DISSERTATION

COLORADO STATE UNIV'S THESIS TEMPLATE

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ABSTRACT

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This document aims to get you started typesetting your thesis or dissertation in LaTeX.

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TABLE OF CONTENTS

ABSTRACT ii
ACKNOWLEDGEMENTS iii
LIST OF TABLES v
LIST OF FIGURES vi

Chapter 1 Introduction 1
1.1 Introduction to Neutrinos 2
1.1.1 Neutrinos in the Standard Model 3
1.1.2 Neutrino Oscillations 4
1.1.2.1 Derivation 4

Bibliography 9

LIST OF TABLES

LIST OF FIGURES

1.1 The Standard Model of particle physics 4
1.2 Depiction of Two Neutrino Flavor Change of Basis 6
1.3 Two Flavor Oscillation Probability 8

Chapter 1

Introduction

Chose trop vue n'est chère tenue

A thing too much seen is little prized

French proverb

This describes the thesis

1.1 Introduction to Neutrinos

The history of the neutrino can be traced back to electron energy spectrum observed in neutron β-decay. While measurements of α- and γ-decay of atomic nuclei showed discrete spectral lines, the electron (β particle) exhibited a continuous energy spectrum. Experimentally, there were two observed particles in each decay process and classical physics dictated that the outgoing daughter particles should have discrete energies. The fact that the β-decay spectrum was not this way posed a fundamental problem for physicists in the mid-1910s and later, was energy conserved? Two solutions were postulated: either the "energy conservation law is only valid statistically in such a process [...] or an additional undetectable new particle [...] carrying away the additional energy and spin [...] is emitted [2]." The latter solution was supported by Wolfgang Pauli in a letter dated 4 December 1930 to a group of physicists meeting in Tübingen, modern Germany, where he first proposed what we would call a neutrino today[1]. Pauli's solution also predicted that the undetected neutrino would have half-integer spin, a quantum mechanical property of matter, since the observed particles in β-decay did not conserve angular momentum. The existence of the neutrino and validation of Pauli's predictions would not experimentally verified for another 20 years.

The neutrino was first discovered in 1953 by Clyde Cowan and Frederick Reines using a nuclear reactor in South Carolina, U.S.A.. Since then three types of neutrinos have been observed and from unique sources like the Sun and a supernova. Neutrino physics continues to be an active region of physics since neutrinos are unique probes to processes otherwise inaccessible in laboratories. For instance in the depths of the Sun's core where fusion occurs

[1] In W Pauli's December 1930 letter, he referred to his proposed particle as the "neutron", which is not the same neutron we know of today. At that point in time, the neutral particles inside the atomic nucleus, also called "neutrons", had not been discovered, let alone understood. The neutron, which was discovered in 1932 by James Chadwick, has been formally associated as the neutral, cousin particle to the proton. It would be Enrico Fermi who would coin the particle in W Pauli's letter and solution to the β-decay spectrum a "neutrino", meaning *little neutral one.*

and neutrinos are created, neutrinos are able to travel through the ultra dense and hot medium of the core (over 10^7 degrees Kelvin) and outer layers of the Sun and reach us on Earth.

Neutrinos rarely interact with normal matter, meaning that they travel essentially unimpeded towards one's particle detector. The rarity of such interactions can be illustrated with the fact that given nearly 7.0×10^{10} neutrinos/cm^2/sec[2] are incident on the Earth from the Sun, statistically one solar neutrino can harmlessly interact with an individual. So this begs the question: how does one detect a neutrino? The short answer is one needs a ultra large volume of matter and a large enough flux of neutrinos in its path just to detect one given today's technology.

Scientists continue to be interested in neutrinos due to properties they exhibit. One of the more recent and surprising aspects about neutrinos is their ability to undergo "flavor oscillations" where a neutrino of definite flavor (type) is created and later observed as a different flavor. The impact of such oscillations could help explain the observed matter and anti-matter asymmetry in the Universe.

