## 1 Assignment 1

# 1.1 Perspective Projection [6 pts]

1. Calculate the coordinates of the endpoints of the projection of the ray onto the image plane. [2pt]

Sol. It is known that

$$Q = \begin{pmatrix} 4 \\ -7 + 2s \\ s \end{pmatrix}, \ s \in (-\infty, -1].$$

With the perspective projection, projected line can be written as

$$Q' = \begin{pmatrix} \frac{4}{-7} \cdot f' \\ -\frac{7}{2} \cdot 2s \\ f' \end{pmatrix} = \begin{pmatrix} \frac{4f'}{s} \\ -\frac{7f'}{s} + 2f' \\ f' \end{pmatrix}, \ s \in (-\infty, -1].$$

Then, one of the closed endpoint at s = -1 is

$$\begin{pmatrix} -4f' \\ 9f' \\ f' \end{pmatrix}.$$

2. Find the equation of a ray  $L \neq Q$  that is parallel to Q. How did you arrive at this equation? [2pt]

Sol. Since parallel rays in 3D space simply means they share the same p2 but may have different p1. So I can simply let

$$L = \begin{pmatrix} 4\\2s\\s \end{pmatrix}, \ s \in (-\infty, -1].$$

3. Can you find the point on the image plane , where the rays L and Q meet? If so, what is that point? [Hint: Think in terms of projective geometry] [2pt]

Sol. Similar to 1., I can write down projected L' as

$$L' = \begin{pmatrix} \frac{4f'}{s} \\ 2f' \\ f' \end{pmatrix}.$$

It is obvious that the line Q' and L' will never intersect with each other, since  $\frac{-7f'}{s} + 2f' \neq 2s$ .

### 1.2 Geometry [6 pt]

1. Using Euclidean coordinates, find the equation of the line perpendicular to the family of lines  $\tilde{y} = \tilde{x} + \lambda, \lambda \in (-\infty, \infty)$  and a distance D from origin. Express your answer in terms of the given parameters. [2pt]

Sol. I need to find those lines that perpendicular to the line family  $\tilde{y} = \tilde{x} + \lambda, \lambda \in (-\infty, \infty)$ . Thus, the line must be in the form  $\tilde{x} + \tilde{y} + c = 0$ .

To calculate the distance between a dot and a line, I have

$$D = \frac{|c-0|}{\sqrt{1+1}}.$$

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$$\tilde{x} + \tilde{y} \pm \sqrt{2}D = 0.$$

- 2. Prove the following two statements (using homogeneous coordinates) that follow from equation 1.
  - (a) The cross product between two points gives us the line joining the two points. [2pt]

Sol. Given two points (x, y, 1) and (x', y', 1), they can be regarded as two vectors in 3D space starting from (0,0,0). It is known that the cross product between two vectors result in a normal vector v that is perpendicular to those two lines. Since judging whether a point lies in a line utilizes exactly the dot product, and the dot product between two lines and v are zero here. According to the definition of perpendicular, it can be regard as their intersection point.

(b) The cross product between two lines gives us their point of intersection. [2pt]

Sol. Given two lines (a, b, c) and (a', b', c'), their cross product result in a vector v that is perpendicular to those two lines. Since the dot product between two lines and v are zero, according to the definition of perpendicular, it can be regard as their intersection point.

3. What is the line, in homogeneous coordinates, joining the points (1,4) and (4,5). [2pt]

Sol. Regard  $x_1 = (1, 4, 1)$  and  $x_2 = (4, 5, 1)$ , their cross product is (-1, 3, -11). 

## Image formation and rigid body transformations [12 points]

1. No rigid body transformation [3 pts]

Sol. Matrix E and I are described as follows, while generated figure and MATLAB code are included in Fig. 1 and Listing 1 in Appendix, respectively.

$$E = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad K = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2. Translation [3 pts]

Sol. Matrix E and I are described as follows, while generated figure and MATLAB code are included in Fig. 1 and Listing 1 in Appendix, respectively.

$$E = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad K = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

