DISSERTATION

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ABSTRACT

COLORADO STATE UNIV'S THESIS TEMPLATE

This document aims to get you started typesetting your thesis or dissertation in LaTeX.

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Chapter 1

Introduction

Chose trop vue n'est chère tenue

A thing too much seen is little prized

French proverb

This describes the thesis

1.1 Introduction to Neutrinos

The history of the neutrino can be traced back to electron energy spectrum observed in neutron β-decay. While measurements of α- and γ-decay of atomic nuclei showed discrete spectral lines, the electron (β particle) exhibited a continuous energy spectrum. Experimentally, there were two observed particles in each decay process and classical physics dictated that the outgoing daughter particles should have discrete energies. The fact that the β-decay spectrum was not this way posed a fundamental problem for physicists in the mid-1910s and later, was energy conserved? Two solutions were postulated: either the "energy conservation law is only valid statistically in such a process [...] or an additional undetectable new particle [...] carrying away the additional energy and spin [...] is emitted [7]." The latter solution was supported by Wolfgang Pauli in a letter dated 4 December 1930 to a group of physicists meeting in Tübingen, modern Germany, where he first proposed what we would call a neutrino today[1]. Pauli's solution also predicted that the undetected neutrino would have half-integer spin, a quantum mechanical property of matter, since the observed particles in β-decay did not conserve angular momentum. The existence of the neutrino and validation of Pauli's predictions would not experimentally verified for another 20 years.

The neutrino was first discovered in 1953 by Clyde Cowan and Frederick Reines using a nuclear reactor in South Carolina, U.S.A.. Since then three types of neutrinos have been observed and from unique sources like the Sun and a supernova. Neutrino physics continues to be an active region of physics since neutrinos are unique probes to processes otherwise inaccessible in laboratories. For instance in the depths of the Sun's core where fusion occurs

[1]In W Pauli's December 1930 letter, he referred to his proposed particle as the "neutron", which is not the same neutron we know of today. At that point in time, the neutral particles inside the atomic nucleus, also called "neutrons", had not been discovered, let alone understood. The neutron, which was discovered in 1932 by James Chadwick, has been formally associated as the neutral, cousin particle to the proton. It would be Enrico Fermi who would coin the particle in W Pauli's letter and solution to the β-decay spectrum a "neutrino", meaning *little neutral one*.

and neutrinos are created, neutrinos are able to travel through the ultra dense and hot medium of the core (over 10^7 degrees Kelvin) and outer layers of the Sun and reach us on Earth.

Neutrinos rarely interact with normal matter, meaning that they travel essentially unimpeded towards one's particle detector. The rarity of such interactions can be illustrated with the fact that given nearly 7.0×10^{10} neutrinos/cm^2/sec^2 are incident on the Earth from the Sun, statistically one solar neutrino can harmlessly interact with an individual. So this begs the question: how does one detect a neutrino? The short answer is one needs a ultra large volume of matter and a large enough flux of neutrinos in its path just to detect one given today's technology.

Scientists continue to be interested in neutrinos due to properties they exhibit. One of the more recent and surprising aspects about neutrinos is their ability to undergo "flavor oscillations" where a neutrino of definite flavor (type) is created and later observed as a different flavor. The impact of such oscillations could help explain the observed matter and anti-matter asymmetry in the Universe.

1.1.1 Neutrinos in the Standard Model

The Standard Model (SM) of particle physics is the theory that describes the electromagnetic, strong nuclear, and weak nuclear forces and the elementary particles therein. These three forces and the gravitational force constitute the four *known* fundamental forces of the Universe. Each force in the SM has at least one "force carrier" particle that mediates the interactions between particles. The force carriers are formally called gauge bosons which indicates they are particles with integer $(0, 1, 2, ...)$ spin. The weak nuclear gauge bosons, the charged $W^{\pm}$ and neutral Z, couple to neutrinos as well as the other fermions, particles

[2]To give some perspective to this number, this means 70 billion neutrinos are travelling every second through an area similar to one's own thumb nail.