1.1.1 Neutrinos in the Standard Model

The Standard Model (SM) of particle physics is the theory that describes the electromagnetic, strong nuclear, and weak nuclear forces and the elementary particles therein. These three forces and the gravitational force constitute the four *known* fundamental forces of the Universe. Each force in the SM has at least one "force carrier" particle that mediates the interactions between particles. The force carriers are formally called gauge bosons which indicates they are particles with interger $(0, 1, 2, ...)$ spin. The weak nuclear gauge bosons, the W and Z, couple to neutrinos as well as the other fermions, particles with half-integer $\left(\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, ...\right)$ spin, in the SM. All the elementary particles of the SM are shown in Figure 1.1.

[2]To give some perspective to this number, this means 70 billion neutrinos are travelling every second through an area similar to one's own thumb nail.

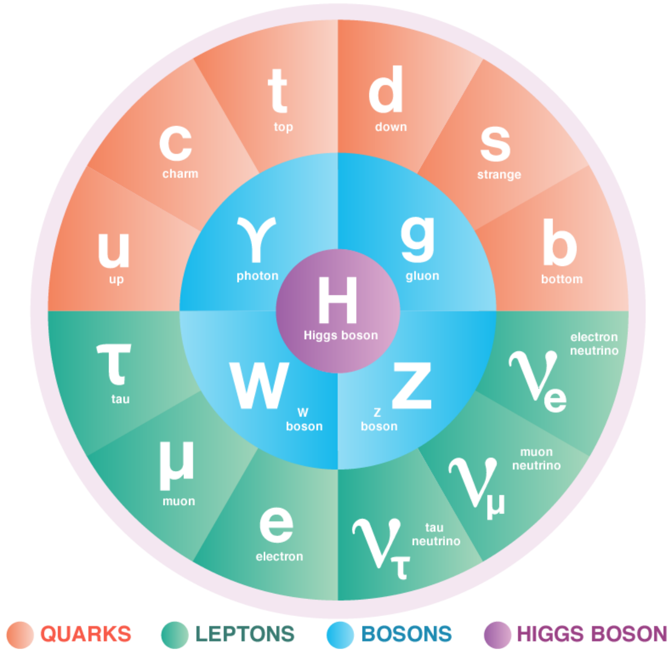

Figure 1.1: The Standard Model of particle physics consists of six quarks, six leptons, and five gauge bosons including the Higgs boson. The focus of this thesis are the neutrinos which are classified according to their charged, more massive Lepton cousins. This graphic was originally submitted on Symmetry Magazine [1].

Neutrinos are electrically neutral, massless particles categorized into three generations based on their charged, more massive Lepton cousins.

1.1.2 Neutrino Oscillations

1.1.2.1 Derivation

The phenomenon of neutrino oscillations can be described with elementary Quantum Mechanics. Beginning with the Schrödinger Equation in Equation 1.1

$$-\frac{\hbar}{i}\frac{\partial}{\partial t}\left|\psi(\mathbf{r},t)\right\rangle = \hat{H}\left|\psi(\mathbf{r},t)\right\rangle \quad , \tag{1.1}$$

where $\hat{H}$ is the Hamiltonian for the physical system. If we consider a massive neutrino of mass m_j in its rest frame, the Hamiltonian acting on $\left|\psi_j\right\rangle$ becomes

$$\hat{H}\left|\psi_j(\mathbf{r},t)\right\rangle = E_j\left|\psi_j(t)\right\rangle, \tag{1.2}$$

where E_j is the energy of the neutrino $\left|\psi_j\right\rangle$. If we substitute Equation 1.2 into Equation 1.1 and solve for $|\psi(\mathbf{r},t)\rangle$, we obtain the following solution

$$\left|\psi_j(\mathbf{r},t)\right\rangle = \left|\psi_j(t)\right\rangle = \left|\psi_j(t=0)\right\rangle e^{-iE_jt/\hbar}, \tag{1.3}$$

where $\left|\psi_j(t=0)\right\rangle$ is the initial position of the neutrino. For demonstration purposes, consider that there are only two eigenstates labeled ν_1 and ν_2 in the "mass" basis with definite mass m_1 and m_2, respectively. Because experiments are only able to probe neutrinos in the "flavor" basis, denoted here by a Greek letter, we can postulate a linear transformation, U, between the basis states given by Equation 1.4.

