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Delta rules

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Rotterdam, Netherlands

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Lecture topics

- Make it pretty: delta rules
- Booleans, boolean logic operators, if-then-else
- Naturals, arithmetic operators, comparison operators

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- We can decide that some specific lambda terms have special meanings
- For example, we could decide that a given lambda term means `TRUE`, another `FALSE`, etc.
- The important thing is that we choose terms that behave as we wish

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As we wish?

- Suppose we define some lambda terms for `TRUE`, `FALSE`, and `AND`
- We expect these terms to reduce[a] following our expectations of boolean logic
- We can use truth tables to encode our expectations

[a] That is, computed according to $\rightarrow_\beta$

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We want to formulate TRUE, FALSE, and AND so that

- $\texttt{TRUE} \wedge \texttt{TRUE} \rightarrow_\beta \texttt{TRUE}$
- $\texttt{TRUE} \wedge \texttt{FALSE} \rightarrow_\beta \texttt{FALSE}$
- $\texttt{FALSE} \wedge \texttt{TRUE} \rightarrow_\beta \texttt{FALSE}$
- $\texttt{FALSE} \wedge \texttt{FALSE} \rightarrow_\beta \texttt{FALSE}$

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Choice terms

- Terms with special meaning essentially make a choice when given parameters
- The choice is expressed by either returning, or applying, the parameters

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Delta rules

- We wish to use special symbols to these terms with special meaning
- We define a series of delta rules, which are transformation from pretty symbols into lambda terms (and vice-versa)

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This means that we will be able to write lambda programs such as 5+3, that will then be translated into the appropriate lambda terms

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Idea

- Boolean operators such as `TRUE` and `FALSE` must be defined so as to identify themselves
- The choice is expressed by returning their identity from a choice of two options

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`TRUE` is defined as a selector of the representative for true, that is the first argument[a]

[a] by arbitrary convention

$(\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t})$

`FALSE` is defined as a selector of the representative for false, that is the second argument[a]

[a] by arbitrary convention, as long as different from the previous

$(\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{f})$

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```
((TRUE bit1) bit0)
```

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```
((TRUE bit1) bit0)
```

```
((TRUE bit1) bit0)
```

Booleans

```
((TRUE bit1) bit0)
```

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```
((TRUE bit1) bit0)
```

```
(((λt→f→t) bit1) bit0)
```

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```
(((λt→f→t) bit1) bit0)
```

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$$(((\lambda t \to f \to t)\ \mathtt{bit1})\ \mathtt{bit0})$$

$$(\underline{((\lambda t \to f \to t)\ \mathtt{bit1})}\ \mathtt{bit0})$$

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$$\boxed{(\underline{((\lambda t \rightarrow f \rightarrow t)\ bit1)}\ bit0)}$$

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$$\texttt{(}\underline{\texttt{((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t) bit1)}}\texttt{ bit0)}$$

$$\texttt{((}\lambda\texttt{f}\rightarrow\texttt{bit1) bit0)}$$

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```
((λf→bit1) bit0)
```

Booleans


```
((λf→bit1) bit0)
```

```
((λf→bit1) bit0)
```

Booleans

$\underline{((\lambda \texttt{f} \rightarrow \texttt{bit1})\ \texttt{bit0})}$

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$\underline{\texttt{((}\lambda\texttt{f}\rightarrow\texttt{bit1) bit0)}}$

`bit1`

Booleans

AND

- The conjunction[a] of two terms is a function that takes as input two booleans and returns a boolean
- Since we just defined booleans to be two-parameter functions, we know that the two input booleans can be applied to each other
- Given two booleans `a` and `b`, their conjunction is `b` if `a` was `true`, or `false` otherwise
-

```
(λa→b→((a b) a))
```

[a] AND, or $\wedge$

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AND

Let us begin to with $\texttt{TRUE} \wedge \texttt{TRUE} \rightarrow_\beta \texttt{TRUE}$

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```
(TRUE ∧ TRUE)
```

Booleans

```
(TRUE ∧ TRUE)
```

```
((∧ TRUE) TRUE)
```

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```
((∧ TRUE) TRUE)
```

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```
((∧ TRUE) TRUE)
```

```
(((λa→b→((a b) a)) TRUE) TRUE)
```

Booleans


```
(((λa→b→((a b) a)) TRUE) TRUE)
```

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$$(((\lambda a \rightarrow b \rightarrow ((a\ b)\ a))\ \mathtt{TRUE})\ \mathtt{TRUE})$$

$$(((\lambda a \rightarrow b \rightarrow ((a\ b)\ a))\ \underline{\mathtt{TRUE}})\ \mathtt{TRUE})$$

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$$\boxed{\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a b) a))} \ \underline{\texttt{TRUE}}\texttt{) TRUE)}}$$

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$(((\lambda a \rightarrow b \rightarrow ((a\ b)\ a))\ \underline{\mathtt{TRUE}})\ \mathtt{TRUE})$

$(((\lambda a \rightarrow b \rightarrow ((a\ b)\ a))\ (\lambda t \rightarrow f \rightarrow t))\ \mathtt{TRUE})$

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$$\boxed{\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a b) a)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)) TRUE)}}$$

Booleans


```
(((λa→b→((a b) a)) (λt→f→t)) TRUE)
```

```
(((λa→b→((a b) a)) (λt→f→t)) TRUE)
```

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$$\boxed{\texttt{((}(\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a b) a))}\ (\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t}))\ \underline{\texttt{TRUE}})}$$

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$$\boxed{(((\lambda \mathtt{a} \rightarrow \mathtt{b} \rightarrow ((\mathtt{a}\ \mathtt{b})\ \mathtt{a}))\ (\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t}))\ \underline{\mathtt{TRUE}})}$$

$$\boxed{(((\lambda \mathtt{a} \rightarrow \mathtt{b} \rightarrow ((\mathtt{a}\ \mathtt{b})\ \mathtt{a}))\ (\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t}))\ (\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t}))}$$

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```
(((λa→b→((a b) a)) (λt→f→t)) (λt→f→t))
```

Booleans

$\boxed{\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a b) a)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}$

$\boxed{\texttt{(}\underline{\texttt{((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a b) a)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}\texttt{ (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}$

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$$\boxed{\texttt{(}\underline{\texttt{((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a b) a)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}\texttt{ (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}$$

Booleans

$$\boxed{\texttt{(}\underline{\texttt{((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a b) a)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}\texttt{ (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}$$

$$\boxed{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t) b) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}$$

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$$\boxed{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t) b) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}$$

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$$\boxed{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{((}(\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t) b) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}$$

$$\boxed{\underline{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{((}(\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t) b) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}}$$

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$$\boxed{\underline{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t) b) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}}$$

Booleans

$$\boxed{\underline{((\lambda \texttt{b} \rightarrow (((\lambda \texttt{t} \rightarrow \texttt{f} \rightarrow \texttt{t})\ \texttt{b})\ (\lambda \texttt{t} \rightarrow \texttt{f} \rightarrow \texttt{t})))\ (\lambda \texttt{t} \rightarrow \texttt{f} \rightarrow \texttt{t}))}}$$

$$\boxed{(((\lambda \texttt{t} \rightarrow \texttt{f} \rightarrow \texttt{t})\ (\lambda \texttt{t} \rightarrow \texttt{f} \rightarrow \texttt{t}))\ (\lambda \texttt{t} \rightarrow \texttt{f} \rightarrow \texttt{t}))}$$

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$$\boxed{\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}$$

Booleans

$$\boxed{(((\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t})\ (\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t}))\ (\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t}))}$$

$$\boxed{(\underline{((\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t})\ (\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t}))}\ (\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t}))}$$

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$$\boxed{(\underline{((\lambda t \rightarrow f \rightarrow t)\ (\lambda t \rightarrow f \rightarrow t))}\ (\lambda t \rightarrow f \rightarrow t))}$$

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$$\texttt{(}\underline{\texttt{((}\lambda\texttt{t}\to\texttt{f}\to\texttt{t)}\ \texttt{(}\lambda\texttt{t}\to\texttt{f}\to\texttt{t))}}\ \texttt{(}\lambda\texttt{t}\to\texttt{f}\to\texttt{t))}$$

$$\texttt{((}\lambda\texttt{f}\to\texttt{t}\to\texttt{f}\to\texttt{t)}\ \texttt{(}\lambda\texttt{t}\to\texttt{f}\to\texttt{t))}$$

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$$\boxed{((\lambda f \rightarrow t \rightarrow f \rightarrow t)\ \ (\lambda t \rightarrow f \rightarrow t))}$$

Booleans

$((\lambda f \rightarrow t \rightarrow f \rightarrow t)\ \ (\lambda t \rightarrow f \rightarrow t))$

$\underline{((\lambda f \rightarrow t \rightarrow f \rightarrow t)\ \ (\lambda t \rightarrow f \rightarrow t))}$

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$\underline{\texttt{((}\lambda\texttt{f}\rightarrow\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)}\ \ \texttt{(}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}$

Booleans

$\underline{((\lambda f \rightarrow t \rightarrow f \rightarrow t)\ (\lambda t \rightarrow f \rightarrow t))}$

$(\lambda t \rightarrow f \rightarrow t)$

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$$\boxed{(\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t})}$$

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$(\lambda t \rightarrow f \rightarrow t)$

$\underline{(\lambda t \rightarrow f \rightarrow t)}$

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$$\boxed{\underline{(\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t})}}$$

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$\underline{(\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t})}$

TRUE

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It works, but it is probably only because of black magic.

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It works, but it is probably only because of black magic.

Or is it? Let's see if we can get lucky again...

OR

- The disjunction[a] of two terms is a function that takes as input two booleans and returns a boolean
- Like with conjunction, remember that the two input booleans can be applied to one another
- Given two booleans `a` and `b`, their disjunction is `true` if `a` was `true`, or `b` otherwise
- $\boxed{\texttt{(}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a a) b))}}$

[a] OR, or $\vee$

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OR

Let us begin to with $\texttt{TRUE} \vee \texttt{TRUE} \rightarrow_\beta \texttt{TRUE}$

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$$\boxed{(\mathtt{TRUE}\ \vee\ \mathtt{TRUE})}$$

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```
(TRUE ∨ TRUE)
```

```
((∨ TRUE) TRUE)
```

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$$\boxed{((\underline{\vee}\ \mathtt{TRUE})\ \mathtt{TRUE})}$$

Booleans

$((\underline{\vee}\ \texttt{TRUE})\ \texttt{TRUE})$

$(((\lambda \texttt{a} \rightarrow \texttt{b} \rightarrow ((\texttt{a}\ \texttt{a})\ \texttt{b}))\ \texttt{TRUE})\ \texttt{TRUE})$

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```
(((λa→b→((a a) b)) TRUE) TRUE)
```

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```
(((λa→b→((a a) b)) TRUE) TRUE)
```

```
(((λa→b→((a a) b)) TRUE) TRUE)
```

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```
(((λa→b→((a a) b)) TRUE) TRUE)
```

Booleans

$$(((\lambda a \rightarrow b \rightarrow ((a\ a)\ b))\ \underline{TRUE})\ TRUE)$$

$$(((\lambda a \rightarrow b \rightarrow ((a\ a)\ b))\ (\lambda t \rightarrow f \rightarrow t))\ TRUE)$$

Booleans