3. Translation and rotation [3 pts]

Sol. Assume that the rotation degree around z-axis, y-axis and x-axis are  $\alpha, \beta, \gamma$ , respectively. Rotation matrix can be written as

$$R = \begin{pmatrix} \cos\alpha \cdot \cos\beta & \cos\alpha \cdot \sin\beta \cdot \sin\gamma - \sin\alpha \cdot \cos\gamma & \cos\alpha \cdot \sin\beta \cdot \cos\gamma + \sin\alpha \cdot \sin\gamma \\ \sin\alpha \cdot \cos\beta & \sin\alpha \cdot \sin\beta \cdot \sin\gamma + \cos\alpha \cdot \cos\gamma & \sin\alpha \cdot \sin\beta \cdot \cos\gamma - \cos\alpha \cdot \sin\gamma \\ -\sin\beta & \cos\beta \cdot \sin\gamma & \cos\beta \cdot \cos\gamma \end{pmatrix}.$$

given setting parameters, I have 
$$R = \begin{pmatrix} \cos\frac{\pi}{3} \cdot \cos\frac{\pi}{4} & \cos\frac{\pi}{3} \cdot \sin\frac{\pi}{4} \cdot \sin0 - \sin\frac{\pi}{3} \cdot \cos0 & \cos\frac{\pi}{3} \cdot \sin\frac{\pi}{4} \cdot \cos0 + \sin\frac{\pi}{3} \cdot \sin0 \\ \sin\frac{\pi}{3} \cdot \cos\frac{\pi}{4} & \sin\frac{\pi}{3} \cdot \sin\frac{\pi}{4} \cdot \sin0 + \cos\frac{\pi}{3} \cdot \cos0 & \sin\frac{\pi}{3} \cdot \sin\frac{\pi}{4} \cdot \cos0 - \cos\frac{\pi}{3} \cdot \sin0 \\ -\sin\frac{\pi}{4} & \cos\frac{\pi}{4} \cdot \sin0 & \cos\frac{\pi}{4} \cdot \cos0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} \cdot \frac{\sqrt{2}}{2} & \frac{1}{2} \cdot \frac{\sqrt{2}}{2} \cdot 0 - \frac{\sqrt{3}}{2} \cdot 1 & \frac{1}{2} \cdot \frac{\sqrt{2}}{2} \cdot 1 + \frac{\sqrt{3}}{2} \cdot 0 \\ -\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} & \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} \cdot 0 + \frac{1}{2} \cdot 1 & \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} \cdot 1 - \frac{1}{2} \cdot 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \cdot 0 & \frac{\sqrt{2}}{2} \cdot 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\sqrt{2}}{4} & -\frac{\sqrt{3}}{2} & \frac{\sqrt{2}}{4} \\ \frac{\sqrt{6}}{4} & \frac{1}{2} & \frac{\sqrt{6}}{4} \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{pmatrix}$$

П

CSE 252A 2 Fall 2016 Matrix E and I are described as follows, while generated figure and MATLAB code are included in Fig. 1 and Listing 1 in Appendix, respectively.

$$E = \begin{pmatrix} \frac{\sqrt{2}}{4} & -\frac{\sqrt{3}}{2} & \frac{\sqrt{2}}{4} & 0\\ \frac{\sqrt{6}}{4} & \frac{1}{2} & \frac{\sqrt{6}}{4} & 0\\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} & 1\\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad K = \begin{pmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1 \end{pmatrix}$$

