Delta rules

The INFDEV@HR Team

Hogeschool Rotterdam
Rotterdam, Netherlands

Introduction

Introduction

Lecture topics

- Make it pretty: delta rules
- Booleans, boolean logic operators, if-then-else
- Naturals, arithmetic operators, comparison operators

Encoding boolean logic

Encoding boolean logic

Introduction

- We can decide that some specific lambda terms have special meanings
- For example, we could decide that a given lambda term means `TRUE`, another `FALSE`, etc.
- The important thing is that we choose terms that behave as we wish

Encoding boolean logic

As we wish?

- Suppose we define some lambda terms for `TRUE`, `FALSE`, and `AND`
- We expect these terms to reduce[a] following our expectations of boolean logic
- We can use truth tables to encode our expectations

[a] That is, computed according to $\rightarrow_\beta$

Encoding boolean logic

We want to formulate TRUE, FALSE, and AND so that

- $\texttt{TRUE} \wedge \texttt{TRUE} \rightarrow_\beta \texttt{TRUE}$
- $\texttt{TRUE} \wedge \texttt{FALSE} \rightarrow_\beta \texttt{FALSE}$
- $\texttt{FALSE} \wedge \texttt{TRUE} \rightarrow_\beta \texttt{FALSE}$
- $\texttt{FALSE} \wedge \texttt{FALSE} \rightarrow_\beta \texttt{FALSE}$

Defining terms with special meaning

Defining terms with special meaning

Choice terms

- Terms with special meaning essentially make a choice when given parameters
- The choice is expressed by either returning, or applying, the parameters

Defining terms with special meaning

Delta rules

- We wish to use special symbols to these terms with special meaning
- We define a series of delta rules, which are transformation from pretty symbols into lambda terms (and vice-versa)

Defining terms with special meaning

Delta rules

This means that we will be able to write lambda programs such as 5+3, that will then be translated into the appropriate lambda terms

Booleans

Booleans

Idea

- Boolean operators such as `TRUE` and `FALSE` must be defined so as to identify themselves
- The choice is expressed by returning their identity from a choice of two options

Booleans

`TRUE` is defined as a selector of the representative for true, that is the first argument[a]

[a] by arbitrary convention

$(\lambda \mathtt{t}\ \ \mathtt{f} \rightarrow \mathtt{t})$

`FALSE` is defined as a selector of the representative for false, that is the second argument[a]

[a] by arbitrary convention, as long as different from the previous

$(\lambda \mathtt{t}\ \ \mathtt{f} \rightarrow \mathtt{f})$

```
((TRUE bit1) bit0)
```

Booleans


```
((TRUE bit1) bit0)
```

```
((TRUE bit1) bit0)
```

Booleans

```
((TRUE bit1) bit0)
```

Booleans

```
(( TRUE  bit1) bit0)
```

```
(( (λt f→t)  bit1) bit0)
```

Booleans


```
(((λt f→t) bit1) bit0)
```

Booleans

$(((\lambda t\ f \to t)\ bit1)\ bit0)$

$(((\lambda t\ f \to t)\ bit1)\ bit0)$

Booleans


```
( ((λt f→t) bit1) bit0)
```

Booleans

$(((\lambda t\ f \rightarrow t)\ \texttt{bit1})\ \ \texttt{bit0})$

$((\lambda f \rightarrow \texttt{bit1})\ \ \texttt{bit0})$

Booleans


```
((λf→bit1) bit0)
```

Booleans


```
((λf→bit1) bit0)
```

```
((λf→bit1) bit0)
```

Booleans

((λf→bit1) bit0)

Booleans


```
((λf→bit1) bit0)
```

```
bit1
```

Booleans

AND

- The conjunction[a] of two terms is a function that takes as input two booleans and returns a boolean
- Since we just defined booleans to be two-parameter functions, we know that the two input booleans can be applied to each other
- Given two booleans p and q, their conjunction is q if p was true, or false otherwise
- `(λp q→((p q) p))`

[a] AND, or $\wedge$

Booleans

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Appendix

AND

Let us begin to with $\texttt{TRUE} \wedge \texttt{TRUE} \rightarrow_\beta \texttt{TRUE}$

Booleans

$\boxed{(\mathtt{TRUE}\ \wedge\ \mathtt{TRUE})}$

Booleans


```
(TRUE ∧ TRUE)
```

```
((∧ TRUE) TRUE)
```

Booleans


```
((∧ TRUE) TRUE)
```

Booleans

$$((\ \boxed{\wedge}\ \ \mathtt{TRUE})\ \ \mathtt{TRUE})$$

$$((\ \boxed{(\lambda p\ q \rightarrow ((p\ q)\ p))}\ \ \mathtt{TRUE})\ \ \mathtt{TRUE})$$

Booleans


```
(((λp q→((p q) p)) TRUE) TRUE)
```

```
(((λp q→((p q) p)) TRUE) TRUE)
```

```
(((λp q→((p q) p)) TRUE) TRUE)
```

Booleans


```
(((λp q→((p q) p)) TRUE) TRUE)
```

Booleans


```
(((λp q→((p q) p)) TRUE) TRUE)
```

```
(((λp q→((p q) p)) (λt f→t)) TRUE)
```

Booleans

$$\boxed{\texttt{(((}\lambda\texttt{p q}\rightarrow\texttt{((p q) p)) (}\lambda\texttt{t f}\rightarrow\texttt{t)) TRUE)}}$$

Booleans

$$(((\lambda p\ q \rightarrow ((p\ q)\ p))\ (\lambda t\ f \rightarrow t))\ \mathtt{TRUE})$$

$$(((\lambda p\ q \rightarrow ((p\ q)\ p))\ (\lambda t\ f \rightarrow t))\ \mathtt{TRUE})$$

Booleans


```
(((λp q→((p q) p)) (λt f→t)) TRUE)
```

Booleans


```
(((λp q→((p q) p)) (λt f→t)) TRUE)
```

```
(((λp q→((p q) p)) TRUE) TRUE)
```

Booleans


```
(((λp q→((p q) p)) TRUE) TRUE)
```

```
(((λp q→((p q) p)) TRUE) TRUE)
```

```
(((λp q→((p q) p)) TRUE) TRUE)
```

Booleans


```
(((λp q→((p q) p)) TRUE) TRUE)
```

Booleans


```
(((λp q→((p q) p)) TRUE) TRUE)
```

```
(((λp q→((p q) p)) TRUE) (λt f→t))
```

Booleans


```
(((λp q→((p q) p)) TRUE) (λt f→t))
```

Booleans

$$(((\lambda p\ q \rightarrow ((p\ q)\ p))\ \mathtt{TRUE})\ (\lambda t\ f \rightarrow t))$$

$$(((\lambda p\ q \rightarrow ((p\ q)\ p))\ \mathtt{TRUE})\ \boxed{(\lambda t\ f \rightarrow t)})$$

Booleans


```
(((λp q→((p q) p)) TRUE) (λt f→t) )
```

Booleans

$(((\lambda p\ q \rightarrow ((p\ q)\ p))\ \mathtt{TRUE})\ (\lambda t\ f \rightarrow t))$

$(((\lambda p\ q \rightarrow ((p\ q)\ p))\ \mathtt{TRUE})\ \mathtt{TRUE})$

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Booleans


```
(((λp q→((p q) p)) TRUE) TRUE)
```

Booleans


```
(((λp q→((p q) p)) TRUE) TRUE)
```

```
(((λp q→((p q) p)) TRUE) TRUE)
```

Booleans


```
(((λp q→((p q) p)) TRUE) TRUE)
```

Booleans


```
(((λp q→((p q) p)) TRUE) TRUE)
```

```
(((λp q→((p q) p)) (λt f→t)) TRUE)
```

Booleans

$$\boxed{(((\lambda p\ q \rightarrow ((p\ q)\ p))\ (\lambda t\ f \rightarrow t))\ \mathtt{TRUE})}$$

Booleans


```
(((λp q→((p q) p)) (λt f→t)) TRUE)
```

```
( ((λp q→((p q) p)) (λt f→t)) TRUE)
```

Booleans


```
( ((λp q→((p q) p)) (λt f→t)) TRUE)
```

Booleans


```
( ((λp q→((p q) p)) (λt f→t)) TRUE)
```

```
((λq→(( (λt f→t) q) (λt f→t) )) TRUE)
```

Booleans


```
((λq→(((λt f→t) q) (λt f→t))) TRUE)
```

Booleans


```
((λq→(((λt f→t) q) (λt f→t))) TRUE)
```

```
((λq→(((λt f→t) q) (λt f→t))) TRUE)
```

Booleans

```
((λq→(((λt f→t) q) (λt f→t))) TRUE)
```

Booleans


```
((λq→(((λt f→t) q) (λt f→t))) TRUE)
```

```
((λq→(((λt f→t) q) (λt f→t))) (λt f→t))
```

Booleans

$$\boxed{((\lambda \mathtt{q} \rightarrow (((\lambda \mathtt{t\ f} \rightarrow \mathtt{t})\ \mathtt{q})\ (\lambda \mathtt{t\ f} \rightarrow \mathtt{t})))\ (\lambda \mathtt{t\ f} \rightarrow \mathtt{t}))}$$

Booleans


```
((λq→(((λt f→t) q) (λt f→t))) (λt f→t))
```

```
((λq→(((λt f→t) q) (λt f→t))) (λt f→t))
```

Booleans

```
((λq→(((λt f→t) q) (λt f→t))) (λt f→t))
```

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Booleans

$$((\lambda q \to (((\lambda t\ f \to t)\ q)\ (\lambda t\ f \to t)))\ (\lambda t\ f \to t))$$

$$((\lambda q \to (((\lambda t\ f \to t)\ q)\ (\lambda t\ f \to t)))\ \mathtt{TRUE})$$

Booleans


```
((λq→(((λt f→t) q) (λt f→t))) TRUE)
```

Booleans


```
((λq→(((λt f→t) q) (λt f→t))) TRUE)
```

```
((λq→(((λt f→t) q) (λt f→t))) TRUE)
```

Booleans


```
((λq→(((λt f→t) q) (λt f→t))) TRUE)
```

Booleans


```
((λq→(((λt f→t) q) (λt f→t))) TRUE)
```

```
((λq→(((λt f→t) q) (λt f→t))) (λt f→t))
```

Booleans


```
((λq→(((λt f→t) q) (λt f→t))) (λt f→t))
```

Booleans


```
((λq→(((λt f→t) q) (λt f→t))) (λt f→t))
```

```
((λq→(((λt f→t) q) (λt f→t))) (λt f→t))
```

Booleans


```
((λq→(((λt f→t) q) (λt f→t))) (λt f→t))
```

Booleans

((λq→(((λt f→t) q) (λt f→t))) (λt f→t))

(((λt f→t) (λt f→t)) (λt f→t))

Booleans

$$\boxed{(((\lambda \texttt{t f} \rightarrow \texttt{t})\ (\lambda \texttt{t f} \rightarrow \texttt{t}))\ (\lambda \texttt{t f} \rightarrow \texttt{t}))}$$

Booleans


```
(((λt f→t) (λt f→t)) (λt f→t))
```

```
( ((λt f→t) (λt f→t)) (λt f→t))
```

Booleans

$(\ ((\lambda t\ f \to t)\ (\lambda t\ f \to t))\ \ (\lambda t\ f \to t))$

Booleans

$$(((\lambda t\ f \rightarrow t)\ (\lambda t\ f \rightarrow t))\ (\lambda t\ f \rightarrow t))$$

$$((\lambda f\ t\ f \rightarrow t)\ (\lambda t\ f \rightarrow t))$$

Booleans


```
((λf t f→t) (λt f→t))
```

Booleans

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$((\lambda \texttt{f t f} \rightarrow \texttt{t})\ (\lambda \texttt{t f} \rightarrow \texttt{t}))$