```
(((λa→b→((a a) b)) (λt→f→t)) TRUE)
```

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$\boxed{\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a a) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)) TRUE)}}$

$\boxed{\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a a) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}\ \underline{\texttt{TRUE}}\texttt{)}}$

Booleans

$$\boxed{(((\lambda \mathtt{a} \rightarrow \mathtt{b} \rightarrow ((\mathtt{a}\ \ \mathtt{a})\ \ \mathtt{b}))\ \ (\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t}))\ \ \underline{\mathtt{TRUE}})}$$

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```
(((λa→b→((a a) b)) (λt→f→t)) TRUE)
```

```
(((λa→b→((a a) b)) (λt→f→t)) (λt→f→t))
```

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```
(((λa→b→((a a) b)) (λt→f→t)) (λt→f→t))
```

Booleans

$$\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a a) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}$$

$$\texttt{(}\underline{\texttt{((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a a) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}\texttt{ (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}$$

Booleans

$$(\underline{((\lambda a \to b \to ((a\ a)\ b))\ (\lambda t \to f \to t))}\ (\lambda t \to f \to t))$$

Booleans

$$\boxed{\texttt{(}\underline{\texttt{((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a a) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}\texttt{ (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}$$

$$\boxed{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}$$

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```
((λb→(((λt→f→t) (λt→f→t)) b)) (λt→f→t))
```

Booleans

```
((λb→(((λt→f→t) (λt→f→t)) b)) (λt→f→t))
```

```
((λb→(((λt→f→t) (λt→f→t)) b)) (λt→f→t))
```

Booleans

$\boxed{\underline{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{((((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)}\ \texttt{(}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}\ \texttt{b))}\ \texttt{(}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}}$

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$\underline{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{((((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}$

$\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}$

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$$\boxed{(((\lambda t \rightarrow f \rightarrow t)\ (\lambda t \rightarrow f \rightarrow t))\ (\lambda t \rightarrow f \rightarrow t))}$$

Booleans

$\boxed{\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)}\ \texttt{(}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}\ \texttt{(}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}$

$\boxed{\texttt{(}\underline{\texttt{((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)}\ \texttt{(}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}\ \texttt{(}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}$

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$$\boxed{(\underline{((\lambda t \to f \to t)\ (\lambda t \to f \to t))}\ (\lambda t \to f \to t))}$$

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$$\boxed{\texttt{(}\underline{\texttt{((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)}\ \ \texttt{(}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}\ \ \texttt{(}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}$$

$$\boxed{\texttt{((}\lambda\texttt{f}\rightarrow\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)}\ \ \texttt{(}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}$$

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$$((\lambda f \rightarrow t \rightarrow f \rightarrow t)\ \ (\lambda t \rightarrow f \rightarrow t))$$

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$$((\lambda f \rightarrow t \rightarrow f \rightarrow t)\ \ (\lambda t \rightarrow f \rightarrow t))$$

$$\underline{((\lambda f \rightarrow t \rightarrow f \rightarrow t)\ \ (\lambda t \rightarrow f \rightarrow t))}$$

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$$\boxed{\underline{((\lambda \texttt{f} \rightarrow \texttt{t} \rightarrow \texttt{f} \rightarrow \texttt{t})\ (\lambda \texttt{t} \rightarrow \texttt{f} \rightarrow \texttt{t}))}}$$

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$\underline{\texttt{((}\lambda\texttt{f}\rightarrow\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}$

$\texttt{(}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)}$

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$$\boxed{(\lambda t \rightarrow f \rightarrow t)}$$

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$$\boxed{(\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t})}$$

$$\boxed{\underline{(\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t})}}$$

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$\underline{(\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t})}$

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if-then-else

- The conditional operator `if-then-else` chooses one of two parameters based on the value of the input condition
- Given a boolean `c` and two values `t` and `e`, the result is `t` if `c` was `true`, or `e` otherwise
- Since `c` is a boolean, it already performs this choice!
-
```
(λc→t→e→((c t) e))
```

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`if-then-else`

Let us try with `if TRUE` $\vee$ `FALSE then A else B` $\rightarrow_\beta$ `A`

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```
if (TRUE ∨ FALSE) then A else B
```

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```
if (TRUE ∨ FALSE) then A else B
```

```
(((if-then-else (TRUE ∨ FALSE)) A) B)
```

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$$\boxed{(((\underline{\texttt{if-then-else}}\ (\texttt{TRUE} \lor \texttt{FALSE}))\ \texttt{A})\ \texttt{B})}$$

Booleans

$$\texttt{(((}\underline{\texttt{if-then-else}}\ \texttt{(TRUE}\ \vee\ \texttt{FALSE))}\ \texttt{A)}\ \texttt{B)}$$

$$\texttt{((((}\lambda\texttt{c}\rightarrow\texttt{t}\rightarrow\texttt{e}\rightarrow\texttt{((c t) e))}\ \texttt{(TRUE}\ \vee\ \texttt{FALSE))}\ \texttt{A)}\ \texttt{B)}$$

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$$\boxed{\texttt{((((}\lambda\texttt{c}\rightarrow\texttt{t}\rightarrow\texttt{e}\rightarrow\texttt{((c t) e)) (TRUE} \vee \texttt{FALSE)) A) B)}}$$

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```
((((λc→t→e→((c t) e)) (TRUE ∨ FALSE)) A) B)
```

```
((((λc→t→e→((c t) e)) ((∨ TRUE) FALSE)) A) B
)
```

Booleans

$$\boxed{((((\lambda c \rightarrow t \rightarrow e \rightarrow ((c\ \ t)\ \ e))\ \ ((\underline{\vee}\ \ \mathtt{TRUE})\ \ \mathtt{FALSE}))\ \ \mathtt{A})\ \ \mathtt{B})}$$

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$\boxed{\texttt{((((}\lambda\texttt{c}\rightarrow\texttt{t}\rightarrow\texttt{e}\rightarrow\texttt{((c t) e)) ((}\underline{\vee}\texttt{ TRUE) FALSE)) A) B)}}$

$\boxed{\texttt{((((}\lambda\texttt{c}\rightarrow\texttt{t}\rightarrow\texttt{e}\rightarrow\texttt{((c t) e)) (((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a a) b)) TRUE) FALSE)) A) B)}}$

Booleans

```
((((λc→t→e→((c t) e)) (((λa→b→((a a) b))
    TRUE) FALSE)) A) B)
```

Booleans

```
((((λc→t→e→((c t) e)) (((λa→b→((a a) b))
    TRUE) FALSE)) A) B)
```

```
((((λc→t→e→((c t) e)) (((λa→b→((a a) b))
    TRUE) FALSE)) A) B)
```

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```
((((λc→t→e→((c  t)  e))  (((λa→b→((a  a)  b))
    TRUE)  FALSE))  A)  B)
```

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```
((((λc→t→e→((c t) e)) (((λa→b→((a a) b))
    TRUE) FALSE)) A) B)
```

```
((((λc→t→e→((c t) e)) (((λa→b→((a a) b)) (λ
    t→f→t)) FALSE)) A) B)
```

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```
((((λc→t→e→((c t) e)) (((λa→b→((a a) b)) (λ
    t→f→t)) FALSE)) A) B)
```

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```
((((λc→t→e→((c t) e)) (((λa→b→((a a) b)) (λ
    t→f→t)) FALSE)) A) B)
```

```
((((λc→t→e→((c t) e)) (((λa→b→((a a) b)) (λ
    t→f→t)) FALSE)) A) B)
```

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```
((((λc→t→e→((c t) e)) (((λa→b→((a a) b)) (λ
    t→f→t)) FALSE)) A) B)
```

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```
((((λc→t→e→((c t) e)) (((λa→b→((a a) b)) (λ
    t→f→t)) FALSE)) A) B)
```

```
((((λc→t→e→((c t) e)) (((λa→b→((a a) b)) (λ
    t→f→t)) (λt→f→f))) A) B)
```

Booleans

```
((((λc→t→e→((c t) e)) (((λa→b→((a a) b)) (λ
    t→f→t)) (λt→f→f))) A) B)
```

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```
((((λc→t→e→((c t) e)) (((λa→b→((a a) b)) (λ
    t→f→t)) (λt→f→f))) A) B)
```

```
((((λc→t→e→((c t) e)) (
    ((λa→b→((a a) b)) (λt→f→t)) (λt→f→f))) A) B
    )
```

Booleans

$$
\texttt{((((}\lambda\texttt{c}\rightarrow\texttt{t}\rightarrow\texttt{e}\rightarrow\texttt{((c t) e)) (}\\
\underline{\texttt{((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a a) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}\ \texttt{(}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))) A) B}\\
\texttt{)}
$$

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$$((((\lambda c \rightarrow t \rightarrow e \rightarrow ((c\ \ t)\ \ e))\ \ (\underline{((\lambda a \rightarrow b \rightarrow ((a\ \ a)\ \ b))\ \ (\lambda t \rightarrow f \rightarrow t))}\ \ (\lambda t \rightarrow f \rightarrow f)))\ \ A)\ \ B)$$

$$((((\lambda c \rightarrow t \rightarrow e \rightarrow ((c\ \ t)\ \ e))\ \ ((\lambda b \rightarrow (((\lambda t \rightarrow f \rightarrow t)\ \ (\lambda t \rightarrow f \rightarrow t))\ \ b))\ \ (\lambda t \rightarrow f \rightarrow f)))\ \ A)\ \ B)$$

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```
((((λc→t→e→((c t) e)) ((λb→(((λt→f→t) (λt→
    f→t)) b)) (λt→f→f))) A) B)
```

Booleans

```
((((λc→t→e→((c t) e)) ((λb→(((λt→f→t) (λt→
    f→t)) b)) (λt→f→f))) A) B)
```

```
((((λc→t→e→((c t) e)) ((λb→(((λt→f→t) (λt→
    f→t)) b)) (λt→f→f))) A) B)
```

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```
((((λc→t→e→((c t) e)) ((λb→(((λt→f→t) (λt→
    f→t)) b)) (λt→f→f))) A) B)
```

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```
((((λc→t→e→((c t) e)) ((λb→(((λt→f→t) (λt→
    f→t)) b)) (λt→f→f))) A) B)
```

```
((((λc→t→e→((c t) e))
    ((λb→(((λt→f→t) (λt→f→t)) b)) (λt→f→f))) A)
     B)
```

Booleans

```
((((λc→t→e→((c t) e))
    ((λb→(((λt→f→t) (λt→f→t)) b)) (λt→f→f))) A)
    B)
```

Booleans

```
((((λc→t→e→((c t) e))
    ((λb→(((λt→f→t) (λt→f→t)) b)) (λt→f→f))) A)
     B)
```

```
(((λt→e→(((((λb→(((λt→f→t) (λt→f→t)) b)) (λ
    t→f→f)) t) e)) A) B)
```

Booleans

$$(((\lambda t \rightarrow e \rightarrow ((((\lambda b \rightarrow (((\lambda t \rightarrow f \rightarrow t)\ (\lambda t \rightarrow f \rightarrow t))\ b))\ (\lambda t \rightarrow f \rightarrow f))\ t)\ e))\ A)\ B)$$

Booleans

```
(((λt→e→((((λb→(((λt→f→t) (λt→f→t)) b)) (λ
    t→f→f)) t) e)) A) B)
```

```
((
    (λt→e→((((λb→(((λt→f→t) (λt→f→t)) b)) (λt→f→f)
    A) B)
```

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```
((
    (λt→e→((((λb→(((λt→f→t) (λt→f→t)) b)) (λt→f→f)
    A) B)
```

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```
((
    (λt→e→((((λb→(((λt→f→t) (λt→f→t)) b)) (λt→f→f)
    A) B)
```

