevant functions are:

1. void Microenvironment::add_dirichlet_node(int voxel_index,

 ${\tt std::vector<double>\&\ value\)}$ adds a Dirichlet node at voxel ${\tt voxel_index},$ so that (for some Microenvironment M):

```
M(voxel_index) = value.
```

2. void Microenvironment::update_dirichlet_node(int voxel_index ,

std::vector<double>& new_value) overwrites the (vector) value of the Dirichlet node at
voxel_index, so that

```
M(voxel_index) = new_value.
```

If the voxel was not previously a Dirichlet node, it is automatically changed to a Dirichlet node.

- 3. void Microenvironment::remove_dirichlet_node(int voxel_index) removes the Dirichlet node at voxel voxel_index.
- 4. void Microenvironment::apply_dirichlet_conditions(void) applies the previously set Dirichlet conditions at all Dirichlet nodes.
- 5. bool& Microenvironment::is_dirichlet_node(int voxel_index) returns true if there is a Dirichlet node at voxel voxel_index.

BioFVM applies these Dirichlet conditions every time it evaluates the diffusion solver.

6.4 Other BioFVM resources

To learn more about using BioFVM, take a look at the tutorials at:

http://mathcancer.org/blog/biofvm-tutorials/

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7 Cells

Each Cell is an extension of BioFVM's Basic_Agent class. As such, it has access to all the parent class's member data and functions. In PhysiCell, Cells have the following major parts:

- 1. std::string type_name: The name of the type of cell.
- 2. Custom_Cell_Data custom_data: Custom data attached the cell, which may differ from other cells. See Section 7.4.1.
- 3. **Cell_Parameters parameters:** A set of standardized parameters (which may eventually be moved into the cell's phenotype). See Section 7.4.2.
- 4. **Cell_Functions functions:** A collection of functions used for updating the cell phenotype, including custom functions. See Section 7.4.3.
- 5. **Cell_State cell_state:** A small set of standard state variables, chiefly (basal-to-apical) orientation. See Section 7.4.4.
- 6. **Phenotype phenotype:** A hierarchical organization of the cell's properties. This data element is discussed in great depth in Section 8.

The following inherited member data and functions (from Basic_Agent) are especially helpful:

- 1. **int ID** is a unique integer identifier for the cell. No other cell now or in the future will have the same ID.
- 2. int index is the cell's current index in std::vector<Basic_Agent*> all_basic_agents, the list of all current Basic_Agents in the simulation.
- 3. **int type** is a user-specified (default 0) integer code to classify the cell's type. Use these to quickly compare if two cells are of the same or different types, or to perform type-specific operations on them.
- 4. int get_current_voxel_index(void) returns the cell's positional index in the BioFVM Microenvironment.
- 5. std::vector<double>& nearest_density_vector(void) allows the user to directly access (i.e., sample or modify) the vector of substrates at the cell's position. This is useful building functions that alter cell phenotype based on the microenvironment.
- 6. std::vector<double>& nearest_gradient(int substrate_index) returns (by reference) the gradient of the substrate with index substrate_index, at the cell's current position. This is useful for things like chemotaxis.
- 7. std::vector<gradient>& nearest_gradient_vector(void) returns a vector of all the substrate gradients at the cell's position. Each gradient is a std::vector<double> of size 3.

Here is how the Cell class is defined in PhysiCell_cell.h:

```
class Cell : public Basic_Agent
private:
   Cell_Container * container;
   int current_mechanics_voxel_index;
   int updated_current_mechanics_voxel_index;
public:
   std::string type_name;
   Custom_Cell_Data custom_data;
   Cell_Parameters parameters;
   Cell_Functions functions;
   Cell_State state;
   Phenotype phenotype;
   void update_motility_vector( double dt_ );
   void advance_bundled_phenotype_functions( double dt_ );
   void add_potentials(Cell*);
   void set_previous_velocity(double xV, double yV, double zV);
   int get_current_mechanics_voxel_index();
   void turn_off_reactions(double);
   bool is_out_of_domain;
   bool is_movable;
   void flag_for_division( void );
   void flag_for_removal( void );
   void start_death( int death_model_index );
   Cell* divide( void );
   void die( void );
   void step(double dt);
   Cell();
   bool assign_position(std::vector<double> new_position);
   bool assign_position(double, double, double);
   void set_total_volume(double);
   double& get_total_volume(void);
   // mechanics
   void update_position( double dt );
   std::vector<double> displacement;
   void assign_orientation();
```

```
void copy_function_pointers(Cell*);

void update_voxel_in_container(void);
void copy_data(Cell *);

void set_phenotype( Phenotype& phenotype );
void update_radius();
Cell_Container * get_container();

std::vector<Cell*>& cells_in_my_container( void );
};
```

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7.1 Other member data

- 1. bool is_out_of_domain is true if the cell is out of the simulation domain boundaries.
- 2. **bool is_movable** indicates whether the cell is in a static position. Set this to **true** if you would like PhysiCell to leave its position fixed and not evaluate the mechanics models for this cell. Note that it can still exert adhesive and repulsive forces on other cells.

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7.2 Member functions

1. void update_motility_vector(double dt_) updates the cell's motility vector, based on the following model:

First, check if the cell should change its direction of motility \mathbf{v}_{mot} within the next Δt time:

- (a) If is_motile == false set $v_{mot} = 0$ and exit.
- (b) Let $s \in U(0,1)$ be a random number from the uniform distribution on [0,1].
- (c) If $s \leq \Delta t/T_{per}$ or if $\Delta t < T_{per}$ (where T_{per} is the cell's mean persistence time), then continue. Otherwise, leave \mathbf{v}_{mot} unchanged and exit.
- (d) Choose a random direction $\boldsymbol{\xi}$ in 3-D:
 - i. Choose $\theta \in U(0, 2\pi)$ to be a random angle in $[0, 2\pi]$.
 - ii. Choose $\phi \in U(0,\pi)$ to be a random angle in $[0,\pi]$. If restrict_to_2D == true, set $\phi = \frac{\pi}{2}$.
 - iii. Set $\boldsymbol{\xi} = [\sin(\phi)\cos(\theta), \sin(\phi)\sin(\theta), \cos(\phi)].$
- (e) Update the motility bias vector **b** and the bias *b* by calling functions.update_migration_bias.
- (f) Set \mathbf{v}_{mot} according to biased random motion:

$$\mathbf{v}_{\text{mot}} = s_{\text{mot}} \frac{(1-b)\boldsymbol{\xi} + b\mathbf{b}}{||(1-b)\boldsymbol{\xi} + b\mathbf{b}||}$$
(1)

Here $0 \le b \le 1$ is the motility bias (phenotype.motility.migration_bias), **b** is the migration bias direction (phenotype.motility.migration_bias_direction), and s_{mot} is the migration speed (phenotype.motility.migration_speed). See Section 9.6.

See Section 7.4.3 for more information on cell functions, and Section 9.6 for further details on cell motility.

- 2. void advance_bundled_phenotype_functions (double dt_) automatically runs the following cell functions once every phenotype time step (by default, once per 6 simulated minutes):
 - (a) Evaluate functions.update_phenotype to update the phenotype based upon the current sampling of the microenvironment and any other parameters (e.g., cell-cell interactions). If update_phenotype == NULL, this is skipped. See Section 8 for more information on the cell phenotype class.
 - (b) Advance the cell volume model by dt_ time by calling functions.volume_update_function. Skip this step if functions.volume_update_function == NULL. See Section 9.3 for more information on cell volume.
 - (c) Update the cell geometry (radius, nuclear radius, surface area, etc.) by calling phenotype.geometry.update. See Section 9.4 for more information on the cell geometry.
 - (d) Check for death events in the next dt_ time (such as apoptosis and necrosis) by calling phenotype.death.check_for_death. If the function returns true, set the cell's current cycle model to the appropriate death model, evaluate any death entry functions, and set motility to zero. Cell secretions are set to zero, and cell uptake is cut by a factor of 10. (Users can make further changes to secretion and uptake using entry_functions that are called at the start of a death model phase. See Section 9.1.1.) See Section 9.2 for more information on cell death, and see Section 9.1 on the general cycle class.
 - (e) Advance the current cycle model, whether it is a (live) cell cycle model or a death cycle model, by calling phenotype.cycle.advance_cycle. See Section 9.1 for more details.
 - (f) If phenotype.flagged_for_removal == true, call flag_for_removal().
 If phenotype.flagged_for_division == true, call flag_for_division().

See Section 7.4.3 for more details on these cell functions, and Section 15 for examples.

- 3. **void add_potentials(Cell*)** is used in the mechanics functions. Users should almost never call this function.
- 4. void set_previous_velocity(double xV, double yV, double zV) sets the cell's previous velocity (for use in Adams-Bashforth evolution of the cell position) to [xV,yV,zV]. Users are not expected to ever need to call this function.
- 5. int get_current_mechanics_voxel_index() returns the index of the Container currently containing the cell.
- 6. void turn_off_reactions (double) turns off all secretions and uptake by the cell.
- 7. **void flag_for_division(void)** adds the cell to the list of cells that should divide at the next opportunity.
- 8. **void flag_for_removal(void)** adds the cell to the list of cells that should be removed from the simulation at the next opportunity.

- 9. **void start_death(int death_model_index)** immediately kills the cell (with the death model indicated by **death_model_index**), sets **cell.phenotype.death.dead = true**, sets motility to zero, sets secretion to zero, cuts all uptake by a factor of ten, and executes any "entry" functions for the associated death model.
- 10. Cell* divide(void) performs cell division, and returns the memory address of the newly-created cell.
- 11. void die(void) removes the cell from the simulation.
- 12. void step(double dt) is neither used nor implemented. It will be deprecated.
- 13. **Cell()** is the default constructor. Users should use the **create_cell** functions rather than this constructor. See Section 7.3.
- 14. bool assign_position(std::vector<double> new_position) safely sets the cell's position to new_position. Always use this rather than directly editing position (inherited from Basic_Agent).
- 15. bool assign_position(double, double, double) performs the same function as above, but supplying the x, y, and z coordinates separately.
- 16. **void set_total_volume(double)** safely sets the cell's total volume. It also sets the cell's subvolumes proportionally.
- 17. double& get_total_volume(void) returns the cell's current total volume, stored in cell.phenotype.volume.total. It is *not* the preferred way to access this data, but it is provided to overwrite the base Basic_Agent::get_total_volume function.
- 18. void update_position(double dt) uses the cell's current and previous velocities to safely update the cell's position, using the second-order Adams-Bashforth algorithm.
- 19. **void assign_orientation()** sets the cell's **state.orientation** according to its current model (if any). See Section 7.4.3.
- 20. void copy_function_pointers(Cell*) overwrites the functions in functions with those from the supplied Cell.
- 21. **void update_voxel_in_container(void)** updates the cell's position within the interaction testing data structure.
- 22. void copy_data(Cell *) copies the member data (including custom_data, parameters, but not phenotype from the Cell at Cell*.
- 23. void set_phenotype(Phenotype& phenotype) is no longer used and will be deprecated. Users can now safely overwrite the cell's phenotype at any time.
- 24. void update_radius() is neither implemented nor used. It will be deprecated.
- 25. Cell_Container* get_container() returns the memory address of the Cell_Container containing the cell.

26. std::vector<Cell*>& cells_in_my_container(void) returns (by reference) a vector of pointers to the cells in the current mechanics voxel. For performance reasons, we are giving users direct access to the underlying data structures, rather than copies. It is critical that users do not alter the underlying array of Cell*. Future releases of PhysiCell will provide more refined access functions, including better lists of neighbors and lists of nearby mechanics voxels.

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7.3 Other key functions

The following are functions in the PhysiCell namespace (and are not members of the Cell class).

1. Cell* create_cell(void) creates a new cell (with default Cell_Definition; see Section 7.4.5) at (0,0,0) and registers it in all the relevant data structures. This is the *safe* way to create a new cell in the simulation.

Example: Let's create a cell (with the default Cell_Definition) at (15,27,-32):

```
Cell* pC = create_cell();
pC->assign_position(15,27,-32);
```

2. Cell* create_cell(Cell_Definition& cd) creates a new cell at (0,0,0) with parameters, phenotype, functions, and other properties specified in cd (See Section 7.4.5), and registers it in all the relevant data structures. This is the *safe* way to create a new cell in the simulation.

Example: Let's define a new type of cell (stem_cell), then create a new cell of this type at (15,27,-32):

```
// create a new cell definition
Cell_Definition stem_cell;

// Operations to set the properties and models
// of the stem_cell type go here.

// create a new stem_cell, and move it.
Cell* pC = create_cell(stem_cell);
pC->assign_position(15,27,-32);
```

3. **void delete_cell(int)** deletes the cell located at all_cells(int) and removes it from all relevant data structures. This is the *safe* way to directly remove a cell from a simulation.

Example: Let's check the 13th cell and delete it if a custom variable "infected" (See Section 7.4.1) indicates that it is infected. (See Section 13.4 for more on the vector all_cells that lists the memory addresses of all cells.)

```
Cell* pCell = (*all_cells)[13];
if( pCell->custom_data[ "infected" ] == true )
{
    delete_cell( 13 );
}
```

4. **void delete_cell(Cell*)** deletes the cell with located at Cell* and removes it from all relevant data structures. This is the *safe* way to directly remove a cell from a simulation.

Example: Let's check the 13th cell and delete it if a custom variable "infected" (See Section 7.4.1) indicates that it is infected. (See Section 13.4 for more on the vector all_cells that lists the memory addresses of all cells.)

```
Cell* pCell = (*all_cells)[13];
if( pCell->custom_data[ "infected" ] == true )
{
    delete_cell( pCell );
}
```

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7.4 Important classes (except Phenotype)

7.4.1 Custom_Cell_Data

This class allows users to dynamically add new custom data to individual cells, or to a Cell_Definition. (See Section 7.4.5.) Here is the full class definition:

```
class Custom_Cell_Data
{
private:
    std::unordered_map<std::string,int> name_to_index_map;
   friend std::ostream& operator<<(std::ostream& os, const Custom_Cell_Data& ccd);
public:
    std::vector<Variable> variables;
    std::vector<Vector_Variable> vector_variables;
    int add_variable( Variable& v );
    int add_variable( std::string name , std::string units , double value );
    int add_variable( std::string name , double value );
   int add_vector_variable( Vector_Variable& v );
    int add_vector_variable( std::string name ,
        std::string units , std::vector<double>& value );
    int add_vector_variable( std::string name , std::vector<double>& value );
    int find_variable_index( std::string name );
   double& operator[]( int i );
   double& operator[]( std::string name );
   Custom_Cell_Data();
   Custom_Cell_Data( const Custom_Cell_Data& ccd );
};
```

Here are the main data elements and member functions, in greater detail:

1. std::vector<Variable> variables is a vector of variables (initially empty). A Variable has the following form:

```
class Variable
{
  private:
    friend std::ostream& operator<<(std::ostream& os, const Variable& v);
  public:
    std::string name;
    double value;
    std::string units;

    Variable();
};</pre>
```

2. std::vector<Vector_Variable> vector_variables is a vector of vector variables (initially empty). A Vector_Variable has the following form:

```
class Vector_Variable
{
  private:
     friend std::ostream& operator<<(std::ostream& os, const Vector_Variable& v);

public:
    std::string name;
    std::vector<double> value;
    std::string units;

    Vector_Variable();
};
```

3. int add_variable(Variable& v) adds the new variable v to the custom data, and returns the index of the newly added variable.

Example:

```
Custom_Cell_Data ccd;
Variable v;
v.name = "sensitivity";
v.value = 0.3;
v.units = "dimensionless";
ccc.add_variable( v );
```

4. int add_variable(std::string name , std::string units , double value) adds a new variable with name name, units units, and value value. It returns the index of the newly added variable.