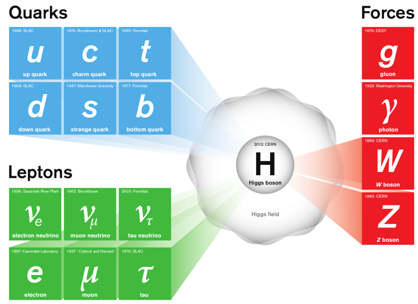

Figure 1.1: The Standard Model of particle physics consists of six quarks (up, down, strange, charm, bottom, and top), six leptons (electron, muon, tau, electron neutrino, muon neutrino, and tau neutrino), four force propagating bosons (gluon, photon, W, and Z), and the Higgs boson. The quarks, electron, muon, tau, W, and Z all gain mass through the Higgs field. The focus of this thesis are the neutrinos which are classified according to their charged, more massive Lepton cousins. Image taken from [1].

with half-integer $\left(\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots\right)$ spin, in the SM. All the elementary particles of the SM are shown in Figure 1.1.

Neutrinos in the SM are electrically neutral, massless particles categorized into three generations based on their charged, more massive Lepton cousins. What follows is a brief introduction to neutrino interactions as well as some of their fundamental properties.

1.1.1.1 Neutrino Weak Interactions

The name "weak force" comes from the fact that this force is has a much smaller effective range than the electromagnetic and strong nuclear forces. This is due to the weak mediating bosons, the W and Z, being massive particles unlike the massless gluon (g) and photon (γ). The W/Z have masses of 80/90 GeV/c^2, which is more massive than all the elementary particles except for the top quark. For weak interactions to occur at energies far below the masses of the W and Z, the interaction time must be infinitesimally small as dictated by the Heisenberg Uncertainty Principle

$$\Delta E \Delta t \gtrsim \hbar \tag{1.1}$$

where ΔE is the energy of the particle and Δt is the time which the particle exists. Interactions involving either of the two charged W bosons are called charged current (CC) interactions since a electrical current is exchanged. The same cannot be said of the neutral Z boson, and so interactions with the Z are called neutral current (NC).

1.1.1.2 Neutrino Chirality

Neutrinos are believed to follow the free-particle Dirac Equation which is given by

$$\left[i\hbar \sum_{\mu=0}^{3} \gamma^{\mu}\partial_{\mu} - mc\right]\psi = 0 \tag{1.2}$$

where

$$\gamma^0 = \begin{bmatrix} I_2 & 0 \\ 0 & -I_2 \end{bmatrix}, \gamma^1 = \begin{bmatrix} 0 & \sigma_x \\ -\sigma_x & 0 \end{bmatrix}, \gamma^2 = \begin{bmatrix} 0 & \sigma_y \\ -\sigma_y & 0 \end{bmatrix}, \gamma^3 = \begin{bmatrix} 0 & \sigma_z \\ -\sigma_z & 0 \end{bmatrix}, \tag{1.3}$$

I_2 is the 2×2 identity matrix, $\sigma_{x,y,z}$ are the Pauli Spin matrices, and

$$\partial_0 = \frac{1}{c}\frac{\partial}{\partial t}, \partial_1 = \frac{\partial}{\partial x}, \partial_2 = \frac{\partial}{\partial y}, \partial_3 = \frac{\partial}{\partial z}. \tag{1.4}$$

The solutions to (1.2) are called Dirac spinors and can be written in terms of left-handed (LH) and right-handed (RH) components. Any spinor can be decomposed in LH and RH projections using a "chiral" operator $\hat{O}$ as

$$\psi = \left(\hat{O}_{\mathrm{LH}} + \hat{O}_{\mathrm{RH}}\right)\psi. \tag{1.5}$$

The chiral operators $\hat{O}_{\mathrm{LH,RH}}$ in (1.5) are defined as

$$\hat{O}_{\mathrm{LH}}\psi = \frac{1}{2}(I_4 - \gamma^5)\psi = \psi_{\mathrm{LH}} \quad \hat{O}_{\mathrm{RH}}\psi = \frac{1}{2}(I_4 + \gamma^5)\psi = \psi_{\mathrm{RH}} \tag{1.6}$$

where I_4 is the 4×4 identity matrix,

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{bmatrix} 0 & I_2 \\ I_2 & 0 \end{bmatrix}, \tag{1.7}$$

and $\psi_{\text{LH,RH}}$ are LH and RH chiral spinors. Using (1.6), the Dirac Equation then becomes after some manipulation

$$\begin{aligned} i\hbar\left(\frac{1}{c}\frac{\partial}{\partial t} - \boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\right)\psi_{\text{RH}} &= m\gamma^0\psi_{\text{LH}} \\ i\hbar\left(\frac{1}{c}\frac{\partial}{\partial t} + \boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\right)\psi_{\text{LH}} &= m\gamma^0\psi_{\text{RH,}} \end{aligned} \tag{1.8}$$

where

$$\boldsymbol{\sigma}\cdot\boldsymbol{\nabla} = \sum_{i=1}^{3}\sigma_i\nabla_i. \tag{1.9}$$

In the limiting case of vanishing mass ($m \to 0$), as in the SM, the free particle equations decouple into

$$\begin{aligned} \left(\frac{E}{c} - \boldsymbol{\sigma}\cdot\boldsymbol{P}\right)\psi_{\text{RH}} &= 0 \\ \left(\frac{E}{c} + \boldsymbol{\sigma}\cdot\boldsymbol{P}\right)\psi_{\text{LH}} &= 0. \end{aligned} \qquad \text{[Momentum basis]} \tag{1.10}$$

We now have enough information to make two very important insights about neutrinos in the SM.