```
((λe→((((λb→(((λt→f→t) (λt→f→t)) b)) (λt→f
    →f)) A) e)) B)
```

Booleans

$$((\lambda e \rightarrow ((((\lambda b \rightarrow (((\lambda t \rightarrow f \rightarrow t)\ (\lambda t \rightarrow f \rightarrow t))\ b))\ (\lambda t \rightarrow f \rightarrow f))\ A)\ e))\ B)$$

Booleans

```
((λe→((((λb→(((λt→f→t) (λt→f→t)) b)) (λt→f
   →f)) A) e)) B)
```

```
((λe→((((λb→(((λt→f→t) (λt→f→t)) b)) (λt→f→f)) A) e
```

Booleans

$\underline{((\lambda\texttt{e}\rightarrow((((\lambda\texttt{b}\rightarrow(((\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t})\ (\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t}))\ \texttt{b}))\ (\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f}))\ \texttt{A})\ \texttt{e}}$

Booleans

```
((λe→((((λb→(((λt→f→t) (λt→f→t)) b)) (λt→f→f)) A) e
```

```
((((λb→(((λt→f→t) (λt→f→t)) b)) (λt→f→f))
    A) B)
```

Booleans

```
((((λb→(((λt→f→t) (λt→f→t)) b)) (λt→f→f))
   A) B)
```

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$$\texttt{((((}\lambda\texttt{b}\rightarrow\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}\ \texttt{A) B)}$$

$$\texttt{((}\underline{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}\texttt{ A) B)}$$

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$$\boxed{\texttt{((}\underline{\texttt{((}\lambda\texttt{b}\to\texttt{(((}\lambda\texttt{t}\to\texttt{f}\to\texttt{t) (}\lambda\texttt{t}\to\texttt{f}\to\texttt{t)) b)) (}\lambda\texttt{t}\to\texttt{f}\to\texttt{f))}}\texttt{ A) B)}}$$

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$$\boxed{((\underline{((\lambda \mathtt{b} \to (((\lambda \mathtt{t} \to \mathtt{f} \to \mathtt{t})\ (\lambda \mathtt{t} \to \mathtt{f} \to \mathtt{t}))\ \mathtt{b}))\ (\lambda \mathtt{t} \to \mathtt{f} \to \mathtt{f}))}\ \mathtt{A})\ \mathtt{B})}$$

$$\boxed{(((((\lambda \mathtt{t} \to \mathtt{f} \to \mathtt{t})\ (\lambda \mathtt{t} \to \mathtt{f} \to \mathtt{t}))\ (\lambda \mathtt{t} \to \mathtt{f} \to \mathtt{f}))\ \mathtt{A})\ \mathtt{B})}$$

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$$\boxed{(((((\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t})\ (\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t}))\ (\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{f}))\ \mathtt{A})\ \mathtt{B})}$$

Booleans

$$(((((\lambda t \to f \to t)\ (\lambda t \to f \to t))\ (\lambda t \to f \to f))\ A)\ B)$$

$$(((\underline{((\lambda t \to f \to t)\ (\lambda t \to f \to t))}\ (\lambda t \to f \to f))\ A)\ B)$$

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$$\boxed{\texttt{(((}\underline{\texttt{((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}\texttt{ (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f)) A) B)}}$$

Booleans

$$\boxed{(((\underline{((\lambda t \rightarrow f \rightarrow t)\ (\lambda t \rightarrow f \rightarrow t))}\ \ (\lambda t \rightarrow f \rightarrow f))\ \ A)\ \ B)}$$

$$\boxed{((((\lambda f \rightarrow t \rightarrow f \rightarrow t)\ \ (\lambda t \rightarrow f \rightarrow f))\ \ A)\ \ B)}$$

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```
((((λf→t→f→t) (λt→f→f)) A) B)
```

Booleans

$$((((\lambda \mathtt{f} \to \mathtt{t} \to \mathtt{f} \to \mathtt{t})\ (\lambda \mathtt{t} \to \mathtt{f} \to \mathtt{f}))\ \mathtt{A})\ \mathtt{B})$$

$$((\underline{((\lambda \mathtt{f} \to \mathtt{t} \to \mathtt{f} \to \mathtt{t})\ (\lambda \mathtt{t} \to \mathtt{f} \to \mathtt{f}))}\ \mathtt{A})\ \mathtt{B})$$

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$$((\underline{((\lambda f \to t \to f \to t)\ (\lambda t \to f \to f))}\ A)\ B)$$

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$$\texttt{(((}\underline{\texttt{(}\lambda\texttt{f}\rightarrow\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}\texttt{ A) B)}$$

$$\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t) A) B)}$$

Booleans


```
(((λt→f→t) A) B)
```

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$$(((\lambda \mathtt{t} \to \mathtt{f} \to \mathtt{t})\ \mathtt{A})\ \mathtt{B})$$

$$(\underline{((\lambda \mathtt{t} \to \mathtt{f} \to \mathtt{t})\ \mathtt{A})}\ \mathtt{B})$$

Booleans

$$\boxed{(\underline{((\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t})\ \mathtt{A})}\ \mathtt{B})}$$

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$(\underline{((\lambda t \rightarrow f \rightarrow t)\ A)}\ B)$

$((\lambda f \rightarrow A)\ B)$

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$$((\lambda f \rightarrow A)\ \ B)$$

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```
((λf→A)  B)
```

```
((λf→A)  B)
```

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$\underline{((\lambda f \rightarrow A)\ \ B)}$

Booleans

$\underline{\texttt{((}\lambda\texttt{f}\rightarrow\texttt{A) B)}}$

$\texttt{A}$

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Idea

- Natural numbers such as 3 and 0 must be defined so as to identify themselves
- Their identity is determined by how many times they perform an action
- The only action we have available is applying a function to a term

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Idea

- We will use unary numbers
- A number is defined by how many times it applies a function to a given term
- Zero applications are also possible, in this case we default to the given term

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0, 1, etc.

A number is defined as an applicator of a term identifying as successor to another term identifying as zero[a]

[a] first and second arguments by arbitrary convention

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0 will thus look like

```
(λs→z→z)
```

1 will look like

```
(λs→z→(s z))
```

7 will look like

```
(λs→z→(s (s (s (s (s (s (s z))))))))
```

etc.

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Addition

- Adding numbers is a function that takes as input two numbers (say `m` and `n`), and returns a number
- The first number applies its first parameter `m` times to its second parameter
- The second number applies its first parameter `n` times to its second parameter
- We can use the second number as the second parameter to the first, therefore obtaining something that applies `m+n` times
-
```
(λm→n→s→z→((m s) ((n s) z)))
```

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Addition

Let us try it out to $2 + 1 \rightarrow_\beta 3$

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```
(2 + 1)
```

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$$(2\ +\ 1)$$

$$((\underline{+}\ 2)\ 1)$$

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```
((+ 2) 1)
```

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```
((+ 2) 1)
```

```
(((λm→n→s→z→((m s) ((n s) z))) 2) 1)
```

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```
(((λm→n→s→z→((m s) ((n s) z))) 2) 1)
```

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```
(((λm→n→s→z→((m s) ((n s) z))) 2) 1)
```

$$\texttt{(((}\lambda\texttt{m}\rightarrow\texttt{n}\rightarrow\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{((m s) ((n s) z))) }\underline{\texttt{2}}\texttt{) 1)}$$

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$$\texttt{(((}\lambda\texttt{m}\rightarrow\texttt{n}\rightarrow\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{((m s) ((n s) z))) }\underline{\texttt{2}}\texttt{) 1)}$$

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```
(((λm→n→s→z→((m s) ((n s) z))) 2) 1)
```

```
(((λm→n→s→z→((m s) ((n s) z))) (λs→z→(s (s
    z)))) 1)
```

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```
(((λm→n→s→z→((m s) ((n s) z))) (λs→z→(s (s
    z)))) 1)
```

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```
(((λm→n→s→z→((m s) ((n s) z))) (λs→z→(s (s
     z)))) 1)
```

$$\texttt{(((}\lambda\texttt{m}\to\texttt{n}\to\texttt{s}\to\texttt{z}\to\texttt{((m s) ((n s) z))) (}\lambda\texttt{s}\to\texttt{z}\to\texttt{(s (s z)))) }\underline{\texttt{1}}\texttt{)}$$

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$$\texttt{(((}\lambda\texttt{m}\rightarrow\texttt{n}\rightarrow\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{((m s) ((n s) z))) (}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{(s (s z)))) }\underline{\texttt{1}}\texttt{)}$$

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```
(((λm→n→s→z→((m s) ((n s) z))) (λs→z→(s (s
    z)))) 1)
```

```
(((λm→n→s→z→((m s) ((n s) z))) (λs→z→(s (s
    z)))) (λs→z→(s z)))
```

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$$\boxed{(((\lambda \mathtt{m} \rightarrow \mathtt{n} \rightarrow \mathtt{s} \rightarrow \mathtt{z} \rightarrow ((\mathtt{m}\ \mathtt{s})\ ((\mathtt{n}\ \mathtt{s})\ \mathtt{z})))\ (\lambda \mathtt{s} \rightarrow \mathtt{z} \rightarrow (\mathtt{s}\ (\mathtt{s}\ \mathtt{z}))))\ (\lambda \mathtt{s} \rightarrow \mathtt{z} \rightarrow (\mathtt{s}\ \mathtt{z})))}$$

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$$\texttt{(((}\lambda\texttt{m}\rightarrow\texttt{n}\rightarrow\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{((m s) ((n s) z))) (}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{(s (s z)))) (}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{(s z)))}$$

$$\texttt{(}\underline{\texttt{((}\lambda\texttt{m}\rightarrow\texttt{n}\rightarrow\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{((m s) ((n s) z))) (}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{(s (s z))))}}\texttt{ (}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{(s z)))}$$

Natural numbers

$$\frac{(((\lambda m \rightarrow n \rightarrow s \rightarrow z \rightarrow ((m\ s)\ ((n\ s)\ z)))\ (\lambda s \rightarrow z \rightarrow (s\ (s\ z))))}{(\lambda s \rightarrow z \rightarrow (s\ z)))}$$

Natural numbers

$$(\underline{((\lambda m \to n \to s \to z \to ((m\ s)\ ((n\ s)\ z)))\ (\lambda s \to z \to (s\ (s\ z))))}\ (\lambda s \to z \to (s\ z)))$$

$$((\lambda n \to s \to z \to (((\lambda s \to z \to (s\ (s\ z)))\ s)\ ((n\ s)\ z)))\ (\lambda s \to z \to (s\ z)))$$

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```
((λn→s→z→(((λs→z→(s (s z))) s) ((n s) z)))
    (λs→z→(s z)))
```

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$$((\lambda n \rightarrow s \rightarrow z \rightarrow (((\lambda s \rightarrow z \rightarrow (s\ (s\ z)))\ s)\ ((n\ s)\ z)))\ (\lambda s \rightarrow z \rightarrow (s\ z)))$$

$$(\underline{(\lambda n \rightarrow s \rightarrow z \rightarrow (((\lambda s \rightarrow z \rightarrow (s\ (s\ z)))\ s)\ ((n\ s)\ z)))}\ \underline{(\lambda s \rightarrow z \rightarrow (s\ z))})$$

Natural numbers

$$\texttt{(}\underline{\texttt{(}\lambda\texttt{n}\to\texttt{s}\to\texttt{z}\to\texttt{(((}\lambda\texttt{s}\to\texttt{z}\to\texttt{(s (s z))) s) ((n s) z)))}}\ \underline{\texttt{(}\lambda\texttt{s}\to\texttt{z}\to\texttt{(s z))}}\texttt{)}$$

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$$\texttt{(}\underline{\texttt{(}\lambda\texttt{n}\rightarrow\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{(((}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{(s (s z))) s) ((n s) z)))}}\ \underline{\texttt{(}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{(s z))}}\texttt{)}$$

$$\texttt{(}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{(((}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{(s (s z))) s) (((}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{(s z)) s) z)))}$$

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```
(λs→z→(((λs→z→(s (s z))) s) (((λs→z→(s z))
     s) z)))
```