4. Translation and rotation, long distance [3 pts]

Sol. Similar to 3., I can write down rotation matrix as

$$\begin{split} R &= \begin{pmatrix} \cos\frac{\pi}{3} \cdot \cos\frac{\pi}{2} & \cos\frac{\pi}{3} \cdot \sin\frac{\pi}{2} \cdot \sin0 - \sin\frac{\pi}{3} \cdot \cos0 & \cos\frac{\pi}{3} \cdot \sin\frac{\pi}{2} \cdot \cos0 + \sin\frac{\pi}{3} \cdot \sin0 \\ \sin\frac{\pi}{3} \cdot \cos\frac{\pi}{2} & \sin\frac{\pi}{3} \cdot \sin\frac{\pi}{2} \cdot \sin0 + \cos\frac{\pi}{3} \cdot \cos0 & \sin\frac{\pi}{3} \cdot \sin\frac{\pi}{2} \cdot \cos0 - \cos\frac{\pi}{3} \cdot \sin0 \\ -\sin\frac{\pi}{2} & \cos\frac{\pi}{2} \cdot \sin0 & \cos\frac{\pi}{2} \cdot \cos0 \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{2} \cdot 0 & \frac{1}{2} \cdot 1 \cdot 0 - \frac{\sqrt{3}}{2} \cdot 1 & \frac{1}{2} \cdot 1 \cdot 1 + \frac{\sqrt{3}}{2} \cdot 0 \\ \frac{\sqrt{3}}{2} \cdot 0 & \frac{\sqrt{3}}{2} \cdot 1 \cdot 0 + \frac{1}{2} \cdot 1 & \frac{\sqrt{3}}{2} \cdot 1 \cdot 1 - \frac{1}{2} \cdot 0 \\ -1 & 0 \cdot 0 & 0 \cdot 1 \end{pmatrix} \\ &= \begin{pmatrix} 0 & -\frac{\sqrt{3}}{2} \cdot 1 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -1 & 0 & 0 \end{pmatrix} \end{split}$$

Matrix E and I are described as follows, while generated figure and MATLAB code are included in Fig. 1 and Listing 1 in Appendix, respectively.

$$E = \begin{pmatrix} 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0\\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & 0\\ -1 & 0 & 0 & 13\\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad K = \begin{pmatrix} 15 & 0 & 0\\ 0 & 15 & 0\\ 0 & 0 & 1 \end{pmatrix}$$

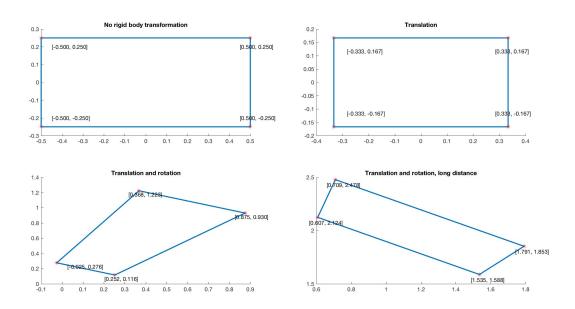


Figure 1: Image Transformation

## 1.4 Rendering [14 points]

1. Plot the face in 2-D [2 pts]

Sol. I simply read in facedata.mat file and render it with imagesc.m function. 2D images for albedo and uniform albedo are shown in Fig 2.

Since in the uniform albedo graph, all the small piece of surface reflect the light in the same magnitude, there is no difference between the small piece of the graph. Therefore, it forms to be in the same color.

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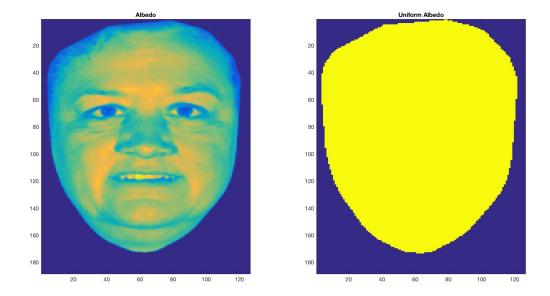


Figure 2: 2D Face

## 2. Plot the face in 3-D [2 pts]

Sol. Similar to 1., I simply read in facedata.mat file and render it with surf.m function. 3D images for albedo and uniform albedo are shown in Fig 3.

Same explanation as written in 1.4.1.

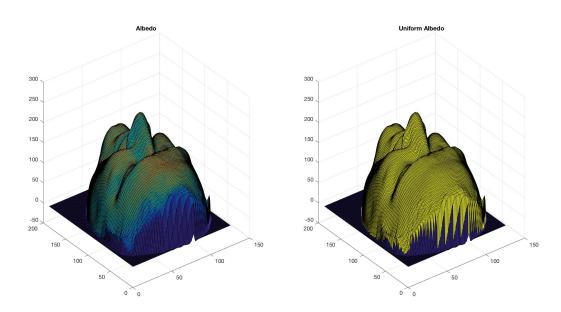


Figure 3: 3D Face

## 3. Surface normals [5 pts]

Sol. Surface normals can be derived from heightmap.mat using the height integration result. To have a easily understanding figure, only 1% of surface vectors are subsampled, and they are shown in Fig 4.