The first important insight is that the chirality and helicity of the neutrino the same for $m \to 0$. A particle's helicity is the projection of the spin vector on its momentum vector given as

$$\mathcal{H} = \frac{\boldsymbol{\sigma}\cdot\mathbf{P}}{|\mathbf{P}|} \tag{1.11}$$

where $\boldsymbol{\sigma}$ is the spin vector and $\mathbf{P}$ is the 3-momentum of the particle. Since (1.10) commutes $\left(\left[\hat{A},\hat{B}\right] = \hat{A}\hat{B} - \hat{B}\hat{A}\right)$ with $\mathcal{H}$, $\psi_{\text{LH,RH}}$ are eigenstates of the helicity operator with eigenvalues ± 1.

The second important fact is that a weakly interacting neutrino has positive energy $E = |\boldsymbol{P}|\, c$ and is always LH as shown in (1.6). The antineutrino is RH and has negative energy

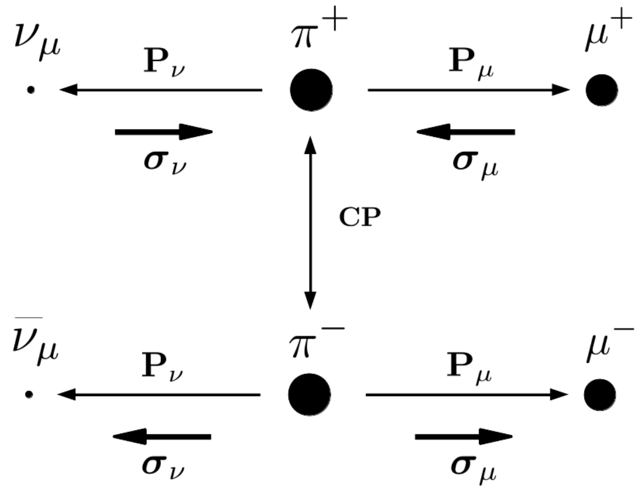

Figure 1.2: Decay of a charged pi meson into a muon and neutrino show the direction of the momentum $\boldsymbol{P}$ and spin $\boldsymbol{\sigma}$ of the outgoing particles. Since a pion at rest has zero (0) angular momentum, the system of daughter particles must have net zero angular momentum as well. A neutrino (antineutrino) is a right- (left-) handed helicity particle since its spin is (anti-)parallel to its momentum. Application of charge and parity (CP) converts all the particles into their respective antiparticles.

$(-E = |\boldsymbol{P}|\, c)$ which is interpreted to mean it travels backwards in time. Since neutrinos are nearly massless, there are no mechanisms for RH neutrinos and LH antineutrinos. The helicity of the neutrino is visualized in Figure 1.2 which shows the decay of a pion at rest into a muon and neutrino. Since a pion has net zero (0) spin, the spin vectors of the daughters must also sum to zero. The neutrino's helicity (-1) and hence LH chirality is inferred by the positively charged muon of which has +1 helicity. To confirm the antineutrino's RH chirality (+1) requires both a charge (C) conjugation and parity (P) transformation. A C conjugation transforms all particles into their corresponding antiparticles while P transformation inverts all spatial coordinates.

The observation of only LH neutrinos and RH antineutrinos are an important feature in the SM. The weak force allows for P and CP violation due to its vector minus axial-vector (V-A) construction which is how a LH neutrino is described in (1.6). Further observation of CP violation is being explored with neutrinos through a process called neutrino oscillations. This will be explained in the next subsection.

1.1.2 Neutrino Oscillations

Neutrino oscillations are the observation of a neutrino produced of definite flavor and later observed as a different flavor. A deficit of neutrinos were observed for a number of atmospheric and solar neutrino experiments and the effect became more pronounced as the distance from the source increased.