Natural numbers

```
(λs→z→(((λs→z→(s (s z))) s) (((λs→z→(s z))
    s) z)))
```

```
(λs→z→(((λs→z→(s (s z))) s) (((λs→z→(s z)) s)
    z)))
```

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$$(\lambda \texttt{s} \rightarrow \texttt{z} \rightarrow (\underline{((\lambda \texttt{s} \rightarrow \texttt{z} \rightarrow (\texttt{s}\ (\texttt{s}\ \texttt{z})))\ \texttt{s})}\ (((\lambda \texttt{s} \rightarrow \texttt{z} \rightarrow (\texttt{s}\ \texttt{z}))\ \texttt{s})\ \texttt{z})))$$

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```
(λs→z→(((λs→z→(s (s z))) s) (((λs→z→(s z)) s)
    z)))
```

```
(λs→z→((λz→(s (s z))) (((λs→z→(s z)) s) z))
    )
```

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```
(λs→z→((λz→(s (s z))) (((λs→z→(s z)) s) z))
    )
```

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$$(\lambda s \to z \to ((\lambda z \to (s\ (s\ z)))\ (((\lambda s \to z \to (s\ z))\ s)\ z)))$$

$$(\lambda s \to z \to ((\lambda z \to (s\ (s\ z)))\ (\underline{((\lambda s \to z \to (s\ z))\ s)}\ z)))$$

Natural numbers

$$\boxed{\texttt{(}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{((}\lambda\texttt{z}\rightarrow\texttt{(s (s z))) (}\underline{\texttt{((}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{(s z)) s)}}\texttt{ z)))}}$$

Natural numbers

$$\boxed{(\lambda s \rightarrow z \rightarrow ((\lambda z \rightarrow (s\ \ (s\ \ z)))\ \ (\underline{((\lambda s \rightarrow z \rightarrow (s\ \ z))\ \ s)}\ \ z)))}$$

$$\boxed{(\lambda s \rightarrow z \rightarrow ((\lambda z \rightarrow (s\ \ (s\ \ z)))\ \ ((\lambda z \rightarrow (s\ \ z))\ \ z)))}$$

Natural numbers

$$\boxed{\texttt{(}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{((}\lambda\texttt{z}\rightarrow\texttt{(s (s z))) ((}\lambda\texttt{z}\rightarrow\texttt{(s z)) z)))}}$$

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$$\texttt{(}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{((}\lambda\texttt{z}\rightarrow\texttt{(s (s z))) ((}\lambda\texttt{z}\rightarrow\texttt{(s z)) z)))}$$

$$\texttt{(}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{((}\lambda\texttt{z}\rightarrow\texttt{(s (s z))) }\underline{\texttt{((}\lambda\texttt{z}\rightarrow\texttt{(s z)) z)}}\texttt{))}$$

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$$\boxed{\texttt{(}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{((}\lambda\texttt{z}\rightarrow\texttt{(s (s z))) }\underline{\texttt{((}\lambda\texttt{z}\rightarrow\texttt{(s z)) z)}}\texttt{))}}$$

Natural numbers

$$\boxed{(\lambda s \to z \to ((\lambda z \to (s\ (s\ z)))\ \underline{((\lambda z \to (s\ z))\ z)}))}$$

$$\boxed{(\lambda s \to z \to ((\lambda z \to (s\ (s\ z)))\ (s\ z)))}$$

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```
(λs→z→((λz→(s (s z))) (s z)))
```

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```
(λs→z→((λz→(s (s z))) (s z)))
```

```
(λs→z→((λz→(s (s z))) (s z)))
```

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$$\boxed{\texttt{(}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{((}\lambda\texttt{z}\rightarrow\texttt{(s (s z))) (s z)))}}$$

Natural numbers

$$(\lambda s \rightarrow z \rightarrow ((\lambda z \rightarrow (s\ (s\ z)))\ (s\ z)))$$

$$(\lambda s \rightarrow z \rightarrow \underline{((\lambda z \rightarrow (s\ (s\ z)))\ (s\ z))})$$

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$$\boxed{\texttt{(}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\underline{\texttt{((}\lambda\texttt{z}\rightarrow\texttt{(s (s z))) (s z))}}\texttt{)}}$$

Natural numbers

$$(\lambda s \to z \to \underline{((\lambda z \to (s\ (s\ z)))\ (s\ z))})$$

$$(\lambda s \to z \to (s\ (s\ (s\ z))))$$

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```
(λs→z→(s (s (s z))))
```

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$(\lambda s \rightarrow z \rightarrow (s\ (s\ (s\ z))))$

$\underline{(\lambda s \rightarrow z \rightarrow (s\ (s\ (s\ z))))}$

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$(\lambda s \rightarrow z \rightarrow (s\ (s\ (s\ z))))$

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$(\lambda \mathtt{s} \rightarrow \mathtt{z} \rightarrow (\mathtt{s}\ (\mathtt{s}\ (\mathtt{s}\ \mathtt{z}))))$

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Multiplication

- Multiplying numbers is a function that takes as input two numbers (say `m` and `n`), and returns a number
- The first number applies its first parameter `m` times to its second parameter
- The second number applies its first parameter `n` times to its second parameter
- We can use the second number as the first parameter to the first, therefore obtaining something that applies `n+ m` times, starting from `z`
- $\boxed{(\lambda m \to n \to s \to z \to ((m\ \ (n\ \ s))\ \ z))}$

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Multiplication

Let us try it out to $2 \times 2 \rightarrow_\beta 4$

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$$\boxed{(2 \times 2)}$$

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(2×2)

$((\underline{\times}\ 2)\ 2)$

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$((\underline{\times}\ 2)\ 2)$

Natural numbers

$$((\underline{\times}\ 2)\ 2)$$

$$(((\lambda m \rightarrow n \rightarrow s \rightarrow z \rightarrow ((m\ (n\ s))\ z))\ 2)\ 2)$$

Natural numbers

$$\texttt{(((}\lambda\texttt{m}\rightarrow\texttt{n}\rightarrow\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{((m (n s)) z)) 2) 2)}$$

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```
(((λm→n→s→z→((m (n s)) z)) 2) 2)
```

$$(((\lambda m \rightarrow n \rightarrow s \rightarrow z \rightarrow ((m\ (n\ s))\ z))\ \underline{2})\ 2)$$

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$$\texttt{(((}\lambda\texttt{m}\rightarrow\texttt{n}\rightarrow\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{((m (n s)) z))}\ \underline{\texttt{2}}\texttt{) 2)}$$

Natural numbers


```
(((λm→n→s→z→((m (n s)) z)) 2) 2)
```

```
(((λm→n→s→z→((m (n s)) z)) (λs→z→(s (s z))
    )) 2)
```

Natural numbers

```
(((λm→n→s→z→((m (n s)) z)) (λs→z→(s (s z))
    )) 2)
```

Natural numbers

```
(((λm→n→s→z→((m (n s)) z)) (λs→z→(s (s z))
    )) 2)
```

$$\texttt{(((}\lambda\texttt{m}\rightarrow\texttt{n}\rightarrow\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{((m (n s)) z)) (}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{(s (s z)))) }\underline{\texttt{2}}\texttt{)}$$

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```
(((λm→n→s→z→((m (n s)) z)) (λs→z→(s (s z))
    )) 2)
```

Natural numbers


```
(((λm→n→s→z→((m (n s)) z)) (λs→z→(s (s z))
    )) 2)
```

```
(((λm→n→s→z→((m (n s)) z)) (λs→z→(s (s z))
    )) (λs→z→(s (s z))))
```

Natural numbers


```
(((λm→n→s→z→((m (n s)) z)) (λs→z→(s (s z))
    )) (λs→z→(s (s z))))
```

Natural numbers

```
(((λm→n→s→z→((m (n s)) z)) (λs→z→(s (s z))
    )) (λs→z→(s (s z)))))
```

```
(((λm→n→s→z→((m (n s)) z)) (λs→z→(s (s z)))) (λs
    →z→(s (s z)))))
```

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```
(((λm→n→s→z→((m (n s)) z)) (λs→z→(s (s z)))) (λs
    →z→(s (s z))))
```

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$$\texttt{(}\underline{\texttt{((}\lambda\texttt{m}\rightarrow\texttt{n}\rightarrow\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{((m (n s)) z)) (}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{(s (s z))))}}\texttt{ (}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{(s (s z))))}$$

$$\texttt{((}\lambda\texttt{n}\rightarrow\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{(((}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{(s (s z))) (n s)) z)) (}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{(s (s z))))}$$

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```
((λn→s→z→(((λs→z→(s (s z))) (n s)) z)) (λs
    →z→(s (s z))))
```

Natural numbers


```
((λn→s→z→(((λs→z→(s (s z))) (n s)) z)) (λs
    →z→(s (s z))))
```

$$(\underline{(\lambda n \to s \to z \to (((\lambda s \to z \to (s\ (s\ z)))\ (n\ s))\ z))}\ \underline{(\lambda s \to z \to (s\ (s\ z)))})$$

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$$\texttt{(}\underline{\texttt{(}\lambda\texttt{n}\to\texttt{s}\to\texttt{z}\to\texttt{(((}\lambda\texttt{s}\to\texttt{z}\to\texttt{(s (s z))) (n s)) z))}}\ \underline{\texttt{(}\lambda\texttt{s}\to\texttt{z}\to\texttt{(s (s z)))}}\texttt{)}$$

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```
((λn→s→z→(((λs→z→(s (s z))) (n s)) z))
    (λs→z→(s (s z))))
```

```
(λs→z→(((λs→z→(s (s z))) ((λs→z→(s (s z)))
     s)) z))
```

Natural numbers

$$\boxed{(\lambda s \rightarrow z \rightarrow (((\lambda s \rightarrow z \rightarrow (s\ (s\ z)))\ ((\lambda s \rightarrow z \rightarrow (s\ (s\ z)))\ s))\ z))}$$

Natural numbers

```
(λs→z→(((λs→z→(s (s z))) ((λs→z→(s (s z)))
      s)) z))
```

```
(λs→z→(((λs→z→(s (s z)))
     ((λs→z→(s (s z))) s)) z))
```

Natural numbers

$$
\begin{array}{l}
(\lambda \mathtt{s} \rightarrow \mathtt{z} \rightarrow (((\lambda \mathtt{s} \rightarrow \mathtt{z} \rightarrow (\mathtt{s}\ \ (\mathtt{s}\ \ \mathtt{z})))) \\
\quad \underline{((\lambda \mathtt{s} \rightarrow \mathtt{z} \rightarrow (\mathtt{s}\ \ (\mathtt{s}\ \ \mathtt{z})))\ \ \mathtt{s})})\ \ \mathtt{z}))
\end{array}
$$

Natural numbers


```
(λs→z→(((λs→z→(s (s z)))
    ((λs→z→(s (s z))) s)) z))
```

```
(λs→z→(((λs→z→(s (s z))) (λz→(s (s z)))) z)
    )
```