$$\begin{bmatrix}\nu_\alpha \\ \nu_\beta\end{bmatrix} = \begin{bmatrix}U_{11} & U_{12} \\ U_{21} & U_{22}\end{bmatrix}\begin{bmatrix}\nu_1 \\ \nu_2\end{bmatrix} \tag{1.4}$$

This linear transformation must be a unitary matrix $\left(U^{-1} = U^*\right)$ since the states $\nu_{1,2}$ constitute a complete orthonormal set in the mass basis. With this unitary property, we can imagine U as a rotation matrix

$$\begin{bmatrix}\nu_\alpha \\ \nu_\beta\end{bmatrix} = \begin{bmatrix}\cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta)\end{bmatrix}\begin{bmatrix}\nu_1 \\ \nu_2\end{bmatrix}, \tag{1.5}$$

where θ is the angle between the two bases. We can imagine this transformation between bases as shown in Figure 1.2. If we create a neutrino of flavor α and observe it after a time $t = T > 0$, then the probability of observing it as flavor $\beta \neq \alpha$ is given by

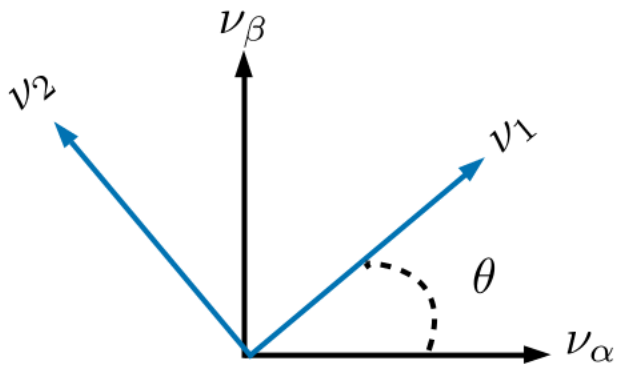

Figure 1.2: The depiction of two neutrino flavor change of basis using a rotation matrix. Compare this with Equation 1.5.

$$
\begin{aligned}
\mathcal{P}\left(\nu_\alpha \to \nu_\beta\right) &= \left|\langle \nu_\alpha(t=0)|\nu_\beta(t=T)\rangle\right|^2 \\
&= |(\cos(\theta)\,\langle \nu_1(t=0)| + \sin(\theta)\,\langle \nu_2(t=0)|) \\
&\quad \times \left(-\sin(\theta)\,|\nu_1(t=T)\rangle + \cos(\theta)\,|\nu_2(t=T)\rangle\right)|^2, \\
&= |\langle \nu_1(0)|\nu_1(T)\rangle\,(-cs) + \langle \nu_1(0)|\nu_2(T)\rangle\,(cc) \\
&\quad + \langle \nu_2(0)|\nu_1(T)\rangle\,(-ss) + \langle \nu_2(0)|\nu_2(T)\rangle\,(sc)|^2
\end{aligned}
\tag{1.6}
$$

where for simplicity $c = \cos(\theta)$ and $s = \sin(\theta)$. Simplifying Equation 1.6 results in Equation 1.7.

$$
\mathcal{P}\left(\nu_\alpha \to \nu_\beta\right) = \sin^2(2\theta)\sin^2\left(\frac{E_1 - E_2}{4\hbar}T\right) \tag{1.7}
$$