$((\lambda \texttt{f t f} \rightarrow \texttt{t})\ (\lambda \texttt{t f} \rightarrow \texttt{t}))$

Booleans

$((\lambda f\ \ t\ \ f{\rightarrow}t)\ \ (\lambda t\ \ f{\rightarrow}t))$

Booleans


```
((λf t f→t) (λt f→t) )
```

```
((λf t f→t) TRUE )
```

Booleans


```
((λf t f→t) TRUE)
```

Booleans


```
((λf t f→t) TRUE)
```

```
((λf t f→t) TRUE)
```

Booleans

$((\lambda \texttt{f t f} \rightarrow \texttt{t})\ \boxed{\texttt{TRUE}})$

Booleans


```
((λf t f→t) TRUE )
```

```
((λf t f→t) (λt f→t) )
```

Booleans

$\boxed{\texttt{((}\lambda\texttt{f t f}\rightarrow\texttt{t) (}\lambda\texttt{t f}\rightarrow\texttt{t))}}$

Booleans


```
((λf t f→t) (λt f→t))
```

```
((λf t f→t) (λt f→t))
```

Booleans

$((\lambda \texttt{f t f} \rightarrow \texttt{t})\ (\lambda \texttt{t f} \rightarrow \texttt{t}))$

Booleans

$((\lambda f\ t\ f \rightarrow t)\ (\lambda t\ f \rightarrow t))$

$(\lambda t\ f \rightarrow t)$

Booleans

$$\boxed{(\lambda t\ \ f \rightarrow t)}$$

Booleans

$(\lambda t\ \ f \rightarrow t)$

$(\lambda t\ \ f \rightarrow t)$

Booleans

$(\lambda$t f$\rightarrow$t)

Booleans

$(\lambda t\ f \rightarrow t)$

TRUE

Booleans

It works, but it is probably only because of black magic.

Booleans

It works, but it is probably only because of black magic.

Or is it? Let's see if we can get lucky again...

Booleans

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OR

- The disjunction[a] of two terms is a function that takes as input two booleans and returns a boolean
- Like with conjunction, remember that the two input booleans can be applied to one another
- Given two booleans `p` and `q`, their disjunction is `true` if `p` was `true`, or `q` otherwise
- $\boxed{(\lambda \texttt{p q} \rightarrow \texttt{((p p) q)})}$

[a] `OR`, or $\vee$

Booleans

OR

Let us begin to with $\texttt{TRUE} \vee \texttt{TRUE} \rightarrow_\beta \texttt{TRUE}$

Booleans

$$\boxed{(\texttt{TRUE} \lor \texttt{TRUE})}$$

Booleans


```
(TRUE ∨ TRUE)
```

```
((∨ TRUE) TRUE)
```

Booleans


```
((∨ TRUE) TRUE)
```

Booleans

$$((\vee\ \mathtt{TRUE})\ \mathtt{TRUE})$$

$$((\ (\lambda p\ q \rightarrow ((p\ p)\ q))\ \ \mathtt{TRUE})\ \mathtt{TRUE})$$

Booleans


```
(((λp q→((p p) q)) TRUE) TRUE)
```

Booleans


```
(((λp q→((p p) q)) TRUE) TRUE)
```

```
(((λp q→((p p) q)) TRUE) TRUE)
```

Booleans


```
(((λp q→((p p) q)) TRUE) TRUE)
```

Booleans


```
(((λp q→((p p) q)) TRUE) TRUE)
```

```
(((λp q→((p p) q)) (λt f→t)) TRUE)
```

Booleans

```
(((λp q→((p p) q)) (λt f→t)) TRUE)
```

Booleans


```
(((λp q→((p p) q)) (λt f→t)) TRUE)
```

```
(((λp q→((p p) q)) (λt f→t)) TRUE)
```

Booleans

$$\texttt{(((}\lambda\texttt{p q}\rightarrow\texttt{((p p) q)) }\boxed{\texttt{(}\lambda\texttt{t f}\rightarrow\texttt{t)}}\texttt{) TRUE)}$$

Booleans


```
(((λp q→((p p) q)) (λt f→t)) TRUE)
```

```
(((λp q→((p p) q)) TRUE) TRUE)
```

Booleans


```
(((λp q→((p p) q)) TRUE) TRUE)
```

Booleans

$(((\lambda p\ q \rightarrow ((p\ p)\ q))\ \mathtt{TRUE})\ \mathtt{TRUE})$

$(((\lambda p\ q \rightarrow ((p\ p)\ q))\ \mathtt{TRUE})\ \boxed{\mathtt{TRUE}})$

Booleans


```
(((λp q→((p p) q)) TRUE) TRUE)
```

Booleans


```
(((λp q→((p p) q)) TRUE) TRUE)
```

```
(((λp q→((p p) q)) TRUE) (λt f→t))
```

Booleans

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```
(((λp q→((p p) q)) TRUE) (λt f→t))
```

Booleans

```
(((λp q→((p p) q)) TRUE) (λt f→t))
```

```
(((λp q→((p p) q)) TRUE) (λt f→t))
```

Booleans

$$(((\lambda \texttt{p q} \rightarrow ((\texttt{p p}) \texttt{ q})) \texttt{ TRUE}) \texttt{ } (\lambda \texttt{t f} \rightarrow \texttt{t}))$$

Booleans


```
(((λp q→((p p) q)) TRUE) (λt f→t))
```

```
(((λp q→((p p) q)) TRUE) TRUE)
```

Booleans

$$\boxed{(((\lambda \mathtt{p}\ \ \mathtt{q} \rightarrow ((\mathtt{p}\ \ \mathtt{p})\ \ \mathtt{q}))\ \ \mathtt{TRUE})\ \ \mathtt{TRUE})}$$

Booleans


```
(((λp q→((p p) q)) TRUE) TRUE)
```

```
(((λp q→((p p) q)) TRUE) TRUE)
```

Booleans

$(((\lambda p\ q \rightarrow ((p\ p)\ q))\ \boxed{\texttt{TRUE}})\ \texttt{TRUE})$

Booleans


```
(((λp q→((p p) q)) TRUE) TRUE)
```

```
(((λp q→((p p) q)) (λt f→t)) TRUE)
```

Booleans


```
(((λp q→((p p) q)) (λt f→t)) TRUE)
```

Booleans


```
(((λp q→((p p) q)) (λt f→t)) TRUE)
```

```
( ((λp q→((p p) q)) (λt f→t)) TRUE)
```

Booleans


```
( ((λp q→((p p) q)) (λt f→t)) TRUE)
```

Booleans


```
( ((λp q→((p p) q)) (λt f→t)) TRUE)
```

```
((λq→(( (λt f→t) (λt f→t) ) q)) TRUE)
```

Booleans

$$\boxed{\texttt{((}\lambda\texttt{q}\rightarrow\texttt{(((}\lambda\texttt{t f}\rightarrow\texttt{t) (}\lambda\texttt{t f}\rightarrow\texttt{t)) q)) TRUE)}}$$

Booleans

```
((λq→((((λt f→t) (λt f→t)) q)) TRUE)
```

```
((λq→((((λt f→t) (λt f→t)) q)) TRUE)
```

Booleans

$$((\lambda \mathtt{q} \rightarrow (((\lambda \mathtt{t}\ \ \mathtt{f} \rightarrow \mathtt{t})\ \ (\lambda \mathtt{t}\ \ \mathtt{f} \rightarrow \mathtt{t}))\ \ \mathtt{q}))\ \ \mathtt{TRUE})$$

Booleans

$$((\lambda q \rightarrow (((\lambda t\ f \rightarrow t)\ (\lambda t\ f \rightarrow t))\ q))\ \texttt{TRUE})$$

$$((\lambda q \rightarrow (((\lambda t\ f \rightarrow t)\ (\lambda t\ f \rightarrow t))\ q))\ (\lambda t\ f \rightarrow t))$$

Booleans


```
((λq→(((λt f→t) (λt f→t)) q)) (λt f→t))
```

Booleans


```
((λq→(((λt f→t) (λt f→t)) q)) (λt f→t))
```

```
((λq→(((λt f→t) (λt f→t)) q)) (λt f→t))
```

Booleans


```
((λq→(((λt f→t) (λt f→t)) q)) (λt f→t))
```

Booleans

$$((\lambda q \rightarrow (((\lambda t\ f \rightarrow t)\ (\lambda t\ f \rightarrow t))\ q))\ (\lambda t\ f \rightarrow t))$$

$$((\lambda q \rightarrow (((\lambda t\ f \rightarrow t)\ (\lambda t\ f \rightarrow t))\ q))\ \mathtt{TRUE})$$

Booleans


```
((λq→(((λt f→t) (λt f→t)) q)) TRUE)
```

Booleans


```
((λq→((((λt f→t) (λt f→t)) q)) TRUE)
```

```
((λq→((((λt f→t) (λt f→t)) q)) TRUE)
```

Booleans


```
((λq→(((λt f→t) (λt f→t)) q)) TRUE)
```

Booleans


```
((λq→(((λt f→t) (λt f→t)) q)) TRUE)
```

```
((λq→(((λt f→t) (λt f→t)) q)) (λt f→t))
```

Booleans

$$\boxed{\texttt{((}\lambda\texttt{q}\rightarrow\texttt{((((}\lambda\texttt{t f}\rightarrow\texttt{t) (}\lambda\texttt{t f}\rightarrow\texttt{t)) q)) (}\lambda\texttt{t f}\rightarrow\texttt{t))}}$$

Booleans


```
((λq→(((λt f→t) (λt f→t)) q)) (λt f→t))
```

```
((λq→(((λt f→t) (λt f→t)) q)) (λt f→t))
```

Booleans

$((\lambda q \rightarrow (((\lambda t\ f \rightarrow t)\ (\lambda t\ f \rightarrow t))\ q))\ (\lambda t\ f \rightarrow t))$

Booleans

```
((λq→(((λt f→t) (λt f→t)) q)) (λt f→t))
```

```
(((λt f→t) (λt f→t)) (λt f→t))
```

Booleans

$$\boxed{(((\lambda \mathtt{t}\ \mathtt{f} \rightarrow \mathtt{t})\ (\lambda \mathtt{t}\ \mathtt{f} \rightarrow \mathtt{t}))\ (\lambda \mathtt{t}\ \mathtt{f} \rightarrow \mathtt{t}))}$$

Booleans

`(((λt f→t) (λt f→t)) (λt f→t))`

`( ((λt f→t) (λt f→t)) (λt f→t))`

Booleans

$$(\; ((\lambda t \;\; f \to t) \;\; (\lambda t \;\; f \to t)) \;\; (\lambda t \;\; f \to t))$$

Booleans


```
( ((λt f→t) (λt f→t)) (λt f→t))
```

```
((λf t f→ t ) (λt f→t))
```

Booleans

$$\boxed{\texttt{((}\lambda\texttt{f t f}\rightarrow\texttt{t) (}\lambda\texttt{t f}\rightarrow\texttt{t))}}$$

Booleans

$$((\lambda f\ t\ f \rightarrow t)\ (\lambda t\ f \rightarrow t))$$

$$((\lambda f\ t\ f \rightarrow t)\ \boxed{(\lambda t\ f \rightarrow t)})$$

Booleans

$$((\lambda f\ \ t\ \ f \rightarrow t)\ \ (\lambda t\ \ f \rightarrow t))$$

Booleans

$$((\lambda f\ \ t\ \ f \rightarrow t)\ \ (\lambda t\ \ f \rightarrow t)\ )$$

$$((\lambda f\ \ t\ \ f \rightarrow t)\ \ \mathtt{TRUE}\ )$$

Booleans