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```
(λs→z→(((λs→z→(s (s z))) (λz→(s (s z)))) z)
    )
```

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```
(λs→z→(((λs→z→(s (s z))) (λz→(s (s z)))) z)
    )
```

```
(λs→z→(((λs→z→(s (s z))) (λz→(s (s z)))) z)
    )
```

Natural numbers


```
(λs→z→(((λs→z→(s (s z))) (λz→(s (s z)))) z)
  )
```

Natural numbers

$$\texttt{(}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{(((}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{(s (s z))) (}\lambda\texttt{z}\rightarrow\texttt{(s (s z)))) z))}$$

$$\texttt{(}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{(}\underline{\texttt{((}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{(s (s z))) (}\lambda\texttt{z}\rightarrow\texttt{(s (s z))))}}\texttt{ z))}$$

Natural numbers

$$\boxed{(\lambda s \to z \to (\underline{((\lambda s \to z \to (s\ (s\ z)))\ (\lambda z \to (s\ (s\ z))))}\ z))}$$

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```
(λs→z→(((λs→z→(s (s z))) (λz→(s (s z)))) z))
```

```
(λs→z→((λz→((λz→(s (s z))) ((λz→(s (s z)))
    z))) z))
```

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```
(λs→z→((λz→((λz→(s (s z))) ((λz→(s (s z)))
    z))) z))
```

Natural numbers

```
(λs→z→((λz→((λz→(s (s z))) ((λz→(s (s z)))
    z))) z))
```

```
(λs→z→
    ((λz→((λz→(s (s z))) ((λz→(s (s z))) z))) z))
```

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$$(\lambda s \rightarrow z \rightarrow \\ \quad \underline{((\lambda z \rightarrow ((\lambda z \rightarrow (s\ (s\ z)))\ ((\lambda z \rightarrow (s\ (s\ z)))\ z)))\ z)})$$

Natural numbers

```
(λs→z→
   ((λz→((λz→(s (s z))) ((λz→(s (s z))) z))) z))
```

```
(λs→z→((λz→(s (s z))) ((λz→(s (s z))) z)))
```

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$$\boxed{\texttt{(}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{((}\lambda\texttt{z}\rightarrow\texttt{(s (s z))) ((}\lambda\texttt{z}\rightarrow\texttt{(s (s z))) z)))}}$$

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$$\boxed{(\lambda s \to z \to ((\lambda z \to (s\ (s\ z)))\ ((\lambda z \to (s\ (s\ z)))\ z)))}$$

$$\boxed{(\lambda s \to z \to ((\lambda z \to (s\ (s\ z)))\ \underline{((\lambda z \to (s\ (s\ z)))\ z)}))}$$

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$$\boxed{\texttt{(}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{((}\lambda\texttt{z}\rightarrow\texttt{(s (s z))) }\underline{\texttt{((}\lambda\texttt{z}\rightarrow\texttt{(s (s z))) z)}}\texttt{))}}$$

Natural numbers

$$\texttt{(}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{((}\lambda\texttt{z}\rightarrow\texttt{(s (s z))) }\underline{\texttt{((}\lambda\texttt{z}\rightarrow\texttt{(s (s z))) z)}}\texttt{))}$$

$$\texttt{(}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{((}\lambda\texttt{z}\rightarrow\texttt{(s (s z))) (s (s z))))}$$

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```
(λs→z→((λz→(s (s z))) (s (s z))))
```

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$$\boxed{(\lambda s \rightarrow z \rightarrow ((\lambda z \rightarrow (s\ (s\ z)))\ (s\ (s\ z))))}$$

$$\boxed{(\lambda s \rightarrow z \rightarrow ((\lambda z \rightarrow (s\ (s\ z)))\ (s\ (s\ z))))}$$

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```
(λs→z→((λz→(s (s z))) (s (s z))))
```

Natural numbers

$$(\lambda \mathtt{s} \rightarrow \mathtt{z} \rightarrow ((\lambda \mathtt{z} \rightarrow (\mathtt{s}\ (\mathtt{s}\ \mathtt{z})))\ (\mathtt{s}\ (\mathtt{s}\ \mathtt{z}))))$$

$$(\lambda \mathtt{s} \rightarrow \mathtt{z} \rightarrow \underline{((\lambda \mathtt{z} \rightarrow (\mathtt{s}\ (\mathtt{s}\ \mathtt{z})))\ (\mathtt{s}\ (\mathtt{s}\ \mathtt{z})))})$$

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$$(\lambda \mathtt{s} \rightarrow \mathtt{z} \rightarrow \underline{((\lambda \mathtt{z} \rightarrow (\mathtt{s}\ (\mathtt{s}\ \mathtt{z})))\ (\mathtt{s}\ (\mathtt{s}\ \mathtt{z})))})$$

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$(\lambda s \rightarrow z \rightarrow \underline{((\lambda z \rightarrow (s\ (s\ z)))\ (s\ (s\ z)))})$

$(\lambda s \rightarrow z \rightarrow (s\ (s\ (s\ (s\ z)))))$

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```
(λs→z→(s (s (s (s z)))))
```

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```
(λs→z→(s (s (s (s z)))))
```

```
(λs→z→(s (s (s (s z)))))
```

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$\boxed{\underline{(\lambda \mathtt{s} \rightarrow \mathtt{z} \rightarrow (\mathtt{s}\ (\mathtt{s}\ (\mathtt{s}\ (\mathtt{s}\ \mathtt{z})))))}}$

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$\underline{(\lambda s \rightarrow z \rightarrow (s\ (s\ (s\ (s\ z)))))}$

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Zero checking

- We might wish to verify whether or not a number is zero
- We can simply pass the number parameters that fail the check (s) and pass it (z)
- ```
  (λm→((m (λx→FALSE)) TRUE))
  ```
  ```

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Zero checking

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$$\boxed{(2 \ = \ 0)}$$

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```
(2 = 0)
```

```
(0? 2)
```

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```
(0? 2)
```

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```
(0? 2)
```

```
((λm→((m (λx→FALSE)) TRUE)) 2)
```

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```
((λm→((m (λx→FALSE)) TRUE)) 2)
```

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$$\boxed{\texttt{((}\lambda\texttt{m}\rightarrow\texttt{((m (}\lambda\texttt{x}\rightarrow\texttt{FALSE)) TRUE)) 2)}}$$

$$\boxed{\texttt{((}\lambda\texttt{m}\rightarrow\texttt{((m (}\lambda\texttt{x}\rightarrow\texttt{FALSE)) TRUE)) }\underline{\texttt{2}}\texttt{)}}$$

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$$\boxed{\texttt{((}\lambda\texttt{m}\rightarrow\texttt{((m (}\lambda\texttt{x}\rightarrow\texttt{FALSE)) TRUE)) }\underline{2}\texttt{)}}$$

Natural numbers

```
((λm→((m (λx→FALSE)) TRUE)) 2)
```

```
((λm→((m (λx→FALSE)) TRUE)) (λs→z→(s (s z)))
   )
```

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```
((λm→((m (λx→FALSE)) TRUE)) (λs→z→(s (s z)))
  )
```

Natural numbers


```
((λm→((m (λx→FALSE)) TRUE)) (λs→z→(s (s z)))
    )
```

```
((λm→((m (λx→FALSE)) TRUE)) (λs→z→(s (s z))))
```

Natural numbers

$$\boxed{\underline{\texttt{((}\lambda\texttt{m}\rightarrow\texttt{((m (}\lambda\texttt{x}\rightarrow\texttt{FALSE)) TRUE)) (}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{(s (s z))))}}}}$$

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$\underline{\texttt{((}\lambda\texttt{m}\rightarrow\texttt{((m (}\lambda\texttt{x}\rightarrow\texttt{FALSE)) TRUE)) (}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{(s (s z))))}}$

$\texttt{(((}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{(s (s z))) (}\lambda\texttt{x}\rightarrow\texttt{FALSE)) TRUE)}$

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```
(((λs→z→(s (s z))) (λx→FALSE)) TRUE)
```

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$$\texttt{(((}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{(s (s z))) (}\lambda\texttt{x}\rightarrow\texttt{FALSE)) TRUE)}$$

$$\texttt{(}\underline{\texttt{((}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{(s (s z))) (}\lambda\texttt{x}\rightarrow\texttt{FALSE))}}\texttt{ TRUE)}$$

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$$\boxed{\texttt{(}\underline{\texttt{((}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{(s (s z))) (}\lambda\texttt{x}\rightarrow\texttt{FALSE))}}\texttt{ TRUE)}}$$

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$$\texttt{(}\underline{\texttt{((}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{(s (s z))) (}\lambda\texttt{x}\rightarrow\texttt{FALSE))}}\texttt{ TRUE)}$$

$$\texttt{((}\lambda\texttt{z}\rightarrow\texttt{((}\lambda\texttt{x}\rightarrow\texttt{FALSE) ((}\lambda\texttt{x}\rightarrow\texttt{FALSE) z))) TRUE)}$$

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$$\boxed{((\lambda z \rightarrow ((\lambda x \rightarrow \texttt{FALSE})\ ((\lambda x \rightarrow \texttt{FALSE})\ z)))\ \texttt{TRUE})}$$

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```
((λz→((λx→FALSE) ((λx→FALSE) z))) TRUE)
```

```
((λz→((λx→FALSE) ((λx→FALSE) z))) TRUE)
```

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$$\boxed{((\lambda\mathtt{z}\rightarrow((\lambda\mathtt{x}\rightarrow\mathtt{FALSE})\ ((\lambda\mathtt{x}\rightarrow\mathtt{FALSE})\ \mathtt{z})))\ \underline{\mathtt{TRUE}})}$$

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$$\boxed{\texttt{((}\lambda\texttt{z}\rightarrow\texttt{((}\lambda\texttt{x}\rightarrow\texttt{FALSE) ((}\lambda\texttt{x}\rightarrow\texttt{FALSE) z))) }\underline{\texttt{TRUE}}\texttt{)}}$$

$$\boxed{\texttt{((}\lambda\texttt{z}\rightarrow\texttt{((}\lambda\texttt{x}\rightarrow\texttt{FALSE) ((}\lambda\texttt{x}\rightarrow\texttt{FALSE) z))) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}$$

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$$\boxed{\texttt{((}\lambda\texttt{z}\rightarrow\texttt{((}\lambda\texttt{x}\rightarrow\texttt{FALSE) ((}\lambda\texttt{x}\rightarrow\texttt{FALSE) z))) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}$$

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$$((\lambda z \rightarrow ((\lambda x \rightarrow \mathtt{FALSE})\ ((\lambda x \rightarrow \mathtt{FALSE})\ z)))\ (\lambda t \rightarrow f \rightarrow t))$$

$$\underline{((\lambda z \rightarrow ((\lambda x \rightarrow \mathtt{FALSE})\ ((\lambda x \rightarrow \mathtt{FALSE})\ z)))\ (\lambda t \rightarrow f \rightarrow t))}$$

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$$\boxed{\underline{((\lambda \mathtt{z} \rightarrow ((\lambda \mathtt{x} \rightarrow \mathtt{FALSE})\ ((\lambda \mathtt{x} \rightarrow \mathtt{FALSE})\ \mathtt{z})))\ (\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t}))}}$$

