

**Input** :  $\lambda_2; v_p = v^{(i,j)}, R^{(i)}, S^{(j)}, \forall 0 \leq i < j \leq T$

**Output**:  $w_p = w^{(i,j)}, \forall 0 \leq i < j \leq T$

1 **Algorithm** 2D Fused Lasso()

2     **Define**  $d(p, p')$  as distance from  $p$  to  $p'$

3     **Define**  $D(k, k') = \min_{\substack{p \in G_k \\ p' \in G'_k}} d(p, p')$

4     **Define**  $c_{k,k'} = \sum_{p \in G_k} \sum_{p' \in G'_k} I[d(p, p') = 1]$

5     **Define**  $N_k = |G_k|$

6     **Define**  $\bar{w}_k = \frac{1}{N_k} \sum_{p \in G_k} w_p$

7     **Define**  $\gamma_k$  as the agreed value in  $G_k$

8     **Init**  $G$ :  $w_p \in G_p$

9     **while**  $w^{(i,j)}$  not converge **do**

10         Descent Cycle

11         Fusion Cycle

12         Smoothing Cycle

13     **end**

1 **Procedure** Descent Cycle

2     Iteratively consider derivatives of  $\gamma_k$  only and fix all other parameters:

$$N_k \cdot (\bar{w}_k - \gamma_k) +$$

$$\lambda_2 \cdot \sum_{D(k,k')=1} c_{k,k'} \cdot \text{sgn}(\gamma_k - \gamma'_k).$$

3     Update  $\gamma_k$  if the model cost reduces.

1 **Procedure** Fusion Cycle

2     Iteratively consider to merge  $G_k$  and  $G'_k$ , where  $D(k, k') = 1$ , and fix all other parameters:

$$N_k \cdot (\bar{w}_k - \gamma_k) + N'_k \cdot (\bar{w}'_k - \gamma'_k) +$$

$$\lambda_2 \cdot \sum_{D(k,l)=1} c_{k,l} \cdot \text{sgn}(\gamma_k - \gamma_l) +$$

$$\lambda_2 \cdot \sum_{D(k',l)=1} c_{k',l} \cdot \text{sgn}(\gamma'_k - \gamma_l).$$

3     If model cost reduces by merging  $G_k$  and  $G'_k$ , we introduce a new group  $G_m = G_k \cup G'_k$ , with  $N_m = N_k + N'_k$ ,  $\bar{w}_m = (N_k \bar{w}_k + N'_k \bar{w}'_k) / N_m$ , and  $c_{m,l} = c_{k,l} + c_{k',l}$ .

1 **Procedure** Smoothing Cycle

2     To prevent model from sticking at local optimal, increase  $\lambda_2$  with slightly, i.e.  $\lambda_2 \leftarrow \lambda_2 + \delta$ .