CSE 250A. Assignment 2

Hao-en Sung (A53204772)
wrangle1005@gmail.com

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2.1 Probabilistic inference

(a) $P(E=1|J=1)$

Sol. For the writing simplicity, I would like to calculate $P(A=1)$ first.

$$\begin{aligned}
P(A=1) &= \sum_{e,b} P(A=1, E=e, B=b) \\
&= \sum_{e,b} P(E=e) \cdot P(B=b) \cdot P(A=1|E=e, B=b) \\
&= \quad P(E=0) \cdot P(B=0) \cdot P(A=1|E=0, B=0) \\
&\quad + P(E=0) \cdot P(B=1) \cdot P(A=1|E=0, B=1) \\
&\quad + P(E=1) \cdot P(B=0) \cdot P(A=1|E=1, B=0) \\
&\quad + P(E=1) \cdot P(B=1) \cdot P(A=1|E=1, B=1) \\
&= \quad (1-0.002) \cdot (1-0.001) \cdot 0.001 + (1-0.002) \cdot 0.001 \cdot 0.94 \\
&\quad + 0.002 \cdot (1-0.001) \cdot 0.29 + 0.002 \cdot 0.001 \cdot 0.95 \\
&\approx 0.002516
\end{aligned}$$

$$\begin{aligned}
P(J=1) &= \sum_a P(J=1, A=a) = \sum_a P(J=1|A=a) \cdot P(A=a) \\
&= P(J=1|A=0) \cdot P(A=0) + P(J=1|A=1) \cdot P(A=1) \\
&= 0.05 \cdot (1-0.002516) + 0.90 \cdot 0.002516 = 0.052139
\end{aligned}$$

$$\begin{aligned}
P(E=1|J=1) &= \frac{P(E=1, J=1)}{P(J=1)} = \frac{\sum_{a,b} P(A=a, B=b, E=1, J=1)}{P(J=1)} \\
&= \frac{\sum_{a,b} P(B=b) \cdot P(E=1) \cdot P(A=a|B=b, E=1) \cdot P(J=1, A=a)}{P(J=1)} \\
&= \quad \frac{P(E=1) \cdot P(B=0) \cdot P(A=0|E=1, B=0) \cdot P(J=1, A=0)}{P(J=1)} \\
&\quad + \frac{P(E=1) \cdot P(B=1) \cdot P(A=0|E=1, B=1) \cdot P(J=1, A=0)}{P(J=1)} \\
&\quad + \frac{P(E=1) \cdot P(B=0) \cdot P(A=1|E=1, B=0) \cdot P(J=1, A=1)}{P(J=1)} \\
&\quad + \frac{P(E=1) \cdot P(B=1) \cdot P(A=1|E=1, B=1) \cdot P(J=1, A=1)}{P(J=1)} \\
&= \quad \frac{0.002 \cdot (1-0.001) \cdot (1-0.29) \cdot 0.05}{P(J=1)} + \frac{0.002 \cdot 0.001 \cdot (1-0.95) \cdot 0.05}{P(J=1)} \\
&\quad + \frac{0.002 \cdot (1-0.001) \cdot 0.29 \cdot 0.90}{P(J=1)} + \frac{0.002 \cdot 0.001 \cdot 0.95 \cdot 0.90}{P(J=1)} \\
&\approx \frac{0.000594}{0.052139} \approx 0.011393
\end{aligned}$$

□

(b) $P(E=1|J=1, B=1)$

Sol. For the writing simplicity, I would like to calculate $P(J=1, B=1)$ first.

$$
\begin{aligned}
P(J=1,B=1) &= \sum_{a,e} P(E=e,B=1,A=a,J=1) \\
&= \sum_{a,e} P(E=e)\cdot P(B=1)\cdot P(A=a|E=e,B=1)\cdot P(J=1|A=a) \\
&= \quad P(E=0)\cdot P(B=1)\cdot P(A=0|E=0,B=1)\cdot P(J=1|A=0) \\
&\quad + P(E=1)\cdot P(B=1)\cdot P(A=0|E=1,B=1)\cdot P(J=1|A=0) \\
&\quad + P(E=0)\cdot P(B=1)\cdot P(A=1|E=0,B=1)\cdot P(J=1|A=1) \\
&\quad + P(E=1)\cdot P(B=1)\cdot P(A=1|E=1,B=1)\cdot P(J=1|A=1) \\
&= \quad (1-0.002)\cdot 0.001\cdot(1-0.94)\cdot 0.05 + 0.002\cdot 0.001\cdot(1-0.95)\cdot 0.05 \\
&\quad + (1-0.002)\cdot 0.001\cdot 0.94\cdot 0.90 + 0.002\cdot 0.001\cdot 0.95\cdot 0.90 \\
&\approx 0.000849
\end{aligned}
$$