## 4. Render images [5 pts]

Sol. For two single-light scenarios, each pixel of image can be calculated with given formula. For the two-light scenarios, I just calculated the average effect caused by two single lights. Note that since the pixel only contains non-negative values, one needs to use truncate it to 0 once it has negative value. Figures with three kinds of light direction for both albedo and uniform albedo are shown in Fig 5.

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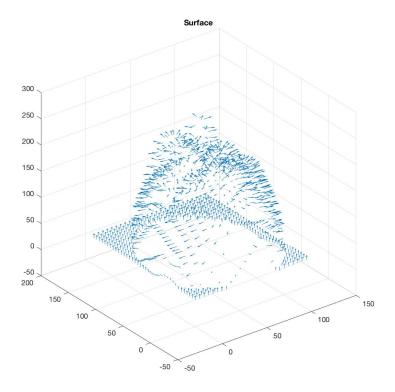
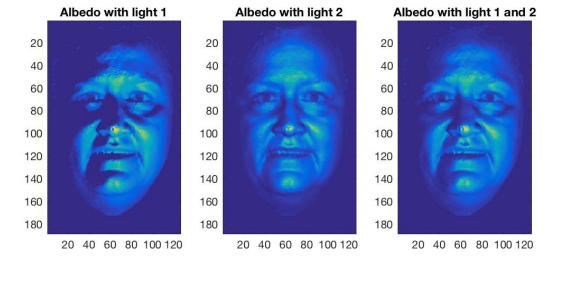


Figure 4: Surface Normals



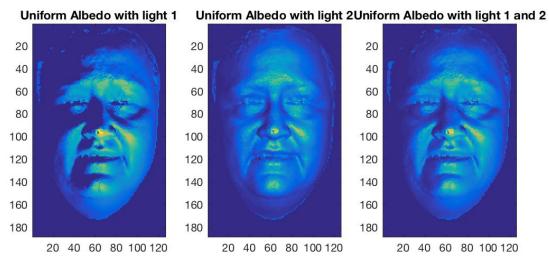


Figure 5: Render Images

## 1.5 Photometric Stereo [18 points]

1. Estimated albedo map [3 pts  $\times 2 = 6$  pts]

Sol. Firstly, I loaded in four images and four corresponding light vectors, and normalize the image matrix to [0,1] by dividing all pixels by 256.

Later, I applied anonymous functions to each element, or said pixel, of image to calculate  $G = Kd \times N = L I$ . It is noticeable that using arrayfun not only increases the code readability but enhances the code efficiency.

After above-said preprocessing, I can render albedo figure easily by calculating Kd = |G| as Fig. 6.

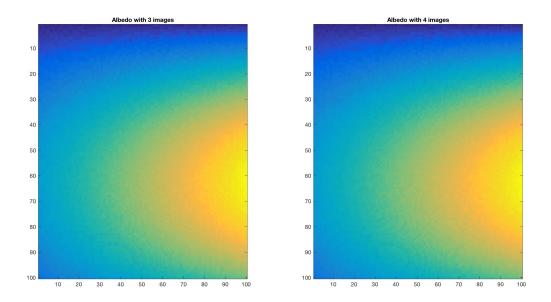


Figure 6: Stereo Albedo

## 2. Estimated surface normals [3 pts $\times 2 = 6$ pts]

Sol. Following 1., surface normals can be obtained by normalizing G with Kd pixel by pixel. I also need to recover depth map here, so that I can add height information into quiver3.m to create correct needle figure. The result of depth map is calculated by integration of  $\frac{N_x}{N_z}$  and  $\frac{N_y}{N_z}$ .

With surface normals  $N_x$ ,  $N_y$ ,  $N_z$  and depth map D given, results created by quiver3 are shown as Fig. 7.