Example:

```
Custom_Cell_Data ccd;
ccd.add_variable( "sensitivity", "dimensionless", 0.3 );
```

5. int add_variable(std::string name, double value) adds a new variable with name name, unspecified units, and value value. It returns the index of the newly added variable.

Example:

```
Custom_Cell_Data ccd;
ccd.add_variable( "sensitivity", 0.3 );
```

6. int add_vector_variable(Vector_Variable& v) adds the new vector variable v to the custom data, and returns the index of the newly added variable.

Example:

```
Custom_Cell_Data ccd;
Vector_Variable v;
v.name = "home";
v.value = {0,1,2};
v.units = "micron";
ccc.add_vector_variable( v );
```

7. int add_vector_variable(std::string name, std::string units,

std::vector<double>& value) adds a new vector variable with name name, units units,
and (vector) value value. It returns the index of the newly added variable.

Example:

```
Custom_Cell_Data ccd;
std::vector<double> myvec = {0,1,2};
ccc.add_vector_variable( "home", "micron", myvec );
```

8. int add_vector_variable(std::string name , std::vector<double>& value) adds a new vector variable with name name, unspecified units, and (vector) value value. It returns the index of the newly added variable.

Example:

```
Custom_Cell_Data ccd;
std::vector<double> myvec = {0,1,2};
ccc.add_vector_variable( "home", myvec );
```

9. int find_variable_index(std::string name) returns the index of the variable with name name, if it exists. Note that this function is case sensitive.

Example:

```
Custom_Cell_Data ccd;

ccd.add_variable( "oxygen" , "mmHg", 38.0 );
ccd.add_variable( "maple syrup" , "megayums", 1.2 );
ccd.add_variable( "VEGF", "dimensionless", 0.01 );

int syrup_index = ccd.find_variable_index( "maple syrup" );
std::cout << ccd.variables[syrup_index].value << std::endl;</pre>
```

10. double& operator[] (int i) access the ith scalar variable.

Example:

```
Custom_Cell_Data ccd;

ccd.add_variable( "oxygen" , "mmHg", 38.0 );

ccd.add_variable( "maple syrup" , "megayums", 1.2 );

ccd.add_variable( "VEGF", "dimensionless", 0.01 );

int syrup_index = ccd.find_variable_index( "maple syrup" );

std::cout << ccd.variables[syrup_index].value << std::endl;

std::cout << ccd[syrup_index] << std::endl;</pre>
```

11. double& operator[](std::string name) access the scalar variable with name name. Note that this function is case sensitive.

Example:

```
Custom_Cell_Data ccd;

ccd.add_variable( "oxygen" , "mmHg", 38.0 );

ccd.add_variable( "maple syrup" , "megayums", 1.2 );

ccd.add_variable( "VEGF", "dimensionless", 0.01 );

int syrup_index = ccd.find_variable_index( "maple syrup" );

std::cout << ccd.variables[syrup_index].value << std::endl;

std::cout << ccd[syrup_index] << std::endl;

std::cout << ccd["maple syrup"] << std::endl;</pre>
```

- 12. Custom_Cell_Data() is the default constructor. It creates a custom cell data structure with no variables and no vector variables.
- 13. Custom_Cell_Data(const Custom_Cell_Data& ccd) is the copy constructor, where the new Custom_Cell_Data is initialized with all contents equal to ccd.

14. **Streaming:** You can display a custom variable to any C++ streaming operator.

Example: We'll create a custom cell data structure called my_custom_data, add some scalar and vector variables, and display it.

```
my_custom_data.add_variable( "spring constant", "1/min" , 0.01 );
my_custom_data.add_variable( "relaxation rate", "1/min" , 7e-5 );
my_custom_data.add_variable( "strain", "micron" , 0.0 );
my_custom_data.add_variable( "integrated strain", "micron*min" , 0.0 );
my_custom_data.add_variable( "max strain", "micron" , 10.0 );
my_custom_data.add_variable( "max integrated strain", "micron*min" , 600.0 );
my_custom_data.add_vector_variable( "home" , "micron" , {0,0,0} );
std::cout << my_custom_data << std::endl;</pre>
The output should look like this:
Custom data (scalar):
0: spring constant: 0.01 1/min
1: relaxation rate: 7.0e-005 1/min
2: strain: 0 micron
3: integrated strain: 0 micron*min
4: max strain: 10 micron
5: max integrated strain: 600 micron*min
Custom data (vector):
0: home: [0,0,0] micron
```

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7.4.2 Cell_Parameters

This class stores standardized parameters for a Cell (or a Cell_Definition; see Section 7.4.5). In future editions of PhysiCell, some of these may be moved into the phenotype. (See Section 8.) Here is the full class definition:

```
class Cell_Parameters
{
  private:
  public:
    double o2_hypoxic_threshold;
    double o2_hypoxic_response;
    double o2_hypoxic_saturation;

    double o2_proliferation_saturation;
    double o2_proliferation_threshold;
```

```
double o2_reference;

double o2_necrosis_threshold;
double o2_necrosis_max;

Phenotype* pReference_live_phenotype;

double max_necrosis_rate;
int necrosis_type;

Cell_Parameters();
}
```

Here are the main data elements and member functions, in greater detail:

1. double o2_hypoxic_threshold is the oxygen value (in mmHg) below which hypoxic signaling starts. (default: 15 mmHg, or about 2% oxygen).

```
o2\_hypoxic\_saturation \le o2\_hypoxic\_response \le o2\_hypoxic\_threshold
```

2. **double o2_hypoxic_response** is the oxygen value (in mmHg) below which hypoxic responses are observed (e.g., omics changes). (default: 8 mmHg)

```
o2\_hypoxic\_saturation \le o2\_hypoxic\_response \le o2\_hypoxic\_threshold
```

3. **double o2_hypoxic_saturation** is the oxygen value (in mmHg) below which hypoxic responses are at a maximum. (default: 4 mmHg)

```
o2_hypoxic_saturation < o2_hypoxic_response < o2_hypoxic_threshold
```

4. double o2_proliferation_saturation is the oxygen value (in mmHg) above which the proliferation rate is maximmized. No further oxygenation benefits the cell. (default: 160 mmHg, or 21% standard culture conditions)

```
o2\_proliferation\_threshold \le o2\_reference \le o2\_proliferation\_saturation
```

5. **double o2_proliferation_threshold** is the oxygen value (in mmHg) below which the proliferation ceases. (default: 5 mmHg)

```
o2\_proliferation\_threshold \le o2\_reference \le o2\_proliferation\_saturation
```

6. **double o2_reference** is the oxygen value that corresponds to the reference phenotype (see below). (default: 160 mmHg)

```
o2\_proliferation\_threshold \le o2\_reference \le o2\_proliferation\_saturation
```

7. double o2_necrosis_threshold is the oxygen value at which necrosis starts. (default: 5 mmHg)

```
o2\_necrosis\_max \le o2\_necrosis\_threshold
```

8. **double o2_necrosis_max** is the oxygen value at which necrosis reaches its maximum rate. (default: 2.5 mmHg)

```
o2_necrosis_max \le o2_necrosis_threshold
```

- 9. Phenotype* pReference_live_phenotype is a pointer to a Phenotype (See Section 8) which will serve as reference values when oxygen is equal to o2_reference.
- 10. **double max_necrosis_rate** is the necrosis rate (in units 1/min) when the oxygen value dips below o2_necrosis_max.
- 11. int necrosis_type is used to decide between deterministic and stochastic necrosis.

```
(Use PhysiCell_constants::deterministic_necrosis or PhysiCell_constants::stochastic_necrosis.)
```

12. Cell_Parameters() is the default constructor.

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7.4.3 Cell_Functions

This data structure attaches user-specified functions for key cell behaviors. Note that in PhysiCell, almost all functions that act on cells take the form

```
void function( Cell* pCell, Phenotype& phenotype, double dt ).
```

Ordinarily, phenotype = pCell->phenotype, but we allow these to be specified separately for cases where pCell = NULL.

Here is the full class definition:

Here are the member data and functions in greater detail:

- 1. Cycle_Model cycle_model is the cell cycle model to be used for this cell (or Cell_Definition—see Section 7.4.5). The Cycle_Model class is discussed in greater detail in Section 9.1.3.
- 2. void (*volume_update_function)(Cell* pCell, Phenotype& phenotype, double dt) is a function pointer to a user-specified model of cell volume regulation, based upon the parameters stored in phenotype.volume. See Section 9.3 for more detail. We recommend the reference model in [3]: standard_volume_update_function.
- 3. void (*update_migration_bias)(Cell* pCell, Phenotype& phenotype, double dt) is a function pointer to a user-specified model to set the migration bias direction, e.g., towards a chemical gradient. See Section 9.6 for more detail. We give an example in Section 15.1.2.
- 4. void (*custom_cell_rule)(Cell* pCell, Phenotype& phenotype, double dt) is a custom function that is executed every time the cell updates its mechanics. We give an example in Section 15.1.3
- 5. void (*update_phenotype) (Cell* pCell, Phenotype& phenotype, double dt) is where users should alter the cell's phenotype based upon microenvironmental conditions. We give an example in Section 15.1.4. We recommend the reference model update_cell_and_death_parameters_02_based found in [3].
- 6. void (*update_velocity)(Cell* pCell, Phenotype& phenotype, double dt) is a pointer to a user-specified model to set the cell's velocity, based upon the mechanics model. We recommend standard_update_cell_velocity from [3].
- 7. void (*add_cell_basement_membrane_interactions)(Cell* pCell,

 Phenotype& phenotype, double dt) modifies the cell's velocity based upon interactions
 with basement membrane given in this function. This function may change in future releases.
- 8. double (*calculate_distance_to_membrane)(Cell* pCell,

 Phenotype& phenotype, double dt); calculates the (signed) distance to the basement membrane (e.g., by an analytic solution, via a level set function, or by other discrete means).
- 9. void (*set_orientation)(Cell* pCell, Phenotype& phenotype, double dt) is a user-supplied function to update the cell's orientation, based upon interactions with other neighbors or other factors.

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7.4.4 Cell_State

The Cell_State is a small collection of cell descriptors that do not neatly belong in the cell Phenotype. Here is the full class definition:

```
class Cell_State
{
  public:
    std::vector<Cell*> neighbors;
    std::vector<double> orientation;

  double simple_pressure;

  Cell_State();
};
```

And here are further details on the member data and functions:

- 1. std::vector<Cell*> neighbors is the memory addresses of all cells that are currently neighbors of the cell (as determined by the adhesion interaction potential). This variable is not currently updated in PhysiCell, but it will be in a future release.
- 2. **std::vector<double> orientation** is the cell's current basal-to-apical orientation, as a unit vector. Please note that if the cell's **phenotype.geometry.polarity** = 0, then this state variable has no meaning. Also note that in 2-D simulations, one should set **orientation** = {0,0,1}.
- 3. **double simple_pressure** is a simple analog for mechanical pressure, nondimensionalized by a scale. We define the dimensional pressure p_i by

$$p_i = \frac{1}{S_i} \sum_{j \in \mathcal{N}_i} \left| \mathbf{F}_{ij}^{\text{ccr}} \right| \tag{2}$$

where \mathcal{N}_i is the set of the neighbor cells of the cell, $\mathbf{F}_{ij}^{\text{ccr}}$ is the cell-cell "repulsive" imparted on cell i by its neighbor j, and S_i is the cell surface area. In [3, 7], we approximated $\mathbf{F}_{ij}^{\text{ccr}}$ by

$$\mathbf{F}_{ij}^{\text{ccr}} = c_j^{\text{ccr}} \left(1 - \frac{d_{ij}}{R_i + R_j} \right)^2 \frac{\mathbf{d}_{ij}}{d_{ij}},\tag{3}$$

where $\mathbf{d}_{ij} = \mathbf{x}_j - \mathbf{x}_i$ and $d_{ij} = |\mathbf{d}_{ij}|$.

To define a scale, suppose the cell is in close packing in 3D by equally-sized cells with total volume V, mean radius R, maximal cross-sectional area A, and equal coefficient $c_{\rm ccr}^j = c_{\rm ccr}^i$ for each neighbor j. In 3D, the cell has 12 neighbors in a tight sphere packing. By [4], the equilibrium cell-cell distance s in a 3-D confluent cell packing is

$$\frac{s}{2R} = \frac{1}{2R}\sqrt{\frac{2A}{\sqrt{3}}} = \sqrt{\frac{2\pi R^2}{4R^2\sqrt{3}}} = \sqrt{\frac{\pi}{2\sqrt{3}}} \approx 0.9523.$$
 (4)

This defines a pressure scale:

$$\bar{p}_i = 12 \frac{c_{\text{ccr}}^i}{S_i} \left(1 - \sqrt{\frac{\pi}{2\sqrt{3}}} \right)^2 \approx 0.02729 \frac{c_{\text{ccr}}^i}{S_i}.$$
(5)

Thus, we define a dimensionless analog of pressure by:

$$\begin{aligned} \text{simple_pressure} &= \frac{p_i}{\overline{p}_i} &= \frac{1}{\left(1 - \sqrt{\frac{\pi}{2\sqrt{3}}}\right)^2} \sum_{j \in \mathcal{N}_i} \frac{c_{\text{ccr}}^j}{c_{\text{ccr}}^i} \left(1 - \frac{d_{ij}}{R_i + R_j}\right)^2 \\ &= \frac{2\sqrt{3}}{\left(\sqrt{2\sqrt{3}} - \sqrt{\pi}\right)^2} \sum_{j \in \mathcal{N}_i} \frac{c_{\text{ccr}}^j}{c_{\text{ccr}}^i} \left(1 - \frac{d_{ij}}{R_i + R_j}\right)^2 \\ &\approx 439.74 \sum_{j \in \mathcal{N}_i} \frac{c_{\text{ccr}}^j}{c_{\text{ccr}}^i} \left(1 - \frac{d_{ij}}{R_i + R_j}\right)^2 \\ &= \frac{439.74}{c_{\text{ccr}}^i} \sum_{j \in \mathcal{N}_i} c_{\text{ccr}}^j \left(1 - \frac{d_{ij}}{R_i + R_j}\right)^2. \end{aligned} \tag{6}$$

4. Cell_State() is the default constructor.

7.4.5 Cell Definition

The Cell_Definition class allows users to define a cell type, including its default phenotype, and all its custom functions. Users can then work on defining multiple cell types at the start of a simulation and using these to initialize many cells at the start of a simulation (or during a simulation).

Here is the full class definition:

```
class Cell_Definition
{
  private:
  public:
    int type;
    std::string name;

    Microenvironment* pMicroenvironment;

    Cell_Parameters parameters;
    Custom_Cell_Data custom_data;
    Cell_Functions functions;
    Phenotype phenotype;

    Cell_Definition();
};
```

Here are the member data and functions in greater detail:

- 1. **int type** is a unique identifier for the cell type. It will be copied to the cell's **type**.
- 2. std::string name is the "plain English" name of the cell type. It is copied to the cell's type_name.
- 3. **pMicroenvironment** is a pointer to the simulation's BioFVM microenvironment, which is always µenvironment.
- 4. parameters is defined in Section 7.4.2. It is copied to the cell's parameters.
- 5. custom_data is defined in Section 7.4.1. It is copied to the cell's custom_data.
- 6. **functions** is defined in Section 7.4.3. It is copied to the cell's functions.
- 7. **phenotype** is defined in Section 8. It is copied to the cell's **phenotype**.
- 8. Cell_Definition() is the default constructor.

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8 Phenotype

The Phenotype is one of the most important components of a cell in PhysiCell, allowing us to specify the current state and properties of a cell. Most functions in PhysiCell are based upon either updating the phenotype or advancing the simulation based upon each cell's current phenotype.