$$
\begin{aligned}
P(E=1|J=1,B=1) &= \frac{P(E=1,J=1,B=1)}{P(J=1,B=1)} = \frac{\sum_a P(E=1,J=1,B=1,A=a)}{P(J=1,B=1)} \\
&= \frac{\sum_a P(B=1)\cdot P(E=1)\cdot P(A=a|B=1,E=1)\cdot P(J=1,A=a)}{P(J=1,B=1)} \\
&= \quad \frac{P(E=1)\cdot P(B=1)\cdot P(A=0|E=1,B=1)\cdot P(J=1,A=0)}{P(J=1,B=1)} \\
&\quad + \frac{P(E=1)\cdot P(B=1)\cdot P(A=1|E=1,B=1)\cdot P(J=1,A=1)}{P(J=1,B=1)} \\
&= \frac{0.002\cdot 0.001\cdot(1-0.95)\cdot 0.05}{P(J=1,B=1)} + \frac{0.002\cdot 0.001\cdot 0.95\cdot 0.90}{P(J=1,B=1)} \\
&\approx \frac{0.000002}{0.000849} \approx 0.002356
\end{aligned}
$$

$\square$

(c) $P(A=1|M=0)$

Sol. For the writing simplicity, I would like to calculate $P(M=0)$ first.

$$
\begin{aligned}
P(M=0) &= \sum_a P(M=0,A=a) \\
&= P(M=0,A=0) + P(M=0,A=1) \\
&= P(M=0|A=0)\cdot P(A=0) + P(M=0|A=1)\cdot P(A=1) \\
&\approx (1-0.01)\cdot(1-0.002516) + (1-0.70)\cdot 0.002516 \approx 0.988264
\end{aligned}
$$

$$
\begin{aligned}
P(A=1|M=0) &= P(M=0|A=1)\cdot\frac{P(A=1)}{P(M=0)} \\
&\approx (1-0.70)\cdot\frac{0.002516}{0.988264} \\
&\approx 0.000764
\end{aligned}
$$

$\square$

(d) $P(A=1|J=0,M=0)$

Sol. For the writing simplicity, I would like to calculate $P(J=0,M=0)$ first.

$$
\begin{aligned}
P(J=0,M=0) &= \sum_a P(J=0,M=0,A=a) \\
&= P(J=0,M=0,A=0) + P(J=0,M=0,A=1) \\
&= \quad P(J=0|A=0)\cdot P(M=0|A=0)\cdot P(A=0) \\
&\quad + P(J=0|A=1)\cdot P(M=0|A=1)\cdot P(A=1) \\
&\approx (1-0.05)\cdot(1-0.01)\cdot(1-0.002516) + (1-0.90)\cdot(1-0.70)\cdot 0.002516 \\
&\approx 0.938209
\end{aligned}
$$

$$
\begin{aligned}
P(A=1|J=0,M=0) &= P(J=0,M=0|A=1)\cdot\frac{P(A=1)}{P(J=0,M=0)}\\
&= P(J=0|A=1)\cdot P(M=0|A=1)\cdot\frac{P(A=1)}{P(J=0,M=0)}\\
&\approx 1-0.90)\cdot(1-0.70)\cdot\frac{0.002516}{0.938209}\\
&\approx 0.000080
\end{aligned}
$$

$\square$

(e) $P(A=1|M=1)$

Sol. From (c) we know that $P(M=1)=1-P(M=0)=0.011736$.

$$
\begin{aligned}
P(A=1|M=1) &= P(M=1|A=1)\cdot\frac{P(A=1)}{P(M=1)}\\
&\approx 0.70\cdot\frac{0.002516}{1-0.988264}\\
&\approx 0.150068
\end{aligned}
$$

$\square$

(f) $P(A=1|M=1,B=0)$

Sol. For the writing simplicity, I would like to calculate $P(A=1|B=0)$first.