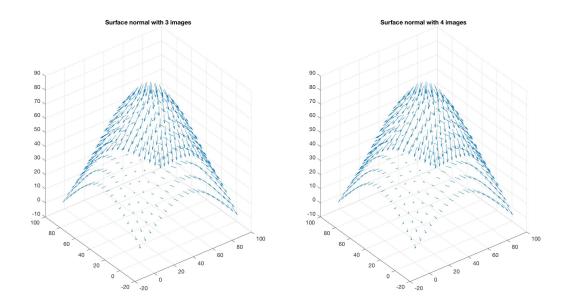


Figure 7: Stereo Surface Normal

3. A wireframe of a depth map [3 pts  $\times 2 = 6$  pts]

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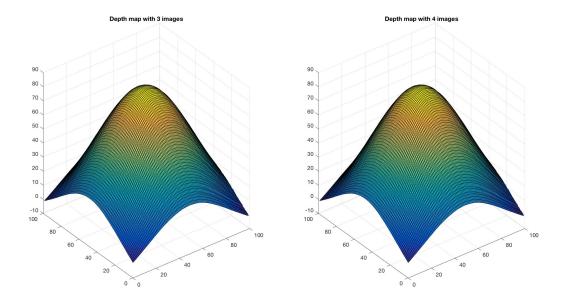


Figure 8: Stereo Depth Map

Sol. Directly utilize the depth matrix obtained in 2., depth map is shown as Fig. 8.

## 1.6 Appendix

### (a) Code for Section 3: Image formation and rigid body transformations

Code to manipulate given plotsquare.m are includued as follows.

```
res = figure('visible','off');
set(res, 'PaperPosition', [0 0 40 20]);
                         % Four 3D points
                          \begin{array}{l} obj \, = \, [-1 \  \  \, -0.5 \  \  \, 2; \  \, 1 \  \  \, -0.5 \  \  \, 2; \  \, 1 \  \  \, 0.5 \  \  \, 2; \  \  \, -1 \  \  \, 0.5 \  \  \, 2]; \\ obj \, (:\, , \  \  \, 4) \, = \, [\, 1 \  \  \, 1 \  \  \, 1\, \, ]\,; \end{array}
                          % Four scenarios
                        E = cell(4);

K = cell(4);

S = cell(4);
  10
 11
  12
                          13
 16
                          \begin{array}{l} K\{2\} = [1 \ 0 \ 0; \ 0 \ 1 \ 0; \ 0 \ 0 \ 1]; \\ E\{2\} = [1 \ 0 \ 0; \ 0 \ 1 \ 0 \ 0; \ 0 \ 0 \ 1 \ 1; \ 0 \ 0 \ 0 \ 1]; \\ S\{2\} = \ 'Translation'; \end{array}
 17
 18
                          \begin{array}{l} {\rm K}\{3\} \,=\, [1\ 0\ 0;\ 0\ 1\ 0;\ 0\ 0\ 1]; \\ {\rm E}\{3\} \,=\, [\, {\rm sqrt}\,(2)\ /\ 4\ -{\rm sqrt}\,(3)\ /\ 2\ {\rm sqrt}\,(2)\ /4\ 0; \\ {\rm sqrt}\,(6)\ /4\ 1/2\ {\rm sqrt}\,(6)\ /4\ 0; \\ \end{array} 
21
 22
 23
                            -\frac{1}{\sqrt{2}} \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \frac{
 26
                         \begin{array}{l} K\{4\} = \begin{bmatrix} 15 & 0 & 0; & 0 & 15 & 0; & 0 & 0 & 1 \end{bmatrix}; \\ E\{4\} = \begin{bmatrix} 0 & -\mathrm{sqrt}\left(3\right)/2 & 1/2 & 0; \\ & 0 & 1/2 & \mathrm{sqrt}\left(3\right)/2 & 0; \\ & -1 & 0 & 0 & 13; & 0 & 0 & 0 & 1 \end{bmatrix}; \\ S\{4\} = \text{'Translation and rotation, long distance'}; \end{array}
 27
 28
 31
 32
                          % Draw subplots accordingly
33
                             for k = 1:4
34
                                                               \begin{array}{l} k = 1:4 \\ & \text{subplot}\,(2\,,\!2\,,\!k)\,; \\ & \text{dat} = K\{k\} * [\text{eye}\,(3) \ \text{zeros}\,(3\,,\ 1)] * E\{k\} * \text{obj'}; \\ & \text{plotsquare}\,(\text{dat})\,; \\ & \text{title}\,(S\{k\})\,; \end{array} 
 35
 36
37
38
                             end
39
 40
                          % Save generated figure
                             saveas(res, '../res/image_transformation.jpg');
```

Listing 1: Code for Section 3

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### (b) Code for Section 4-1: Plot the face in 2-D