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$$\boxed{\underline{((\lambda z \rightarrow ((\lambda x \rightarrow \texttt{FALSE})\ ((\lambda x \rightarrow \texttt{FALSE})\ z)))\ (\lambda t \rightarrow f \rightarrow t))}}$$

$$\boxed{((\lambda x \rightarrow \texttt{FALSE})\ ((\lambda x \rightarrow \texttt{FALSE})\ (\lambda t \rightarrow f \rightarrow t)))}$$

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```
((λx→FALSE) ((λx→FALSE) (λt→f→t)))
```

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```
((λx→FALSE) ((λx→FALSE) (λt→f→t)))
```

$$\texttt{((}\lambda\texttt{x}\rightarrow\texttt{FALSE) }\underline{\texttt{((}\lambda\texttt{x}\rightarrow\texttt{FALSE) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}\texttt{)}$$

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$$\boxed{\texttt{((}\lambda\texttt{x}\rightarrow\texttt{FALSE)}\ \underline{\texttt{((}\lambda\texttt{x}\rightarrow\texttt{FALSE)}\ \texttt{(}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}\texttt{)}}$$

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$$((\lambda x \to \mathtt{FALSE})\ \underline{((\lambda x \to \mathtt{FALSE})\ (\lambda t \to f \to t))})$$

$$((\lambda x \to \mathtt{FALSE})\ \mathtt{FALSE})$$

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```
((λx→FALSE) FALSE)
```

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```
((λx→FALSE) FALSE)
```

```
((λx→FALSE) FALSE)
```

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```
((λx→FALSE) FALSE)
```

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```
((λx→FALSE) FALSE)
```

```
((λx→FALSE) FALSE)
```

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$$\boxed{\underline{((\lambda x \rightarrow \texttt{FALSE})\ \texttt{FALSE})}}$$

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$\underline{((\lambda\texttt{x}\rightarrow\texttt{FALSE})\ \texttt{FALSE})}$

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```
FALSE
```

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```
FALSE
```

```
FALSE
```

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```
FALSE
```

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<u>FALSE</u>

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$(\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{f})$

$\underline{(\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{f})}$

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$\underline{(\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{f})}$

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$\underline{(\lambda t \rightarrow f \rightarrow f)}$

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$$\boxed{(0 \;=\; 0)}$$

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```
(0 = 0)
```

```
(0? 0)
```

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```
(0? 0)
```

Natural numbers


```
(0? 0)
```

```
((λm→((m (λx→FALSE)) TRUE)) 0)
```

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Natural numbers

$$\boxed{\texttt{((}\lambda\texttt{m}\rightarrow\texttt{((m (}\lambda\texttt{x}\rightarrow\texttt{FALSE)) TRUE)) 0)}}$$

Natural numbers

$$\boxed{((\lambda m \rightarrow ((m\ (\lambda x \rightarrow \texttt{FALSE}))\ \texttt{TRUE}))\ \texttt{0})}$$

$$\boxed{((\lambda m \rightarrow ((m\ (\lambda x \rightarrow \texttt{FALSE}))\ \texttt{TRUE}))\ \underline{\texttt{0}})}$$

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$$\boxed{\texttt{((}\lambda\texttt{m}\rightarrow\texttt{((m (}\lambda\texttt{x}\rightarrow\texttt{FALSE)) TRUE)) }\underline{\texttt{0}}\texttt{)}}$$

Natural numbers


```
((λm→((m (λx→FALSE)) TRUE)) 0)
```

```
((λm→((m (λx→FALSE)) TRUE)) (λs→z→z))
```

Natural numbers


```
((λm→((m (λx→FALSE)) TRUE)) (λs→z→z))
```

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$\boxed{\texttt{((}\lambda\texttt{m}\rightarrow\texttt{((m (}\lambda\texttt{x}\rightarrow\texttt{FALSE)) TRUE)) (}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{z))}}$

$\boxed{\underline{\texttt{((}\lambda\texttt{m}\rightarrow\texttt{((m (}\lambda\texttt{x}\rightarrow\texttt{FALSE)) TRUE)) (}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{z))}}}$

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$\underline{\texttt{((}\lambda\texttt{m}\rightarrow\texttt{((m (}\lambda\texttt{x}\rightarrow\texttt{FALSE)) TRUE)) (}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{z))}}$

Natural numbers

$\underline{((\lambda m \rightarrow ((m\ (\lambda x \rightarrow \mathtt{FALSE}))\ \mathtt{TRUE}))\ (\lambda s \rightarrow z \rightarrow z))}$

$(((\lambda s \rightarrow z \rightarrow z)\ (\lambda x \rightarrow \mathtt{FALSE}))\ \mathtt{TRUE})$

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```
(((λs→z→z) (λx→FALSE)) TRUE)
```

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```
(((λs→z→z) (λx→FALSE)) TRUE)
```

$(\underline{((\lambda s \rightarrow z \rightarrow z)\ (\lambda x \rightarrow \mathtt{FALSE}))}\ \mathtt{TRUE})$

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$$\texttt{(}\underline{\texttt{((}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{z)\ (}\lambda\texttt{x}\rightarrow\texttt{FALSE))}}\ \texttt{TRUE)}$$

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$$\texttt{(}\underline{\texttt{((}\lambda\texttt{s}\rightarrow\texttt{z}\rightarrow\texttt{z) (}\lambda\texttt{x}\rightarrow\texttt{FALSE))}}\texttt{ TRUE)}$$

$$\texttt{((}\lambda\texttt{z}\rightarrow\texttt{z) TRUE)}$$

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$$((\lambda z \rightarrow z)\ \ \mathtt{TRUE})$$

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```
((λz→z) TRUE)
```

$((\lambda z \to z)\ \underline{\texttt{TRUE}})$

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$$((\lambda z \rightarrow z)\ \ \underline{\texttt{TRUE}})$$

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`((λz→z) TRUE)`

`((λz→z) (λt→f→t))`

Natural numbers


```
((λz→z) (λt→f→t))
```

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$((\lambda z \rightarrow z)\ \ (\lambda t \rightarrow f \rightarrow t))$

$\underline{((\lambda z \rightarrow z)\ \ (\lambda t \rightarrow f \rightarrow t))}$

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$$\underline{((\lambda z \to z)\ (\lambda t \to f \to t))}$$

Natural numbers

$$\underline{((\lambda z \rightarrow z)\ \ (\lambda t \rightarrow f \rightarrow t))}$$

$$(\lambda t \rightarrow f \rightarrow t)$$

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```
(λt→f→t)
```

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$(\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t})$

$\underline{(\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t})}$

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$\underline{(\lambda t \rightarrow f \rightarrow t)}$

TRUE

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Other arithmetic operators

- Division, subtraction, and all manners of comparison operators can be defined similarly
- The level of detail of the specification can be compared to that of a very high level CPU
- This means that we are, to an extent, programming in a sort of assembly
- This is the reason why the traces have been so verbose so far

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Other arithmetic operators

- We could also define numbers in base two instead of base one
- This would save processing time, but would result in a slighter more complex specification
- We will just ignore these engineering details: we only focus on **what** can be done, not the best way to do it

Conclusion

Conclusion

Recap

- Lambda terms can be used to encode arbitrary basic data types
- The terms are always lambda expression which, when they get parameters passed in, identify themselves somehow
- Identification can be done by applying something (possibly even a given number of times), or returning one of the parameters

Conclusion

Recap

- There are many encodings of data types, but they all behave in the same way by producing the same outputs for the same inputs
- From now on we will start ignoring the reduction steps for simple terms such as 3+3
- We will instead focus on more complex data structures, such as tuples, discriminated unions, and even lists

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```
((FALSE bit1) bit0)
```

Appendix

```
((FALSE bit1) bit0)
```

```
((FALSE bit1) bit0)
```

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```
((FALSE bit1) bit0)
```

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```
((FALSE bit1) bit0)
```

```
(((λt→f→f) bit1) bit0)
```

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$$\boxed{(((\lambda \texttt{t} \rightarrow \texttt{f} \rightarrow \texttt{f})\ \texttt{bit1})\ \texttt{bit0})}$$

Appendix

$$\texttt{(((}\lambda\texttt{t}\to\texttt{f}\to\texttt{f) bit1) bit0)}$$

$$\texttt{(}\underline{\texttt{((}\lambda\texttt{t}\to\texttt{f}\to\texttt{f) bit1)}}\texttt{ bit0)}$$

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$$\boxed{\texttt{(}\underline{\texttt{((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f) bit1)}}\texttt{ bit0)}}$$

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$\texttt{(}\underline{\texttt{((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f) bit1)}}\texttt{ bit0)}$

$\texttt{((}\lambda\texttt{f}\rightarrow\texttt{f) bit0)}$

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```
((λf→f) bit0)
```

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$((\lambda f \rightarrow f)\ \ \texttt{bit0})$

$\underline{((\lambda f \rightarrow f)\ \ \texttt{bit0})}$

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$\underline{\texttt{((}\lambda\texttt{f}\rightarrow\texttt{f) bit0)}}$

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$\underline{\texttt{((}\lambda\texttt{f}\rightarrow\texttt{f) bit0)}}$

`bit0`

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Remaining and derivations

Let us move to $\texttt{TRUE} \wedge \texttt{FALSE} \rightarrow_\beta \texttt{FALSE}$

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```
(TRUE ∧ FALSE)
```

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```
(TRUE ∧ FALSE)
```

```
((∧ TRUE) FALSE)
```

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```
((∧ TRUE) FALSE)
```

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```
((∧ TRUE) FALSE)
```

```
(((λa→b→((a b) a)) TRUE) FALSE)
```

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```
(((λa→b→((a b) a)) TRUE) FALSE)
```

Appendix


```
(((λa→b→((a b) a)) TRUE) FALSE)
```

```
(((λa→b→((a b) a)) TRUE) FALSE)
```

Appendix

$(((\lambda \mathtt{a} \rightarrow \mathtt{b} \rightarrow ((\mathtt{a}\ \ \mathtt{b})\ \ \mathtt{a}))\ \ \underline{\mathtt{TRUE}})\ \ \mathtt{FALSE})$

Appendix

$$\boxed{\texttt{(((}\lambda\texttt{a}\to\texttt{b}\to\texttt{((a b) a)) }\underline{\texttt{TRUE}}\texttt{) FALSE)}}$$

$$\boxed{\texttt{(((}\lambda\texttt{a}\to\texttt{b}\to\texttt{((a b) a)) (}\lambda\texttt{t}\to\texttt{f}\to\texttt{t)) FALSE)}}$$

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```
(((λa→b→((a b) a)) (λt→f→t)) FALSE)
```

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```
(((λa→b→((a b) a)) (λt→f→t)) FALSE)
```

```
(((λa→b→((a b) a)) (λt→f→t)) FALSE)
```

Appendix

$$(((\lambda a \rightarrow b \rightarrow ((a\ b)\ a))\ (\lambda t \rightarrow f \rightarrow t))\ \underline{\mathtt{FALSE}})$$

Appendix

$$\boxed{\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a b) a)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)) }\underline{\texttt{FALSE}}\texttt{)}}$$

$$\boxed{\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a b) a)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$$

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```
(((λa→b→((a b) a)) (λt→f→t)) (λt→f→f))
```

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```
(((λa→b→((a b) a)) (λt→f→t)) (λt→f→f))
```

$$(\underline{((\lambda a \to b \to ((a\ b)\ a))\ (\lambda t \to f \to t))}\ (\lambda t \to f \to f))$$

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$$\boxed{(\underline{((\lambda \mathtt{a} \rightarrow \mathtt{b} \rightarrow ((\mathtt{a}\ \mathtt{b})\ \mathtt{a}))\ (\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t}))}\ (\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{f}))}$$