```
((λf t f→t) TRUE)
```

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Booleans


```
((λf t f→t) TRUE)
```

```
((λf t f→t) TRUE)
```

Booleans


```
((λf t f→t) TRUE )
```

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Booleans


```
((λf t f→t) TRUE )
```

```
((λf t f→t) (λt f→t) )
```

Booleans

$$\boxed{\texttt{((}\lambda\texttt{f t f}\rightarrow\texttt{t) (}\lambda\texttt{t f}\rightarrow\texttt{t))}}$$

Booleans

$((\lambda f\ \ t\ \ f \rightarrow t)\ \ (\lambda t\ \ f \rightarrow t))$

$((\lambda f\ \ t\ \ f \rightarrow t)\ \ (\lambda t\ \ f \rightarrow t))$

Booleans

$((\lambda \texttt{f t f} \rightarrow \texttt{t})\ (\lambda \texttt{t f} \rightarrow \texttt{t}))$

Booleans

$((\lambda \mathtt{f\ t\ f} \rightarrow \mathtt{t})\ (\lambda \mathtt{t\ f} \rightarrow \mathtt{t}))$

$(\lambda \mathtt{t\ f} \rightarrow \mathtt{t})$

Booleans


```
(λt  f→t)
```

Booleans

$(\lambda t\ \ f \rightarrow t)$

$(\lambda t\ \ f \rightarrow t)$

Booleans

$(\lambda t\ f \rightarrow t)$

Booleans

$(\lambda t\ f \rightarrow t)$

TRUE

Booleans

`if-then-else`

- The conditional operator `if-then-else` chooses one of two parameters based on the value of the input condition
- Given a boolean `c` and two values `th` and `el`, the result is `th` if `c` was `true`, or `el` otherwise
- Since `c` is a boolean, it already performs this choice!
- $\boxed{(\lambda \texttt{p th el} \rightarrow ((\texttt{p th})\ \texttt{el}))}$

Booleans

`if-then-else`

Let us try with `if TRUE` $\vee$ `FALSE then A else B` $\rightarrow_\beta$ `A`

Booleans

```
if TRUE then A else B
```

Booleans


```
if TRUE then A else B
```

```
(((if-then-else TRUE) A) B)
```

Booleans


```
(((if-then-else TRUE) A) B)
```

Booleans


```
((( if-then-else TRUE) A) B)
```

```
((( (λp th el→((p th) el)) TRUE) A) B)
```

Booleans


```
((((λp th el→((p th) el)) TRUE) A) B)
```

Booleans

$$((((\lambda p\ th\ el \rightarrow ((p\ th)\ el))\ TRUE)\ A)\ B)$$

$$((((\lambda p\ th\ el \rightarrow ((p\ th)\ el))\ \boxed{TRUE})\ A)\ B)$$

Booleans


```
((((λp th el→((p th) el)) TRUE) A) B)
```

Booleans


```
((((λp th el→((p th) el)) TRUE ) A) B)
```

```
((((λp th el→((p th) el)) (λt f→t) ) A) B)
```

Booleans

$$\boxed{\texttt{((((}\lambda\texttt{p th el}\rightarrow\texttt{((p th) el)) (}\lambda\texttt{t f}\rightarrow\texttt{t)) A) B)}}$$

Booleans

$\boxed{\texttt{((((}\lambda\texttt{p th el}\rightarrow\texttt{((p th) el)) (}\lambda\texttt{t f}\rightarrow\texttt{t)) A) B)}}$

$\boxed{\texttt{((((}\lambda\texttt{p th el}\rightarrow\texttt{((p th) el)) (}\lambda\texttt{t f}\rightarrow\texttt{t)) A) B)}}$

Booleans


```
((((λp th el→((p th) el)) (λt f→t)) A) B)
```

Booleans


```
((((λp th el→((p th) el)) (λt f→t)) A) B)
```

```
((((λp th el→((p th) el)) TRUE) A) B)
```

Booleans

$$\boxed{\texttt{((((}\lambda\texttt{p th el}\rightarrow\texttt{((p th) el)) TRUE) A) B)}}$$

Booleans


```
((((λp th el→((p th) el)) TRUE) A) B)
```

```
((((λp th el→((p th) el)) TRUE) A) B)
```

Booleans

$$(((((\lambda \mathtt{p\ th\ el} \rightarrow ((\mathtt{p\ th})\ \mathtt{el}))\ \boxed{\mathtt{TRUE}})\ \mathtt{A})\ \mathtt{B})$$

Booleans

$((((\lambda p\ th\ el \rightarrow ((p\ th)\ el))\ \boxed{TRUE})\ A)\ B)$

$((((\lambda p\ th\ el \rightarrow ((p\ th)\ el))\ \boxed{(\lambda t\ f \rightarrow t)})\ A)\ B)$

Booleans

$$\boxed{\texttt{((((}\lambda\texttt{p th el}\rightarrow\texttt{((p th) el)) (}\lambda\texttt{t f}\rightarrow\texttt{t)) A) B)}}$$

Booleans

$$((((\lambda p\ th\ el \rightarrow ((p\ th)\ el))\ (\lambda t\ f \rightarrow t))\ A)\ B)$$

$$((((\lambda p\ th\ el \rightarrow ((p\ th)\ el))\ \boxed{(\lambda t\ f \rightarrow t)})\ A)\ B)$$

Booleans


```
((((λp th el→((p th) el)) (λt f→t)) A) B)
```

Booleans

$$((((\lambda \texttt{p th el} \rightarrow ((\texttt{p th})\ \texttt{el}))\ (\lambda \texttt{t f} \rightarrow \texttt{t}))\ \texttt{A})\ \texttt{B})$$

$$((((\lambda \texttt{p th el} \rightarrow ((\texttt{p th})\ \texttt{el}))\ \texttt{TRUE})\ \texttt{A})\ \texttt{B})$$

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Booleans

$$\boxed{\texttt{((((}\lambda\texttt{p th el}\rightarrow\texttt{((p th) el)) TRUE) A) B)}}$$

Booleans


```
((((λp th el→((p th) el)) TRUE) A) B)
```

```
((((λp th el→((p th) el)) TRUE) A) B)
```

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```
((((λp th el→((p th) el)) TRUE) A) B)
```

Booleans

$$\texttt{((((}\lambda\texttt{p th el}\rightarrow\texttt{((p th) el)) TRUE) A) B)}$$

$$\texttt{((((}\lambda\texttt{p th el}\rightarrow\texttt{((p th) el)) (}\lambda\texttt{t f}\rightarrow\texttt{t)) A) B)}$$

Booleans

$$\boxed{\texttt{((((}\lambda\texttt{p th el}\rightarrow\texttt{((p th) el)) (}\lambda\texttt{t f}\rightarrow\texttt{t)) A) B)}}$$

Booleans

$$((((\lambda \texttt{p th el} \rightarrow ((\texttt{p th})\ \texttt{el}))\ (\lambda \texttt{t f} \rightarrow \texttt{t}))\ \texttt{A})\ \texttt{B})$$

$$((((\lambda \texttt{p th el} \rightarrow ((\texttt{p th})\ \texttt{el}))\ (\lambda \texttt{t f} \rightarrow \texttt{t}))\ \texttt{A})\ \texttt{B})$$

Booleans

$$((\; ((\lambda p\ th\ el \rightarrow ((p\ th)\ el))\ (\lambda t\ f \rightarrow t)) \; A)\ B)$$

Booleans

$$((\ ((\lambda p\ th\ el \rightarrow ((p\ th)\ el))\ (\lambda t\ f \rightarrow t))\ \ A)\ \ B)$$

$$(((\lambda th\ \ el \rightarrow ((\ (\lambda t\ f \rightarrow t)\ \ th)\ \ el))\ \ A)\ \ B)$$

```
(((λth el→(((λt f→t) th) el)) A) B)
```

Booleans


```
(((λth el→(((λt f→t) th) el)) A) B)
```

```
( ((λth el→(((λt f→t) th) el)) A) B)
```

Booleans

$$(\; ((\lambda \texttt{th el} \rightarrow (((\lambda \texttt{t f} \rightarrow \texttt{t}) \; \texttt{th}) \; \texttt{el})) \; \texttt{A}) \; \texttt{B})$$

Booleans


```
( ((λth el→(((λt f→t) th) el)) A)  B)
```

```
((λel→(((λt f→t)  A ) el)) B)
```

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$$\boxed{((\lambda \texttt{el} \rightarrow (((\lambda \texttt{t f} \rightarrow \texttt{t})\ \texttt{A})\ \texttt{el}))\ \texttt{B})}$$

Booleans


```
((λel→(((λt f→t) A) el)) B)
```

```
((λel→(((λt f→t) A) el)) B)
```

Booleans


```
((λel→(((λt f→t) A) el)) B)
```

Booleans


```
((λel→(((λt f→t) A) el)) B)
```

```
(((λt f→t) A) B)
```

Booleans

$$(((\lambda t\ f \rightarrow t)\ A)\ B)$$

Booleans


```
(((λt f→t) A) B)
```

```
( ((λt f→t) A) B)
```

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Booleans

```
( ((λt f→t) A)  B)
```

Booleans


```
( ((λt f→t) A)  B)
```

```
((λf→ A )  B)
```

Booleans


```
((λf→A) B)
```

Booleans

$((\lambda f \rightarrow A)\ \ B)$

$((\lambda f \rightarrow A)\ \ B)$

Booleans


```
((λf→A) B)
```

Booleans


```
((λf→A) B)
```

```
A
```

Natural numbers

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Natural numbers

Idea

- Natural numbers such as 3 and 0 must be defined so as to identify themselves
- Their identity is determined by how many times they perform an action
- The only action we have available is applying a function to a term

Natural numbers

Idea

- We will use unary numbers
- A number is defined by how many times it applies a function to a given term
- Zero applications are also possible, in this case we default to the given term

Natural numbers

0, 1, etc.

A number is defined as an applicator of a term identifying as successor to another term identifying as zero[a]

[a] first and second arguments by arbitrary convention

0 will thus look like

```
(λs z→z)
```

1 will look like

```
(λs z→(s z))
```

7 will look like

```
(λs z→(s (s (s (s (s (s (s z))))))))
```

etc.

