Augmented reality
Exercises

Note: These exercises are **possible** exercises that could be asked at the written exam. They are divided into sections for each topic of the course. For all the solutions show the steps necessary to compute the result. If the steps are not provided the exercise is considered wrong even if the result is correct. The formulas that you need are provided in the exam but you must know the meaning of the terms.

1 Filters

RGB to YUV and YUV to RGB conversions:

$$W_R = 0.299,\ W_B = 0.114,\ W_G = 1 - W_R - W_B$$
$$U_{max} = 0.436,\ V_{max} = 0.615$$
$$R = Y + V\frac{1 - W_R}{V_{max}}$$
$$B = Y + U\frac{1 - W_B}{U_{max}}$$
$$G = \frac{Y - W_R R - W_B B}{W_G}$$

$$W_R = 0.299,\ W_B = 0.114,\ W_G = 1 - W_R - W_B$$
$$U_{max} = 0.436,\ V_{max} = 0.615$$
$$Y = W_R R + W_G G + W_B B$$
$$U = U_{max}\frac{B - Y}{1 - W_B}$$
$$V = V_{max}\frac{R - Y}{1 - W_R}$$

Exercise 1: Consider the following pixel $p = (150, 100, 20)$ in RGB encoding and the conversion formulas from RGB to YUV colour space, compute the equivalent pixel in the YUV space.
Exercise 2: Consider the following pixel $p = (0.573901951, 0.373823553, -0.0894366354)$ in YUV encoding and the conversion formulas from YUV to RGB colour space, compute the equivalent pixel in the RGB space.
Exercise 3: Consider the pixel $p = (15, 100, 100)$ in RGB encoding. Compute the RGB components of the output of the Black & White filter for p.

Exercise 4: Consider the pixel $p = (15, 100, 100)$ in RGB encoding . Compute the YUV components of the output of the Black & White filter for p.

2 Interpolation

Bilinear interpolation formulas:

$$
\begin{aligned}
r =& \lfloor r_f \rfloor, \quad c = \lfloor c_f \rfloor \\
\Delta r =& r_f - r \\
\Delta c =& c_f - c \\
I(r', c') =& I(r, c) \cdot (1 - \Delta r) \cdot (1 - \Delta c) + \\
& I(r + 1, c) \cdot \Delta r \cdot (1 - \Delta c) + \\
& I(r, c + 1) \cdot (1 - \Delta r) \cdot \Delta c + \\
& I(r + 1, c + 1) \cdot \Delta r \cdot \Delta c
\end{aligned}
$$

Exercise 1: Given the four pixels in RGB encoding $p_{1,1} = (100, 100, 100)$, $p_{1,2} = (200, 100, 200)$, $p_{2,1} = (50, 50, 100)$, and $p_{2,2} = (50, 50, 100)$, where $p_{i,j}$ is the pixel at row i and column j in the encoded representation of the image (for example $p_{1,1}$ is the pixel at row 1 and column 1), compute the interpolation with, and assume that $r_f = 1.3$ and $c_f = 1.65$.

1. The nearest neighbour interpolation technique.
2. The bilinear interpolation technique.

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Exercise 2: Assume that an image is being rescaled by a factor of 3 on both width and height.

1. Find the coordinates of the equivalent fractional pixel in the original image for the pixel at position $(4, 15)$ in the rescaled image.
2. Consider the pixels $p_{1,5} = (15, 50, 40)$, $p_{2,5} = (40, 40, 40)$, $p_{1,6} = (10, 15, 50)$, and $p_{2,6} = (25, 25, 25)$, where $p_{i,j}$ is the pixel at row i and column j in the encoded representation of the image (for example $p_{1,1}$ is the pixel at row 1 and column 1), find the interpolated pixel colours by using the bilinear interpolation technique for the fractional pixel obtained at the previous step.

Exercise 3: Consider the following transformation function, that transform the row and column position of a pixel i and j.

$$
f(i) = \frac{i}{\sqrt{2i}}
$$
$$
f(j) = \frac{j}{3\sqrt{j+3}}
$$

1. Find the fractional pixel coordinates for the pixel with coordinates $(8, 6)$ after applying the transformation function above.

2. Consider the pixels $p_{2,0} = (15, 15, 15)$, $p_{2,1} = (10, 10, 10)$, $p_{3,0} = (15, 15, 15)$, and $p_{3,1} = (20, 20, 20)$, where $p_{i,j}$ is the pixel at row i and column j in the encoded representation of the image (for example $p_{1,1}$ is the pixel at row 1 and column 1), find the interpolated pixel colours by using the bilinear interpolation technique for the fractional pixel obtained at the previous step.

3 Clustering and segmentation

Exercise 1: Given the cluster formed by the following pixels encoded in RGB format obtained after running the K-Means algorithm:

$$\begin{aligned} p_1 &= (200, 75, 50) \\ p_2 &= (200, 80, 80) \\ p_3 &= (150, 100, 90) \\ p_4 &= (150, 125, 100) \end{aligned}$$

Find the colour that the segmented image would contain for the pixels of that cluster.

Exercise 2: Assume that the centroid for the cluster obtained at the previous iteration $i-1$ is $(150, 100, 50)$, and that in the current iteration i the cluster contains now the following pixels.

$$\begin{aligned} p_1 &= (200, 75, 50) \\ p_2 &= (200, 80, 80) \\ p_3 &= (100, 100, 90) \\ p_4 &= (100, 135, 100) \end{aligned}$$

Assuming that all the other clusters are unchanged, does the algorithm execute a further iteration $i+1$ or will it stop? Motivate your answer.

Exercise 3: Given the cluster defined by the centroids $c_1 = (50, 45, 100)$, $c_2 = (100, 75, 100)$, and $c_3 = (75, 75, 125)$, find the cluster that the pixel $p = (110, 100, 90)$ will be assigned to.

4 Unwarping algorithm

Omnidirectional camera formulas:

$$\begin{cases} \theta = -2\pi x_1 \\ r = r_{in} + (1 - y_1)(r_{out} - r_{in}) \end{cases}$$

$$\begin{cases} x_p = r \cdot \cos(\theta) + c_x \\ y_p = r \cdot \sin(\theta) + c_y \end{cases}$$

Exercise 1: Consider the pixel at coordinates $p = (35, 20)$ in the panoramic image (the output of the unwarping algorithm that contains the perspective visualization), and an omnidirectional camera whose blind spot radius is 20 pixels, the centre at $(100, 100)$, and the outer lens radius is 180 pixels. What are the coordinates in the omnidirectional image (the image captured by the omnidirectional camera) that will be read for pixel p in order to fill in the colour in the panoramic image.