Code for plot 2D-face images are included as follows.

```
1 load('../dat/facedata.mat');
2
3 res = figure('visible','off');
4 set(res, 'PaperPosition', [0 0 40 20]);
5
6 subplot(1,2,1);
7 imagesc(albedo);
8 title('Albedo');
9
10 subplot(1,2,2);
11 imagesc(uniform_albedo);
12 title('Uniform Albedo');
13
14 saveas(res, '../res/face_2D.jpg');
```

Listing 2: Code for Section 4-1

### (c) Code for Section 4-2: Plot the face in 3-D

Code for plot 3D-face images are included as follows.

```
load('../dat/facedata.mat');

res = figure('visible','off');
set(res, 'PaperPosition', [0 0 40 20]);

subplot(1,2,1);
surf(heightmap, albedo);
title('Albedo');

subplot(1,2,2);
surf(heightmap, uniform_albedo);
title('Uniform Albedo');
saveas(res, '../res/face_3D.jpg');
```

Listing 3: Code for Section 4-2

#### (d) Code for Section 4-3: Surface normals

Code for plot surface normals are included as follows.

```
load('../dat/facedata.mat');
    % Draw Surface Normal
    res = figure('visible','off');
set(res, 'PaperPosition', [0 0 20 20]);
5
     [n, m] = size(heightmap);
    nx = zeros(n, m);

ny = zeros(n, m);
10
     nz = zeros(n, m);
11
13
     for i = 1:n
           \begin{array}{ccc} \textbf{for} & \textbf{j} & = 1 : \textbf{m} \\ & \textbf{if} & \textbf{i} & < \ \textbf{n} \end{array}
14
15
                        b = heightmap(i,j) - heightmap(i+1,j);
16
17
                   _{\rm else}
                        b = heightmap(i,j);
                   end
19
                  \begin{array}{ccc} \textbf{if} & \textbf{j} & < m \end{array}
20
                        a = heightmap(i,j) - heightmap(i,j+1);
21
                   else
22
23
                        a = heightmap(i, j);
                  end
24
                  nz\,(\,i\,\,,j\,)\,\,=\,\,\mathbf{sqrt}\,(\,1\,/\,(\,a\!*\!a\!+\!b\!*\!b\!+\!1)\,)\,;
25
                  nx(i,j) = a * nz(i,j);

ny(i,j) = b * nz(i,j);
26
27
           \quad \text{end} \quad
28
     end
30
     x = 1:5:130;
31
     y = 1:5:190;

[X,Y] = meshgrid(x, y);
32
33
     ss_h = imresize(heightmap, 0.2);
     ss_nx = imresize(nx, 0.2);
```

```
37     ss_ny = imresize(ny, 0.2);
38     ss_nz = imresize(nz, 0.2);
39
40     quiver3(X, Y, ss_h, ss_nx, ss_ny, ss_nz);
41     title('Surface');
42
43     saveas(res, '../res/surface_normal.jpg');
```