$$
\begin{aligned}
P(A=1|B=0) &= \sum_e\frac{P(A=1,B=0,E=e)}{P(B=0)}=\sum_e\frac{P(B=0)\cdot P(E=e)\cdot P(A=1|B=0,E=e)}{P(B=0)}\\
&= P(E=0)\cdot P(A=1|E=0,B=0)+P(E=1)\cdot P(A=1|E=1,B=0)\\
&= (1-0.002)\cdot 0.001+0.002\cdot 0.29\\
&= 0.001578
\end{aligned}
$$

Later, I can utilize $P(A=0|B=0)$ and $P(A=1|B=0)$ to derive $P(M=1,B=0)$ to finally solve $P(A=1|M=1,B=0)$.

$$
\begin{aligned}
P(M=1,B=0) &= \sum_a P(M=1,B=0,A=a)\\
&= P(M=1,B=0,A=0)+P(M=1,B=0,A=1)\\
&= \quad P(M=1|A=0)\cdot P(A=0|B=0)\cdot P(B=0)\\
&\quad + P(M=1|A=1)\cdot P(A=1|B=0)\cdot P(B=0)\\
&= 0.01\cdot(1-0.001578)\cdot(1-0.001)+0.70\cdot 0.001578\cdot(1-0.001)\\
&\approx 0.011078
\end{aligned}
$$

$$
\begin{aligned}
P(A=1|M=1,B=0) &= P(M=1,B=0|A=1)\cdot\frac{P(A=1)}{P(M=1,B=0)}\\
&= P(M=1|A=1)\cdot P(A=1|B=0)\cdot\frac{P(B=0)}{P(A=1)}\cdot\frac{P(A=1)}{P(M=1,B=0)}\\
&\approx 0.70\cdot 0.001578\cdot\frac{1-0.001}{0.011078}\\
&\approx 0.099611
\end{aligned}
$$

$\square$

2.2 Probabilistic reasoning

(a) Compute the ratio r_k as a function of k. How does the doctor's diagnosis depend on the day of the month? Show your work.

Sol. I firstly focus on the numerator and I have derivations as follows.

$$
\begin{aligned}
P(D=1|S_1=1,...,S_k=1) &= P(S_1=1,...,S_k=1|D=1)\cdot\frac{P(D=1)}{P(S_1=1,...,S_k=1)}\\
&= \prod_{i=1}^{k} P(S_i=1|D=1)\cdot\frac{P(D=1)}{P(S_1=1,...,S_k=1)}\\
&= \prod_{i=1}^{k} P(S_i=1|D=1)\cdot\frac{P(D=1)}{\sum_d P(D=d,S_1=1,...,S_k=1)}\\
&= \prod_{i=1}^{k} P(S_i=1|D=1)\cdot\frac{P(D=1)}{\sum_d P(D=d)\cdot P(S_1=1,...,S_k=1|D=d)}\\
&= \prod_{i=1}^{k} P(S_i=1|D=1)\cdot\frac{1}{\sum_d \prod_{i=1}^{k} P(S_i=1|D=d)}\\
&= \frac{\prod_{i=1}^{k} P(S_i=1|D=1)}{\prod_{i=1}^{k} P(S_i=1|D=0)+\prod_{i=1}^{k} P(S_i=1|D=1)}\\
&= \frac{\frac{1}{2^k+(-1)^k}}{\frac{1}{2^k}+\frac{1}{2^k+(-1)^k}}
\end{aligned}
$$

Then, we can calculate r_k as follows.

$$
\begin{aligned}
r_k &= \frac{P(D=1|S_1=1,...,S_k=1)}{P(D=0|S_1=1,...,S_k=1)}\\
&= \frac{\frac{\frac{1}{2^k+(-1)^k}}{\frac{1}{2^k}+\frac{1}{2^k+(-1)^k}}}{1-\frac{\frac{1}{2^k+(-1)^k}}{\frac{1}{2^k}+\frac{1}{2^k+(-1)^k}}}\\
&= \frac{\frac{1}{2^k+(-1)^k}}{\frac{1}{2^k}}\\
&= \frac{2^k}{2^k+(-1)^k}
\end{aligned}
$$

One can tell that $r_k > 1$ if k is odd and $r_k < 1$ if k is even. □

(b) Does the diagnosis become more or less certain as more symptoms are observed? Explain.