The Phenotype is divided into multiple functional groups:

- 1. **Cycle** gives cell cycle information, including the graph structure of the cycle model, associated parameter values (transition rates), and information on the current cycle phase. See Section 9.1. Note that for a dead cell, the "cycle model" is set to a death model.
- 2. **Death** gives current cell death rates for one or more death models. See Section 9.2.
- 3. **Volume** gives the cell's current total volume, as well as nuclear, cytoplasmic, fluid, and other subvolumes. See Section 9.3.
- 4. **Geometry** records the cell's radius, nuclear radius, and surface area (using spherical approximations). See Section 9.4.
- 5. **Mechanics** records the cell's adhesion and repulsion strength parameters for interactions with cells and the basement membrane, as well as the maximum adhesive interaction distance (a multiple of the cell's equivalent radius). Future releases of PhysiCell will likely expand this aspect of the cell phenotype. See Section 9.5.
- 6. **Motility** records phenotype information on motility, currently including persistence, a direction for biased random walks (e.g., chemotaxis), and a scalar bias parameter that can vary motility between purely Brownian and completely deterministic along the current bias direction. See Section 9.6.
- 7. **Secretion** records cell secretion and uptake rates, as well as a saturation density (at which secretion ends). See Section 9.7.

Here is how the Phenotype class is defined in PhysiCell_phenotype.h:

```
class Phenotype
{
private:
public:
    bool flagged_for_division;
    bool flagged_for_removal;
    Cycle cycle;
    Death death;
    Volume volume;
    Geometry geometry;
    Mechanics mechanics;
    Motility motility;
    Secretion secretion;
    Phenotype();
    void sync_to_functions( Cell_Functions& functions );
    void sync_to_default_functions( void );
};
```

In the Section 9, we give further details on the data elements within the Phenotype class.

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8.1 Member functions

- 1. **Phenotype()** is the default constructor.
- 2. void sync_to_functions (Cell_Functions & functions) makes sure that phenotype.cycle is matched to the cycle model and parameters in functions.cycle_model. See Section 9.1 for more details on the cycle model.
- 3. void sync_to_default_functions(void) is neither implemented nor used, because default constructors take care of this. The function will be deprecated.

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9 Phenotype details

9.1 Cycle Models

Cycle stores the graph structure and parameters of the current cycle model, which can either be a cell cycle model or a death cycle model. It also includes member functions to progress through the cycle, and call appropriate cell division or removal functions. Before introducing Cycle (Section 9.1.5), we will detail the sub-classes necessary to work with a cycle model. All these data structures have been introduced with the

goal of separating the graph structure of a cycle model (Cycle_Model: Section 9.1.3) from its parameter values (Cycle_Data: section 9.1.4), while still bundling them in a simple structure (Cycle: Section 9.1.5) for use in a Phenotype.

A Cycle_Model is constructed of one or more Phases (Section 9.1.1) that are connected into a directed graph by Phase_Links (Section 9.1.2).

```
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```

9.1.1 Phase

The Phase class defines the phase in a cycle model, including the following data elements:

- 1. **int index** is an internal unique index within a cycle model.
- 2. **int code** is a unique global identifier for the phase, using constants defined in PhysiCell_constants.h using PhysiCell_constants. See Section 13.7.
- 3. std::string name is the "plain English" name of the phase (e.g., G0/G1).
- 4. **bool division_at_phase_exit** is a Boolean variable that indicates whether the cell should divide at the end of this phase (e.g., at the end of M phase).
- 5. **bool removal_at_phase_exit** is a Boolean variable that indicates whether the cell should be removed at the end of this phase (e.g., at the end of apoptosis).
- 6. void (*entry_function)(Cell* pCell, Phenotype& phenotype, double dt) is a pointer to a function that is executed at the start of the phase. This would be a good place to write a function to perform mutations (e.g., at the start of G0/G1, randomly choose selected phenotype parameters according to a distribution).

Here is the formal class declaration:

```
class Phase
{
  public:
    int index;
    int code;
    std::string name;

    bool division_at_phase_exit;
    bool removal_at_phase_exit;

    void (*entry_function)( Cell* pCell, Phenotype& phenotype, double dt );

    Phase();
};
```

Member functions

1. **Phase()** is the default constructor. It sets index = 0, code = 0, name = "unnamed", all flags to false, and the entry function to NULL.

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9.1.2 Phase_Link

The Phase_Link class is a link between one Phase and another (e.g., progression from G0/G1 to S phase). It includes the following data elements:

- 1. int start_phase_index is the unique index of the starting phase in the phase transition (a link from phase start_phase_index to phase end_phase_index).
- 2. **int end_phase_index** is the unique index of the ending phase in the phase transition (a link from phase start_phase_index to phase end_phase_index).
- 3. **bool fixed_duration** is a Boolean variable that indicates whether the cell spends a fixed amount of time before transitioning from phase start_phase_index to phase end_phase_index. Note that this variable only makes sense if there is a single, unique phase transition starting from start_phase_index.
- 4. void (*arrest_function)(Cell* pCell, Phenotype& phenotype, double dt) is a function pointer that allows you to set an arrest condition for the phase transition (e.g., only progress from M to G0/G1 if the cell is of sufficient volume). Its return value is true if the transition is arrested, and false if the transition is allowed to proceed. Set this pointer to NULL to bypass checking for arrest in this phase link.
- 5. void (*exit_function)(Cell* pCell, Phenotype& phenotype, double dt) is a function pointer to a function that is executed at the end of the phase transition. This would be a good place to write a function to perform mutations (e.g., at the transition from M to G0/G1, randomly choose selected phenotype parameters according to a distribution).

Here is the formal class declaration:

```
class Phase_Link
{
  public:
    int start_phase_index;
    int end_phase_index;

    bool fixed_duration;

    bool (*arrest_function)( Cell* pCell, Phenotype& phenotype, double dt );
    void (*exit_function)( Cell* pCell, Phenotype& phenotype, double dt );

    Phase_Link();
};
```

Member functions

1. **Phase_Link()** is the default constructor. It sets all indices to 0, all flags to false, and it sets arrest_function = NULL.

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9.1.3 Cycle_Model

The Cycle_Model connects one or more Phases through Phase_Links, and stores this entire graph structure along with parameter values. (See Cycle_Data in Section 9.1.4.) It also contains member functions to assist with constructing and executing a cycle model, as well as to readily access transition rates. Here are the key data elements:

- 1. std::vector< std::unordered_map<int,int> > inverse_index_maps is an internal (private) data structure for easily accessing specific phase transitions. For reference, index_inverse_map[i][j] = k helps us find the Phase_Link from Phase i to Phase j, which is stored in phase_links[i][k], and whose transition rate is stored in data.transition_rates[i][k].
- 2. std::string name is the "plain English" name of the cycle model (e.g., Ki67 (Basic)).
- 3. **int code** is a unique global (integer) identifier for the cycle model, using constants defined in PhysiCell_constants.h using PhysiCell_constants. (See Section 13.7.)
- 4. std::vector<Phases> phases is a vector of Phases.
- 5. std::vector< std::vector<Phase_Link> > phase_links is a vector of vectors of Phase_Links. Note that phase_links[i] is the vector of all phase links starting at phase i. (This is why we need the inverse map above.)
- 6. int default_phase_index is the index of the default phase in the cycle model (usually 0). If a new Phenotype instance is created and assigned this cycle model, it should start out in this phase.
- 7. Cycle_Data data (of type Cycle_Data) contains a template of the key parameter values for this cycle model. Place default values for your cycle model here, as a copy of data is passed to any new Phenotype instance containing the cycle model. See Section 9.1.4.

Here is the formal class declaration:

```
class Cycle_Model
{
  private:
    std::vector< std::unordered_map<int,int> > inverse_index_maps;

public:
    std::string name;
    int code;
```

```
std::vector<Phase> phases;
     std::vector< std::vector<Phase_Link> > phase_links;
     int default_phase_index;
     Cycle_Data data;
     Cycle_Model();
     void advance_model( Cell* pCell, Phenotype& phenotype, double dt );
     int add_phase( int code, std::string name );
     int add_phase_link( int start_index, int end_index ,
          bool (*arrest_function)( Cell* pCell, Phenotype& phenotype, double dt ) );
     int add_phase_link( int start_index, int end_index , double rate ,
          bool (*arrest_function)( Cell* pCell, Phenotype& phenotype, double dt ) );
     int find_phase_index( int code );
     int find_phase_index( std::string );
     double& transition_rate( int start_index , int end_index );
     Phase_Link& phase_link(int i,int j);
     std::ostream& display( std::ostream& os );
};
```

Member functions

- 1. Cycle_Model() is the default constructor. It sets name = "unnamed", sets code = PhysiCell_constants::custom_cycle_model, sets the model to an empty one with no phases, and the links data.pCycle_Model to this (the Cycle_Model under construction). See Section 9.1.4.
- 2. int add_phase(int code, std::string name) adds a new Phase to the cycle model, with the supplied code and name. It returns the index (new_index) of the new phase, so that you can subsequently access it with phases[new_index].
- 3. int add_phase_link(int start_index, int end_index, bool (*arrest_function)(Cell* pCell, Phenotype& phenotype, double dt)) adds a new Phase_Link to phase_links[start_index] joining phase start_index to phase end_index, with the arrest function arrest_function. It resizes internal data structures including data (and its transition_rates) automatically, and sets the transition rate to 0.0. It returns the index (new_index) so that the phase link can be directly accessed as phase_links[start_index] [new_index].
- 4. int add_phase_link(int start_index, int end_index, double rate bool (*arrest_function)(Cell* pCell, Phenotype& phenotype, double dt)) adds a new Phase_Link to phase_links[start_index] joining phase start_index to phase end_index, with the arrest function arrest_function. It resizes internal data structures including data (and

- its transition_rates) automatically, and sets the transition rate to rate. It returns the index (new_index) so that the phase link can be directly accessed as phase_links[start_index] [new_index].
- 5. int find_phase_index(int code) finds the index i such that phases[i].code = code. Please note that this returns 0 if there is no exact match!
- 6. int find_phase_index(std::string name) finds the index i such that phases[i].name = name. Please note that this returns 0 if there is no exact match! Note that this function is case sensitive.
- 7. double& transition_rate(int start_index , int end_index) is a user-friendly interface function to access (by reference) the transition rate from phase start_index to phase end_index.
- 8. Phase_Link& phase_link(int start_index , int end_index) is a user-friendly interface function to access (by reference) the Phase_Link from phase start_index to phase end_index.
- 9. void advance_model(Cell* pCell, Phenotype& phenotype, double dt) advances the the cycle model by dt time (assumed minutes in current PhysiCell versions). For the current phase in phenotype.cycle.data.current_phase(), it evaluates the probability of advancing to all linked phases within dt time (see [3]) and changes the model (and the state of phenotype.cycle.data) accordingly. It will call cell division and removal functions as needed.
- 10. std::ostream& display(std::ostream& os) allows streaming of a basic visual output of the cycle model. I recommend calling display(std::cout).

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9.1.4 Cycle_Data

The Cycle_Data class contains key parameters and state variables for a cell cycle model (see Section 9.1.3), as well as member functions to easily access transition rates and the current cycle phase. It includes the following data elements:

- 1. std::vector< std::unordered_map<int,int> > inverse_index_maps is an internal (private) data structure for easily accessing specific phase transitions. For reference, index_inverse_map[i][j] = k helps us find the Phase_Link from Phase i to Phase j in the pCycle_Model, which is stored in pCycle_Model->phase_links[i][k], and whose transition rate is stored in transition_rates[i][k].
- 2. Cycle_Model* pCycle_Model is a pointer to the appropriate Cycle_Model.
- 3. **std::string time_units** is the units of time for the cycle model. (PhysiCell currently assumes all time units are in minutes.)
- 4. std::vector< std::vector<double> > transition_rates is a vector of vectors of cell phase transition rates. For each i, transition_rates[i] is the vector of transition rates from Phase i to any other linked phases. For safety, we might make this data element private in the future, as it can be more intuitively accessed via double& transition_rate(int,int) (below).
- 5. int current_phase_index indicates the current phase in the Cycle_Model.
- 6. double elapsed_time_in_phase records how long the cell has been in the current phase.

Here is the formal class declaration:

```
class Cycle_Data
{
  private:
    std::vector< std::unordered_map<int,int> > inverse_index_maps;

public:
    Cycle_Model* pCycle_Model;

    std::string time_units;

    std::vector< std::vector<double> > transition_rates;

    int current_phase_index;
    double elapsed_time_in_phase;

    Cycle_Data();

    Phase& current_phase( void );

    void sync_to_cycle_model( void );

    double& transition_rate(int start_phase_index, int end_phase_index );
    double& exit_rate(int phase_index );
};
```

Member functions

- 1. Cycle_Data() is the default constructor. It resizes transition_rates to zero, sets pCycle_Model = NULL, and defaults time_units = "min".
- 2. Phase& current_phase(void) is a user-friendly interface function to access (by reference) the current Phase in the cycle. Use this to get the name and other structural information.
- 3. **void sync_to_cycle_model(void)** resizes the internal data structures for consistency with the pCycle_Model, if it is non-NULL.
- 4. double& transition_rate(int start_phase_index , int end_phase_index) is a user-friendly interface function to access (by reference) the transition rate from phase start_phase_index to phase end_phase_index.
- 5. double& exit_rate(int phase_index) is a user-friendly interface function to access (by reference) the rate of exiting the phase_index phase, in the case where there is only one Phase_Link from that phase to another. (e.g., for a cycle model where the S phase only links to the G2 phase.) In this case, the cell spends (in the mean)

$$\frac{1}{\texttt{exit_rate}(\texttt{phase_index})} \tag{7}$$

time in phase phase_index.

9.1.5 Cycle

The Cycle bundles a Cycle_Model with Cycle_Data for simpler inclusion in a Phenotype. Its main data elements are:

- 1. Cycle_Model* pCycle_Model is a pointer to a Cycle_Model.
- 2. Cycle_Data data is Cycle_Data associated with the Cycle_Model. Note that this is an *independent* copy of the pCycle_Model->data, so that a single cell's phenotype can be updated without modifying the defaults for the underlying cycle model.

Here is the formal class declaration:

```
class Cycle
{
  private:
  public:
    Cycle_Model* pCycle_Model;
    Cycle_Data data;

    Cycle();

    void advance_cycle( Cell* pCell, Phenotype& phenotype, double dt );

    Cycle_Model& model( void );
    Phase& current_phase( void );
    int& current_phase_index( void );

    void sync_to_cycle_model( Cycle_Model& cm );
};
```

Member functions

- 1. Cycle() is the default constructor. It sets pCycle_Model = NULL and uses the default constructor for the Cycle_Data.
- 2. void advance_cycle(Cell* pCell, Phenotype& phenotype, double dt advances the cycle model by dt time, executing any arrest, entry, or other functions according to the supplied phenotype. (It is implemented by calling pCycle_Model->advance_model(pCell,phenotype,dt).
- 3. Cycle_Model& model(void) returns (by reference) the cycle model pointed to by pCycle_Model.
- 4. Phase& current_phase(void) returns (by reference) the current phase (as given in data) in the cycle model.

- 5. int& current_phase_index(void) returns (by reference) the current phase index (as given in data) in the cycle model.
- 6. void sync_to_cycle_model(Cycle_Model& cm) sets pCycle_Model = &cm and then overwrites data with cm.data.

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9.2 Death models

A death model is a Cycle_Model (Section 9.1.3), where one of the phases is marked to trigger cell removal. We define Death_Parameters (Section 9.2.1) to include key parameters needed in most death models, and bundle these as Death (Section 9.2.2) within the phenotype, similarly to Cycle (Section 9.1.5).

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9.2.1 Death_Parameters

Death_Parameters bundles key parameters needed to initialize a cell death model, particularly changes in a cell's volume model. The key data elements include:

- 1. std::string time_units gives the time units (by default minutes throughout PhysiCell).
- 2. double unlysed_fluid_change_rate is the rate of fluid change (in min⁻¹) prior to cell lysis.
- 3. double lysed_fluid_change_rate is the rate of fluid change (in min⁻¹) after cell lysis.
- 4. double cytoplasmic_biomass_change_rate is the degradation rate for cytoplasmic solids (in min⁻¹).
- 5. double nuclear_biomass_change_rate is the degradation rate for nuclear solids (in min⁻¹).
- 6. double calcification_rate is the rate of cell calcification (in min^{-1}).
- 7. **double relative_rupture_volume** is the relative amount by which the cell must swell (compared to the volume at the onset of cell death) before it bursts or lyses.

Here is the formal class declaration:

```
class Death_Parameters
{
  public:
    std::string time_units;

    double unlysed_fluid_change_rate;
    double lysed_fluid_change_rate;

    double cytoplasmic_biomass_change_rate;
```

```
double nuclear_biomass_change_rate;

double calcification_rate;

double relative_rupture_volume;

Death_Parameters();
};
```

Member functions

1. **Death_Parameters()** is the default constructor. It sets all parameter values to the reference apoptosis values for a generic breast epithelium line (calibrated to MCF-10A measurements), as in [3].

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9.2.2 Death

In PhysiCell, we allow the cells to evaluate multiple death models (e.g., apoptosis and necrosis). Death stores the cell's death rates, the corresponding death models, and associated parameters. Here are the data elements:

- 1. std::vector<double> rates is a vector of death rates, one per death model.
- 2. **std::vector<Cycle_Model*> models** is a vector of pointers to death models, which are of type Cycle_Model.
- 3. std::vector<Death_Parameters> parameters is a vector of Death_Parameterss, one for each Cycle_Model.
- 4. **bool** dead is a Boolean variable that is true if the cell is dead.
- 5. int current_death_model_index is the index of the current death model, when dead == true.

Here is the formal class declaration:

```
class Death
{
  private:
  public:
    std::vector<double> rates;
    std::vector<Cycle_Model*> models;
    std::vector<Death_Parameters> parameters;

  bool dead;
  int current_death_model_index;
```

Member functions

- 1. **Death()** is the default construtor. It resizes all the vectors to size zero, sets **dead = false**, and current_death_model_index = 0.
- 2. int add_death_model(double rate , Cycle_Model* pModel) adds the cycle model at pModel and sets its corresponding death rate to rate. It also resizes the parameters with values in the default constructor. It returns the index of the newly added death model.
- 3. int add_death_model(double rate, Cycle_Model* pModel,

Death_Parameters& death_parameters) adds the cycle model at pModel and sets its corresponding death rate to rate. It also increases the size of parameters by one by appending death_parameters. It returns the index of the newly added death model.

- 4. int find_death_model_index(int code) returns an integer i such that models[i]->code == code. Note that if no exact match is found, this returns 0.
- 5. int find_death_model_index(std::string name) returns an integer i such that models[i]->name == name. Note that if no exact match is found, this returns 0. Note that this function is case sensitive.
- 6. bool check_for_death(double dt) checks for each type of cell death in the next dt time. If the cell dies by any of the models between now and dt time in the future, this code sets dead = true, sets current_death_model_index to the index of the corresponding death model, and returns true.
- 7. void trigger_death(int death_model_index) immediately sets the cell to dead = true and current_death_model_index = death_model_index.
- 8. Cycle_Model& current_model(void) returns (by reference) the current cell death model (if dead == true).
- 9. Death_Parameters& current_parameters(void) returns (by reference) the Death_Parameters for the current death model (if dead == true).

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9.3 Volume

Volume stores the cell's total volume, its various sub-volume elements, and critical parameters. (PhysiCell supplies reasonable defaults.) Here are the main elements (all volume units in PhysiCell are currently assumed to be μ m³.)

- 1. **double total** is the cell's total volume.
- 2. **double solid** is the solid component of the cell's total volume. It is in dimensional units of volumenot a fraction or ratio. Note that volume.total = volume.solid + volume.fluid.
- 3. **double fluid** is the fluid component of the cell's total volume. It is in dimensional units of volumenot a fraction or ratio. Note that volume.total = volume.solid + volume.fluid.
- 4. double fluid_fraction is the fraction of the cell that is fluid, defined as:

```
volume.fluid_fraction = volume.fluid / volume.total.
```

Note that $0 \leq fluid_fraction \leq 1$.

5. double nuclear is the total nuclear volume. Note that:

volume.total = volume.nuclear + volume.cytoplasmic.

6. **double nuclear_solid** is the solid component of the cell's nuclear volume. It is in dimensional units of volume—not a fraction or ratio. Note that:

volume.nuclear = volume.nuclear_solid + volume.nuclear_fluid.

7. **double nuclear_fluid** is the fluid component of the cell's nuclear volume. It is in dimensional units of volume—not a fraction or ratio. Note that:

volume.nuclear = volume.nuclear_solid + volume.nuclear_fluid.

8. double cytoplasmic is the total cytoplasmic volume. Note that:

volume.total = volume.nuclear + volume.cytoplasmic.

9. double cytoplasmic_solid is the solid component of the cell's cytoplasmic volume. It is in dimensional units of volume—not a fraction or ratio. Note that:

volume.cytoplasmic = volume.cytoplasmic_solid + volume.cytoplasmic_fluid.

10. double cytoplasmic_fluid is the fluid component of the cell's cytoplasmic volume. It is in dimensional units of volume—not a fraction or ratio. Note that:

volume.cytoplasmic = volume.cytoplasmic_solid + volume.cytoplasmic_fluid.

11. **double calcified_fraction** is the fraction of the cell that is calcified. (This is particularly useful to simulations of ductal carcinoma in situ of the breast.) Note that

$$0 \le \text{calcified_fraction} \le 1.$$
 (8)

12. double cytoplasmic_to_nuclear_ratio is volume.cytoplasmic / volume.nuclear. Note that

$$0 \le \text{cytoplasmic_to_nuclear_ratio} \le 1.$$
 (9)

- 13. double rupture_volume is the (dimensional) volume at which a cell will burst or lyse.
- 14. double cytoplasmic_biomass_change_rate is the rate that cytoplasmic solid material can be synthesized to reach the target cytoplasmic solid volume. It is assumed to be written in units of min⁻¹.
- 15. **double nuclear_biomass_change_rate** is the rate that nuclear solid material can be synthesized to reach the target nuclear solid volume. It is assumed to be written in units of min⁻¹.
- 16. **double fluid_change_rate** is the rate that fluid can enter or leave the cell to reach the target fluid fraction. It is assumed to be written in units of min⁻¹.
- 17. **double calcification_rate** is the rate that the cell calcifies. It is assumed to be written in units of min⁻¹.
- 18. double target_solid_cytoplasmic is the cell's target ("desired" or "goal") solid cytoplasmic volume.
- 19. double target_solid_nuclear is the cell's target ("desired" or "goal") solid nuclear volume.
- 20. double target_fluid_fraction is the cell's target ("desired" or "goal") fluid fraction.
- 21. **double relative_rupture_volume** is the relative volume at which a cell, written as a multiple of the cell's total volume at the onset of a swelling process. (For example, at the start of necrosis.)

 Note that

$$0 \le \text{relative_rupture_volume} \le 1.$$
 (10)

Here is the class declaration:

```
class Volume
{
  public:
    double total;
    double solid;
    double fluid;
    double fluid_fraction;

    double nuclear;
    double nuclear_fluid;
    double nuclear_solid;

    double cytoplasmic;
    double cytoplasmic_fluid;
    double cytoplasmic_solid;

    double calcified_fraction;

    double rupture_volume;
```

```
double cytoplasmic_biomass_change_rate;
double nuclear_biomass_change_rate;
double fluid_change_rate;

double fluid_change_rate;

double calcification_rate;

double target_solid_cytoplasmic;
double target_solid_nuclear;
double target_fluid_fraction;

double target_cytoplasmic_to_nuclear_ratio;

double relative_rupture_volume;

Volume();

void divide( void );
void multiply_by_ratio(double);

void update( Cell* pCell, Phenotype& phenotype, double dt );
};
```

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9.3.1 Member functions

- 1. **Volume()** is the default constructor. It sets the variables to the reference values of a generic breast epithelium line (calibrated to MCF-10A).
- 2. **void divide(void)** cuts the volume, all sub-volumes, and the target volumes by one half. (It is used during cell division.)
- 3. **void multiply_by_ratio(double ratio)** multiplies the volume, all sub-volumes, and the target volumes by ratio.
 - void update(Cell* pCell, Phenotype& phenotype, double dt) is neither used nor implemented. It will be deprecated.

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9.4 Geometry

Geometry stores critical aspects of cell geometry other than volume. In the current implementation, we use spherical approximations for the cell geometry, since PhysiCell tracks cell volume but not cell morphology. The main data elements are:

1. double radius is the cell's equivalent radius, based upon the spherical approximation

$$volume.total = \frac{4}{3}\pi \text{ radius}^3. \tag{11}$$

In the current version of PhysiCell, spatial units are assumed to be μm .

2. double nuclear_radius is the nucleus' equivalent radius, based upon the spherical approximation

$$volume.nuclear = \frac{4}{3}\pi \text{ nuclear_radius}^3. \tag{12}$$

Its units are assumed μ m.

3. double surface_area is the cell's equivalent surface area, based upon the spherical approximation

$$surface_area = 4\pi radius^2$$
. (13)

4. **double polarity** is a dimensionless number between 0 and 1 to indicate how polarized the cell is along its basal-to-apical axis. If the polarity is zero, the cell has no discernible polarity. Note that polarity should be set to one for 2-D simulations.

Its units are assumed μm^2 .

Here is the class definition in PhysiCell_phenotype.h:

```
class Geometry
{
  public:
    double radius;
    double nuclear_radius;
    double surface_area;

    double polarity;

    Geometry();

    void update_radius( Cell* pCell, Phenotype& phenotype, double dt );
    void update_nuclear_radius( Cell* pCell, Phenotype& phenotype, double dt );
    void update_surface_area( Cell* pCell, Phenotype& phenotype, double dt );
    void update( Cell* pCell, Phenotype& phenotype, double dt );
};
```

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9.4.1 Member functions

- 1. **Geometry()** is the default constructor. It sets the variables to the reference values of a generic breast epithelium line (calibrated to MCF-10A).
- 2. void update_radius(Cell* pCell, Phenotype& phenotype, double dt) sets radius according to the spherical approximation in Section 9.4, using phenotype.volume.total.
- 3. void update_nuclear_radius(Cell* pCell, Phenotype& phenotype, double dt) sets nuclear_radius according to the spherical approximation in Section 9.4, using phenotype.volume.nuclear.
- 4. void update_surface_area(Cell* pCell, Phenotype& phenotype, double dt) sets surface_area according to the spherical approximation in Section 9.4, using cell.phenotype.volume.total.
- 5. void update(Cell* pCell, Phenotype& phenotype, double dt) sets radius, nuclear_radius, and surface_area according to phenotype.volume.

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9.5 Mechanics

Mechanics stores the main mechanics parameters for a cell. The main data elements are:

- 1. **double cell_cell_adhesion_strength** is the parameter C_{cca} in the default PhysiCell mechanics model, written as a multiple of the drag coefficient ν ; see [3]. It regulates the relative strength of cell-cell adhesive forces. Future releases of PhysiCell will allow this to be defined for interactions with multiple cell types.
- 2. **double cell_BM_adhesion_strength** is the parameter C_{cba} in the default PhysiCell mechanics model, written as a multiple of the drag coefficient ν ; see [3]. It regulates the relative strength of adhesion of cells to the basement membrane, when present.
- 3. **double cell_cell_repulsion_strength** is the parameter C_{ccr} in the default PhysiCell mechanics model, written as a multiple of the drag coefficient ν ; see [3]. It regulates the relative strength of cell-cell "repulsive" forces (resistance to deformation and compression).
- 4. **double cell_BM_repulsion_strength** is the parameter C_{cbr} in the default PhysiCell mechanics model, written as a multiple of the drag coefficient ν ; see [3]. It regulates the relative strength of cell-BM "repulsive" forces (resistance of cells to deformation and compression, and resistance of basement membranes to penetration and deformation by cells).
- 5. double relative_maximum_adhesion_distance is the maximum distance of cell adhesion to other cells or a basement membrane, given as a (dimensionless) multiple of geometry.radius.
- 6. **double maximum_adhesion_distance** is the absolute maximum distance at which a cell interacts adhesively with other cells or basement membrane. We tend not to use this variable in PhysiCell functions, so it may be deprecated in future releases.

As future releases of PhysiCell may include additional mechanics models, this class may be expanded in the future. In particular, we anticipate models to allow varying strengths of adhesion between different cell types, and improved adhesion models.

Here is the class definition in PhysiCell_phenotype.h:

```
class Mechanics
{
  public:
    double cell_cell_adhesion_strength;
    double cell_BM_adhesion_strength;
    double cell_cell_repulsion_strength;
    double cell_BM_repulsion_strength;

    double relative_maximum_adhesion_distance;
    double maximum_adhesion_distance;

    Mechanics();
};
```

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9.5.1 Member functions

1. **Mechanics()** is the default constructor function. It sets the parameters to the reference value for a generic breast epithelium line.

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9.6 Motility

Motility stores motility parameters and the current speed/direction of motility. It was designed to be sufficiently generic to allow recovery of purely Brownian motion, deterministic taxis, and combinations of these. Here are the main data elements:

- 1. bool is motile is a Boolean variable that can be used to enable/disable cell motility.
- 2. **double persistence_time** is the mean time cell continues at its current speed and direction before re-evaluating and choosing a new motility vector. It is assumed to have units minutes.
- 3. **double migration_speed** is the speed of motility, in the absence of other forces (e.g., cell-cell adhesion). It is assumed to have units of μ m/min.
- 4. **std::vector<double> migration_bias_direction** is the 3-D vector giving the cell's preferred direction of motility for biased Brownian motion. If the user modifies this vector, they must ensure it is a *unit vector*:

$$|| migration_bias_direction || = 1.$$
 (14)

- 5. **double migration_bias** (with a value in [0,1]) sets the degree to which cell motility is biased along migration_bias_direction. If 0, then motion is completely Brownian. If 1, it is completely determinists along the bias direction.
- 6. **bool restrict_to_2D** is a Boolean variable that is set to **true** is we are restricting cell motility to 2D.
- 7. **std::vector<double> motility_vector** is the velocity vector for cell motility, based upon the all the variables and parameters defined above. See also Section 9.6.2.

Here is the class definition:

```
class Motility
{
  public:
    bool is_motile;

    double persistence_time;
    double migration_speed;
    std::vector<double> migration_bias_direction;
    double migration_bias;
    bool restrict_to_2D;
    std::vector<double> motility_vector;

    Motility();
};
```

The main parameters are further defined and related in Section 9.6.2

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9.6.1 Member functions

1. Motility() is the default constructor function. Note that it sets:

```
is_motile = false,
update_migration_bias_vector = NULL, and
restrict_to_2D = false.
```

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9.6.2 Motility definitions

The direction of (biased random) motility is given by

$$\mathbf{d}_{\text{mot}} = \frac{b \, \mathbf{d}_{\text{bias}} + (1 - b) \boldsymbol{\xi}}{||b \, \mathbf{d}_{\text{bias}} + (1 - b) \boldsymbol{\xi}||}$$
(15)

where b (migration_bias) is the level of bias, ξ is a random unit vector (length 1, uniformly random direction), and \mathbf{d}_{bias} (migration_bias_direction) is the directional bias for motility. See Section 7.2 for more information on how the Motility class is used when updating a cell's velocity.

9.7 Secretion

Secretion collects the cell's biotransport parameters for interfacing with BioFVM [2]. Its main data elements include:

- 1. **Microenvironment* pMicroenvironment** is a pointer to the correct microenvironment, where substrates have already been declared.
- 2. std::vector<double> secretion_rates is a vector of secretion rates for the substrates in the microenvironment.
- 3. **std::vector<double> uptake_rates** is a vector of uptake rates for the substrates in the microen-vironment.
- 4. std::vector<double> saturation_densities is a vector of densities at which secretions saturate. See [2].

Here is the full class declaration:

```
class Secretion
{
private:
public:
   Microenvironment* pMicroenvironment;
    std::vector<double> secretion_rates;
    std::vector<double> uptake_rates;
    std::vector<double> saturation_densities;
   Secretion();
   void sync_to_current_microenvironment( void );
   void advance( Basic_Agent* pCell, Phenotype& phenotype , double dt );
   void sync_to_microenvironment( Microenvironment* pNew_Microenvironment );
   void set_all_secretion_to_zero( void );
   void set_all_uptake_to_zero( void );
   void scale_all_secretion_by_factor( double factor );
   void scale_all_uptake_by_factor( double factor );
};
```

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9.7.1 Member functions

- 1. **Secretion()** is the default constructor. If a default Microenvironment has already been set in BioFVM, then pMicroenvironment is set to this (otherwise NULL). Thereafter, secretion_rates, uptake_rates, and saturation_densities are resized for consistency with pMicroenvironment, with all values set to 0.0.
- 2. **void sync_to_current_microenvironment(void)** resizes secretion_rates, uptake_rates, and saturation_densities consistency with pMicroenvironment and sets all vector entries to 0.0.
- 3. void sync_to_microenvironment(Microenvironment* pNew_Microenvironment) sets pMicroenvironment = pNew_Microenvironment, resizes secretion_rates, uptake_rates, and saturation_densities for consistency with pMicroenvironment, and sets all vector entries to 0.0.
- 4. void advance (Basic_Agent* pCell, Phenotype& phenotype, double dt) evaluates the BioFVM secretion and uptake functions for this individual cell. Consistency checks with BioFVM (including those from volume changes) are fully automated.
- 5. **void set_all_secretion_to_zero(void)** sets all the secretion rates to zero. (Please note that settings pCell->is_active = false is the most efficient way to set all secretion and uptake to zero in a cell.)
- 6. **void set_all_uptake_to_zero(void)** sets all the uptake rates to zero. (Please note that settings pCell->is_active = false is the most efficient way to set all secretion and uptake to zero in a cell.)
- 7. **void scale_all_secretion_by_factor(double factor)** multiplies all the secretion rates by factor.
- 8. void scale_all_uptake_by_factor(double factor) multiplies all the uptake rates by factor.

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10 Cell Containers

PhysiCell uses Cell_Containers to help organize and search for cells within the simulation (spatial) domain, and to accelerate cell-cell mechanics. It is overloaded with much of the core functionality of PhysiCell, so we highly recommend that you **avoid direct operations on cell containers!** For reference, here is the full class definition:

```
class Cell_Container : public BioFVM::Agent_Container
{
  private:
    std::vector<Cell*> cells_ready_to_divide;
    std::vector<Cell*> cells_ready_to_die;
    int boundary_condition_for_pushed_out_agents;
    bool initialzed = false;

public:
```

```
BioFVM::Cartesian_Mesh underlying_mesh;
    std::vector<double> max_cell_interactive_distance_in_voxel;
    int num_divisions_in_current_step;
    int num_deaths_in_current_step;
    double last_diffusion_time =0.0;
    double last_cell_cycle_time = 0.0;
    double last_mechanics_time = 0.0;
    Cell_Container();
    void initialize(double x_start, double x_end,
        double y_start, double y_end, double z_start, double z_end,
        double voxel_size);
    void initialize(double x_start, double x_end,
        double y_start, double y_end, double z_start, double z_end,
        double dx, double dy, double dz);
    std::vector<std::vector<Cell*> > agent_grid;
    std::vector<std::vector<Cell*> > agents_in_outer_voxels;
    void update_all_cells(double t);
    void update_all_cells(double t, double dt);
    void update_all_cells(double t, double phenotype_dt, double mechanics_dt);
    void update_all_cells(double t, double phenotype_dt, double mechanics_dt,
        double diffusion_dt );
    void register_agent( Cell* agent );
    void add_agent_to_outer_voxel(Cell* agent);
    void remove_agent(Cell* agent );
    void remove_agent_from_voxel(Cell* agent, int voxel_index);
    void add_agent_to_voxel(Cell* agent, int voxel_index);
    void flag_cell_for_division( Cell* pCell );
    void flag_cell_for_removal( Cell* pCell );
    bool contain_any_cell(int voxel_index);
};
Users may want to use the following function to ensure that the mechanics data structures are set up
consistently with BioFVM microenvironment's domain:
Cell_Container* create_cell_container_for_microenvironment( BioFVM::Microenvironment& m,
    double mechanics_voxel_size );
Here is an example use:
// Set mechanics voxel size.
double mechanics_voxel_size = 30;
// Assume microenvironment is defined above somewhere.
```

```
// Set up the PhysiCell mechanics data structure.
Cell_Container* cell_container = create_cell_container_for_microenvironment(
    microenvironment, mechanics_voxel_size );
```

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11 PhysiCell Outputs

PhysiCell supports several methods of output, including SVG files (allowing virtual pathology through a fixed cross-section), MultiCellDS digital snapshots (a single-time save file), and other output methods.

Over the next several releases, we plan further improvements to PhysiCell outputs.

11.1 Virtual Pathology

PhysiCell can simulate transmitted light microscopy to create virtual H&E (hematoxylin and eosin) images, as well as false-colored images. These images are saved as SVG (scalable vector graphics) files, which allow lossless rescaling of the image. Moreover, because SVG files are a specialized XML, users can change labels and other image aspects long after image processing, using simple text editors.

We also note that the SVG functions provided in PhysiCell (./modules/PhysiCell_SVG.h) can be compiled independently of PhysiCell.

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11.1.1 SVG functions

SVG functions (defined in ./modules/PhysiCell_SVG.h) provide basic functionality for creating SVG files. In all the functions below, please note that the origin (0,0) in an SVG file is in the upper left corner.

1. bool Write_SVG_start(std::ostream& os, double width, double height) creates the header information for an SVG file of width width, height height, and writes the stream in os.

Example: Starting a 640×480 image:

```
// open the file, write a basic "header"
std::ofstream os( "sample_SVG.svg" , std::ios::out );
Write_SVG_start( os, 640, 480 );
```

2. bool Write_SVG_end(std::ostream& os) writes the end of an SVG file to the stream in os.

Example: Starting a 640×480 image:

```
// open the file, write a basic "header"
std::ofstream os( "sample_SVG.svg" , std::ios::out );
Write_SVG_start( os, 640, 480 );
```

```
// operations on the SVG file
  // close and save the file
  Write_SVG_end(os);
  os.close():
3. bool Write_SVG_text( std::ostream& os, const char* str,
        double position_x, double position_y, double font_size,
        const char* color, const char* font) places the text str at (upper left) position
  [position_x,position_y] with text height font_size, color color, and font font. It writes to
  the stream os.
  Example: Placing a dark red "hello world" at (30,60), with font height 17 and the Arial font.
  // open the file, write a basic "header"
  std::ofstream os( "sample_SVG.svg" , std::ios::out );
  Write_SVG_start( os, 640, 480 );
  std::string my_text = "Hello world!";
  std::string my_color = "rgb(128,0,0)";
  std::string my_font = "Arial";
  Write_SVG_text( os, my_text.c_str(), 30, 60, 17, my_color.c_str(),
      my_font.c_str() );
  // close and save the file
  Write_SVG_end(os);
  os.close();
4. bool Write_SVG_circle( std::ostream& os,
        double center_x, double center_y, double radius,
        double stroke_size, std::string stroke_color,
        std::string fill_color ) places a circle with center at [center_x,center_y] and radius
  radius. The circle is filled with color fill_color, and an outline of thickness stroke_size and
  color stroke color. It writes to the stream os.
  Example: Placing a cyan circle with black outline at (100,90), with radius 8.6, and outline width 1.
  // open the file, write a basic "header"
  std::ofstream os( "sample_SVG.svg" , std::ios::out );
  Write_SVG_start( os, 640, 480 );
  std::string my_fill_color = "rgb(0,255,255)";
  std::string my_outline_color = "black";
  Write_SVG_circle(os,100,90,8.6,1,my_outline_color.c_str(),my_fill_color.c_str());
  // close and save the file
  Write_SVG_end(os);
```

os.close();

```
double UL_corner_x, double UL_corner_y,
        double width, double height, double stroke_size,
        std::string stroke_color , std::string fill_color ) places a rectangle with upper-
  left corner [UL_corner_x, UL_corner_y], width width, height height, filled with color fill_color,
  and outlined with color stroke_color and line thickness stroke_size. It writes to the stream os.
  Example: Placing a black border with no fill around the SVG image.
  // open the file, write a basic "header"
  std::ofstream os( "sample_SVG.svg" , std::ios::out );
  Write_SVG_start( os, 640, 480 );
  std::string my_fill_color = "none";
  std::string my_outline_color = "black";
  Write_SVG_rect(OS,0,0,640,480,1,my_outline_color.c_str(),my_fill_color.c_str());
  // close and save the file
  Write_SVG_end(os);
  os.close();
6. bool Write_SVG_line( std::ostream& os,
        double start_x, double start_y, double end_x, double end_y,
        double thickness, std::string stroke_color ) draws a line with color stroke_color
  and thickness thickness from [start_x,start_y] to [end_x,end_y]. It writes to the stream os.
  Example: Placing a thin dark blue line from (0,0) to (640,480).
  // open the file, write a basic "header"
  std::ofstream os( "sample_SVG.svg" , std::ios::out );
  Write_SVG_start( os, 640, 480 );
  std::string my_stroke_color = "rgb(0,0,64)";
  Write_SVG_line(os,0,0,640,480,1.5,my_stroke_color.c_str());
  // close and save the file
  Write_SVG_end(os);
  os.close();
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```

11.1.2 Pathology functions

5. bool Write_SVG_rect(std::ostream& os,

PhysiCell can create fast virtual pathology images through a cross section. Here are the main functions:

 void SVG_plot(std::string filename, Microenvironment& M, double z_slice, double time, $std::vector < std::string > (*cell_coloring_function) (Cell*))$ creates an SVG plot through $z = z_slice$, using the coloring function cell_coloring_function (See Section 11.1.3), with labeling for time time, in the microenvironment M, saved to filename.

This function checks all cells for intersection with the plane through $z = \mathbf{z}_{-}\mathbf{slice}$, and plots the intersecting part of the cell cytoplasm and nucleus (using circular approximations). In the plot, 1 pixel is 1 μ m.

Example: H&E plot through $z = 0\mu m$, followed by a false-colored Ki-67 plot through $z = 10\mu m$:

```
SVG_plot( "initial_HE.svg" , microenvironment, 0.0 , t,
    hematoxylin_and_eosin_cell_coloring );
SVG_plot( "initial_Ki67.svg" , microenvironment, 10.0 , t,
    false_cell_coloring_Ki67 );
```

The SVG plotting options are set by PhysiCell_SVG_options, which is discussed further in Section 13.6.

- 2. std::string formatted_minutes_to_DDHHMM(double minutes) creates a nicely formated string (in days, hours, and minutes) based upon minutes. It is used extensively in SVG_plot.
- 3. std::vector<double> transmission(std::vector<double>& incoming_light, std::vector<double>& absorb_color, double thickness, double stain) simulates transmission of light of color incoming_light through a tissue of thickness thickness, stained at relative intensity stain (with range from 0 to 1), which absorbs light of color absorb_color. Its output is the transmitted color, as an RGB vector.

We use a Lambert-Beer [5] light transmission model for each color channel (Red, Green, Blue):

$$\begin{aligned} \text{Transmitted}_{\text{C}} &= & \text{Incoming}_{\text{C}} \cdot \exp \left(- \left(\text{thickness} \cdot \text{stain} \cdot \frac{1}{255.0} \text{Absorb}_{\text{C}} \right) \right), \\ & \text{C} \in \{ \text{Red}, \text{Green}, \text{Blue} \} \end{aligned}$$

Note that we use 24-bit color, so red, green, and blue values should vary from 0 to 255. Outputs to SVG files will be rounded to the nearest integer value. This function is available for use in custom coloring functions, and PhysiCell uses it for virtual H&E stains.

Future releases will include functions to color the background according to data values in the BioFVM microenvironment.

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11.1.3 Cell coloring functions

PhysiCell's virtual pathology functions are built upon choosing a coloring function for the cells. A coloring function takes the form:

```
std::vector<std::string> some_coloring_function( Cell* pCell )
```

and it returns a vector of four strings: the cytoplasm fill color, cytoplasm outline color, nuclear color, and nuclear outline color.

Colors in SVG can be specified as:

- 1. **RGB colors:** Use a string of the form "rgb(R,G,B)", where R, G, and B are the red, green, and blue values, and they are integers between 0 and 255.
- 2. **Standard web colors:** Use a string like "black" or "red", or use "none" for no color (transparent). See https://www.w3.org/TR/SVG11/types.html#ColorKeywords for a list of valid SVG colors.

The following cell coloring functions are provided in PhysiCell:

- 1. $simple_cell_coloring$ colors the cytoplasm red (255,0,0), the nucleus blue (0,0,255), and all outlines black.
- 2. false_cell_coloring_Ki67 is recommended for the Ki67_Basic and Ki67_Advanced cycle models. (See Section 9.1 and Section 14.1.) Pre-mitotic Ki67+ cells are colored green (0,255,0), with a darker green nucleus (0,125,0). Post-mitotic Ki67+ cells are colored magenta (255,0,255), with a darker magenta nucleus (125,0,125). (In the Ki67_Basic model, all Ki67+ cells are green.) Ki67- cells are colored blue (40,200,255) with a darker blue nucleus (20,100,255).
 - Apoptotic cells are colored red (255,0,0) with a darker red nucleus (125,0,0). Necrotic cells are colored brown (250,138,38) with a darker brown nucleus (139,69,19). All outlines are black.
- 3. **false_cell_coloring_live_dead** colors live cells green, apoptotic cells red, and necrotic cells brown, with colors defined as the Ki67 coloring function. All outlines are black.
- 4. hematoxylin_and_eosin_cell_coloring colors the cytoplasm by first using the transmission function, with white (255,255,255) incoming light, an eosin absorb color (2.55,33.15,2.55) [10], a thickness of 20, and a stain intensity given by

$$\mathtt{stain} = \frac{\mathtt{pCell}\text{-}\mathtt{phenotype.volume.cytoplasmic_solid}}{\mathtt{pCell}\text{-}\mathtt{phenotype.volume.cytoplasmic} + 10^{-10}}, \tag{17}$$

which approximates the process of staining cytoplasmic solids with eosin, and the water fraction remaining unstained. The result of this simulated transmission is then fed back through the transmission function (as the incoming light color), with an hematoxylin absorb color (49.90,51.00,20.40) [10], a thickness of 20, and a stain intensity given by

$$stain = pCell \rightarrow phenotype.volume.calcified_fraction$$
 (18)

which approximates the process of staining calcified cytoplasmic solids with hematoxylin.

The nucleus is colored by virtual hematoxylin staining, with incoming light color white (255,255,255), hematoxylin absorb color (49.90,51.00,20.40) [10], and a stain intensity given by

$$\mathtt{stain} = \frac{\mathtt{pCell}\text{-}\mathtt{phenotype.volume.nuclear}\text{-}\mathtt{solid}}{\mathtt{pCell}\text{-}\mathtt{phenotype.volume.nuclear} + 10^{-10}}, \tag{19}$$

which approximates the process of staining nuclear solids with hematoxylin, and the water fraction remaining unstained.

All outlines match the corresponding fill colors.