Listing 4: Code for Section 4-3

### (e) Code for Section 4-4: Render images

Code for rendering images are included as follows.

```
load('../dat/facedata.mat');
      res = figure('visible','off');
set(res, 'PaperPosition', [0 0 20 20]);
       [n, m] = size(heightmap);
      img_a.1 = zeros(n, m);

img_a.2 = zeros(n, m);

img_a.12 = zeros(n, m);

img_ua.12 = zeros(n, m);

img_ua.1 = zeros(n, m);

img_ua.2 = zeros(n, m);
11
       img_ua_12 = zeros(n, m);
       for i = 1:n
14
               \begin{array}{ccc} \textbf{for} & \textbf{j} = 1 : \textbf{m} \\ & \textbf{if} & \textbf{i} < \textbf{n} \end{array}
15
16
                                b = heightmap(i,j) - heightmap(i+1,j);
17
                        b = heightmap(i,j);
end
19
20
                        i\,f\ j\ <\,m
21
                                a = heightmap(i,j) - heightmap(i,j+1);
22
23
                                a = heightmap(i, j);
                        end
25
                        nz \; = \; \mathbf{sqrt} \; (1/(\, a\! *\! a\! +\! b\! *\! b\! +\! 1))\; ;
26
                       nx = a * nz;

ny = b * nz;
27
28
                        \begin{array}{l} dist\_1 = norm(lightsource(1,:) - [i, j, heightmap(i,j)]);\\ img\_a\_1(i,j) = max(0, albedo(i,j) * [nx ny nz] ...\\ * lightsource(1,:)' * 1.0 / (dist\_1^2));\\ img\_ua\_1(i,j) = max(0, uniform\_albedo(i,j) * [nx ny nz] ...\\ * lightsource(1,:)' * 1.0 / (dist\_1^2)); \end{array}
30
31
32
33
34
35
                        \begin{array}{l} dist\_2 = norm(lightsource(2\,,:) - [i\,,\,j\,,\,heightmap(i\,,j)])\,;\\ img\_a\_2(i\,,j) = max(0\,,\,albedo(i\,,j)\,*\,[nx\,\,ny\,\,nz]\,\ldots\\ & *\,lightsource(2\,,:)\,'\,*\,1.0\,/\,(dist\_1\,^2))\,;\\ img\_ua\_2(i\,,j) = max(0\,,\,uniform\_albedo(i\,,j)\,*\,[nx\,\,ny\,\,nz]\,\ldots\\ & *\,lightsource(2\,,:)\,'\,*\,1.0\,/\,(dist\_1\,^2))\,; \end{array}
36
37
38
39
41
                        \begin{array}{l} img_-a_-12\,(i\,,j\,) \, = \, img_-a_-1\,(i\,,j\,) \, + \, img_-a_-2\,(i\,,j\,)\,; \\ img_-ua_-12\,(i\,,j\,) \, = \, img_-ua_-1\,(i\,,j\,) \, + \, img_-ua_-2\,(i\,,j\,)\,; \end{array}
42
43
44
47
       subplot(2,3,1);
       imagesc(img-a-1);
title('Albedo with light 1');
48
49
       subplot(2,3,2);
       imagesc(img_a_2);
title('Albedo with light 2');
53
54
       subplot(2,3,3);
55
       imagesc(img_a_12);
       title ('Albedo with light 1 and 2');
       subplot (2,3,4);
59
       imagesc(img_ua_1);
60
       title ('Uniform Albedo with light 1');
61
63
       subplot(2,3,5);
       imagesc(img_ua_2);
title('Uniform Albedo with light 2');
64
65
66
       subplot(2,3,6);
       imagesc(img_ua_12);
68
       title ('Uniform Albedo with light 1 and 2');
69
70
       saveas(res, '../res/render.jpg');
```