Sol. If the higher certainty is defined as the larger value of $|r_k - 1|$, one can find out that certainty reduces while k gets larger. A easy example is to consider $k = 1$ and $k = 2$.

$$
\begin{aligned}
r_1 &= \frac{2}{2-1} = 2\\
r_2 &= \frac{4}{4+1} = \frac{4}{5}
\end{aligned}
$$

One can tell that $|r_1 - 1| > |r_2 - 1|$. □

2.3 Sigmoid function

(a) $\sigma'(z) = \sigma(z)\cdot\sigma(-z)$

Sol.

$$
\begin{aligned}
\sigma'(z) &= \frac{0+1\cdot -e^{-z}}{(1+e^{-z})^2} = \frac{1}{1+e^{-z}}\cdot\frac{-e^{-z}}{1+e^{-z}}\\
&= \frac{1}{1+e^{-z}}\cdot\frac{1}{e^{z}+1} = \sigma(z)\cdot\sigma(-z)
\end{aligned}
$$

□

(b) $\sigma(-z) + \sigma(z) = 1$

Sol.

$$
\sigma(-z)+\sigma(z) = \frac{1}{1+e^{z}} + \frac{1}{1+e^{-z}} = \frac{1+e^{-z}+1+e^{z}}{1+e^{z}+e^{-z}+e^{0}} = 1
$$

□

(c) $L(\sigma(z)) = z$, where $L(p) = \log\left(\frac{p}{1-p}\right)$ is the *log-odds* function

Sol.

$$L(\sigma(z)) = \log\left(\frac{\sigma(z)}{1-\sigma(z)}\right) = \log\left(\frac{\frac{1}{1+e^{-z}}}{\frac{e^{-z}}{1+e^{-z}}}\right) = \log\left(\frac{1}{e^{-z}}\right) = \log(e^z) = z$$

□

(d) $w_i = L(p_i)$, where $p_i = P(Y=1|X_i=1, X_j=0,\ \forall j \neq i)$

Sol.

$$L(p_i) = \log\left(\frac{P(Y=1|X_i=1,X_j=0,\ \forall j\neq i)}{1-P(Y=1|X_i=1,X_j=0,\ \forall j\neq i)}\right) = \log\left(\frac{\sigma(w_i)}{1-\sigma(w_i)}\right) = L(\sigma(w_i))$$

According the conclusion from (c), $L(\sigma(w_i)) = w_i$. □

2.4 Conditional independence

Sol. Response and simple reason is described in Table 1.

$$\begin{aligned}
P(M,W|S,R) &= P(M|S,R)\cdot P(W|S,R)\\
P(M,W|S,R,A) &= P(M|S,R,A)\cdot P(W|S,R,A)\\
P(M,A|W) &= P(M|W)\cdot P(A|W)\\
P(M,A|R,W) &= P(M|R,W)\cdot P(A|R,W)\\
P(M,A|S,W) &= P(M|S,W)\cdot P(A|S,W)\\
P(M,A|S,R,W) &= P(M|S,R,W)\cdot P(A|S,R,W)\\
P(M,A|S,R) &= P(M|S,R)\cdot P(A|S,R)\\
P(R,S|M) &= P(R|M)\cdot P(S|W)\\
P(R,A|W) &= P(R1W)\cdot P(A|W)\\
P(R,A|M,W) &= P(R|M,W)\cdot P(A|M,W)\\
P(R,A|S,W) &= P(R|S,W)\cdot P(A|S,W)\\
P(R,A|M,S,W) &= P(R|M,S,W)\cdot P(A|M,S,W)\\
P(S,A|W) &= P(S|W)\cdot P(A|W)\\
P(S,A|M,W) &= P(S|M,W)\cdot P(A|M,W)\\
P(S,A|R,W) &= P(S|R,W)\cdot P(A|R,W)\\
P(S,A|M,R,W) &= P(S|M,R,W)\cdot P(A|M,R,W)
\end{aligned}$$

Table 1: Conditional Independence

□

2.5 Markov blanket

Case 1. *Via a parent of a parent of X, i.e. node 1 in the hint figure.*

Sol. Since the parent of X is given, it applies to type-1 d-separate. □

Case 2. *Via a parent of a parent of a child of X, i.e. node 2 in the hint figure.*

Sol. Denote node 2 as N_2, the child of N_2 as A and the co-child of A and X as B. We would like to prove $P(X,N_2|A,B) = P(X|A,B)\cdot P(N_2|A,B)$.