Natural numbers

Addition

- Adding numbers is a function that takes as input two numbers (say `m` and `n`), and returns a number
- The first number applies its first parameter `m` times to its second parameter
- The second number applies its first parameter `n` times to its second parameter
- We can use the second number as the second parameter to the first, therefore obtaining something that applies `m+n` times
- $(\lambda \texttt{m n} \rightarrow (\lambda \texttt{s z} \rightarrow \texttt{((m s) ((n s) z))))}$

Natural numbers

Addition

Let us try it out to $2 + 1 \rightarrow_\beta 3$

Natural numbers


```
(2 + 1)
```

Natural numbers


```
(2 + 1)
```

```
((+ 2) 1)
```

```
((+ 2) 1)
```

```
((+ 2) 1)
```

```
(((λm n→ (λs z→((m s) ((n s) z)))) 2) 1)
```

Natural numbers

$$\texttt{(((}\lambda\texttt{m n}\rightarrow\texttt{ (}\lambda\texttt{s z}\rightarrow\texttt{((m s) ((n s) z)))) 2) 1)}$$

Natural numbers


```
(((λm n→ (λs z→((m s) ((n s) z))))) 2) 1)
```

```
(((λm n→ (λs z→((m s) ((n s) z))))) 2) 1)
```

Natural numbers


```
(((λm n→ (λs z→((m s) ((n s) z)))) 2) 1)
```

Natural numbers


```
(((λm n→ (λs z→((m s) ((n s) z)))) 2) 1)
```

```
(((λm n→ (λs z→((m s) ((n s) z))))
    (λs z→(s (s z)))) 1)
```

Natural numbers


```
(((λm n→ (λs z→((m s) ((n s) z)))) (λs z→(s
    (s z)))) 1)
```

```
(((λm n→ (λs z→((m s) ((n s) z)))) (λs z→(s
    (s z)))) 1)
```

```
(((λm n→ (λs z→((m s) ((n s) z))))
    (λs z→(s (s z)))) 1)
```

Natural numbers


```
(((λm n→ (λs z→((m s) ((n s) z))))
    (λs z→(s (s z)))) 1)
```

Natural numbers


```
(((λm n→ (λs z→((m s) ((n s) z))))
     (λs z→(s (s z))) ) 1)
```

```
(((λm n→ (λs z→((m s) ((n s) z)))) 2) 1)
```

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$\boxed{\texttt{((($\lambda$m n$\rightarrow$ (λs z$\rightarrow$((m s) ((n s) z)))) 2) 1)}}$

Natural numbers

$$(((\lambda m\ n \rightarrow (\lambda s\ z \rightarrow ((m\ s)\ ((n\ s)\ z))))\ 2)\ 1)$$

$$(((\lambda m\ n \rightarrow (\lambda s\ z \rightarrow ((m\ s)\ ((n\ s)\ z))))\ 2)\ \boxed{1})$$

Natural numbers


```
(((λm n→ (λs z→((m s) ((n s) z)))) 2) 1)
```

Natural numbers

$$(((\lambda m\ n \to (\lambda s\ z \to ((m\ s)\ ((n\ s)\ z))))\ 2)\ \boxed{1})$$

$$(((\lambda m\ n \to (\lambda s\ z \to ((m\ s)\ ((n\ s)\ z))))\ 2)\ \boxed{(\lambda s\ z \to (s\ z))})$$

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Natural numbers

$$\texttt{(((}\lambda\texttt{m n}\rightarrow\texttt{ (}\lambda\texttt{s z}\rightarrow\texttt{((m s) ((n s) z)))) 2) (}\lambda\texttt{s z}\rightarrow\texttt{ (s z)))}$$

```
(((λm n→ (λs z→((m s) ((n s) z)))) 2) (λs z→
    (s z)))
```

```
(((λm n→ (λs z→((m s) ((n s) z)))) 2)
    (λs z→(s z)) )
```

```
(((λm n→ (λs z→((m s) ((n s) z)))) 2)
    (λs z→(s z)) )
```

Natural numbers


```
(((λm n→ (λs z→((m s) ((n s) z)))) 2)
    (λs z→(s z)) )
```

```
(((λm n→ (λs z→((m s) ((n s) z)))) 2) 1 )
```

Natural numbers

$$\boxed{(((\lambda m\ n \rightarrow\ (\lambda s\ z \rightarrow ((m\ s)\ ((n\ s)\ z))))\ 2)\ 1)}$$

Natural numbers


```
(((λm n→ (λs z→((m s) ((n s) z)))) 2) 1)
```

```
(((λm n→ (λs z→((m s) ((n s) z)))) 2) 1)
```

Natural numbers

```
(((λm n→ (λs z→((m s) ((n s) z)))) 2) 1)
```

Natural numbers

```
(((λm n→ (λs z→((m s) ((n s) z)))) 2) 1)
```

```
(((λm n→ (λs z→((m s) ((n s) z))))
    (λs z→(s (s z)))) 1)
```

Natural numbers


```
(((λm n→ (λs z→((m s) ((n s) z)))) (λs z→(s
    (s z)))) 1)
```

Natural numbers


```
(((λm n→ (λs z→((m s) ((n s) z)))) (λs z→(s
    (s z)))) 1)
```

```
(
    ((λm n→ (λs z→((m s) ((n s) z)))) (λs z→(s (s z))))
    1)
```

Natural numbers


```
(
     ((λm n→ (λs z→((m s) ((n s) z)))) (λs z→(s (s z)))
     1)
```

Natural numbers


```
(
    ((λm n→ (λs z→((m s) ((n s) z)))) (λs z→(s (s z))))
    1)
```

```
((λn s z→(( (λs z→(s (s z))) s) ((n s) z))) 1)
```

Natural numbers

$$\boxed{\texttt{((}\lambda\texttt{n s z}\rightarrow\texttt{((((}\lambda\texttt{s z}\rightarrow\texttt{(s (s z))) s) ((n s) z))) 1)}}$$

Natural numbers


```
((λn s z→(((λs z→(s (s z))) s) ((n s) z))) 1)
```

```
((λn s z→(((λs z→(s (s z))) s) ((n s) z)))
    1)
```

Natural numbers


```
((λn s z→(((λs z→(s (s z))) s) ((n s) z)))
    1 )
```

Natural numbers

```
((λn s z→(((λs z→(s (s z))) s) ((n s) z)))
    1 )
```

```
((λn s z→(((λs z→(s (s z))) s) ((n s) z)))
    (λs z→(s z)) )
```

Natural numbers

$$\boxed{\texttt{((}\lambda\texttt{n s z}\rightarrow\texttt{((((}\lambda\texttt{s z}\rightarrow\texttt{(s (s z))) s) ((n s) z))) (}\lambda\texttt{ s z}\rightarrow\texttt{(s z)))}}$$

Natural numbers


```
((λn s z→(((λs z→(s (s z))) s) ((n s) z))) (λ
    s z→(s z)))
```

```
((λn s z→(((λs z→(s (s z))) s) ((n s) z)))
    (λs z→(s z)))
```

Natural numbers


```
((λn s z→(((λs z→(s (s z))) s) ((n s) z)))
    (λs z→(s z)) )
```

Natural numbers


```
((λn s z→(((λs z→(s (s z))) s) ((n s) z)))
    (λs z→(s z)) )
```

```
((λn s z→(((λs z→(s (s z))) s) ((n s) z)))
    1 )
```

Natural numbers

$$\boxed{\texttt{((}\lambda\texttt{n s z}\rightarrow\texttt{(((}\lambda\texttt{s z}\rightarrow\texttt{(s (s z))) s) ((n s) z))) 1)}}$$

```
((λn s z→(((λs z→(s (s z))) s) ((n s) z))) 1)
```

```
((λn s z→(((λs z→(s (s z))) s) ((n s) z)))
    1 )
```

Natural numbers


```
((λn s z→(((λs z→(s (s z))) s) ((n s) z)))
     1 )
```

Natural numbers


```
((λn s z→(((λs z→(s (s z))) s) ((n s) z)))
    1 )
```

```
((λn s z→(((λs z→(s (s z))) s) ((n s) z)))
    (λs z→(s z)) )
```

Natural numbers


```
((λn s z→(((λs z→(s (s z))) s) ((n s) z))) (λ
    s z→(s z)))
```

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```
((λn s z→(((λs z→(s (s z))) s) ((n s) z))) (λ
    s z→(s z)))
```

```
((λn s z→(((λs z→(s (s z))) s) ((n s) z))) (λs z→(s z))
```

Natural numbers

((λn s z→(((λs z→(s (s z))) s) ((n s) z))) (λs z→(s z))

```
((λn s z→(((λs z→(s (s z))) s) ((n s) z))) (λs z→(s z)
```

```
(λs z→(((λs z→(s (s z))) s) (( (λs z→(s z)) s)
    z)))
```

Natural numbers


```
(λs z→(((λs z→(s (s z))) s) (((λs z→(s z)) s
   ) z)))
```

Natural numbers


```
(λs z→(((λs z→(s (s z))) s) (((λs z→(s z)) s
    ) z)))
```

```
(λs z→( ((λs z→(s (s z))) s)  (((λs z→(s z)) s)
    z)))
```

Natural numbers

```
(λs z→( ((λs z→(s (s z))) s) (((λs z→(s z)) s)
   z)))
```

Natural numbers


```
(λs z→( ((λs z→(s (s z))) s) (((λs z→(s z)) s)
    z)))
```

```
(λs z→((λz→(s (s z))) (((λs z→(s z)) s) z))
    )
```

```
(λs z→((λz→(s (s z))) (((λs z→(s z)) s) z)))
```

```
(λs z→((λz→(s (s z))) (((λs z→(s z)) s) z)))
```

```
(λs z→((λz→(s (s z))) ( ((λs z→(s z)) s)  z)))
```

```
(λs z→((λz→(s (s z))) (((λs z→(s z)) s) z)))
```

```
(λs z→((λz→(s (s z))) ( ((λs z→(s z)) s) z)))
```

```
(λs z→((λz→(s (s z))) ((λz→( s z)) z)))
```

Natural numbers

$$\boxed{(\lambda s\ z \rightarrow ((\lambda z \rightarrow (s\ (s\ z)))\ ((\lambda z \rightarrow (s\ z))\ z)))}$$

```
(λs z→((λz→(s (s z))) ((λz→(s z)) z)))
```

```
(λs z→((λz→(s (s z))) ((λz→(s z)) z)))
```

```
(λs z→((λz→(s (s z))) ((λz→(s z)) z)))
```

Natural numbers

$(\lambda s\ z \rightarrow ((\lambda z \rightarrow (s\ (s\ z)))\ ((\lambda z \rightarrow (s\ z))\ z)))$

$(\lambda s\ z \rightarrow ((\lambda z \rightarrow (s\ (s\ z)))\ (s\ z)))$

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```
(λs z→((λz→(s (s z))) (s z)))
```

Natural numbers

$(\lambda \texttt{s z} \rightarrow ((\lambda \texttt{z} \rightarrow (\texttt{s (s z)})) \texttt{ (s z)}))$

$(\lambda \texttt{s z} \rightarrow ((\lambda \texttt{z} \rightarrow (\texttt{s (s z)})) \texttt{ (s z)}))$

Natural numbers

$(\lambda s\ \ z \rightarrow ((\lambda z \rightarrow (s\ \ (s\ \ z)))\ \ (s\ \ z)))$

Natural numbers

$$(\lambda s\ z \rightarrow ((\lambda z \rightarrow (s\ (s\ z)))\ (s\ z)))$$

$$(\lambda s\ z \rightarrow (s\ (s\ (s\ z))))$$

Natural numbers


```
(λs z→(s (s (s z))))
```

Natural numbers


```
(λs z→(s (s (s z))))
```

```
(λs z→(s (s (s z))))
```

Natural numbers


```
(λs z→(s (s (s z))))
```

Natural numbers


```
(λs z→(s (s (s z))))
```

```
3
```

Natural numbers

Multiplication

- Multiplying numbers is a function that takes as input two numbers (say m and n), and returns a number
- The first number applies its first parameter m times to its second parameter
- The second number applies its first parameter n times to its second parameter
- We can use the second number as the first parameter to the first, therefore obtaining something that applies n+ m times, starting from z
- $\boxed{(\lambda m\ n \rightarrow (\lambda s\ z \rightarrow ((m\ (n\ s))\ z)))}$

Multiplication

Let us try it out to $2 \times 2 \rightarrow_\beta 4$

$$\boxed{(2 \times 2)}$$

Natural numbers

(2×2)