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11.1.4 Examples of custom cell coloring functions

Example: Black nucleus and oxygen-based cytoplasmic coloring

```
std::vector<std::string> oxygen_coloring( Cell* pCell )
   std::vector< std::string > output( 4 , "black" );
   // nucleus and both outlines are already black.
   // cytoplasm color
   // first, get the oxygenation
   // determine the o2 index the first time the function is run.
   static int o2_index = microenvironment.find_density_index( "oxygen" );
   // sample the microenvironment at the cell's locaiton
   double o2 = pCell->nearest_density_vector()[o2_index];
   // set the o2 max scale by the cell's parameters.
   static double max_o2 = pCell->parameters.o2_proliferation_saturation;
   // O2 goes from blue (anoxic, 0 mmHg) to red (fully oxygenated).
   int color = (int) round( o2 * 255.0 / max_o2 );
    char szTempString [128];
    sprintf( szTempString , "rgb(%u,0,%u)", color, 255-color );
    output[0].assign( szTempString );
   return output;
}
```

Example: Simulated immunohistochemistry with DAB and a hematoxylin counterstain As in H&E, we assume that hematoxylin stains all nuclear solids, and here we assume that it stains cytoplasmic solids at 10% of the nuclear intensity. We assume that DAB stains a nuclear protein (in custom data). Based on these virtual stains, we use simulated light transmission.

For reference, we use a DAB absorb color of (25.50,53.55,73.95) [10].
std::vector<std::string> nuclear_immunostain(Cell* pCell)
{
 std::vector< std::string > output(4 , "black");

 // absorb colors
 static std::vector<double> hematoxylin_absorb = {45.90,51.00,20.40};
 static std::vector<double> DAB_absorb = {25.50,53.55,73.95};

// cytoplasm colors

```
double solid_fraction = 1.0 - pCell->phenotype.fluid_fraction;
double cyto_stain_intensity = 0.1 * solid_fraction;
std::vector<double> = color(3, 255.0); // start with white light
color = transmission(color,hematoxylin_absorb,20,cyto_stain_intensity);
char szTempString [128];
sprintf( szTempString , "rgb(%u,%u,%u)",
    (int)round(color[0]), (int)round(color[1]), (int)round(color[2]) );
output[0].assign( szTempString );
output[1].assign( szTempString );
// nuclear colors
// determine the index of the nuclear protein (assumed between 0 and 1)
static int protein_index =
    pCell->custom_data.find_variable_index( "nuclear_protein" );
double nuclear_DAB_stain_intensity = pCell->custom_data[protein_index];
double nuclear_H_stain_intensity = solid_fraction;
color = \{255.0, 255.0, 255.0\};
color = transmission(color,hematoxylin_absorb,20,nuclear_H_stain_intensity);
color = transmission(color,DAB_absorb,20,nuclear_DAB_stain_intensity);
sprintf( szTempString , "rgb(%u,%u,%u)",
    (int)round(color[0]), (int)round(color[1]), (int)round(color[2]) );
output[2].assign( szTempString );
output[3].assign( szTempString );
return output;
```

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11.2 MultiCellDS digital simulation snapshots

PhysiCell saves its data as specialized MultiCellDS digital snapshots [1]. These snapshots save key metadata (See Section 13.5), the microenvironment, and a compact cell readout in an XML file. See [1] and http://multicellds.org/Standards.php for detailed information on the data standard.

To save a MultiCellDS simulation snapshot, use:

Here, the filebase determines how all the snapshot sub-files will be named: filebase.xml, filebase_cells.mat filebase_mesh0.mat, etc.

Example:

}

```
double t = 0.0;
double dt = 0.01;
double t_max = 30.0 * 24.0 * 60.0; // 30 days
double tolerance = 0.01 * dt;
// other initialization code
// save the initial data
save_PhysiCell_to_MultiCellDS_xml_pugi( "initial" , microenvironment , t );
double next_save_time = t;
double save_interval = 60.0; // save every 60 minutes
output_index = 0;
while( t < t_max + tolerance )</pre>
{
    // save if it's time
    if( fabs( t - next_save_time ) < tolerance )</pre>
    {
        // make an appropriate file name
        char filename[1024];
        sprintf( filename , "output%08u" , output_index );
        // save
        save_PhysiCell_to_MultiCellDS_xml_pugi( filename , microenvironment , t );
        next_save_time += save_interval;
        output_index++;
    }
    // more simulation steps
    t += dt;
}
```

Here is the overall structure of PhysiCell snapshot

- 1. XML headers in the <XML> tag.
- 2. MultiCellDS tag, indicating a digital simulation snapshot: <MultiCellDS version="0.5" type="snapshot/simulation">.
 - (a) Metadata in a <metadata> tag, including simulation provenance (who ran it, with what software, citation information, etc.), and other notable elements including:
 - i. The current simulation time, saved in a tag like <current_time units="min">0.000000</current_time>.

- (b) The BioFVM microenvironment, in a <microenvironment> tag structured as:
 - i. <domain> (MultiCellDS can support multiple domains, although we only use one.)
 - A. <mesh> describes the BioFVM mesh. The spatial units are given as an XML attribute of this element. Here is the overall (truncated) structure

The voxel information (in this case, in initial_mesh0.mat) is stored with one voxel per column. The first three rows give the x-, y-, and z-coordinates of each voxel's center, respectively. The fourth row gives each voxel's volume.

B. <variables> gives a list of variables, including the name (XML attributes), units (XML attributes), and some physical parameters. Here is a (truncated) example:

<variables>

C. Microenvironment data, typically stored compactly in a MATLAB file, like this:

```
<data type="matlab">
     <filename>initial_microenvironment0.mat</filename>
</data>
```

The data file (here, initialize_microenvironment0.mat) stores the microenvironment in one column per voxel, and if there are n substrates, n+3 rows, where rows one to three are the (x,y,z) coordinates of the voxel, and each subsequent row is a variable value (as defined in $\$ above).

ii. Cellular information, here in a customized and compact MultiCellDS format:

```
<label index="4" size="1">total_volume</label>
                  <label index="5" size="1">cell_type</label>
                  <label index="6" size="1">cycle_model</label>
                  <label index="7" size="1">current_phase</label>
                  <label index="8" size="1">elapsed_time_in_phase</label>
                  <label index="9" size="1">nuclear_volume</label>
                  <label index="10" size="1">cytoplasmic_volume</label>
                  <label index="11" size="1">fluid_fraction</label>
                  <label index="12" size="1">calcified_fraction</label>
                  <label index="13" size="3">orientation</label>
                  <label index="16" size="1">polarity</label>
               </labels>
               <filename>initial_cells_physicell.mat</filename>
            </simplified_data>
         </custom>
      </cell_population>
   </cell_populations>
</cellular_information>
```

In the indicated MATLAB file, each cell is stored in a separate column, with cell-scale data given in separate rows as defined in <labels> above. We anticipate improvements in this MultiCellDS output in future editions, but the data will always be structured as above with labels. We will also always preserve the ordering of the first 17 data elements for compatibility.

In the case above, the first row is the cell ID, the next three rows are the cell (x,y,z) position, the next row is the cell's (integer) type, the next row gives the cell's cycle model (as a PhysiCell::constants integer), and so forth.

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11.2.1 Reading PhysiCell snapshots in MATLAB

In the ./matlab directory, we include a few MATLAB functions to parse the PhysiCell MultiCellDS XML files and associated .mat files. We recommend copying them to your path, or to the same directory as your data.

Here is an example of loading a snapshot (in MATLAB), plotting a microenvironment variable, and then plotting some cells.

```
MCDS = read_MultiCellDS_xml( 'initial.xml' );
% output looks like this
% Elapsed time is 0.522643 seconds.
%
% Summary for file initial.xml:
% Voxels: 40000
% Substrates: 1
% oxygen (mmHg)
% Cells: 21845
```

```
%
contour( MCDS.mesh.X , MCDS.mesh.Y, MCDS.continuum_variables(1).data(:,:,1) )
axis ii
axis square
% label using MCDS metadata
xlabel( sprintf('x (%s)', MCDS.metadata.spatial_units) , 'fontsize', 13 )
ylabel( sprintf('y (%s)', MCDS.metadata.spatial_units) , 'fontsize', 13 )
% more labeling
title( sprintf('%s at time %3.2f %s', MCDS.continuum_variables(1).name , ...
MCDS.metadata.current_time , MCDS.metadata.time_units) , 'fontsize' , 14 );
% add a colorbar, and label
c = colorbar;
c.Label.String = sprintf('%s (%s)', MCDS.continuum_variables(1).name, ...
MCDS.continuum_variables(1).units );
c.FontSize = 13;
%
%
% Now, plot the cell positions in 3D
figure
plot3( MCDS.discrete_cells.state.position(:,1) , ...
MCDS.discrete_cells.state.position(:,2), ...
MCDS.discrete_cells.state.position(:,3) , 'ko' );
% labeling
axis equal
xlabel( sprintf('x (%s)', MCDS.metadata.spatial_units) , 'fontsize', 13 )
ylabel(sprintf('y (%s)', MCDS.metadata.spatial_units), 'fontsize', 13)
zlabel( sprintf('z (%s)', MCDS.metadata.spatial_units) , 'fontsize', 13 )
```

We anticipate further refinements to MATLAB plotting, and improvements to importing this data beyond MATLAB. Please also note that the current matlab read script uses the reduced matlab file stored in <simplified_data type="matlab" source="BioFVM">. Future PhysiCell releases will use the more complete <simplified_data type="matlab" source="PhysiCell">.

Unfortunately, Octave does not yet ship with xmlread, so the example above will not fully run. You must get the xerces java binaries and xmlread (in the IO octave forge package). There are instructions there on how to get your xerxes JAR files in the path. Please note that xmlread is about one order of magnitude slower on Octave than in Matlab.

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11.3 Other outputs

PhysiCell also has limited POV-ray support (visualization by the open source POV-Ray raytracer), and some additional logging. This will be further documented soon.

12 Key initialization functions

The following initialization functions should be called (in this order):

- 1. omp_set_num_threads(omp_num_threads); to set the number of threads in the simulation.
- 2. **SeedRandom()**; to initialize the random number generator. (It generates a random seed based on the current system time.) You can also use **long SeedRandom(long input)**; to specify the random seed, such as to continue a simulation.
- 3. initialize_microenvironment(); to initialize the microenvironment, after setting any options in default_microenvironment_options. See Section 6.1.
- 5. initialize_default_cell_definition(); sets up the default Cell_Definition and makes sure it is self-consistent. It also automatically runs setup functions to create standard cell cycle and death models. Users creating new cell definitions are encouraged to copy the default cell definition and then modify it.

All these initialization functions are included in the 2-D and 3-D project templates. See Section 5.

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13 Key global data structures

PhysiCell (and BioFVM) have numerous global data structures to help configure options. Here, we detail the most useful of these.

13.1 Global strings

The BioFVM and PhysiCell versions and strings can be accessed via

```
std::string BioFVM_version;
std::string BioFVM_URL;
std::string PhysiCell_version;
std::string PhysiCell_URL;
```

13.2 Default microenvironment

The name of the default microenvironment in PhysiCell is microenvironment, which is declared in ./core/PhysiCell_phenotype.cpp. This strcture is initialized with the function

```
initialize_microenvironment()
```

based upon the settings in default_microenvironment_options (of type Microenvironment_Options). Here is the definition of that data structure:

```
class Microenvironment_Options
{
private:
public:
    Microenvironment* pMicroenvironment;
    std::string name;
    std::string time_units;
    std::string spatial_units;
    double dx;
    double dy;
    double dz;
    bool outer_Dirichlet_conditions;
    std::vector<double> Dirichlet_condition_vector;
    bool simulate_2D;
    std::vector<double> X_range;
    std::vector<double> Y_range;
    std::vector<double> Z_range;
    Microenvironment_Options();
    bool calculate_gradients;
};
```

See Section 6.1 for more details on these options.

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13.3 Default cell definition

In Section 7.4.5, we documented the data structure for a Cell Definition (similar to a MultiCellDS digital cell line [1]).

PhysiCell creates a default Cell_Definition called cell_defaults. Once the user calls initialize_default_cell_definition() (Section 12), this default definition is set to parameter values for the a generic breast epithelium line (calibrated to MCF-10A).

- 1. It uses the Ki-67 Advanced model. (See Section 14.1.3.)
- 2. It defaults to apoptosis and necrosis death models, with parameters for a generic breast epithelium line (calibrated to MCF-10A) as given in [3].

- 3. It uses general breast epithelial parameters (calibrated to MCF-10A) for volume, and the standard volume regulation model standard_volume_update_function. (See Section 14.3.)
- 4. It sets the cell to be non-motile.
- 5. It sets the custom rule (custom_cell_rule) to NULL and update phenotype function (update_phenotype) to update_cell_and_death_parameters_02_based, so the cell changes its cycle entry rate and necrosis rate according to its local oxygenation conditions. (See Section 14.6.)
- 6. It uses the default mechanics model in [3], by setting update_velocity equal to standard_update_cell_velocity. (See Section 14.4.)

When create_cell() or the default Cell constructor is called, this default definition is used. (See Section 7.3.)

Users can modify cell_defaults at any time; the changes will only apply to cells created after that point in time.

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13.4 A list of all cells

PhysiCell keeps a list of all current cells in the simulation:

```
std::vector<Cell*> *all_cells;
```

It is critical that users **do not modify this data structure**, but it is available for performing an operation on all cells.

Example: Dislay the cycle phase of all cells

```
Cell* pC = NULL;
for( int i=0; i < (*all_cells).size(); i++ )
{
    pC = (*all_cells)[i];
    std::cout << pC->ID << ": " << pC->phenotype.cycle.current_phase().name << std::endl;
}</pre>
```

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13.5 MultiCellDS options

MultiCellDS embeds program and user metadata each time it saves. You can modify this information. BioFVM (and hence PhysiCell) saves the default metadata in BioFVM_metadata (of type MultiCellDS_Metadata). Here is the full class definition:

```
class MultiCellDS_Metadata
{
private:
public:
    std::string MultiCellDS_type;
    Software_Metadata program;
    Citation_Metadata data_citation;
    // scientific information
    std::string spatial_units;
    std::string time_units;
    std::string runtime_units;
    double current_time;
    double current_runtime;
    std::string description; // any optional text -- not implemented
    MultiCellDS_Metadata();
    void display_information( std::ostream& os);
    void sync_to_microenvironment( Microenvironment& M );
    void restart_runtime( void );
    void add_to_open_xml_pugi( double current_simulation_time,
        pugi::xml_document& xml_dom );
};
```

The various member functions are used by BioFVM/PhysiCell when saving a simulation snapshot, and are not needed for users. We define Software_Metadata, Citation_Metadata, and Person_Metadata (currently unused) below. The other metadata elements (spatial_units, etc.) are as expected.

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13.5.1 Software metadata

```
class Software_Metadata
{
  private:
  public:
    // basic program information
    std::string program_name;
    std::string program_version;
    std::string program_URL;

    Person_Metadata creator;
    Person_Metadata user;
    Citation_Metadata citation;
```

```
Software_Metadata();

void display_information( std::ostream& os );
void insert_in_open_xml_pugi( pugi::xml_node& insert_here );
};
```

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13.5.2 Citation metadata

```
class Citation_Metadata
{
  private:
  public:
    std::string DOI;
    std::string PMID;
    std::string PMCID;
    std::string text;
    std::string text;
    std::string notes;
    std::string URL;

    Citation_Metadata();
    void display_information( std::ostream& os );
    void insert_in_open_xml_pugi( pugi::xml_node& insert_here );
};
```

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13.5.3 Person metadata

```
class Person_Metadata
{
private:
    bool is_empty;
public:
    std::string type; // author, creator, user, curator
    std::string surname;
    std::string given_names;
    std::string email;
    std::string URL;
    std::string organization;
    std::string department;
    std::string ORCID;
    Person_Metadata( );
    void display_information( std::ostream& os );
    void insert_in_open_xml_pugi( pugi::xml_node& insert_here );
};
```

13.6 SVG options

PhysiCell keeps a simple data structure named PhysiCell_SVG_options (of type PhysiCell_SVG_options_struct) to easily set SVG plotting options (particularly in digital pathology; see Section 11.1). Here is how that data structure is defined (with its default values):

```
struct PhysiCell_SVG_options_struct
{
   bool plot_nuclei = true;

   std::string simulation_time_units = "min";
   std::string mu = "μ";
   std::string simulation_space_units = "μm";

   std::string label_time_units = "days";

   double font_size = 200;
   std::string font_color = "black";
   std::string font = "Arial";

   double length_bar = 100;
};
```

All plots are performed in the simulation units, so a font of height 200 is 200 μ m tall.