$$\begin{aligned}
P(X,N_2|A,B) &= \frac{P(X,3,A,B)}{P(A,B)} = \frac{P(X)\cdot P(N_2)\cdot P(A|N_2)\cdot P(B|A,X)}{P(A,B)}\\
&= \frac{P(X)\cdot P(N_2)\cdot P(A|N_2)\cdot P(B|A,X)\cdot \frac{P(A,X)}{P(A,X)}\cdot}{P(A,B)}\\
&= P(X|A,B)\cdot\frac{P(X)\cdot P(N_2)\cdot P(A|N_2)}{P(A)\cdot P(X)}\\
&= P(X|A,B)\cdot P(N_2|A)\\
&= P(X|A,B)\cdot P(N_2|A,B)
\end{aligned}$$

□

Case 3. *Via a child of a parent of a child of X, i.e. node 3 in the hint figure.*

Sol. Denote node 3 as N_3, the parent of N_3 as A and the co-child of A and X as B. We would like to prove $P(X, N_3|A, B) = P(X|A, B) \cdot P(N_3|A, B)$.

$$\begin{aligned}
P(X, N_3|A, B) &= \frac{P(X, 3, A, B)}{P(A, B)} = \frac{P(X) \cdot P(A) \cdot P(B|A, X) \cdot P(N_3|A)}{P(A, B)} \\
&= \frac{\frac{P(X) \cdot P(A)}{P(X,A)} \cdot P(X, A) \cdot P(B|A, X) \cdot P(N_3|A)}{P(A, B)} \\
&= 1 \cdot P(X|A, B) \cdot P(N_3|A) \\
&= P(X|A, B) \cdot P(N_3|A, B)
\end{aligned}$$

□

Case 4. *Via a child of a child of X, i.e. node 4 in the hint figure.*

Sol. Since the child of X is given, it applies to type-1 d-separate. □

Case 5. *Via a child of a parent of X, i.e. node 5 in the hint figure.*

Sol. Since the parent of X is given, it applies to type-2 d-separate. □

2.6 True or false

Sol. Response and simple reason is described in Table 2.

True	$P(C, D\|A)$	$=$	$P(C\|A) \cdot P(D\|A)$
False	$P(A\|D)$	$=$	$P(A\|B, D)$
True	$P(C, E)$	$=$	$P(C) \cdot P(E)$
False	$P(C, D, E)$	$=$	$P(C) \cdot P(D) \cdot P(E)$
False	$P(F, G)$	$=$	$P(F) \cdot P(G)$
False	$P(F, G\|D)$	$=$	$P(F\|D) \cdot P(G\|D)$
False	$P(A, D, G)$	$=$	$P(A) \cdot P(D\|A) \cdot P(G\|D)$
False	$P(B\|E)$	$=$	$P(B\|E, G)$
False	$P(C\|E)$	$=$	$P(C\|E, G)$

Table 2: True or false

□

2.7 Subsets

Sol. Response is recorded in the Table 3.

$P(A)$	$=$	$P(A\|S)$	$S = \{B, D, G\}$
$P(A\|C)$	$=$	$P(A\|S)$	$S = \{B, C, D, E, F, G\}$
$P(C)$	$=$	$P(C\|S)$	$S = \{B, D, G\}$
$P(C\|A)$	$=$	$P(C\|S)$	$S = \{A, B, D, G\}$
$P(C\|A, E)$	$=$	$P(C\|S)$	$S = \{A, B, D, E, G\}$
$P(C\|A, E, F)$	$=$	$P(C\|S)$	$S = \{A, E, F\}$
$P(C\|A, D, E, F)$	$=$	$P(C\|S)$	$S = \{A, B, D, E, F, G\}$
$P(F)$	$=$	$P(F\|S)$	$S = \{\}$
$P(F\|C)$	$=$	$P(F\|S)$	$S = \{A, C, E\}$
$P(F\|C, D)$	$=$	$P(F\|S)$	$S = \{A, B, C, D, E, G\}$
$P(B, G)$	$=$	$P(B, G\|S)$	$S = \{A, C, E\}$

Table 3: Subsets

□

2.8 Noisy-OR

Sol. Response is recorded in the Table 4. Since I have no intuition for last three comparisons, I assumed $P(X = 1) = a$ and $P(Y = 1) = b$ and extend all probabilistic formulas...