The first indication of neutrino oscillations was from the Ray Davis Homesteak Mine experiment which began in the 1960s. Ray Davis was an expert Chemist and designed a radiochemical experiment to measure the flux of neutrinos from Sun. The purpose of this experiment was to test John Bahcall's prediction of the fusion rate in and neutrino flux from the Sun. Measurements continued into the 1980s and showed that the flux of neutrinos as measured at Homesteak was about 1/3 the expected rate and became known as the "Solar Neutrino Problem." The primary solutions were either the neutrino capture cross section is lacking or the solar model was incorrect. The Sudbury Neutrino Observatory (SNO) was able to resolve this by making a model-independent measurement of the solar neutrino flux. The SNO ν_eCC/NC ratio is given by 0.301 ± 0.033, which confirmed that only 30% of neutrinos arrive as ν_e on Earth, firmly established that the majority of neutrinos arrive as the wrong flavor.

Another outstanding problem emerged with measurements of atmospheric neutrinos, in particular muon and electron types. Atmospheric neutrinos are produced when high energy cosmic rays strike atmospheric particles. These cosmic ray collisions generate mostly pions and kaons that decay into neutrinos. When trying to measure the ν_μ/ν_e ratio and comparing that with expected ratio, there was a significant deficit. This was particularly a problem as a function of the zenith angle for the Super-Kamiokande experiment.

While the phenomenon of neutrino oscillations has understood for decades, it is not incorporated into the SM since oscillations require the neutrino has mass. The reasons why neutrino oscillations require massive neutrinos is explained in the next subsection.

1.1.2.1 Two Flavor Derivation

The phenomenon of neutrino oscillations can be described with elementary Quantum Mechanics. Beginning with the Schrödinger Equation in (1.12)

$$-\frac{\hbar}{i}\frac{\partial}{\partial t}\left|\nu(\mathbf{r},t)\right\rangle = \hat{H}\left|\nu(\mathbf{r},t)\right\rangle \tag{1.12}$$

where $\hat{H}$ is the Hamiltonian for the physical system. If we consider a massive neutrino of mass m_j in its rest frame (free particle), the Hamiltonian becomes diagonal which acting on $\left|\nu_j\right\rangle$ results in the eigenvalue equation

$$\hat{H}\left|\nu_j(\mathbf{r},t)\right\rangle = E_j\left|\nu_j(\mathbf{r},t)\right\rangle \tag{1.13}$$

where E_j is the energy of the neutrino $\left|\nu_j\right\rangle$. If we substitute (1.13) into (1.12) and solve for $\left|\nu(\mathbf{r},t)\right\rangle$, we obtain the following

$$\left|\nu_j(\mathbf{r},t)\right\rangle = e^{-iE_jt/\hbar}\left|\nu_j(\mathbf{r},t=0)\right\rangle \tag{1.14}$$

where $\left|\nu_j(\mathbf{r},t=0)\right\rangle$ is created with momentum $\mathbf{p}$ at the origin $\mathbf{r}=0$. The time-independent solution to (1.12) is a plane-wave given by

$$\left|\nu_j(\mathbf{r},t=0)\right\rangle = e^{i\mathbf{p}\cdot\mathbf{r}/\hbar}\left|\nu_j\right\rangle. \tag{1.15}$$

Before being able to describe neutrino oscillations, we must define our basis states. For this example, consider that there are only two eigenstates, labeled ν_1 and ν_2, in the "mass" basis with definite mass m_1 and m_2, respectively. However, experiments can produce neutrinos, as well as probe them, only of definite "flavor", denoted by a Greek letter subscript λ. Let the generated neutrino, which is a linear superposition of mass states 1 and 2, have momentum $\mathbf{p}$ and flavor α. Since both mass eigenstates share the same momentum momentum $\mathbf{p}$ (but

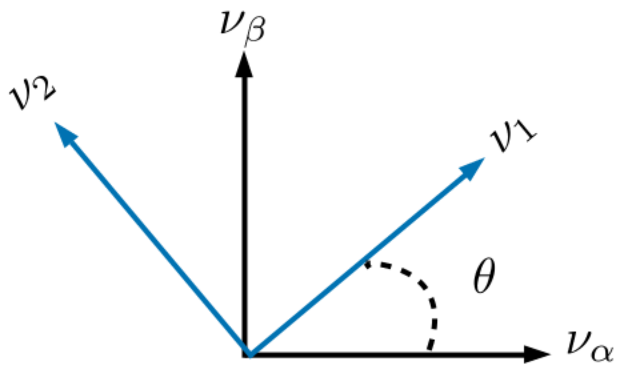

Figure 1.3: The depiction of two neutrino flavor change of basis using a rotation matrix. Compare this with (1.17).