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13.7 PhysiCell Constants

The structure PhysiCell_constants (in ./core/PhysiCell_constants.h) defines many standardized constants and integer identifiers. They can be accessed with syntax like this:

```
}
Here is the list of all current constants (in Version 1.2.0):
class PhysiCell_constants
{
public:
    static constexpr double pi=3.1415926535897932384626433832795;
    static constexpr double cell_removal_threshold_volume = 20;
        // 20 cubic microns -- about 1% of typical cell
    static const int keep_pushed_out_cells_in_outer_voxel=1;
    static const int solid_boundary = 2;
    static const int default_boundary_condition_for_pushed_out_agents
        = keep_pushed_out_cells_in_outer_voxel;
    static const int deterministic_necrosis = 0;
    static const int stochastic_necrosis = 1;
    static const int oxygen_index = 0; // deprecate
    static const int glucose_index = 1; // deprecate
    static const int TUMOR_TYPE=0; // deprecate
    static const int VESSEL_TYPE=1; // deprecate
    static const int mesh_min_x_index=0;
    static const int mesh_min_y_index=1;
    static const int mesh_min_z_index=2;
    static const int mesh_max_x_index=3;
    static const int mesh_max_y_index=4;
    static const int mesh_max_z_index=5;
    static const int mesh_lx_face_index=0;
    static const int mesh_ly_face_index=1;
    static const int mesh_lz_face_index=2;
    static const int mesh_ux_face_index=3;
    static const int mesh_uy_face_index=4;
    static const int mesh_uz_face_index=5;
    // currently recognized cell cycle models
    static const int advanced_Ki67_cycle_model= 0;
    static const int basic_Ki67_cycle_model=1;
    static const int flow_cytometry_cycle_model=2;
    static const int live_apoptotic_cycle_model=3;
    static const int total_cells_cycle_model=4;
    static const int live_cells_cycle_model = 5;
```

```
// currently recognized death models
   static const int apoptosis_death_model = 6;
    static const int necrosis_death_model = 7;
    static const int autophagy_death_model = 8;
   static const int custom_cycle_model=9999;
   // currently recognized cell cycle and death phases
   // cycle phases
   static const int Ki67_positive_premitotic=0;
   static const int Ki67_positive_postmitotic=1;
    static const int Ki67_positive=2;
   static const int Ki67_negative=3;
    static const int GOG1_phase=4;
    static const int GO_phase=5;
   static const int G1_phase=6;
   static const int G1a_phase=7;
    static const int G1b_phase=8;
   static const int G1c_phase=9;
   static const int S_phase=10;
   static const int G2M_phase=11;
    static const int G2_phase=12;
   static const int M_phase=13;
    static const int live=14;
   static const int custom_phase = 9999;
   // death phases
   static const int apoptotic=100;
   static const int necrotic_swelling=101;
   static const int necrotic_lysed=102;
    static const int necrotic=103;
    static const int debris=104;
};
static std::string time_units = "min";
static std::string space_units = "micron";
static double diffusion_dt = 0.01;
static double mechanics_dt = 0.1;
static double phenotype_dt = 6.0;
```

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14 Standard models

PhysiCell includes several models for easier out-of-the-box simulation. We document here the underlying mathematics of these models, as also presented in [3].

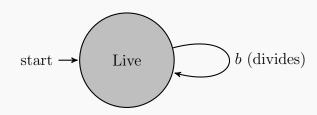


Figure 1: The graph structure of the Live cell cycle model. The default phase is marked "start."

14.1 Cell Cycle Models

PhysiCell includes several pre-built cell cycle models. More models will be added in future releases.

14.1.1 Live (live)

code: PhysiCell_constants::live_cells_cycle_model)

This Cycle_Mode has the following Phases:

1. Phase 0: Named live with code PhysiCell_constants::live.

In this model, live cells can divide into two live cells, with birth rate b. See Fig. 1. The population-scale model is given by:

$$\frac{dL}{dt} = bL. (20)$$

In PhysiCell, the transition rate from the Live state to the Live state is

$$transition_rate(0,0) = b. (21)$$

Further details on the *biology* of this model (including details the placement of daughter cells and changes in cell volume) and reference parameter values can be found in [3].

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14.1.2 Ki-67 Basic (Ki67_basic)

(Code: PhysiCell_constants::basic_Ki67_cycle_model)

This Cycle_Mode has the following Phases:

- 1. Phase 0: Named Ki67- with code PhysiCell_constants::Ki67_negative.
- 2. Phase 1: Named Ki67+ with code PhysiCell_constants::Ki67_positive.

In this model, Ki67- cells (those staining negative for the cell proliferation marker Ki67) can enter the cell cycle to become Ki67+ cells, at transition rate r_{01} . Ki67+ cells divide into two Ki67- cells at rate r_{10} . See

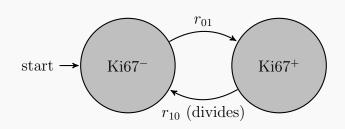


Figure 2: The graph structure of the Ki67 Basic cell cycle model. The default phase is marked "start."

Fig. 2. The population-scale model is given by:

$$\frac{d \left[\text{Ki67-}\right]}{dt} = -r_{01} \left[\text{Ki67-}\right] + 2r_{10} \left[\text{Ki67+}\right]
\frac{d \left[\text{Ki67+}\right]}{dt} = r_{01} \left[\text{Ki67-}\right] - r_{10} \left[\text{Ki67+}\right]$$
(22)

$$\frac{d \left[\text{Ki}67+\right]}{dt} = r_{01} \left[\text{Ki}67-\right] - r_{10} \left[\text{Ki}67+\right]$$
(23)

Further details on the biology of this model (including details the placement of daughter cells and changes in cell volume) and reference parameter values can be found in [3].

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14.1.3 Ki-67 Advanced (Ki67_advanced) (code: PhysiCell_constants::advanced_Ki67_cycle_model)

This Cycle_Mode has the following Phases:

- 1. Phase 0: Named Ki67- with code PhysiCell_constants::Ki67_negative. This is the default phase of the model.
- 2. Phase 1: Named Ki67+ (premitotic) with code PhysiCell_constants::Ki67_positive_premitotic.
- 3. Phase 2: Named Ki67+ (postmitotic) with code PhysiCell_constants::Ki67_positive_postmitotic.

In this model, Ki67- cells (those staining negative for the cell proliferation marker Ki67) can enter the cell cycle to become premitotic Ki67+ cells, at transition rate r_{01} . Ki67+ premitotic cells divide into two Ki67+ postmitotic cells at rate r_{12} . Postmitotic Ki67+ cells become Ki67- cells at rate r_{20} . See Fig. 3. The population-scale model is given by:

$$\frac{d \left[\text{Ki}67 - \right]}{dt} = -r_{01} \left[\text{Ki}67 - \right] + r_{10} \left[\text{Ki}67 + (\text{post}) \right]$$
 (24)

$$\frac{d \left[\text{Ki}67 - \right]}{dt} = -r_{01} \left[\text{Ki}67 - \right] + r_{10} \left[\text{Ki}67 + (\text{post}) \right]
\frac{d \left[\text{Ki}67 + (\text{pre}) \right]}{dt} = r_{01} \left[\text{Ki}67 - \right] - r_{12} \left[\text{Ki}67 + (\text{pre}) \right]$$
(24)

$$\frac{d\left[\text{Ki}67 + (\text{post})\right]}{dt} = 2r_{12}\left[\text{Ki}67 + (\text{pre})\right] - r_{20}\left[\text{Ki}67 + (\text{post})\right]$$
(26)

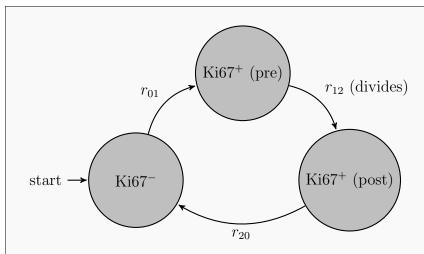


Figure 3: The graph structure of the Ki67 Advanced cell cycle model. The default phase is marked "start."

Further details on the *biology* of this model (including details the placement of daughter cells and changes in cell volume) and reference parameter values can be found in [3].

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14.2 Death Cycle Models

PhysiCell currently includes two pre-built death cycle models. More models may be added in future releases, such as autophagy.

14.2.1 Apoptosis (apoptosis) (code: PhysiCell_constants::apoptosis_death_model)

This Cycle_Mode has the following Phases:

1. Phase 0: Named Apoptotic with code PhysiCell_constants::apoptotic.

In this model, apoptotic cells shrink and exit the phase at rate r_{01} (with fixed duration $1/r_{01}$). Cells are removed from the simulation at the end of the apoptotic phase. (PhysiCell currently uses a dummy "debris" phase to avoid coding a phase transition from the apoptotic phase to the apoptotic phase; this may be removed in future releases.) See Fig. 4. The population-scale model is given by:

$$\frac{d\left[\text{Apoptotic}\right]}{dt} = -r_{01}\left[\text{Apoptotic}\right]. \tag{27}$$

Further details on the *biology* of this model (including details on changes in cell volume) and reference parameter values can be found in [3].

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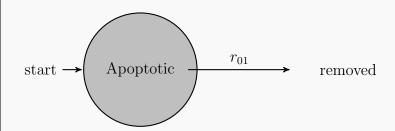


Figure 4: The graph structure of the Apoptosis death cycle model. The default phase is marked "start," and the cell is removed as marked.

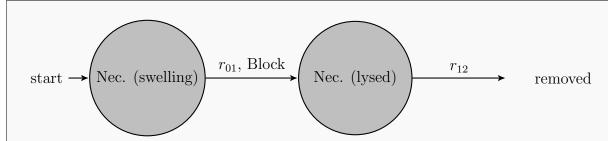


Figure 5: The graph structure of the Necrosis death cycle model. The default phase is marked "start," and the cell is removed as marked. The transition from the unlysed to lysed state is blocked until the cell volume reaches sufficient volume.

14.2.2 Necrosis (necrosis) (code: PhysiCell_constants::necrosis_death_model)

This Cycle_Mode has the following Phases:

- 1. Phase 0: Named Necrotic (swelling) with code PhysiCell_constants::necrotic_swelling.
- 2. Phase 1: Named Necrotic (lysed) with code PhysiCell_constants::necrotic_lysed.

In this model, unlysed necrotic cells swell and attempt to transition to the lysed necrotic phase. There is a block on the transition until the cell reaches a sufficient total volume. Lysed cells gradually shrink (and calcify, if enable) and exit the phase at rate r_{12} . Cells are removed from the simulation at the end of the lysed necrotic phase. (PhysiCell currently uses a dummy "debris" phase to avoid coding a phase transition from the lysed necrotic phase phase to another necrotic phase; this may be removed in future releases.) See Fig. 5.

Further details on the *biology* of this model (including details on changes in cell volume) and reference parameter values can be found in [3].

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14.3 Volume model (standard_volume_update_function)

The standard volume model separately evolves the total fluid volume, cytoplasmic solid volume, and nuclear solid volume, as a system of ODEs (given in [3]). The standard model also updates cell calcification (but the default rate parameter is zero). See Section 9.3 for more information on the Volume class in the Phenotype, and [3] for the biological details of this model.

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14.4 Cell velocity model (standard_update_cell_velocity)

In this model, the cell uses the mechanics interaction data structure to find nearby cells, adds the contributions from adhesion and "repulsion,", adds the effects of basement membrane interactions (by calling functions.add_cell_basement_membrane_interactions), then computes the contribution to cell velocity by the motility model (by calling update_motility_vector, which in turn calls functions.update_migration_bias). See Section 9.6 for details on motility, Section 9.5 for key cell mechanics phenotype parameters, and Section 7 for more information on Cells and their member functions.

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14.5 Up orientation model (up_orientation)

By default 3-D cells have no preferred orientation, and 2-D cells have an "up" orientation (set to [0,0,1]) to ensure they stay in the z=0 plane. This supplied function is up_orientation.

14.6 Oxygen-dependent phenotype (update_cell_and_death_parameters_02_based)

Oxygen-dependent cell proliferation and death rates are so commonly needed in cancer models that we include a model function as standard: update_cell_and_death_parameters_02_based. Here is the overall model for proliferation:

- 1. Sample the microenvironment for the oxygen concentration σ at the cell center.
- 2. If $\sigma \geq$ [parameters.o2_proliferation_saturation], then the cycle entry rate is set to 100% of the entry rate in the pReference_live_phenotype reference phenotype.
- 3. If $\sigma \leq [parameters.o2_proliferation_threshold]$, then the cycle entry rate is set to 0% of the entry rate in the pReference_live_phenotype reference phenotype.
- 4. Otherwise, the reference cycle entry rate is scaled by

$$\frac{\sigma - [\text{parameters.o2_proliferation_threshold}]}{[\text{parameters.o2_proliferation_saturation}] - [\text{parameters.o2_proliferation_threshold}]}$$
 (28)

Here is the overall model for the necrosis death rate:

- 1. Sample the microenvironment for the oxygen concentration σ at the cell center.
- 2. If $\sigma \geq [parameters.o2_necrosis_threshold]$, then the necrotic death rate is zero.
- 3. If $\sigma \leq [parameters.o2_necrosis_max]$, then the necrosis death rate is set to 100% of parameters.max_necrosis_rate.
- 4. Otherwise, the necrotic death rate is scaled by

$$\frac{\left[\text{parameters.o2_necrosis_threshold}\right] - \sigma}{\left[\text{parameters.o2_necrosis_threshold}\right] - \left[\text{parameters.o2_necrosis_max}\right]} \tag{29}$$

This function currently works for the following cell cycle models:

- 1. The Ki67 advanced model (PhysiCell_constants::advanced_Ki67_cycle_model). See Section 14.1.3.
- 2. The Ki67 basic model (PhysiCell_constants::basic_Ki67_cycle_model). See Section 14.1.2.
- 3. The Live model (PhysiCell_constants::live_cells_cycle_model). See Section 14.1.1.

See Section 9.1 for more on the cell cycle portion of the phenotype, Section refsec:Death for more information on the death phenotype elements, Section 14.2 on standard death models, and Section 14.1 on standard cell cycle models. See [3] for more on the biology of these models.