The terminology of "neutrino oscillations" should be more apparent now since Equation 1.7 demonstrates that the probability changes sinusoidally. Equation 1.7 is not, however, terribly useful in the laboratory frame since it is hard to make an experiment where the travel time an individual neutrino is well known. Instead, we can make useful approximations that are accessible in the laboratory frame. Since neutrinos are nearly massless, they travel very close to the speed of light. Therefore we can replace time T with L/c where L is the distance between the neutrino origin and detection and c is now the speed of light in vacuum. We can also approximate the energy of the mass eigenstate as

$$\begin{aligned} E_j = \left(m_j^2 c^4 + p_j^2 c^2\right)^{\frac{1}{2}} &= p_j c \left(1 + \frac{m_j^2 c^2}{p_j^2}\right)^{\frac{1}{2}} \\ &\approx p_j c \left(1 + \frac{m_j^2 c^2}{2p_j^2} + \mathcal{O}\left(\frac{m_j c}{p_j}\right)^4\right) \\ &\approx E_\nu + \frac{m_j^2 c^4}{2E_\nu}, \end{aligned} \tag{1.8}$$

where we have used the fact that $p_j \ll m_j c$ and $p_j c \approx E_\nu$ where E_ν is the neutrino energy as measured in the laboratory. Substituting all of our assumptions in Equation 1.7, we get Equation 1.9

$$\begin{aligned} \mathcal{P}\left(\nu_\alpha \to \nu_\beta\right) &= \sin^2(2\theta)\sin^2\left(\frac{\Delta m_{21}^2 c^3}{8\hbar}\frac{L}{E_\nu}\right) \\ &= \sin^2(2\theta)\sin^2\left(\frac{2\pi}{4.959}\frac{\Delta m_{21}^2}{\left[\text{eV}^2\right]}\frac{{}^{L}/_{E_\nu}}{\left[{}^{\text{km}}/_{\text{GeV}}\right]}\right), \end{aligned} \tag{1.9}$$

where $\Delta m_{21}^2 = m_2^2 - m_1^2$ is the mass-squared difference between the mass states, and all the physical constants have been evaluated in natural units ($c = \hbar = 1$). We can compare Equation 1.9 with a time-varying sinusoid, $A\sin^2(2\pi f t)$ of amplitude A and frequency f. The oscillation probability has amplitude $\sin^2(2\theta)$ and varies with frequency proportional to Δm_{21}^2 as illustrated in Figure 1.3.

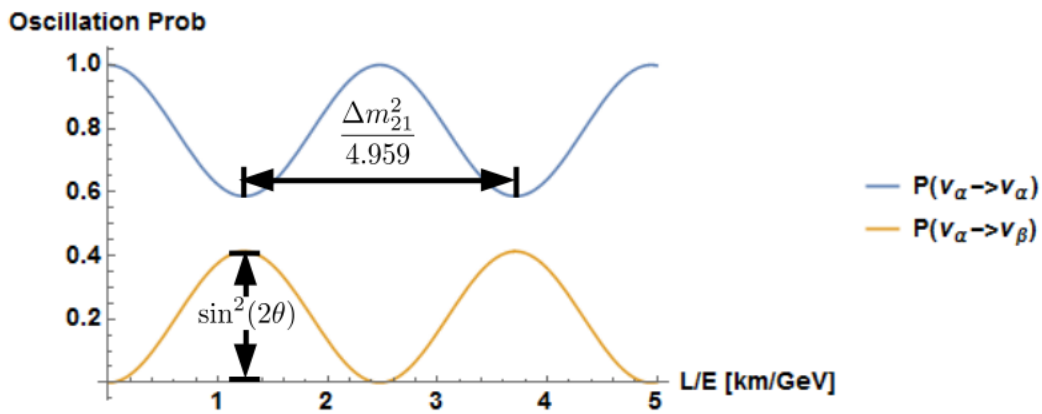

Figure 1.3: The two flavor oscillation probability as a function L/E is shown using an appropriate choice of Δm^2_{21}.

Bibliography

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