not energy!), the exponential term in (1.15) is an overall phase that will cancel out later. We can postulate a linear transformation, U, between the basis states given by (1.16).

$$\begin{bmatrix} \nu_\alpha \\ \nu_\beta \end{bmatrix} = \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} \tag{1.16}$$

This linear transformation must be a unitary matrix $\left(U^{-1} = U^\dagger, \dagger = \text{transpose conjugate}\right)$ since the states $\nu_{1,2}$ constitute a complete orthonormal set in the mass basis. With this unitary property, we can imagine U as a rotation matrix

$$\begin{bmatrix} \nu_\alpha \\ \nu_\beta \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix}, \tag{1.17}$$

where θ is the angle between the two bases. We can imagine this transformation between bases as shown in Figure 1.3. If we create a neutrino of flavor α and observe it after a time $t = T > 0$, then the probability of observing it as flavor $\beta \neq \alpha$ is given by

$$
\begin{aligned}
\mathcal{P}\left(\nu_\alpha \to \nu_\beta\right) &= \left|\langle \nu_\alpha(t=0) | \nu_\beta(t=T)\rangle\right|^2 \\
&= |(\cos(\theta)\,\langle \nu_1(t=0)| + \sin(\theta)\,\langle \nu_2(t=0)|) \\
&\quad \times (-\sin(\theta)\,|\nu_1(t=T)\rangle + \cos(\theta)\,|\nu_2(t=T)\rangle)|^2 \\
&= |\langle \nu_1(0)|\nu_1(T)\rangle\,(-cs) + \langle \nu_1(0)|\nu_2(T)\rangle\,(cc) \\
&\quad + \langle \nu_2(0)|\nu_1(T)\rangle\,(-ss) + \langle \nu_2(0)|\nu_2(T)\rangle\,(sc)|^2
\end{aligned}
\tag{1.18}
$$

where for simplicity $c = \cos(\theta)$ and $s = \sin(\theta)$. Evaluating all inner products and simplifying terms in (1.18) results in (1.19) below.

$$
\mathcal{P}\left(\nu_\alpha \to \nu_\beta\right) = \sin^2(2\theta)\sin^2\left(\frac{E_1 - E_2}{2\hbar}T\right)
\tag{1.19}
$$

The terminology of "neutrino oscillations" should be more apparent now since (1.19) demonstrates that the probability changes sinusoidally. This equation is not, however, terribly useful in the laboratory frame since it is hard to make an experiment where the travel time an individual neutrino is well known. Instead, we can make useful approximations that are accessible in the laboratory frame. Since neutrinos are nearly massless, they travel very close to the speed of light. Therefore we can replace time T with L/c where L is the distance between the neutrino origin and detection and c is now the speed of light in vacuum. We can also approximate the energy of the mass eigenstate as

$$
\begin{aligned}
E_j = \left(m_j^2 c^4 + p_j^2 c^2\right)^{\frac{1}{2}} &= p_j c \left(1 + \frac{m_j^2 c^2}{p_j^2}\right)^{\frac{1}{2}} \\
&\approx p_j c \left(1 + \frac{m_j^2 c^2}{2p_j^2} + \mathcal{O}\left(\frac{m_j c}{p_j}\right)^4\right) \\
&\approx E_\nu + \frac{m_j^2 c^4}{2E_\nu},
\end{aligned}
\tag{1.20}
$$

where we have used the fact that $p_j \gg m_j c$ and $p_j c \approx E_\nu$ where E_ν is the neutrino energy as measured in the laboratory. Substituting all of our assumptions in (1.19), we get (1.21)

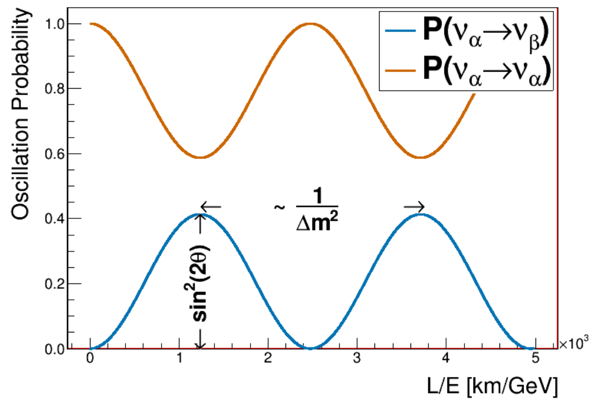

Figure 1.4: Two flavor oscillation probability as a function L/E is shown using $\theta = 20^{\circ}$ and $\Delta m^2 = [\,10^{-3}]eV^2$. The spacing between adjacent peaks/troughs is proportional to the inverse of Δm^2. Note that $\mathcal{P}\left(\nu_\alpha \rightarrow \nu_\alpha\right) = 1 - \mathcal{P}\left(\nu_\alpha \rightarrow \nu_\beta\right)$ since the oscillation probability must always sum to 1.