Appendix

$$\boxed{(\underline{((\lambda a \to b \to ((a\ b)\ a))\ (\lambda t \to f \to t))}\ (\lambda t \to f \to f))}$$

$$\boxed{((\lambda b \to (((\lambda t \to f \to t)\ b)\ (\lambda t \to f \to t)))\ (\lambda t \to f \to f))}$$

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$$\boxed{((\lambda \mathtt{b} \rightarrow (((\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t})\ \mathtt{b})\ (\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t})))\ (\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{f}))}$$

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$$\boxed{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t) b) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$$

$$\boxed{\underline{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t) b) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}}$$

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$$\boxed{\underline{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t) b) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}}$$

Appendix

$$\boxed{\underline{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t) b) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}}$$

$$\boxed{\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}$$

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$$\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)}\ \texttt{(}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}\ \texttt{(}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}$$

Appendix

$$(((\lambda t \rightarrow f \rightarrow t)\ (\lambda t \rightarrow f \rightarrow f))\ (\lambda t \rightarrow f \rightarrow t))$$

$$(\underline{((\lambda t \rightarrow f \rightarrow t)\ (\lambda t \rightarrow f \rightarrow f))}\ (\lambda t \rightarrow f \rightarrow t))$$

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$$\texttt{(}\underline{\texttt{((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}\texttt{ (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}$$

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$$(\underline{((\lambda t \to f \to t)\ (\lambda t \to f \to f))}\ (\lambda t \to f \to t))$$

$$((\lambda f \to t \to f \to f)\ (\lambda t \to f \to t))$$

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$$((\lambda \mathtt{f} \rightarrow \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{f})\ \ (\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t}))$$

Appendix

$$\boxed{((\lambda \mathtt{f} \rightarrow \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{f})\ \ (\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t}))}$$

$$\boxed{\underline{((\lambda \mathtt{f} \rightarrow \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{f})\ \ (\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t}))}}$$

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$\underline{((\lambda \texttt{f} \rightarrow \texttt{t} \rightarrow \texttt{f} \rightarrow \texttt{f})\ (\lambda \texttt{t} \rightarrow \texttt{f} \rightarrow \texttt{t}))}$

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$$\boxed{(\lambda t \rightarrow f \rightarrow f)}$$

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(λt $\rightarrow$ f $\rightarrow$ f)

$\underline{(\lambda t \rightarrow f \rightarrow f)}$

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$\underline{(\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{f})}$

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$\underline{(\lambda t \rightarrow f \rightarrow f)}$

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```
(FALSE ∧ TRUE)
```

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(FALSE ∧ TRUE)
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```
((∧ FALSE) TRUE)
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```
((∧ FALSE) TRUE)
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$$((\underline{\triangle}\ \texttt{FALSE})\ \texttt{TRUE})$$

$$(((\lambda a \rightarrow b \rightarrow ((a\ b)\ a))\ \texttt{FALSE})\ \texttt{TRUE})$$

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```
(((λa→b→((a b) a)) FALSE) TRUE)
```

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```
(((λa→b→((a b) a)) FALSE) TRUE)
```

(((λa→b→((a b) a)) <u>FALSE</u>) TRUE)

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$$\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a b) a))}\ \underline{\texttt{FALSE}}\texttt{) TRUE)}$$

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$\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a b) a))}\ \underline{\texttt{FALSE}}\texttt{) TRUE)}$

$\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a b) a))}\ \texttt{(}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}\ \texttt{TRUE)}$

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```
(((λa→b→((a b) a)) (λt→f→f)) TRUE)
```

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```
(((λa→b→((a b) a)) (λt→f→f)) TRUE)
```

$\boxed{\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a b) a)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f)) }\underline{\texttt{TRUE}}\texttt{)}}$

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$$\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a b) a)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f)) }\underline{\texttt{TRUE}}\texttt{)}$$

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$\boxed{(((\lambda a \rightarrow b \rightarrow ((a\ b)\ a))\ (\lambda t \rightarrow f \rightarrow f))\ \underline{\texttt{TRUE}})}$

$\boxed{(((\lambda a \rightarrow b \rightarrow ((a\ b)\ a))\ (\lambda t \rightarrow f \rightarrow f))\ (\lambda t \rightarrow f \rightarrow t))}$

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$$\boxed{\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a b) a)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}$$

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$$\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a b) a)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}$$

$$\texttt{(}\underline{\texttt{((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a b) a)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}\texttt{ (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}$$

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$$\boxed{(\underline{((\lambda \texttt{a} \to \texttt{b} \to ((\texttt{a}\ \texttt{b})\ \texttt{a}))\ (\lambda \texttt{t} \to \texttt{f} \to \texttt{f}))}\ (\lambda \texttt{t} \to \texttt{f} \to \texttt{t}))}$$

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$$(((\lambda a \rightarrow b \rightarrow ((a\ b)\ a))\ (\lambda t \rightarrow f \rightarrow f))\ (\lambda t \rightarrow f \rightarrow t))$$

$$((\lambda b \rightarrow (((\lambda t \rightarrow f \rightarrow f)\ b)\ (\lambda t \rightarrow f \rightarrow f)))\ (\lambda t \rightarrow f \rightarrow t))$$

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$$\boxed{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{((((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f) b) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}$$

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$$\boxed{((\lambda b \to (((\lambda t \to f \to f)\ b)\ (\lambda t \to f \to f)))\ (\lambda t \to f \to t))}$$

$$\boxed{\underline{((\lambda b \to (((\lambda t \to f \to f)\ b)\ (\lambda t \to f \to f)))\ (\lambda t \to f \to t))}}$$

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$\boxed{\underline{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f) b) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}}$

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$\boxed{\underline{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{((((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f) b) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}}$

$\boxed{\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$

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$$\boxed{(((\lambda t \to f \to f)\ (\lambda t \to f \to t))\ (\lambda t \to f \to f))}$$

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$\boxed{(((\lambda \texttt{t} \rightarrow \texttt{f} \rightarrow \texttt{f})\ (\lambda \texttt{t} \rightarrow \texttt{f} \rightarrow \texttt{t}))\ (\lambda \texttt{t} \rightarrow \texttt{f} \rightarrow \texttt{f}))}$

$\boxed{(\underline{((\lambda \texttt{t} \rightarrow \texttt{f} \rightarrow \texttt{f})\ (\lambda \texttt{t} \rightarrow \texttt{f} \rightarrow \texttt{t}))}\ (\lambda \texttt{t} \rightarrow \texttt{f} \rightarrow \texttt{f}))}$

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$$\boxed{\texttt{(}\underline{\texttt{((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f)}\ \texttt{(}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}\ \texttt{(}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$$

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$$\texttt{(}\underline{\texttt{((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f)}\ \texttt{(}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}\ \texttt{(}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}$$

$$\texttt{((}\lambda\texttt{f}\rightarrow\texttt{f)}\ \texttt{(}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}$$

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```
((λf→f) (λt→f→f))
```

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$$\boxed{((\lambda \mathtt{f} \rightarrow \mathtt{f})\ (\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{f}))}$$

$$\boxed{\underline{((\lambda \mathtt{f} \rightarrow \mathtt{f})\ (\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{f}))}}$$

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$\boxed{\underline{((\lambda f \rightarrow f)\ \ (\lambda t \rightarrow f \rightarrow f))}}$

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$\underline{((\lambda f \rightarrow f)\ (\lambda t \rightarrow f \rightarrow f))}$

$(\lambda t \rightarrow f \rightarrow f)$

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$$\boxed{(\lambda t \to f \to f)}$$

Appendix

$(\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{f})$

$\underline{(\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{f})}$

Appendix

$(\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{f})$

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$\underline{(\lambda t \rightarrow f \rightarrow f)}$

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Let us move to $\texttt{FALSE} \wedge \texttt{FALSE} \rightarrow_\beta \texttt{FALSE}$

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```
(FALSE ∧ FALSE)
```

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```
(FALSE ∧ FALSE)
```

```
((∧ FALSE) FALSE)
```

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((∧ FALSE) FALSE)
```

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$$\texttt{((}\underline{\wedge}\ \texttt{FALSE) FALSE)}$$

$$\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a b) a)) FALSE) FALSE)}$$

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```
(((λa→b→((a b) a)) FALSE) FALSE)
```

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$(((\lambda a \rightarrow b \rightarrow ((a\ b)\ a))\ \text{FALSE})\ \text{FALSE})$

$(((\lambda a \rightarrow b \rightarrow ((a\ b)\ a))\ \underline{\text{FALSE}})\ \text{FALSE})$

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$$\boxed{\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a b) a))}\ \underline{\texttt{FALSE}}\texttt{) FALSE)}}$$

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$$\boxed{(((\lambda a \rightarrow b \rightarrow ((a\ b)\ a))\ \underline{\mathtt{FALSE}})\ \mathtt{FALSE})}$$

$$\boxed{(((\lambda a \rightarrow b \rightarrow ((a\ b)\ a))\ (\lambda t \rightarrow f \rightarrow f))\ \mathtt{FALSE})}$$

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```
(((λa→b→((a b) a)) (λt→f→f)) FALSE)
```

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```
(((λa→b→((a b) a)) (λt→f→f)) FALSE)
```

$\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a b) a)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f)) }\underline{\texttt{FALSE}}\texttt{)}$

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$$(((\lambda a \to b \to ((a\ \ b)\ \ a))\ \ (\lambda t \to f \to f))\ \ \underline{\mathtt{FALSE}})$$

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$\boxed{\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a b) a)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f)) }\underline{\texttt{FALSE}}\texttt{)}}$

$\boxed{\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a b) a)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$

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(((λa→b→((a b) a)) (λt→f→f)) (λt→f→f))
```

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$\boxed{\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a b) a)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$

$\boxed{\texttt{(}\underline{\texttt{((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a b) a)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}\texttt{ (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$

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$$\boxed{\texttt{(}\underline{\texttt{((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a b) a)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}\ \texttt{(}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$$

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$$\texttt{(}\underline{\texttt{((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a b) a)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}\texttt{ (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}$$

$$\texttt{((}\lambda\texttt{b}\rightarrow\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f) b) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}$$

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$\boxed{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f) b) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$

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$$\boxed{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f) b) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$$

$$\boxed{\underline{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f) b) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}}$$

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$$\boxed{\underline{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f) b) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}}$$

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$\underline{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f) b) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$

$\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}$

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$$\boxed{(((\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{f})\ (\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{f}))\ (\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{f}))}$$

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$$\boxed{\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$$

$$\boxed{\texttt{(}\underline{\texttt{((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}\texttt{ (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$$

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$$\boxed{(\underline{((\lambda t \to f \to f)\ (\lambda t \to f \to f))}\ (\lambda t \to f \to f))}$$

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```
((λf→f) (λt→f→f))
```

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$$\boxed{\texttt{((}\lambda\texttt{f}\rightarrow\texttt{f) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$$

$$\boxed{\underline{\texttt{((}\lambda\texttt{f}\rightarrow\texttt{f) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}}$$

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$$\underline{((\lambda f \rightarrow f)\ (\lambda t \rightarrow f \rightarrow f))}$$

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(TRUE ∨ FALSE)
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```
(TRUE ∨ FALSE)
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```
((∨ TRUE) FALSE)
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```
((⊻ TRUE) FALSE)
```

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```
((∨ TRUE) FALSE)
```

```
(((λa→b→((a a) b)) TRUE) FALSE)
```