$((\times\ 2)\ 2)$

```
(( × 2) 2)
```

Natural numbers

$((\times\ 2)\ 2)$

$(((\lambda m\ n \rightarrow (\lambda s\ z \rightarrow ((m\ (n\ s))\ z)))\ 2)\ 2)$

$$(((\lambda \texttt{m n} \rightarrow (\lambda \texttt{s z} \rightarrow ((\texttt{m (n s)) z)))}\ \texttt{2})\ \texttt{2})$$

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```
(((λm n→ (λs z→((m (n s)) z))) 2) 2)
```

```
(((λm n→ (λs z→((m (n s)) z))) 2) 2)
```

Natural numbers

$$(((\lambda \mathtt{m\ n} \rightarrow (\lambda \mathtt{s\ z} \rightarrow ((\mathtt{m\ (n\ s)})\ \mathtt{z})))\ \boxed{\mathtt{2}})\ \mathtt{2})$$

Natural numbers


```
(((λm n→ (λs z→((m (n s)) z))) 2) 2)
```

```
(((λm n→ (λs z→((m (n s)) z)))
    (λs z→(s (s z)))) 2)
```

```
(((λm n→ (λs z→((m (n s)) z))) (λs z→(s (s z
    )))) 2)
```

```
(((λm n→ (λs z→((m (n s)) z))) (λs z→(s (s z
    )))) 2)
```

```
(((λm n→ (λs z→((m (n s)) z)))
    (λs z→(s (s z))) ) 2)
```

Natural numbers


```
(((λm n→ (λs z→((m (n s)) z)))
    (λs z→(s (s z)))) 2)
```

Natural numbers


```
(((λm n→ (λs z→((m (n s)) z)))
    (λs z→(s (s z)))) 2)
```

```
(((λm n→ (λs z→((m (n s)) z))) 2) 2)
```

Natural numbers


```
(((λm n→ (λs z→((m (n s)) z))) 2) 2)
```

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```
(((λm n→ (λs z→((m (n s)) z))) 2) 2)
```

```
(((λm n→ (λs z→((m (n s)) z))) 2) 2)
```

Natural numbers


```
(((λm n→ (λs z→((m (n s)) z))) 2) 2)
```

Natural numbers

```
(((λm n→ (λs z→((m (n s)) z))) 2) 2)
```

```
(((λm n→ (λs z→((m (n s)) z))) 2)
    (λs z→(s (s z))) )
```

Natural numbers

```
(((λm n→ (λs z→((m (n s)) z))) 2) (λs z→(s (
    s z))))
```

Natural numbers

```
(((λm n→ (λs z→((m (n s)) z))) 2) (λs z→(s (
    s z)))))
```

```
(((λm n→ (λs z→((m (n s)) z))) 2)
    (λs z→(s (s z))) )
```

```
(((λm n→ (λs z→((m (n s)) z))) 2)
    (λs z→(s (s z))) )
```

Natural numbers


```
(((λm n→ (λs z→((m (n s)) z))) 2)
    (λs z→(s (s z))) )
```

```
(((λm n→ (λs z→((m (n s)) z))) 2) 2 )
```

Natural numbers

$$\boxed{\texttt{(((}\lambda\texttt{m n}\rightarrow\texttt{ (}\lambda\texttt{s z}\rightarrow\texttt{((m (n s)) z))) 2) 2)}}$$

Natural numbers


```
(((λm n→ (λs z→((m (n s)) z))) 2) 2)
```

```
(((λm n→ (λs z→((m (n s)) z))) 2) 2)
```

Natural numbers


```
(((λm n→ (λs z→((m (n s)) z))) 2) 2)
```

Natural numbers


```
(((λm n→ (λs z→((m (n s)) z))) 2) 2)
```

```
(((λm n→ (λs z→((m (n s)) z)))
    (λs z→(s (s z)))) 2)
```

```
(((λm n→ (λs z→((m (n s)) z))) (λs z→(s (s z
    )))) 2)
```

```
(((λm n→ (λs z→((m (n s)) z))) (λs z→(s (s z
    )))) 2)
```

```
( ((λm n→ (λs z→((m (n s)) z))) (λs z→(s (s z))))
    2)
```

Natural numbers

$$(\boxed{((\lambda m\ n \rightarrow (\lambda s\ z \rightarrow ((m\ (n\ s))\ z)))\ (\lambda s\ z \rightarrow (s\ (s\ z))))}\ 2)$$

Natural numbers


```
( ((λm n→ (λs z→((m (n s)) z))) (λs z→(s (s z))))
   2)
```

```
((λn s z→(( (λs z→(s (s z))) (n s)) z)) 2)
```

Natural numbers

$$\boxed{\texttt{((}\lambda\texttt{n s z}\rightarrow\texttt{((((}\lambda\texttt{s z}\rightarrow\texttt{(s (s z))) (n s)) z)) 2)}}$$

Natural numbers


```
((λn s z→(((λs z→(s (s z))) (n s)) z)) 2)
```

```
((λn s z→(((λs z→(s (s z))) (n s)) z)) 2)
```

Natural numbers

$$\texttt{((}\lambda\texttt{n s z}\rightarrow\texttt{(((}\lambda\texttt{s z}\rightarrow\texttt{(s (s z))) (n s)) z))}\ \boxed{2}\texttt{)}$$

Natural numbers


```
((λn s z→(((λs z→(s (s z))) (n s)) z)) 2)
```

```
((λn s z→(((λs z→(s (s z))) (n s)) z))
     (λs z→(s (s z))) )
```

Natural numbers


```
((λn s z→(((λs z→(s (s z))) (n s)) z)) (λs z
    →(s (s z))))
```

Natural numbers

```
((λn s z→(((λs z→(s (s z))) (n s)) z)) (λs z
    →(s (s z))))
```

```
((λn s z→(((λs z→(s (s z))) (n s)) z))
    (λs z→(s (s z))) )
```

Natural numbers


```
((λn s z→(((λs z→(s (s z))) (n s)) z))
    (λs z→(s (s z))) )
```

Natural numbers

```
((λn s z→(((λs z→(s (s z))) (n s)) z))
    (λs z→(s (s z))) )
```

```
((λn s z→(((λs z→(s (s z))) (n s)) z)) 2 )
```

$$\boxed{\texttt{((}\lambda\texttt{n s z}\rightarrow\texttt{(((}\lambda\texttt{s z}\rightarrow\texttt{(s (s z))) (n s)) z)) 2)}}$$

Natural numbers


```
((λn s z→(((λs z→(s (s z))) (n s)) z)) 2)
```

```
((λn s z→(((λs z→(s (s z))) (n s)) z)) 2)
```

Natural numbers

$$((\lambda \mathtt{n\ s\ z} \rightarrow (((\lambda \mathtt{s\ z} \rightarrow (\mathtt{s\ (s\ z)}))\ (\mathtt{n\ s}))\ \mathtt{z}))\ \boxed{2})$$

Natural numbers

```
((λn s z→(((λs z→(s (s z))) (n s)) z)) 2)
```

```
((λn s z→(((λs z→(s (s z))) (n s)) z))
    (λs z→(s (s z))) )
```

Natural numbers

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```
((λn s z→(((λs z→(s (s z))) (n s)) z)) (λs z
   →(s (s z))))
```

Natural numbers


```
((λn s z→(((λs z→(s (s z))) (n s)) z)) (λs z
    →(s (s z))))
```

```
((λn s z→(((λs z→(s (s z))) (n s)) z)) (λs z→(s (s z)))
```

```
((λn s z→(((λs z→(s (s z))) (n s)) z)) (λs z→(s (s z))
```

Natural numbers


```
((λn s z→(((λs z→(s (s z))) (n s)) z)) (λs z→(s (s z)))
```

```
(λs z→(((λs z→(s (s z))) ( (λs z→(s (s z))) s)
    ) z))
```

Natural numbers

```
(λs z→(((λs z→(s (s z))) ((λs z→(s (s z))) s
   )) z))
```

```
(λs z→(((λs z→(s (s z))) ((λs z→(s (s z))) s
   )) z))
```

```
(λs z→( ((λs z→(s (s z))) ((λs z→(s (s z))) s))  z)
   )
```

```
(λs z→( ((λs z→(s (s z))) ((λs z→(s (s z))) s)) z)
    )
```

Natural numbers


```
(λs  z→( ((λs z→(s (s z))) ((λs z→(s (s z))) s))  z)
    )
```

```
(λs  z→((λz→( ((λs z→(s (s z))) s)  (
     ((λs z→(s (s z))) s)  z)))  z))
```

Natural numbers


```
(λs z→((λz→(((λs z→(s (s z))) s) (((λs z→(s
    (s z))) s) z))) z))
```

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```
(λs z→((λz→(((λs z→(s (s z))) s) (((λs z→(s
    (s z))) s) z))) z))
```

```
(λs z→
    ((λz→(((λs z→(s (s z))) s) (((λs z→(s (s z))) s) z)
```

```
(λs  z→
     ((λz→(((λs  z→(s  (s  z)))  s)  (((λs  z→(s  (s  z)))  s)  z)
```

Natural numbers


```
(λs z→
    ((λz→(((λs z→(s (s z))) s) (((λs z→(s (s z))) s) z)
```

```
(λs z→(((λs z→(s (s z))) s) (((λs z→(s (s z)
    )) s) z)))
```

```
(λs z→(((λs z→(s (s z))) s) (((λs z→(s (s z)
    )) s) z)))
```

Natural numbers


```
(λs z→(((λs z→(s (s z))) s) (((λs z→(s (s z)
    )) s) z)))
```

```
(λs z→( ((λs z→(s (s z))) s)  (((λs z→(s (s z)))
     s) z)))
```

```
(λs z→( ((λs z→(s (s z))) s) (((λs z→(s (s z)))
     s) z)))
```

Natural numbers


```
(λs z→( ((λs z→(s (s z))) s) (((λs z→(s (s z)))
    s) z)))
```

```
(λs z→((λz→(s (s z))) (((λs z→(s (s z))) s)
    z)))
```

Natural numbers

```
(λs z→((λz→(s (s z))) (((λs z→(s (s z))) s)
    z)))
```

Natural numbers


```
(λs z→((λz→(s (s z))) (((λs z→(s (s z))) s)
    z)))
```

```
(λs z→((λz→(s (s z))) ( ((λs z→(s (s z))) s)  z)
    ))
```

```
(λs z→((λz→(s (s z))) ( ((λs z→(s (s z))) s) z)
   ))
```

Natural numbers


```
(λs z→((λz→(s (s z))) ( ((λs z→(s (s z))) s) z)
    ))
```

```
(λs z→((λz→(s (s z))) ((λz→( s ( s z))) z)))
```

$$\boxed{\texttt{(}\lambda\texttt{s z}\rightarrow\texttt{((}\lambda\texttt{z}\rightarrow\texttt{(s (s z))) ((}\lambda\texttt{z}\rightarrow\texttt{(s (s z))) z)))}}$$

`(λs z→((λz→(s (s z))) ((λz→(s (s z))) z)))`

`(λs z→((λz→(s (s z))) ((λz→(s (s z))) z)))`

Natural numbers

$(\lambda \texttt{s z} \rightarrow ((\lambda \texttt{z} \rightarrow (\texttt{s (s z)})) \; ((\lambda \texttt{z} \rightarrow (\texttt{s (s z)})) \; \texttt{z})))$

Natural numbers


```
(λs z→((λz→(s (s z))) ((λz→(s (s z))) z)))
```

```
(λs z→((λz→(s (s z))) (s (s z))))
```

```
(λs z→((λz→(s (s z))) (s (s z))))
```