$$\mathcal{P}\left(\nu_\alpha \rightarrow \nu_\beta\right) = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 c^3}{4\hbar}\frac{L}{E_\nu}\right), \tag{1.21}$$

where $\Delta m^2 = m_2^2 - m_1^2$ is the mass-squared difference between the mass states. If we evaluate all the physical constants in natural units ($c = \hbar = 1$) and choose appropriate units for Δm^2, L, and E_ν, we arrive at the following expression

$$\mathcal{P}\left(\nu_\alpha \rightarrow \nu_\beta\right) = \sin^2(2\theta)\sin^2\left(1.27\frac{\Delta m^2}{\left[\mathrm{eV}^2\right]}\frac{L/E_\nu}{[\mathrm{km}/\mathrm{GeV}]}\right) \text{ [natural units]} \tag{1.22}$$

which more clearly demonstrates the Physics in oscillations. The oscillation probability has an amplitude of $\sin^2(2\theta)$ and varies with frequency inversely proportional to Δm^2 as illustrated in Figure 1.4. Since L and E_ν are the only controllable parameters for an oscillation experiment, probing θ or Δm^2 can be difficult unless the experiment can probe a large range of L/E_ν as shown in Figure 1.5.

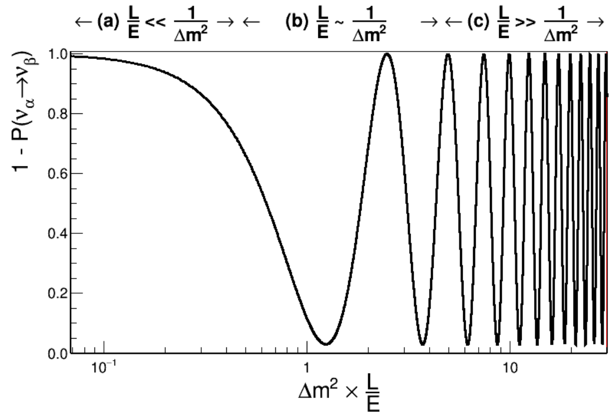

Figure 1.5: Logarithmic plot of the survival probability $(1 - \mathcal{P}(\nu_\alpha \to \nu_\beta) = \mathcal{P}(\nu_\alpha \to \nu_\alpha))$ over a wide range of L/E values for $\theta = 40^\circ$. The arrows above the plot roughly denote three possible cases: (a) no oscillations $(L/E \ll 1/\Delta m^2)$; (b) sensitivity to oscillations $(L/E \sim 1/\Delta m^2)$; (c) only average measurement $(L/E \gg 1/\Delta m^2)$. Image originally inspired by [5].

1.1.2.2 Three Flavor Oscillations

In the general case of oscillations using a $n \times n$ mixing matrix, the unitary transformation can be written as a rotation matrix with $\frac{n}{2}(n-1)$ weak mixing angles with $\frac{1}{2}(n-2)(n-1)$ Charge-Parity (CP) violating phases. In addition, oscillations are dictated by a total of $n-1$ mass-squared splittings [7]. This all assumes that neutrinos obey the Dirac Equation, or that they are not their own antiparticles. The flavored mixing model is the 3×3 matrix since there are three known neutrino flavors, ν_e, ν_μ, and ν_τ, which means there are three (3) mixing angles, one (1) CP violating phase, and two (2) mass-squared splittings.