$$
\begin{array}{rcl}
P(Z=1|X=0,Y=0) & < & P(Z=1|X=0,Y=1) \\
P(Z=1|X=1,Y=0) & < & P(Z=1|X=0,Y=1) \\
P(Z=1|X=1,Y=0) & < & P(Z=1|X=1,Y=1) \\
P(X=1) & = & P(X=1|Y=0) \\
P(X=1) & < & P(X=1|Z=1) \\
P(X=1|Z=1) & > & P(X=1|Y=1,Z=1) \\
P(X=1)P(Y=1)P(Z=1) & < & P(X=1,Y=1,Z=1)
\end{array}
$$

Table 4: Noisy-OR

□

2.9 Polytree inference

(a) Consider just the subgraph of the DAG containing the nodes A and B. Compute the marginal probability $P(B)$ from the CPTs in this subgraph.

Sol.

$$P(B) = \sum_a P(A, B) = \sum_a P(A) \cdot P(B|A)$$

□

(b) Consider just the subgraph of the DAG containing the nodes A, B, C, and D. Compute the conditional probability $P(D|C)$ from the CPTs in this subgraph.

Sol.

$$
\begin{aligned}
P(D|C) &= \frac{P(C,D)}{P(C)} = \frac{\sum_{a,b} P(A,B,C,D)}{P(C)} \\
&= \frac{\sum_{a,b} P(A) \cdot P(B|A) \cdot P(C) \cdot P(D|B,C)}{P(C)} \\
&= \sum_b \sum_a P(A,B) \cdot P(D|B,C) \\
&= \sum_b P(B) \cdot P(D|B,C)
\end{aligned}
$$

□

(c) Compute the conditional probability $P(F|C,E)$.

Sol.

$$
\begin{aligned}
P(F|C,E) &= \frac{P(C,E,F)}{P(C,E)} = \frac{\sum_{b,d} P(B,C,D,E,F)}{P(C) \cdot P(E)} \\
&= \frac{\sum_{b,d} P(B) \cdot P(C) \cdot P(D|B,C) \cdot P(E) \cdot P(F|D,E)}{P(C) \cdot P(E)} \\
&= \frac{\sum_{b,d} \frac{P(B) \cdot P(C)}{P(B,C)} \cdot P(B,C) \cdot P(D|B,C) \cdot P(E) \cdot P(F|D,E)}{P(C) \cdot P(E)} \\
&= \frac{\sum_{b,d} 1 \cdot P(B,C,D) \cdot P(F|D,E)}{P(C)} \\
&= \sum_d \frac{P(C,D)}{P(C)} \cdot P(F|D,E) \\
&= \sum_d P(D|C) \cdot P(F|D,E)
\end{aligned}
$$

□

(d) Compute the posterior probability $P(E|C,F)$.

Sol.

$$
\begin{aligned}
P(E|C,F) &= P(F|C,E) \cdot \frac{P(C,E)}{P(C,F)} \\
&= P(F|C,E) \cdot \frac{P(C) \cdot P(E)}{\sum_{d,e} P(C) \cdot P(D|C) \cdot P(E) \cdot P(F|D,E)} \\
&= P(F|C,E) \cdot \frac{P(E)}{\sum_{d,e} P(D|C) \cdot P(E) \cdot P(F|D,E)}
\end{aligned}
$$

□

(e) Suppose that each node in the belief network can take on n possible values. Consider the complexity of your overall calculation for $P(E = e|C = c, F = f)$, assuming that it need only be done for one particular triplet of values $\{e, c, f\}$. Is it linear, polynomial, or exponential in n? If polynomial, what is the degree? Justify your answer.

Sol. Given e, c, f, I break down the overall procedures into several steps.

1. According to (a): calculate $P(B = b')$ takes $O(n)$ time to sum up over a
2. Follow step 1: calculate $P(B = b')$, $\forall b'$ takes $O(n^2)$ time
3. According to (b): calculate $P(D = d'|C = c)$ takes $O(n)$ time
4. Follow step 3: calculate $P(D = d'|C = c)$, $\forall d'$ takes $O(n^2)$ time to sum up over b
5. According to (c): calculate $P(F = f|C = c, E = e)$ takes $O(n)$ time to sum up over d
6. According to (d): calculate $P(E = e|C = c, F = f)$ takes $O(n^2)$ time to sum up over d and e

With step 2, step 4, step 5 and step 6, the time complexity is $O(n^2 + n^2 + n + n^2) = O(n^2)$. In other words, this algorithm is a polynomial one with degree 2. □