Due Time : 11:59pm, Monday October 23, 2017 Submit to Gradescope

The homework is about convex functions. The first four exercises cover the definition, information inequality, convex conditions and convexity of probabilities. The four exercises worth 1 point each and are graded by completion. The first assignment checks the convexity of a log-sum-exponential function which approximates the range of vector elements x_i. The last assignments use a mass motion as an example to derive the Taylor's expansion and test CVX code. The two assignments worth 3 points each and are graded by content.

II.2 (1) is optional, and II.2 (2) is required.

I Exercises from textbook Chapter Three: 3.2, 3.13, 3.16, 3.24a-f

II Assignments

II.1 [log-sum-exp] We have a log-sum-exponential function,

$$f(x) = \log(\sum_i e^{x_i}) + \log(\sum_i e^{-x_i}),\ x \in \mathbb{R}^n$$

Is function f convex or concave? Show your explanation.

II.2 [Least Action]

(1) Suppose that we throw a ball up into the air, we want to describe the height of the ball as a function of time, $x = f(t),\ f : \mathbb{R} \to \mathbb{R}$, considering only the gravity effect. Given the mass of the ball m_o, gravitational acceleration g, the starting height at x_1, the starting time at t_1, the ending height at x_2, and the ending time at t_2. Following the least action principle, which means the least energy consumption in the action, could f be convex, concave, or neither? Show your derivation.

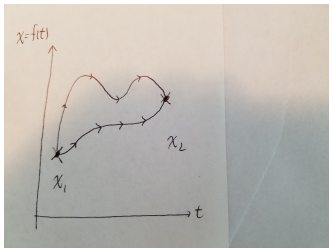

FIGURE 1. Two examples of $x = f(t)$

Hint 1 : action $S \equiv \int_{t_1}^{t_2}(KE - PE)dt$, KE: kinetic energy, PE: potential energy, both KE and PE are functions of time
Hint 2 : Jensen' s Inequality
Hint 3 : Approximation to the first order derivative is sufficient, second or higher order derivative can be neglected

(2) In this problem, you have to use cvx program to solve it (http://cvxr.com/cvx/). Following (1), we change the gravitational field to a more general form, $V(x) = \frac{-k}{x}, V'(x) = -\frac{k}{x^2}$. Suppose $m_o = 0.1, k = 0.01$, what is the magnitude of the force applied to the ball at the positions $\{x|1 \le x \le 10, x \in \mathbb{N}\}$? You should turn in both your code and the solutions.

Hint 1 : Use Hint 1 to 3 in II.2
Hint 2 : Use Taylor Expansion on PE

Hint 3 : Once the objective function is found, plug it into cvx program.
Hint 4 : Feel free to use the file *hw2.m* .