The most frequently used matrix parameterization is the Maki-Nakagawa-Sakata-Pontecorvo (MNSP) matrix. Pontecorvo is accredited for first conceiving of neutrino oscillations, albeit between neutrino and anti-neutrinos [4]. It was Maki, Nakagawa, and Sakata who conceived of the parameterization based off the ideas of Pontecorvo [3]. The MNSP matrix is decomposed into separate rotation matrices as given by 1.23

Source	Species	$\overline{E}$ [GeV]	L [km]	$\min\left(\Delta m^2\right)$ $\left[\mathrm{eV}^2\right]$
Reactor	$\overline{\nu}_e$	$\sim 10^{-3}$	1	$\sim 10^{-3}$
Reactor	$\overline{\nu}_e$	$\sim 10^{-3}$	100	$\sim 10^{-5}$
Accelerator	$\nu_\mu, \overline{\nu}_\mu$	~ 1	1	~ 1
Accelerator	$\nu_\mu, \overline{\nu}_\mu$	~ 1	10^3	$\sim 10^{-3}$
Atmospheric ν's	$\nu_{e,\mu}, \overline{\nu}_{\mu,e}$	~ 1	10^4	$\sim 10^{-4}$
Sun	ν_e	$\sim 10^{-3}$	1.5×10^8	$\sim 10^{-11}$

Table 1.1: Sensitivity of different oscillation experiments originally published in [6].

$$U_{\mathrm{MNSP}} = \overbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{32} & s_{32} \\ 0 & -s_{32} & c_{32} \end{bmatrix}}^{U_{\mathrm{atm}}} \times \overbrace{\begin{bmatrix} c_{31} & 0 & s_{31}e^{i\delta_{\mathrm{CP}}} \\ 0 & 1 & 0 \\ -s_{31}e^{-i\delta_{\mathrm{CP}}} & 0 & c_{31} \end{bmatrix}}^{U_{\mathrm{rea}}} \times \overbrace{\begin{bmatrix} c_{21} & s_{21} & 0 \\ -s_{21} & c_{21} & 0 \\ 0 & 0 & 1 \end{bmatrix}}^{U_{\mathrm{sol}}} \tag{1.23}$$

where

$$c_{ij} = \cos\theta_{ij}, s_{ij} = \sin\theta_{ij}, \tag{1.24}$$

and δ_{CP} represents the CP violating phase. Each rotation matrix represents the different sources for neutrino oscillations experiments with "atm", "rea", and "sol" representing atmospheric ν's, nuclear reactor ν's, and Solar ν's, respectively. The sensitivity of neutrino oscillations for different sources is given in Table 1.1.

If neutrinos are their own antiparticles, they do not follow the Dirac Equation but do follow the Majorana Equation. This adds two (in general $n-1$) more CP violating phases to the MNSP matrix

$$U_{\mathrm{MNSP}} \rightarrow (1.23) \times \overbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{bmatrix}}^{U_{\mathrm{Majorana}}} \tag{1.25}$$

Unfortunately, neutrino oscillations are not able to probe the Majorana phases since the Majorana matrix is diagonal. The question of if neutrinos are Majorana ($\nu = \overline{\nu}$) or Dirac

$(\nu \neq \overline{\nu})$ particles is an open question and is being explored by non-oscillation experiments. The full three flavor oscillation probability is given by

$$\begin{aligned}\mathcal{P}\left(\nu_\alpha \to \nu_\beta\right) = \delta_{\alpha\beta} - 4\sum_{j=1}^{3}\left[\sum_{i>j}^{3} \mathrm{Re}\left(K_{\alpha\beta,ij}\right)\sin^2\left(\phi_{ij}\right)\right] \\ +4\sum_{j=1}^{3}\left[\sum_{i>j}^{3} \mathrm{Im}\left(K_{\alpha\beta,ij}\right)\sin\left(\phi_{ij}\right)\cos\left(\phi_{ij}\right)\right]\end{aligned} \tag{1.26}$$

where

$$K_{\alpha\beta,ij} = U_{\alpha i}U^*_{\beta_i}U^*_{\alpha j}U_{\beta j} \tag{1.27}$$

encapsulates the MNSP matrix elements and

$$\phi_{ij} = \frac{\Delta m^2_{ij}c^3}{4\hbar}\frac{L}{E_\nu}. \tag{1.28}$$

CP violating terms in the oscillation probability between neutrino and antineutrino oscillations requires that any terms with δ_{CP} must be an odd function. As an example, consider the survival probability of a muon type neutrinos is given by

$$\begin{aligned}\mathcal{P}\left(\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_\mu\right) = 1 - 4s^2_{23}c^2_{13}\left(V_{\cos\delta_{\mathrm{CP}}}\right)\sin^2\phi_{31} \\ -4s^2_{23}c^2_{13}\left(W_{\cos\delta_{\mathrm{CP}}}\right)\sin^2\phi_{32} \\ -4\left(V_{\cos\delta_{\mathrm{CP}}}\right)\left(W_{\cos\delta_{\mathrm{CP}}}\right)\sin^2\phi_{21}\end{aligned} \tag{1.29}$$