Natural numbers


```
(λs z→((λz→(s (s z))) (s (s z))))
```

```
(λs z→((λz→(s (s z))) (s (s z))))
```

Natural numbers

`(λs z→ ((λz→(s (s z))) (s (s z))) )`

Natural numbers


```
(λs z→((λz→(s (s z))) (s (s z))))
```

```
(λs z→(s (s (s (s z)))))
```

Natural numbers


```
(λs z→(s (s (s (s z)))))
```

Natural numbers


```
(λs z→(s (s (s (s z)))))
```

```
(λs z→(s (s (s (s z)))))
```

$(\lambda \texttt{s z} \rightarrow (\texttt{s (s (s (s z))))})$

Natural numbers

$(\lambda \mathtt{s\ z} \rightarrow (\mathtt{s\ (s\ (s\ (s\ z))))})$

4

Zero checking

- We might wish to verify whether or not a number is zero
- We can simply pass the number parameters that fail the check (s) and pass it (z)
- ```
  (λm n→((m (λx→FALSE)) TRUE))
  ```
  ```

Natural numbers

Zero checking

Let us try it out to $0 \; \texttt{=} \; 2 \rightarrow_\beta \texttt{FALSE}$

Natural numbers


```
(2 = 0)
```

Natural numbers


```
(2 = 0)
```

```
( 0?  2)
```

Natural numbers


```
( 0?  2)
```

Natural numbers


```
( 0?  2)
```

```
( (λm n→((m (λx→FALSE)) TRUE))  2)
```

Natural numbers


```
((λm n→((m (λx→FALSE)) TRUE)) 2)
```

Natural numbers

$((\lambda m\ n \rightarrow ((m\ (\lambda x \rightarrow FALSE))\ TRUE))\ 2)$

$((\lambda m\ n \rightarrow ((m\ (\lambda x \rightarrow FALSE))\ TRUE))\ \boxed{2})$

Natural numbers


```
((λm n→((m (λx→FALSE)) TRUE)) 2)
```

Natural numbers


```
((λm n→((m (λx→FALSE)) TRUE)) 2)
```

```
((λm n→((m (λx→FALSE)) TRUE))
    (λs z→(s (s z))) )
```

Natural numbers


```
((λm n→((m (λx→FALSE)) TRUE)) (λs z→(s (s z)
    )))
```

Natural numbers


```
((λm n→((m (λx→FALSE)) TRUE)) (λs z→(s (s z)
    )))
```

```
((λm n→((m (λx→FALSE)) TRUE))
    (λs z→(s (s z))) )
```

```
((λm n→((m (λx→FALSE)) TRUE))
    (λs z→(s (s z))) )
```

Natural numbers


```
((λm n→((m (λx→FALSE)) TRUE))
    (λs z→(s (s z))) )
```

```
((λm n→((m (λx→FALSE)) TRUE))  2 )
```

Natural numbers

$$\boxed{\texttt{((}\lambda\texttt{m n}\rightarrow\texttt{((m (}\lambda\texttt{x}\rightarrow\texttt{FALSE)) TRUE)) 2)}}$$

Natural numbers


```
((λm n→((m (λx→FALSE)) TRUE)) 2)
```

```
((λm n→((m (λx→FALSE)) TRUE)) 2)
```

Natural numbers


```
((λm n→((m (λx→FALSE)) TRUE)) 2)
```

Natural numbers


```
((λm n→((m (λx→FALSE)) TRUE)) 2)
```

```
((λm n→((m (λx→FALSE)) TRUE))
    (λs z→(s (s z))) )
```

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```
((λm n→((m (λx→FALSE)) TRUE)) (λs z→(s (s z)
    )))
```

Natural numbers


```
((λm n→((m (λx→FALSE)) TRUE)) (λs z→(s (s z)
    )))
```

```
((λm n→((m (λx→FALSE)) TRUE)) (λs z→(s (s z))))
```

Natural numbers


```
((λm n→((m (λx→FALSE)) TRUE)) (λs z→(s (s z))))
```

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```
((λm n→((m (λx→FALSE)) TRUE)) (λs z→(s (s z))))
```

```
(λn→(( (λs z→(s (s z))) (λx→FALSE)) TRUE))
```

Natural numbers

$$\boxed{\texttt{(}\lambda\texttt{n}\rightarrow\texttt{((}(\lambda\texttt{s z}\rightarrow\texttt{(s (s z))) (}\lambda\texttt{x}\rightarrow\texttt{FALSE)) TRUE))}}$$

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```
(λn→(((λs z→(s (s z))) (λx→FALSE)) TRUE))
```

```
(λn→(((λs z→(s (s z))) (λx→FALSE)) TRUE))
```

Natural numbers


```
(λn→(((λs z→(s (s z))) (λx→FALSE)) TRUE))
```

Natural numbers


```
(λn→(((λs z→(s (s z))) (λx→FALSE)) TRUE))
```

```
(λn→(((λs z→(s (s z))) (λx→(λt f→f))) TRUE))
```

Natural numbers


```
(λn→(((λs z→(s (s z))) (λx→ (λt f→f))) TRUE)
    )
```

Natural numbers


```
(λn→(((λs z→(s (s z))) (λx→ (λt f→f))) TRUE)
    )
```

```
(λn→(((λs z→(s (s z))) (λx t f→f)) TRUE))
```

Natural numbers

```
(λn→(((λs z→(s (s z))) (λx t f→f)) TRUE))
```

Natural numbers


```
(λn→(((λs z→(s (s z))) (λx t f→f)) TRUE))
```

```
(λn→(((λs z→(s (s z))) (λx t f→f)) (λt f→t))
    )
```

Natural numbers


```
(λn→(((λs z→(s (s z))) (λx t f→f)) (λt f→t))
    )
```

Natural numbers


```
(λn→(((λs z→(s (s z))) (λx t f→f)) (λt f→t))
    )
```

```
(λn→(((λs z→(s (s z))) (λx t f→f)) (λt f→t))
    )
```

Natural numbers


```
(λn→(((λs z→(s (s z))) (λx t f→f)) (λt f→t))
   )
```

Natural numbers


```
(λn→(((λs z→(s (s z))) (λx t f→f)) (λt f→t))
    )
```

```
(λn→(((λs z→(s (s z))) (λx t f→f)) TRUE))
```

Other arithmetic operators

- Division, subtraction, and all manners of comparison operators can be defined similarly
- The level of detail of the specification can be compared to that of a very high level CPU
- This means that we are, to an extent, programming in a sort of assembly
- This is the reason why the traces have been so verbose so far

Other arithmetic operators

- We could also define numbers in base two instead of base one
- This would save processing time, but would result in a slighter more complex specification
- We will just ignore these engineering details: we only focus on **what** can be done, not the best way to do it

Conclusion

Conclusion

Recap

- Lambda terms can be used to encode arbitrary basic data types
- The terms are always lambda expression which, when they get parameters passed in, identify themselves somehow
- Identification can be done by applying something (possibly even a given number of times), or returning one of the parameters

Conclusion

Recap

- There are many encodings of data types, but they all behave in the same way by producing the same outputs for the same inputs
- From now on we will start ignoring the reduction steps for simple terms such as 3+3
- We will instead focus on more complex data structures, such as tuples, discriminated unions, and even lists

Appendix

```
((FALSE bit1) bit0)
```

Appendix


```
((FALSE bit1) bit0)
```

```
((FALSE bit1) bit0)
```

Appendix


```
((FALSE bit1) bit0)
```

Appendix


```
(( FALSE  bit1)  bit0)
```

```
(( (λt f→f)  bit1)  bit0)
```

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```
(((λt f→f) bit1) bit0)
```

Appendix


```
(((λt f→f) bit1) bit0)
```

```
( ((λt f→f) bit1)  bit0)
```

Appendix


```
( ((λt f→f) bit1)  bit0)
```

Appendix

$$(\; ((\lambda t\ f \rightarrow f)\ \texttt{bit1}) \;\; \texttt{bit0})$$

$$((\lambda f \rightarrow f)\ \ \texttt{bit0})$$

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```
((λf→f) bit0)
```

Appendix


```
((λf→f) bit0)
```

```
((λf→f) bit0)
```

Appendix


```
((λf→f) bit0)
```

Appendix


```
((λf→f) bit0)
```

```
bit0
```

Remaining and derivations

Let us move to $\texttt{TRUE} \wedge \texttt{FALSE} \rightarrow_\beta \texttt{FALSE}$

Appendix


```
(TRUE ∧ FALSE)
```

Appendix


```
(TRUE ∧ FALSE)
```

```
((∧ TRUE) FALSE)
```

Appendix


```
((∧ TRUE) FALSE)
```

Appendix


```
(( ∧  TRUE) FALSE)
```

```
(( (λp q→((p q) p))  TRUE) FALSE)
```

Appendix


```
(((λp q→((p q) p)) TRUE) FALSE)
```

Appendix


```
(((λp q→((p q) p)) TRUE) FALSE)
```

```
(((λp q→((p q) p)) TRUE) FALSE)
```

Appendix


```
(((λp q→((p q) p)) TRUE) FALSE)
```

Appendix


```
(((λp q→((p q) p)) TRUE) FALSE)
```

```
(((λp q→((p q) p)) (λt f→t)) FALSE)
```

Appendix


```
(((λp q→((p q) p)) (λt f→t)) FALSE)
```

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$(((\lambda p\ q \rightarrow ((p\ q)\ p))\ (\lambda t\ f \rightarrow t))\ \mathtt{FALSE})$

$(((\lambda p\ q \rightarrow ((p\ q)\ p))\ \boxed{(\lambda t\ f \rightarrow t)})\ \mathtt{FALSE})$

Appendix


```
(((λp q→((p q) p)) (λt f→t)) FALSE)
```

Appendix


```
(((λp q→((p q) p)) (λt f→t)) FALSE)
```

```
(((λp q→((p q) p)) TRUE) FALSE)
```

```
(((λp q→((p q) p)) TRUE) FALSE)
```

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$(((\lambda p\ q \rightarrow ((p\ q)\ p))\ \mathtt{TRUE})\ \mathtt{FALSE})$

$(((\lambda p\ q \rightarrow ((p\ q)\ p))\ \mathtt{TRUE})\ \boxed{\mathtt{FALSE}})$

Appendix


```
(((λp q→((p q) p)) TRUE) FALSE)
```

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```
(((λp q→((p q) p)) TRUE) FALSE)
```

```
(((λp q→((p q) p)) TRUE) (λt f→f))
```

```
(((λp q→((p q) p)) TRUE) (λt f→f))
```

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```
(((λp q→((p q) p)) TRUE) (λt f→f))
```

```
(((λp q→((p q) p)) TRUE) (λt f→f))
```

Appendix


```
(((λp q→((p q) p)) TRUE) (λt f→f))
```

Appendix


```
(((λp q→((p q) p)) TRUE) (λt f→f))
```

```
(((λp q→((p q) p)) TRUE) FALSE)
```

Appendix


```
(((λp q→((p q) p)) TRUE) FALSE)
```

Appendix


```
(((λp q→((p q) p)) TRUE) FALSE)
```

```
(((λp q→((p q) p)) TRUE) FALSE)
```

Appendix


```
(((λp q→((p q) p)) TRUE) FALSE)
```

Appendix


```
(((λp q→((p q) p)) TRUE) FALSE)
```

```
(((λp q→((p q) p)) (λt f→t)) FALSE)
```