Listing 5: Code for Section 4-4

#### (f) Code for Section 5: Photometric Stereo

Code for estimated albedo map, surface normals and depth map from three or four images are included as follows.

```
load('../dat/synthetic_data.mat');
      % Preparation
 3
      \% Prepare image matrix
 5
     img = zeros(4, 10000);

img(1,:) = im1(:);
      \begin{array}{l} \operatorname{img}(2\,;) &= \operatorname{im2}(:)\,;\\ \operatorname{img}(3\,;) &= \operatorname{im3}(:)\,;\\ \operatorname{img}(4\,;) &= \operatorname{im4}(:)\,;\\ \operatorname{img} &= \operatorname{arrayfun}(@(x) \ \operatorname{double}(x) \ / \ 256\,, \ \operatorname{img})\,; \end{array}
      imgc = num2cell(img, 1);
     % Prepare small image matrix s_img = img([1 \ 2:4],:);
14
15
      s_imgc = num2cell(s_img, 1);
16
     % Prepare light direction sources lgt = zeros(4, 3); lgt(1,:) = l1(:) / norm(l1(:)); lgt(2,:) = l2(:) / norm(l2(:)); lgt(3,:) = l3(:) / norm(l3(:)); lgt(4,:) = l4(:) / norm(l4(:));
19
20
^{21}
24
      % Calculate pesudo inverse of light matrix
25
      plgt = pinv(lgt);
26
      \% Calculate G = Kd * N = L \setminus I
      \begin{aligned} G &= \operatorname{arrayfun}(@(x) \ \operatorname{plgt} \ * \ x\{:\}, \ \operatorname{imgc}, \ '\operatorname{UniformOutput'}, \ \operatorname{false}); \\ s\_G &= \operatorname{arrayfun}(@(x) \ \operatorname{plgt} \ * \ x\{:\}, \ s\_\operatorname{imgc}, \ '\operatorname{UniformOutput'}, \ \operatorname{false}); \end{aligned}
30
31
      G = reshape(G, [100, 100]);

s_{-}G = reshape(s_{-}G, [100, 100]);
32
35
36
      % (1) Draw Albedo
37
38
      % Calculate Kd = |G|
Kd = arrayfun(@(x) norm(x{:}), G, 'UniformOutput', false);
s_Kd = arrayfun(@(x) norm(x{:}), s_G, 'UniformOutput', false);
41
42
      res = figure('visible','off');
set(res, 'PaperPosition', [0 0 40 20]);
43
       subplot(1,2,1);
imagesc(cell2mat(s_Kd));
title('Albedo with 3 images');
46
47
48
      {
m subplot} (1,2,2); \\ {
m imagesc} ({
m cell2mat} ({
m Kd}));
52
       title('Albedo with 4 images');
53
       saveas(res, '../res/stereo_albedo.jpg');
54
57
      % (2) Draw Surface Normal
58
59
60
     N = cellfun(@(x,y) x/y, G, Kd, 'UniformOutput', false);
N = reshape(cell2mat(N), [3, 100, 100]);
Nx = squeeze(N(1,:,:));
Ny = squeeze(N(2,:,:));
63
64
      Nz = squeeze(N(3,:,:));
65
      69
70
      s_Nz = squeeze(s_N(3,:,:));
      % Calculate D
73
      D = zeros(100, 100);
74
      s_D = zeros(100, 100);
75
       for i = 2:100
76
              \begin{array}{l} D(i\;,1) \; = \; D(\;i\;-1\;,1) \; - \; Nx(\;i\;,1) \; / Nz(\;i\;,1) \; ; \\ s_{-}D(\;i\;,1) \; = \; s_{-}D(\;i\;-1\;,1) \; - \; s_{-}Nx(\;i\;,1) \; / s_{-}Nz(\;i\;,1) \; ; \end{array} 
79
       end
       for i = 1:100
80
               for j = 2:100

D(i,j) = D(i,j-1) - Ny(i,j)/Nz(i,j);
81
82
                       s_{-}D\,(\,i\,\,,\,j\,)\,\,=\,\,s_{-}D\,(\,i\,\,,\,j\,-1)\,\,-\,\,s_{-}Ny\,(\,i\,\,,\,j\,)\,/\,s_{-}Nz\,(\,i\,\,,\,j\,)\,;
```

```
end
 86
      % Subsample graph

x = 1:5:100;

y = 1:5:100;

[X,Y] = meshgrid(x, y);

ss_Nx = imresize(Nx, 0.2);

ss_Ny = imresize(Ny, 0.2);

ss_Nz = imresize(Nz, 0.2);
 87
 88
 92
 93
       ss_D = imresize(D, 0.2);
       res = figure('visible','off');
set(res, 'PaperPosition', [0 0 40 20]);
97
 98
       \begin{array}{l} \textbf{subplot}\left(1\,,2\,,1\right)\,;\\ \textbf{quiver3}\left(X,\ Y,\ ss\_D\,,\ ss\_Nx\,,\ ss\_Ny\,,\ ss\_Nz\,\right)\,;\\ \textbf{title}\left(\,\,^{\mathsf{L}}\textbf{Surface normal with 3 images}\,\,^{\mathsf{L}}\right)\,;\\ \end{array}
99
100
101
102
       subplot(1,2,2);
quiver3(X, Y, ss_D, ss_Nx, ss_Ny, ss_Nz);
title('Surface normal with 4 images');
103
104
105
        saveas(res, '../res/stereo_surface_normal.jpg');
107
108
109
110
       \% (3) Draw depth map
111
112
       res = figure('visible','off');
set(res, 'PaperPosition', [0 0 40 20]);
113
114
115
       subplot (1,2,1);
116
       x = 1:1:100;

y = 1:1:100;
117
118
       [X,Y] = meshgrid(x, y);
surf(X, Y, D);
title('Depth map with 3 images');
119
120
121
123
        subplot(1,2,2);
       surf(s_D);
title('Depth map with 4 images');
124
125
126
        saveas(res , '../res/stereo_depth_map.jpg');
127
```

Listing 6: Code for Section 5