where

$$V_{\cos\delta_{\mathrm{CP}}} = s^2_{12}c^2_{23} + s^2_{13}s^2_{23}c^2_{12} + 2s_{12}s_{13}s_{23}c_{12}c_{23}\cos\delta_{\mathrm{CP}} \tag{1.30}$$

$$W_{\cos\delta_{\mathrm{CP}}} = c^2_{12}c^2_{23} + s^2_{13}s^2_{23}s^2_{12} - 2s_{12}s_{13}s_{23}c_{12}c_{23}\cos\delta_{\mathrm{CP}} \tag{1.31}$$

and $\overset{(-)}{\nu}_\mu$ represents either ν_μ or $\overline{\nu}_\mu$. The complementary appearance probability of tau and electron neutrinos types are given below

$$
\begin{aligned}
\mathcal{P}\left(\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e \right) =& 4c_{13}^2 s_{23}^2 s_{13}^2 \sin^2 \phi_{31} \left(1 + \frac{2\alpha}{\Delta m_{31}^2} \left(1 - 2s_{13}^2 \right) \right) \\
&+ \left(X_{\cos \delta_{\mathrm{CP}}} \right) \cos \phi_{23} \sin \phi_{31} \sin \phi_{21} \\
&\overset{(+)}{-} \left(Y_{\sin \delta_{\mathrm{CP}}} \right) \sin \phi_{32} \sin \phi_{31} \sin \phi_{21} \\
&+ 4s_{12}^2 c_{13}^2 \left(W_{\cos \delta_{\mathrm{CP}}} \right) \sin^2 \phi_{21} \\
&- 8c_{13}^2 s_{13}^2 s_{23}^2 \left(1 - 2s_{13}^2 \right) \frac{\overset{(-)}{+} \alpha L}{4E} \cos \phi_{32} \sin \phi_{31}
\end{aligned} \tag{1.32}
$$

where

$$
X_{\cos \delta_{\mathrm{CP}}} = 8c_{13}^2 s_{12} s_{12} s_{23} \left(c_{12} c_{23} \cos \delta_{\mathrm{CP}} - s_{12} s_{13} \right) \tag{1.33}
$$

$$
Y_{\sin \delta_{\mathrm{CP}}} = 8c_{13}^2 c_{12} c_{23} s_{12} s_{13} s_{23} \sin \delta_{\mathrm{CP}} \tag{1.34}
$$

The oscillations of muon neutrinos into other flavors are of primary importance in accelerator and atmospheric neutrino oscillation experiments. However, traveling through matter introduces sensitivity for CP violation.

The presence of matter alters the probability for any oscillations involving electron neutrinos. This is due to coherent forward scattering of ν_e with electrons in the media. This a function of the path length. This is the Mikheyev-Smirnov-Wolfenstein effect which

$$
\mathcal{P}\left(\nu_\mu \to \nu_e \right) \cong \mathcal{P}\left(\nu_e \to \nu_\mu \right) \cong P_0 + \underbrace{P_{\sin \delta_{\mathrm{CP}}}}_{\text{CP violating}} + P_{\cos \delta_{\mathrm{CP}}} + P_3
$$

where Best measurements of the oscillations parameters is given in Table

Parameter	Value		Units
	Normal Hierarchy	Inverted Hierarchy	
$\Delta m^2_{32} = \Delta m^2_{\text{atm}}$	2.51 ± 0.05	-2.56 ± 0.04	10^{-3} eV^2
$\Delta m^2_{21} = \Delta m^2_{\text{sol}}$	7.53 ± 0.18		10^{-5} eV^2
$\sin^2(\theta_{21})$=$\sin^2(\theta_{\text{sol}})$	$0.307^{+0.013}_{-0.012}$		1
$\sin^2(\theta_{32})$=$\sin^2(\theta_{\text{atm}})$	Q1 $0.592^{+0.023}_{-0.030}$ Q2 $0.597^{+0.024}_{-0.030}$	Q1 $0.421^{+0.033}_{-0.025}$ Q2 $0.417^{+0.025}_{-0.028}$	1
$\sin^2(\theta_{31})$	2.12 ± 0.08		10^{-2}
$\delta_{\text{CP}}{}^{\dagger}$	217^{+40}_{-28}	280^{+25}_{-28}	degrees

Table 1.2: Table of best fit MNSP parameters split by normal and inverted hierarchy. All values are combined values from the Particle Data Group [6]. †Taken from [2] since no best fit value is available from the Particle Data Group.

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