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15 Examples

15.1 Working with Cell functions

15.1.1 Example: a custom volume model

Here, we will define a simpler volume model that only upates the total volume. We use the mathematical model:

$$\frac{d}{dt}[\text{volume.total}] = r\left(V^* - [\text{volume.total}]\right), \tag{30}$$

where

$$V^* = \left(\frac{1 + [\texttt{target_cytoplasmic_to_nuclear_ratio}]}{1 - [\texttt{volume.fluid_fraction}]}\right) \cdot [\texttt{volume.target_solid_nuclear}] \quad (31)$$

$$r = \ln\left(\frac{1 - 0.95}{0.5}\right) [phenotype.cycle.model.transition_rate(0,0)]$$
 (32)

(This uses the built-in phenotype to get the "target" total volume in terms of the built-in targets, and it sets r so that a divided cell reaches 95% of its target within the mean duration of the cell's interdivision time.)

```
void simple_volume_function( Cell* pCell, Phenotype& phenotype, double dt )
{
    double V_target = phenotype.volume.target_solid_nuclear *
        (1.0 + phenotype.volume.target_cytoplasmic_to_nuclear_ratio) /
        (1-phenotype.volume.fluid_fraction);
    double rate = phenotype.cycle.model().transition_rate(0,0) * log(0.1);

    double addme = V_target;
    addme -= phenotype.volume.total;
    addme *= rate;
    addme *= att; // dt*rate*( V_target - V )

    phenotype.volume.total += addme;
    return;
}
```

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15.1.2 Example: a custom migration bias

In this example, cells migrate along oxygen gradients. The migration speed is fastest in regions of low oxygen, and the migration is least stochastic (most biased) as the oxygenation increases.

```
void chemotaxis_bias_function( Cell* pCell, Phenotype& phenotype , double dt )
    // quickly find 02
    static int O2_index = microenvironment.find_density_index( "oxygen" );
    // sample 02
    double o2 = pCell->nearest_density_vector()[02_index];
    // set direction along 02 gradients
    phenotype.motility.migration_bias_direction = pCell->nearest_gradient(02_index);
    normalize( &( phenotype.motility.migration_bias_direction ) );
    // set speed proportional to 02, scaled by normoxic 02 ( 160 mmHg);
    // with a maximum of 1.2 micron per minute
    double theta = 02 / 160.0;
    phenotype.motility.migration_speed = 1.2*(1.0-theta);
    if( phenotype.motility.migration_speed > 1.2 )
    { phenotype.motility.migration_speed = 1.2; }
    // the greater the oxygen, the more biased the motion
    phenotype.motility.migration_bias = theta;
    return;
}
```

15.1.3 Example: a custom cell rule

In this example, an immune cell tests nearby cells first for contact, and then for expression of an antigen (a custom variable). The immune cell initiates apoptosis in the target cell with probability that scales with the antigen expression.

```
void immune_cell_rule( Cell* pCell, Phenotype& phenotype, double dt )
{
    static int antigen_index =
        pCell->custom_data.find_variable_index( "antigen" );
    static int activation_index =
        pCell->custom_data.find_variable_index( "activation" );
    static int apoptosis_model_index =
        pCell->phenotype.death.find_death_model_index( "apoptosis" );
    // exit if immune function is not yet stimulated, or dead
    if( pCell->custom_data[activation_index] < 1e-3</pre>
        || pCell->phenotype.death.dead == true )
    { return; }
    std::vector<Cell*> nearby = pCell->cells_in_my_container();
    // test antigen on nearby cells. stop if you find one
    // Don't try to kill dead cells.
    // Don't kill yourself
    Cell* pC = NULL;
    bool stop = false;
    int i=0;
    while( !stop && i < nearby.size() )</pre>
    ₹
        pC = nearby[i];
        if( pC->custom_data[antigen_index] > 1e-3 &&
            pC->phenotype.death.dead == false &&
            pC != pCell )
        { stop = true; }
        i++;
        if( stop == false )
        \{ pC = NULL; \}
    }
    // if we found a cell, attempt to kill it
    if(pC)
    {
```

```
double probability = 1.0; // dt*pC->custom_data[antigen_index];
        if( UniformRandom() < probability )</pre>
        ₹
            std::cout << "death!" << std::endl;</pre>
             system("pause");
            pC->start_death( apoptosis_model_index );
        }
    }
    return;
}
Here is the code to add the custom variables to a specific cell:
pCell->custom_data.add_variable( "antigen", "dimensionless", 0.0 );
pCell->custom_data.add_variable( "activation", "dimensionless", 0.0 );
Better still, you could make cell definitions for tumor cells and immune cells:
Cell_Definition immune_cell;
// operations to define this type
immune_cell.functions.custom_cell_rule = immune_cell_rule;
immune_cell.custom_data.add_variable( "activation", "dimensionless", 0.0 );
Cell_Definition MCF7 = cell_defaults;
MCF7.custom_data.add_variable( "antigen", "dimensionless", 0.0 );
```

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15.1.4 Example: a custom phenotype update function

Let's create a custom phenotype update rule that uses the standard oxygen-based cell birth and death rates, but also increases cell motility and decreases cell-cel adhesion in low oxygen conditions.

Note that these effects gradually turn on and then saturate based on oxygen values in the cell's parameters, and we use the cell's pReference_live_phenotype.

```
void custom_o2_phenotype_rule( Cell* pCell, Phenotype& phenotype, double dt )
{
    // don't bother if you're dead
    if( pCell->phenotype.death.dead == true )
        { return; }

    // first, call the standard function
        update_cell_and_death_parameters_02_based(pCell,phenotype,dt);
```

```
// next, let's evaluate the oxygen
static int o2_index = microenvironment.find_density_index("oxygen");
double o2 = pCell->nearest_density_vector()[o2_index];
if( o2 > pCell->parameters.o2_hypoxic_response )
{ return; }
// interpolation variable
double theta = ( pCell->parameters.o2_hypoxic_response - o2 )/
   (pCell->parameters.o2_hypoxic_response - pCell->parameters.o2_hypoxic_saturation);
if( theta > 1.0 )
{ theta = 1.0; }
// increase the speed of motiltiy up to a max of 1.5 micron/min
phenotype.motility.is_motile = true;
phenotype.motility.migration_speed = 1.5;
phenotype.motility.migration_speed *= theta;
phenotype.mechanics.cell_cell_adhesion_strength = (1.0-theta);
phenotype.mechanics.cell_cell_adhesion_strength *=
   pCell->parameters.pReference_live_phenotype->mechanics.cell_cell_adhesion_strength;
return;
```

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15.1.5 Example: a custom velocity update function

We'll add an example soon. For now, use the default, along with the default motility functions (which allow considerable customization!).

15.1.6 Example: analytical basement membrane functions

This space held in reserve.

}

15.1.7 Example: a custom cell orientation function

This space held in reserve.

15.2 Cell cycle models

15.2.1 Creating a custom cell cycle model

Let's create a simple cycle model, with three phases: Red, Green, and Blue. Red cells become Green. Green cells become Blue. Blue cells divide into two Red cells. Here's how we accomplish this using the

Cycle_Model class. Notice that we're saving the indices of the newly-created phases for easier reference.

```
Cycle_Model model;
// set up name information
model.code = PhysiCell_constants::custom_cycle_model;
mode.name = "Red Green Blue";
// add the phases
int red_index = model.add_phase( 0 , "Red" );
int green_index = model.add_phase( 1 , "Green" );
int blue_index = model.add_phase( 2 , "Blue" );
// set the Blue phase to have division at its end
model.phases[blue_index].division_at_phase_exit = true;
// set up the phase links
model.add_phase_link( red_index , green_index , NULL ); // red -> green
model.add_phase_link( green_index , blue_index , NULL ); // green -> blue
model.add_phase_link( blue_index , red_index , NULL ); // blue -> red
// set the transition rates
model.transition_rate(red_index,green_index) = 1.0/(60.0 * 5.0);
     // mean duration: 5 hours
model.transition_rate(green_index,blue_index) = 1.0/( 60.0 * 8.0 );
     // mean duration: 8 hours
model.transition_rate(blue_index,red_index) = 1.0/(60.0 * 2.0);
     // mean duration: 2 hours
// display the model
model.display( std::cout );
And this is how we would register that model in a cell definition:
immune_cell.phenotype.cycle.sync_to_cycle_model( rgb_model );
Or in a single cell:
pCell->phenotype.cycle.sync_to_cycle_model( rgb_model );
                                [Return to Table of Contents.]
```

15.2.2 Adding an arrest function

Suppose now that we only want to allow Blue cells to proceed to Red and divide if they have a fluid fraction over 50%. (See Section 9.3.) Then we can define an arrest function:

```
bool fluid_arrest_function( Cell* pCell, Phenotype& phenotype, double dt )
{
     if( phenotype.volume.fluid_fraction < 0.5 )</pre>
     { return true; }
     return false;
}
We then assign this arrest function to the transition from the Blue phase to the Red phase:
rgb_model.phase_link(blue_index,red_index).arrest_function = fluid_arrest_function;
                                 [Return to Table of Contents.]
15.2.3
        Adding a custom phase entry function
Suppose now that we want the cell to "mutate" its transition rates whenever it re-enters the Red phase.
We can do this with an entry function:
void my_mutation_function( Cell* pCell, Phenotype& phenotype, double dt )
{
     // mutate all transition rates by a uniformly-distributed
     // number within 10\% of the current value
     double multiplier = 0.9 + 0.2*uniform_random();
     phenotype.cycle.data.transition_rate(0,1) *= multiplier;
     multiplier = 0.9 + 0.2*uniform_random();
     phenotype.cycle.data.transition_rate(1,2) *= multiplier;
     multiplier = 0.9 + 0.2*uniform_random();
     phenotype.cycle.data.transition_rate(2,0) *= multiplier;
     return;
}
```

Then, we assign this as the entry function for the Red phase:

rgb_model.phases[red_index].entry_function = my_mutation_function;

Alternatively, we can assign this as the exit function for the Blue-Red transition:

rgb_model.phase_link(blue_index,red_index).exit_function = my_mutation_function;

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16 Future

Several features are planned for upcoming PhysiCell releases:

- 1. We will further refine the MultiCellDS output functions to capture more of the cell's state, including custom variable values.
- 2. We will add functions to read in a saved simulation state.
- 3. We will add an XML-based configuration file.
- 4. We will add a function like void contact_interaction_function(Cell*,Phenotype&,double) to allow cell contact-based signaling and behavior changes.
- 5. We plan to start actively tracking the cell's list of mechanically interacting neighbors in cell.state.neighbors
- 6. We will further develop cell.state.simple_pressure and provide better examples.
- 7. We will create a standard standard_update_orientation function to let cells rotate towards a preferred orientation (which will be based upon its neighbors).
- 8. We will generalize the cell-cell adhesion model to allow differing levels of adhesion between different types of cells.
- 9. We will merge the Vector_Variable and Variable types for a more unified Custom_Cell_Data struture.
- 10. We may update motilty and associated update functions to build in some hysteresis (bias towards the last directoin of the day).
- 11. We will consider partial SBML support for importing molecular-scale models.

17 Some notes on parameter values

We chose reference proliferation and apoptosis rates for the generic breast epithelium line (calibrated to MCF-10A) so that the apoptotic fraction is approximately 2% [11], and the net proliferation rate (for the total cell population) is on the order of 0.04 hr⁻¹ [1, 8, 13].

17.1 Live cycle model

For the Live cell model (Section 14.1.1), we fit the system of ODEs

$$\frac{d}{dt}[\text{Live}] = (b-d)[\text{Live}] \tag{33}$$

$$\frac{d}{dt}[\text{Apoptotic}] = d[\text{Live}] - \frac{1}{T_A}[\text{Apoptotic}], \tag{34}$$

and we note that

$$\begin{split} \frac{d}{dt}[\texttt{Total}] &= b[\texttt{Live}] - \frac{1}{T_A}[\texttt{Apoptotic}] \\ &= \left(b(1-\texttt{AI}) + \frac{\texttt{AI}}{T_A}\right)[\texttt{Total}] \\ &= r[\texttt{Total}] \end{split} \tag{35}$$

Here, $r = 0.04 \text{ hr}^{-1}$, AI = 0.02 is the apoptotic index, and $T_A = 8.6 \text{ hour (Section 14.2.1)}$. Thus,

$$b = \frac{r + \frac{1}{T_A} \text{AI}}{1 - \text{AI}} \sim 0.0432 \text{ hr}^{-1}.$$
 (36)

To get the death rate d, we use a simple iterative fitting method (see ./documentation/matlab/tune_death_in_live_model) to get $d \sim 0.00319 \text{ hr}^{-1}$.

17.2 Ki67 Basic model

For the Ki67 Basic model (Section 14.1.2), we fit the system of ODEs

$$\frac{d}{dt}[\text{Ki67-}] = -\left(\frac{1}{T_Q} + d\right)[\text{Ki67-}] + \frac{2}{T_K}[\text{Ki67+}]$$
(37)

$$\frac{d}{dt}[\text{Ki67+}] = \frac{1}{T_Q}[\text{Ki67-}] - \left(\frac{1}{T_K} + d\right)[\text{Ki67+}]$$
(38)

$$\frac{d}{dt}[\texttt{Apoptotic}] = d\left([\texttt{Ki67-}] + [\texttt{Ki67+}]\right) - \frac{1}{T_A}[\texttt{Apoptotic}], \tag{39}$$

and we note that

$$\begin{split} \frac{d}{dt}[\text{Total}] &= \frac{1}{t_K}[\text{Ki67+}] - \frac{1}{T_A}[\text{Apoptotic}] \\ &= \left(\frac{\text{KI}}{T_K} + \frac{\text{AI}}{T_A}\right)[\text{Total}] \\ &= r[\text{Total}] \end{split} \tag{40}$$

We set $T_K = 13 + 2.5$ hr (the duration of the two phases in the Ki67 Advanced model), r = 0.04 hr⁻¹, and we set AI = 0.02 as before and keep d = 0.00319 hr⁻¹ from the prior estimate. We thus need to fit T_Q . We use a simple iterative fitting method (see ./documentation/matlab/tune_Ki67_basic) to get $T_Q \sim 4.59$ hr.

17.3 Ki67 Advanced model

For the Ki67 Advanced model (Section 14.1.3), we fit the system of ODEs

$$\frac{d}{dt}[\text{Ki67-}] = -\left(\frac{1}{T_Q} + d\right)[\text{Ki67-}] + \frac{1}{T_{K2}}[\text{Ki67+}]_2 \tag{41}$$

$$\frac{d}{dt}[\text{Ki67+}]_1 = \frac{1}{T_Q}[\text{Ki67-}] - \left(\frac{1}{T_{K1}} + d\right)[\text{Ki67+}]_1 \tag{42}$$

$$\frac{d}{dt}[\text{Ki67+}]_2 = \frac{2}{T_{K1}}[\text{Ki67+}]_1 - \left(\frac{1}{T_{K2}} + d\right)[\text{Ki67+}]_2 \tag{43}$$

$$\frac{d}{dt}[\texttt{Apoptotic}] \ = \ d\left([\texttt{Ki67-}] + [\texttt{Ki67+}]_1 + [\texttt{Ki67+}]_2\right) - \frac{1}{T_A}[\texttt{Apoptotic}], \tag{44}$$

and we note that

$$\begin{split} \frac{d}{dt}[\texttt{Total}] &= \frac{1}{T_{K1}}[\texttt{Ki67+}]_1 - \frac{1}{T_A}[\texttt{Apoptotic}] \\ &= \left(\frac{\texttt{KI}_1}{T_{K1}} + \frac{\texttt{AI}}{T_A}\right)[\texttt{Total}] \\ &= r[\texttt{Total}] \end{split} \tag{45}$$

We set $T_{K1} = 13$ hr, $T_{K2} = 2.5$ hr, $T_A = 8.6$ hr, and r = 0.04 hr⁻¹, and we set AI = 0.02 as before and keep d = 0.00319 hr⁻¹ from the prior estimate. We thus need to fit T_Q . We use a simple iterative fitting method (see ./documentation/matlab/tune_Ki67_advanced) to get $T_Q \sim 3.62$ hr for this model. [12].

18 Acknowledgements

We thank the Breast Cancer Research Foundation, the Jayne Koskinas Ted Giovanis Foundation for Health and Policy, and the National Cancer Institute for past and present funding for PhysiCell. We gratefully acknowledge the encouragement and suport of the multiscale modeling community as we developed and refined MultiCellDS. We hope the community finds this software useful!

Paul Macklin thanks the Chaste, CompuCell3D, COPASI, and Morpheus communities and developers for Open Source leadership and inspiration.

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