Appendix


```
(((λp q→((p q) p)) (λt f→t)) FALSE)
```

Appendix


```
(((λp q→((p q) p)) (λt f→t)) FALSE)
```

```
( ((λp q→((p q) p)) (λt f→t)) FALSE)
```

Appendix

(((λp q→((p q) p)) (λt f→t)) FALSE)

Appendix


```
( ((λp q→((p q) p)) (λt f→t)) FALSE)
```

```
((λq→(( (λt f→t) q) (λt f→t) )) FALSE)
```

Appendix


```
((λq→(((λt f→t) q) (λt f→t))) FALSE)
```

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```
((λq→(((λt f→t) q) (λt f→t))) FALSE)
```

```
((λq→(((λt f→t) q) (λt f→t))) FALSE)
```

Appendix


```
((λq→(((λt f→t) q) (λt f→t))) FALSE)
```

Appendix


```
((λq→(((λt f→t) q) (λt f→t))) FALSE)
```

```
((λq→(((λt f→t) q) (λt f→t))) (λt f→f))
```

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```
((λq→(((λt f→t) q) (λt f→t))) (λt f→f))
```

Appendix


```
((λq→((((λt f→t) q) (λt f→t))) (λt f→f))
```

```
((λq→((((λt f→t) q) (λt f→t))) (λt f→f))
```

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```
((λq→(((λt f→t) q) (λt f→t))) (λt f→f) )
```

Appendix

```
((λq→(((λt f→t) q) (λt f→t))) (λt f→f) )
```

```
((λq→(((λt f→t) q) (λt f→t))) FALSE )
```

Appendix


```
((λq→(((λt f→t) q) (λt f→t))) FALSE)
```

Appendix


```
((λq→(((λt f→t) q) (λt f→t))) FALSE)
```

```
((λq→(((λt f→t) q) (λt f→t))) FALSE)
```

Appendix


```
((λq→(((λt f→t) q) (λt f→t))) FALSE)
```

Appendix


```
((λq→(((λt f→t) q) (λt f→t))) FALSE)
```

```
((λq→(((λt f→t) q) (λt f→t))) (λt f→f))
```

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```
((λq→(((λt f→t) q) (λt f→t))) (λt f→f))
```

Appendix

```
((λq→(((λt f→t) q) (λt f→t))) (λt f→f))
```

```
((λq→(((λt f→t) q) (λt f→t))) (λt f→f))
```

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$$((\lambda q \rightarrow (((\lambda t\ f \rightarrow t)\ q)\ (\lambda t\ f \rightarrow t)))\ (\lambda t\ f \rightarrow f))$$

Appendix


```
((λq→(((λt f→t) q) (λt f→t))) (λt f→f))
```

```
(((λt f→t) (λt f→f)) (λt f→t))
```

Appendix

$$(((\lambda t\ \ f \rightarrow t)\ \ (\lambda t\ \ f \rightarrow f))\ \ (\lambda t\ \ f \rightarrow t))$$

Appendix


```
(((λt f→t) (λt f→f)) (λt f→t))
```

```
( ((λt f→t) (λt f→f)) (λt f→t))
```

Appendix


```
( ((λt f→t) (λt f→f)) (λt f→t))
```

Appendix


```
( ((λt f→t) (λt f→f)) (λt f→t))
```

```
((λf t f→ f ) (λt f→t))
```

Appendix


```
((λf t f→f) (λt f→t))
```

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```
((λf t f→f) (λt f→t))
```

```
((λf t f→f) (λt f→t))
```

Appendix

$((\lambda \texttt{f t f} \rightarrow \texttt{f})\ (\lambda \texttt{t f} \rightarrow \texttt{t}))$

Appendix


```
((λf t f→f) (λt f→t))
```

```
((λf t f→f) TRUE)
```

Appendix


```
((λf t f→f) TRUE)
```

```
((λf t f→f) TRUE)
```

```
((λf t f→f) TRUE)
```

Appendix

$((\lambda \texttt{f t f} \rightarrow \texttt{f})\ \boxed{\texttt{TRUE}})$

Appendix


```
((λf t f→f) TRUE )
```

```
((λf t f→f) (λt f→t) )
```

```
((λf t f→f) (λt f→t))
```

```
((λf t f→f) (λt f→t))
```

```
((λf t f→f) (λt f→t))
```

Appendix

$$((\lambda\mathtt{f\ t\ f}\rightarrow\mathtt{f})\ (\lambda\mathtt{t\ f}\rightarrow\mathtt{t}))$$

Appendix

$((\lambda \texttt{f t f} \rightarrow \texttt{f})\ (\lambda \texttt{t f} \rightarrow \texttt{t}))$

$(\lambda \texttt{t f} \rightarrow \texttt{f})$

```
(λt f→f)
```

Appendix

$(\lambda t\ \ f \rightarrow f)$

$(\lambda t\ \ f \rightarrow f)$

Appendix

$(\lambda$t f$\rightarrow$f)

$(\lambda$t f$\rightarrow$f)

FALSE

Remaining and derivations

Let us move to $\texttt{FALSE} \wedge \texttt{TRUE} \rightarrow_\beta \texttt{FALSE}$

Appendix


```
(FALSE ∧ TRUE)
```

Appendix


```
(FALSE ∧ TRUE)
```

```
((∧ FALSE) TRUE)
```

Appendix


```
(( ∧  FALSE)  TRUE)
```

Appendix

$$((\wedge \ \texttt{FALSE}) \ \texttt{TRUE})$$

$$((\ (\lambda p \ q \rightarrow ((p \ q) \ p)) \ \texttt{FALSE}) \ \texttt{TRUE})$$

Appendix


```
(((λp q→((p q) p)) FALSE) TRUE)
```

Appendix


```
(((λp q→((p q) p)) FALSE) TRUE)
```

```
(((λp q→((p q) p)) FALSE) TRUE)
```

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```
(((λp q→((p q) p)) FALSE) TRUE)
```

Appendix


```
(((λp q→((p q) p)) FALSE) TRUE)
```

```
(((λp q→((p q) p)) (λt f→f)) TRUE)
```

Appendix

$$\boxed{(((\lambda\texttt{p q}\rightarrow((\texttt{p q})\ \texttt{p}))\ (\lambda\texttt{t f}\rightarrow\texttt{f}))\ \texttt{TRUE})}$$

Appendix

```
(((λp q→((p q) p)) (λt f→f)) TRUE)
```

```
(((λp q→((p q) p)) (λt f→f)) TRUE)
```

```
(((λp q→((p q) p)) (λt f→f)) TRUE)
```

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```
(((λp q→((p q) p)) (λt f→f)) TRUE)
```

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(((λp q→((p q) p)) FALSE) TRUE)
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Appendix


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(((λp q→((p q) p)) FALSE) TRUE)
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(((λp q→((p q) p)) FALSE) TRUE)
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(((λp q→((p q) p)) FALSE) TRUE)
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(((λp q→((p q) p)) FALSE) TRUE)
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Appendix


```
(((λp q→((p q) p)) FALSE) TRUE)
```

```
(((λp q→((p q) p)) FALSE) (λt f→t))
```

Appendix


```
(((λp q→((p q) p)) FALSE) (λt f→t))
```

Appendix


```
(((λp q→((p q) p)) FALSE) (λt f→t))
```

```
(((λp q→((p q) p)) FALSE) (λt f→t))
```

Appendix


```
(((λp q→((p q) p)) FALSE) (λt f→t) )
```

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```
(((λp q→((p q) p)) FALSE) (λt f→t))
```

```
(((λp q→((p q) p)) FALSE) TRUE)
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Appendix

```
(((λp q→((p q) p)) FALSE) TRUE)
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Appendix


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(((λp q→((p q) p)) FALSE) TRUE)
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(((λp q→((p q) p)) FALSE) TRUE)
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(((λp q→((p q) p)) FALSE) TRUE)
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Appendix


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(((λp q→((p q) p)) FALSE) TRUE)
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(((λp q→((p q) p)) (λt f→f)) TRUE)
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Appendix


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(((λp q→((p q) p)) (λt f→f)) TRUE)
```

Appendix


```
(((λp q→((p q) p)) (λt f→f)) TRUE)
```

```
( ((λp q→((p q) p)) (λt f→f))  TRUE)
```

Appendix

$(((\lambda p\ q \rightarrow ((p\ q)\ p))\ (\lambda t\ f \rightarrow f))\ \ \texttt{TRUE})$

Appendix


```
( ((λp q→((p q) p)) (λt f→f)) TRUE)
```

```
((λq→(( (λt f→f) q) (λt f→f) )) TRUE)
```

Appendix

```
((λq→(((λt f→f) q) (λt f→f))) TRUE)
```

Appendix


```
((λq→(((λt f→f) q) (λt f→f))) TRUE)
```

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((λq→(((λt f→f) q) (λt f→f))) TRUE)
```

```
((λq→(((λt f→f) q) (λt f→f))) TRUE)
```

Appendix

$((\lambda q \rightarrow (((\lambda t\ f \rightarrow f)\ q)\ (\lambda t\ f \rightarrow f)))\ \texttt{TRUE})$

$((\lambda q \rightarrow (((\lambda t\ f \rightarrow f)\ q)\ (\lambda t\ f \rightarrow f)))\ (\lambda t\ f \rightarrow t))$

Appendix

```
((λq→(((λt f→f) q) (λt f→f))) (λt f→t))
```

Appendix

$$((\lambda q \rightarrow (((\lambda t\ \ f \rightarrow f)\ \ q)\ \ (\lambda t\ \ f \rightarrow f)))\ \ (\lambda t\ \ f \rightarrow t))$$

$$((\lambda q \rightarrow (((\lambda t\ \ f \rightarrow f)\ \ q)\ \ (\lambda t\ \ f \rightarrow f)))\ \ \boxed{(\lambda t\ \ f \rightarrow t)})$$

Appendix


```
((λq→(((λt f→f) q) (λt f→f))) (λt f→t))
```

```
((λq→(((λt f→f) q) (λt f→f))) (λt f→t))
```

```
((λq→(((λt f→f) q) (λt f→f))) TRUE)
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Appendix

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((λq→(((λt f→f) q) (λt f→f))) TRUE)
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((λq→(((λt f→f) q) (λt f→f))) TRUE)
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((λq→(((λt f→f) q) (λt f→f))) TRUE)
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Appendix


```
((λq→(((λt f→f) q) (λt f→f))) TRUE)
```

Appendix

$((\lambda q \rightarrow (((\lambda t\ f \rightarrow f)\ q)\ (\lambda t\ f \rightarrow f)))\ \texttt{TRUE})$

$((\lambda q \rightarrow (((\lambda t\ f \rightarrow f)\ q)\ (\lambda t\ f \rightarrow f)))\ (\lambda t\ f \rightarrow t))$

Appendix

$$\boxed{\texttt{((}\lambda\texttt{q}\rightarrow\texttt{((((}\lambda\texttt{t f}\rightarrow\texttt{f) q) (}\lambda\texttt{t f}\rightarrow\texttt{f))) (}\lambda\texttt{t f}\rightarrow\texttt{t))}}$$

Appendix