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$$\boxed{(((\lambda \texttt{a} \rightarrow \texttt{b} \rightarrow ((\texttt{a}\ \ \texttt{a})\ \ \texttt{b}))\ \ \texttt{TRUE})\ \ \texttt{FALSE})}$$

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```
(((λa→b→((a a) b)) TRUE) FALSE)
```

```
(((λa→b→((a a) b)) TRUE) FALSE)
```

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$\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a a) b)) }\underline{\texttt{TRUE}}\texttt{) FALSE)}$

$\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a a) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)) FALSE)}$

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$$\boxed{\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a a) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)) FALSE)}}$$

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$$\boxed{(((\lambda a \rightarrow b \rightarrow ((a\ a)\ b))\ (\lambda t \rightarrow f \rightarrow t))\ \texttt{FALSE})}$$

$$\boxed{(((\lambda a \rightarrow b \rightarrow ((a\ a)\ b))\ (\lambda t \rightarrow f \rightarrow t))\ \underline{\texttt{FALSE}})}$$

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$$\boxed{\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a a) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)) }\underline{\texttt{FALSE}}\texttt{)}}$$

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$\boxed{\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a a) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)) }\underline{\texttt{FALSE}}\texttt{)}}$

$\boxed{\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a a) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$

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$$\boxed{\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a a) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$$

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$$\texttt{(}\underline{\texttt{((}\lambda\texttt{a}\to\texttt{b}\to\texttt{((a a) b)) (}\lambda\texttt{t}\to\texttt{f}\to\texttt{t))}}\texttt{ (}\lambda\texttt{t}\to\texttt{f}\to\texttt{f))}$$

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$$\boxed{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$$

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$$\boxed{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$$

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$$\texttt{((}\lambda\texttt{b}\rightarrow\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}$$

$$\underline{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$$

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$$\underline{((\lambda \texttt{b} \rightarrow (((\lambda \texttt{t} \rightarrow \texttt{f} \rightarrow \texttt{t})\ (\lambda \texttt{t} \rightarrow \texttt{f} \rightarrow \texttt{t}))\ \texttt{b}))\ (\lambda \texttt{t} \rightarrow \texttt{f} \rightarrow \texttt{f}))}$$

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$\underline{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$

$\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}$

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$$(\underline{((\lambda t \rightarrow f \rightarrow t)\ (\lambda t \rightarrow f \rightarrow t))}\ (\lambda t \rightarrow f \rightarrow f))$$

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$$\boxed{\texttt{((}\lambda\texttt{f}\rightarrow\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)}\ \texttt{(}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$$

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$$\boxed{((\lambda \texttt{f} \rightarrow \texttt{t} \rightarrow \texttt{f} \rightarrow \texttt{t})\ \ (\lambda \texttt{t} \rightarrow \texttt{f} \rightarrow \texttt{f}))}$$

$$\boxed{\underline{((\lambda \texttt{f} \rightarrow \texttt{t} \rightarrow \texttt{f} \rightarrow \texttt{t})\ \ (\lambda \texttt{t} \rightarrow \texttt{f} \rightarrow \texttt{f}))}}$$

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$\underline{((\lambda \texttt{f} \rightarrow \texttt{t} \rightarrow \texttt{f} \rightarrow \texttt{t})\ (\lambda \texttt{t} \rightarrow \texttt{f} \rightarrow \texttt{f}))}$

$(\lambda \texttt{t} \rightarrow \texttt{f} \rightarrow \texttt{t})$

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```
(FALSE ∨ TRUE)
```

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```
(FALSE ∨ TRUE)
```

$$((\underline{\vee}\ \texttt{FALSE})\ \texttt{TRUE})$$

Appendix


```
((∨ FALSE) TRUE)
```

Appendix


```
((∨ FALSE) TRUE)
```

```
(((λa→b→((a a) b)) FALSE) TRUE)
```

Appendix


```
(((λa→b→((a a) b)) FALSE) TRUE)
```

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```
(((λa→b→((a a) b)) FALSE) TRUE)
```

$\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a a) b))}\ \underline{\texttt{FALSE}}\texttt{) TRUE)}$

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$$\boxed{\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a a) b)) }\underline{\texttt{FALSE}}\texttt{) TRUE)}}$$

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```
(((λa→b→((a a) b)) FALSE) TRUE)
```

```
(((λa→b→((a a) b)) (λt→f→f)) TRUE)
```

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```
(((λa→b→((a a) b)) (λt→f→f)) TRUE)
```

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$$(((\lambda a \rightarrow b \rightarrow ((a\ a)\ b))\ (\lambda t \rightarrow f \rightarrow f))\ \mathtt{TRUE})$$

$$(((\lambda a \rightarrow b \rightarrow ((a\ a)\ b))\ (\lambda t \rightarrow f \rightarrow f))\ \underline{\mathtt{TRUE}})$$

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$$\boxed{(((\lambda \texttt{a} \rightarrow \texttt{b} \rightarrow ((\texttt{a}\ \texttt{a})\ \texttt{b}))\ (\lambda \texttt{t} \rightarrow \texttt{f} \rightarrow \texttt{f}))\ \underline{\texttt{TRUE}})}$$

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$$\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a a) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f)) }\underline{\texttt{TRUE}}\texttt{)}$$

$$\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a a) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}$$

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```
(((λa→b→((a a) b)) (λt→f→f)) (λt→f→t))
```

Appendix

$$\boxed{(((\lambda \texttt{a} \to \texttt{b} \to ((\texttt{a}\ \texttt{a})\ \texttt{b}))\ (\lambda \texttt{t} \to \texttt{f} \to \texttt{f}))\ (\lambda \texttt{t} \to \texttt{f} \to \texttt{t}))}$$

$$\boxed{(\underline{((\lambda \texttt{a} \to \texttt{b} \to ((\texttt{a}\ \texttt{a})\ \texttt{b}))\ (\lambda \texttt{t} \to \texttt{f} \to \texttt{f}))}\ (\lambda \texttt{t} \to \texttt{f} \to \texttt{t}))}$$

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$$\boxed{\texttt{(}\underline{\texttt{((}\lambda\texttt{a}\to\texttt{b}\to\texttt{((a a) b)) (}\lambda\texttt{t}\to\texttt{f}\to\texttt{f))}}\ \texttt{(}\lambda\texttt{t}\to\texttt{f}\to\texttt{t))}}$$

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$$\boxed{\texttt{(}\underline{\texttt{((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a a) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}\texttt{ (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}$$

$$\boxed{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f)) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}$$

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$$\boxed{((\lambda b \rightarrow (((\lambda t \rightarrow f \rightarrow f)\ (\lambda t \rightarrow f \rightarrow f))\ b))\ (\lambda t \rightarrow f \rightarrow t))}$$

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$((\lambda b \rightarrow (((\lambda t \rightarrow f \rightarrow f)\ (\lambda t \rightarrow f \rightarrow f))\ b))\ (\lambda t \rightarrow f \rightarrow t))$

$\underline{((\lambda b \rightarrow (((\lambda t \rightarrow f \rightarrow f)\ (\lambda t \rightarrow f \rightarrow f))\ b))\ (\lambda t \rightarrow f \rightarrow t))}$

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$$\boxed{\underline{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f)) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}}$$

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$\underline{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f)) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}$

$\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}$

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$$\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}$$

$$\texttt{(}\underline{\texttt{((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}\texttt{ (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}$$

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$$\boxed{(\underline{((\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{f})\ (\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{f}))}\ (\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{t}))}$$

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$$\texttt{((}\lambda\texttt{f}\rightarrow\texttt{f) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}$$

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$\underline{\texttt{((}\lambda\texttt{f}\rightarrow\texttt{f) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t))}}$

$\texttt{(}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{t)}$

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```
(λt→f→t)
```

Appendix

$\boxed{(\lambda \texttt{t} \rightarrow \texttt{f} \rightarrow \texttt{t})}$

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$\boxed{\texttt{(FALSE} \lor \texttt{FALSE)}}$

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(((λa→b→((a a) b)) FALSE) FALSE)
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(((λa→b→((a a) b)) FALSE) FALSE)
```

$$\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a a) b)) }\underline{\texttt{FALSE}}\texttt{) FALSE)}$$

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$$\boxed{\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a a) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f)) }\underline{\texttt{FALSE}}\texttt{)}}$$

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$$\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a a) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f)) }\underline{\texttt{FALSE}}\texttt{)}$$

$$\texttt{(((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a a) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}$$

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```
(((λa→b→((a a) b)) (λt→f→f)) (λt→f→f))
```

Appendix

$$(((\lambda a \rightarrow b \rightarrow ((a\ a)\ b))\ (\lambda t \rightarrow f \rightarrow f))\ (\lambda t \rightarrow f \rightarrow f))$$

$$(\underline{((\lambda a \rightarrow b \rightarrow ((a\ a)\ b))\ (\lambda t \rightarrow f \rightarrow f))}\ (\lambda t \rightarrow f \rightarrow f))$$

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$$\boxed{\texttt{(}\underline{\texttt{((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a a) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}\ \texttt{(}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$$

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$$\boxed{\texttt{(}\underline{\texttt{((}\lambda\texttt{a}\rightarrow\texttt{b}\rightarrow\texttt{((a a) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}\texttt{ (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$$

$$\boxed{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f)) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$$

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$$\boxed{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f)) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$$

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$$\boxed{((\lambda b\to(((\lambda t\to f\to f)\ (\lambda t\to f\to f))\ b))\ (\lambda t\to f\to f))}$$

$$\boxed{\underline{((\lambda b\to(((\lambda t\to f\to f)\ (\lambda t\to f\to f))\ b))\ (\lambda t\to f\to f))}}$$

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$\boxed{\underline{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f)) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}}$

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$\underline{\texttt{((}\lambda\texttt{b}\rightarrow\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f)) b)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$

$\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}$

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$$\boxed{\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$$

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$$\boxed{\texttt{(((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f)) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$$

$$\boxed{\texttt{(}\underline{\texttt{((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}\texttt{ (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$$

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$$\boxed{\texttt{(}\underline{\texttt{((}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}\ \texttt{(}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$$

Appendix

$$(\underline{((\lambda t \rightarrow f \rightarrow f)\ (\lambda t \rightarrow f \rightarrow f))}\ (\lambda t \rightarrow f \rightarrow f))$$

$$((\lambda f \rightarrow f)\ (\lambda t \rightarrow f \rightarrow f))$$

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$$((\lambda \mathtt{f} \rightarrow \mathtt{f}) \; (\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{f}))$$

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$\boxed{\texttt{((}\lambda\texttt{f}\rightarrow\texttt{f) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$

$\boxed{\underline{\texttt{((}\lambda\texttt{f}\rightarrow\texttt{f) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}}$

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$\underline{\texttt{((}\lambda\texttt{f}\rightarrow\texttt{f) (}\lambda\texttt{t}\rightarrow\texttt{f}\rightarrow\texttt{f))}}$

Appendix

$\underline{((\lambda f \to f)\ (\lambda t \to f \to f))}$

$(\lambda t \to f \to f)$

Appendix

$$\boxed{(\lambda t \rightarrow f \rightarrow f)}$$

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$$(\lambda t \rightarrow f \rightarrow f)$$

$$\underline{(\lambda t \rightarrow f \rightarrow f)}$$

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$\underline{(\lambda t \rightarrow f \rightarrow f)}$

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$\underline{(\lambda \mathtt{t} \rightarrow \mathtt{f} \rightarrow \mathtt{f})}$

`FALSE`

This is it!

The best of luck, and thanks for the attention!