```
((λq→(((λt f→f) q) (λt f→f))) (λt f→t))
```

```
((λq→(((λt f→f) q) (λt f→f))) (λt f→t))
```

Appendix

$((\lambda q \to (((\lambda t\ f \to f)\ q)\ (\lambda t\ f \to f)))\ (\lambda t\ f \to t))$

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$((\lambda q \rightarrow (((\lambda t\ f \rightarrow f)\ q)\ (\lambda t\ f \rightarrow f)))\ (\lambda t\ f \rightarrow t))$

$(((\lambda t\ f \rightarrow f)\ (\lambda t\ f \rightarrow t))\ (\lambda t\ f \rightarrow f))$

Appendix

$$(((\lambda \texttt{t f} \rightarrow \texttt{f})\ (\lambda \texttt{t f} \rightarrow \texttt{t}))\ (\lambda \texttt{t f} \rightarrow \texttt{f}))$$

Appendix

$(((\lambda t\ f \rightarrow f)\ (\lambda t\ f \rightarrow t))\ (\lambda t\ f \rightarrow f))$

$(\ ((\lambda t\ f \rightarrow f)\ (\lambda t\ f \rightarrow t))\ \ (\lambda t\ f \rightarrow f))$

Appendix

(((λt f$\rightarrow$f) (λt f$\rightarrow$t)) (λt f$\rightarrow$f))

```
( ((λt f→f) (λt f→t)) (λt f→f))
```

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((λf→f) (λt f→f))
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Appendix


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((λf→f) (λt f→f))
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Appendix


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((λf→f) (λt f→f))
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((λf→f) (λt f→f))
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```
((λf→f) (λt f→f))
```

Appendix

((λf→f) (λt f→f))

((λf→f) FALSE)

Appendix


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((λf→f) FALSE)
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((λf→f) FALSE)
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((λf→f) FALSE)
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((λf→f) FALSE)
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((λf→f) FALSE)
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((λf→f) (λt f→f) )
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((λf→f) (λt f→f))
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Appendix


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((λf→f) (λt f→f))
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((λf→f) (λt f→f))
```

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```
((λf→f) (λt f→f))
```

Appendix

$((\lambda \mathtt{f} \rightarrow \mathtt{f})\ (\lambda \mathtt{t}\ \mathtt{f} \rightarrow \mathtt{f}))$

$(\lambda \mathtt{t}\ \mathtt{f} \rightarrow \mathtt{f})$

Appendix

$\boxed{(\lambda t\ \ f \rightarrow f)}$

Appendix


```
(λt  f→f )
```

```
(λt  f→f)
```

Appendix

$(\lambda t\ f \rightarrow f)$

Appendix

$(\lambda t\ f \rightarrow f)$

FALSE

Appendix

Remaining and derivations

Let us move to $\texttt{FALSE} \wedge \texttt{FALSE} \rightarrow_{\beta} \texttt{FALSE}$

Appendix


```
(FALSE ∧ FALSE)
```

Appendix


```
(FALSE ∧ FALSE)
```

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((∧ FALSE) FALSE)
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Appendix

$$((\boxed{\wedge}\ \texttt{FALSE})\ \texttt{FALSE})$$

```
(( ∧ FALSE) FALSE)
```

```
(( (λp q→((p q) p)) FALSE) FALSE)
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(((λp q→((p q) p)) FALSE) FALSE)
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(((λp q→((p q) p)) FALSE) FALSE)
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(((λp q→((p q) p)) FALSE) FALSE)
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(((λp q→((p q) p)) FALSE) FALSE)
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(((λp q→((p q) p)) FALSE) FALSE)
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(((λp q→((p q) p)) (λt f→f)) FALSE)
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Appendix


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(((λp q→((p q) p)) (λt f→f)) FALSE)
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(((λp q→((p q) p)) (λt f→f)) FALSE)
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(((λp q→((p q) p)) (λt f→f)) FALSE)
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(((λp q→((p q) p)) (λt f→f)) FALSE)
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(((λp q→((p q) p)) (λt f→f)) FALSE)
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(((λp q→((p q) p)) FALSE) FALSE)
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Appendix

$$\texttt{(((}\lambda\texttt{p q}\rightarrow\texttt{((p q) p)) FALSE) FALSE)}$$

Appendix


```
(((λp q→((p q) p)) FALSE) FALSE)
```

```
(((λp q→((p q) p)) FALSE) FALSE)
```

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```
(((λp q→((p q) p)) FALSE) FALSE)
```

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```
(((λp q→((p q) p)) FALSE) FALSE)
```

```
(((λp q→((p q) p)) FALSE) (λt f→f))
```

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```
(((λp q→((p q) p)) FALSE) (λt f→f))
```

Appendix


```
(((λp q→((p q) p)) FALSE) (λt f→f))
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(((λp q→((p q) p)) FALSE) (λt f→f))
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Appendix


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(((λp q→((p q) p)) FALSE) (λt f→f))
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Appendix


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(((λp q→((p q) p)) FALSE) (λt f→f) )
```

```
(((λp q→((p q) p)) FALSE) FALSE )
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Appendix


```
(((λp q→((p q) p)) FALSE) FALSE)
```

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Appendix


```
(((λp q→((p q) p)) FALSE) FALSE)
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(((λp q→((p q) p)) FALSE) FALSE)
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Appendix


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(((λp q→((p q) p)) FALSE) FALSE)
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(((λp q→((p q) p)) FALSE) FALSE)
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```
(((λp q→((p q) p)) (λt f→f)) FALSE)
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Appendix


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(((λp q→((p q) p)) (λt f→f)) FALSE)
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(((λp q→((p q) p)) (λt f→f)) FALSE)
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( ((λp q→((p q) p)) (λt f→f)) FALSE)
```

Appendix

(((λp q→((p q) p)) (λt f→f)) FALSE)

Appendix

```
( ((λp q→((p q) p)) (λt f→f)) FALSE)
```

```
((λq→(( (λt f→f) q) (λt f→f) )) FALSE)
```

Appendix


```
((λq→(((λt f→f) q) (λt f→f))) FALSE)
```

Appendix


```
((λq→(((λt f→f) q) (λt f→f))) FALSE)
```

```
((λq→(((λt f→f) q) (λt f→f))) FALSE)
```

$$((\lambda q \rightarrow (((\lambda t\ \ f \rightarrow f)\ \ q)\ \ (\lambda t\ \ f \rightarrow f)))\ \ \boxed{\texttt{FALSE}})$$

Appendix


```
((λq→(((λt f→f) q) (λt f→f))) FALSE)
```

```
((λq→(((λt f→f) q) (λt f→f))) (λt f→f))
```

Appendix


```
((λq→(((λt  f→f)  q)  (λt  f→f)))  (λt  f→f))
```

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((λq→(((λt f→f) q) (λt f→f))) (λt f→f))

((λq→(((λt f→f) q) (λt f→f))) (λt f→f))

Appendix

$$((\lambda q \rightarrow (((\lambda t\ f \rightarrow f)\ q)\ (\lambda t\ f \rightarrow f)))\ (\lambda t\ f \rightarrow f))$$

Appendix

$$\texttt{((}\lambda\texttt{q}\rightarrow\texttt{((((}\lambda\texttt{t f}\rightarrow\texttt{f) q) (}\lambda\texttt{t f}\rightarrow\texttt{f))) (}\lambda\texttt{t f}\rightarrow\texttt{f))}$$

$$\texttt{((}\lambda\texttt{q}\rightarrow\texttt{((((}\lambda\texttt{t f}\rightarrow\texttt{f) q) (}\lambda\texttt{t f}\rightarrow\texttt{f))) FALSE)}$$

Appendix

$$\boxed{\texttt{((}\lambda\texttt{q}\rightarrow\texttt{((((}\lambda\texttt{t f}\rightarrow\texttt{f) q) (}\lambda\texttt{t f}\rightarrow\texttt{f))) FALSE)}}$$

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```
((λq→(((λt f→f) q) (λt f→f))) FALSE)
```

```
((λq→(((λt f→f) q) (λt f→f))) FALSE)
```

Appendix


```
((λq→((((λt f→f) q) (λt f→f))) FALSE )
```

Appendix


```
((λq→(((λt f→f) q) (λt f→f))) FALSE)
```

```
((λq→(((λt f→f) q) (λt f→f))) (λt f→f))
```

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$$\boxed{((\lambda q \rightarrow (((\lambda t\ \ f \rightarrow f)\ \ q)\ \ (\lambda t\ \ f \rightarrow f)))\ \ (\lambda t\ \ f \rightarrow f))}$$

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Delta rules

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```
((λq→(((λt f→f) q) (λt f→f))) (λt f→f))
```

```
((λq→(((λt f→f) q) (λt f→f))) (λt f→f))
```

Appendix

`((λq→(((λt f→f) q) (λt f→f))) (λt f→f))`

```
((λq→(((λt f→f) q) (λt f→f))) (λt f→f))
```

```
(((λt f→f) (λt f→f)) (λt f→f))
```

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$$\texttt{(((}\lambda\texttt{t f}\rightarrow\texttt{f) (}\lambda\texttt{t f}\rightarrow\texttt{f)) (}\lambda\texttt{t f}\rightarrow\texttt{f))}$$

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```
(((λt f→f) (λt f→f)) (λt f→f))
```

```
( ((λt f→f) (λt f→f)) (λt f→f))
```

Appendix

($\boxed{\texttt{((}\lambda\texttt{t f}\rightarrow\texttt{f) (}\lambda\texttt{t f}\rightarrow\texttt{f))}}$ (λt f$\rightarrow$f))

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Appendix


```
( ((λt f→f) (λt f→f)) (λt f→f))
```

```
((λf→f) (λt f→f))
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Appendix


```
((λf→f) (λt f→f))
```

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```
((λf→f) (λt f→f))
```

```
((λf→f) (λt f→f))
```

Appendix

$((\lambda f \rightarrow f)\ \ \boxed{(\lambda t\ f \rightarrow f)}\ )$

Appendix

$((\lambda f \rightarrow f) \; (\lambda t \; f \rightarrow f))$

$((\lambda f \rightarrow f) \; \texttt{FALSE})$

Appendix


```
((λf→f) FALSE)
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Appendix


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((λf→f) FALSE)
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((λf→f) FALSE)
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Appendix


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((λf→f) FALSE)
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Appendix


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((λf→f) FALSE)
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((λf→f) (λt f→f) )
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Appendix


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((λf→f) (λt f→f))
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Appendix


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((λf→f) (λt f→f))
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((λf→f) (λt f→f))
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Appendix


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((λf→f) (λt f→f))
```

Appendix

$((\lambda \mathtt{f} \rightarrow \mathtt{f})\ (\lambda \mathtt{t}\ \mathtt{f} \rightarrow \mathtt{f}))$

$(\lambda \mathtt{t}\ \mathtt{f} \rightarrow \mathtt{f})$

Appendix


```
(λt f→f)
```

Appendix

$(\lambda t\ \ f \rightarrow f)$

$(\lambda t\ \ f \rightarrow f)$

Appendix

$(\lambda \texttt{t f} \rightarrow \texttt{f})$

Appendix

`(λt f→f)`

`FALSE`

Appendix

Remaining or derivations

Let us begin to with $\texttt{TRUE} \vee \texttt{FALSE} \rightarrow_\beta \texttt{TRUE}$

Appendix


```
(TRUE ∨ FALSE)
```

Appendix


```
(TRUE ∨ FALSE)
```

```
((∨ TRUE) FALSE)
```

Appendix


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((∨ TRUE) FALSE)
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Appendix


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((∨ TRUE) FALSE)
```

```
(( (λp q→((p p) q)) TRUE) FALSE)
```

Appendix


```
(((λp q→((p p) q)) TRUE) FALSE)
```

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```
(((λp q→((p p) q)) TRUE) FALSE)
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(((λp q→((p p) q)) TRUE) FALSE)
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Appendix


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(((λp q→((p p) q)) TRUE) FALSE)
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Appendix


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(((λp q→((p p) q)) TRUE) FALSE)
```

```
(((λp q→((p p) q)) (λt f→t)) FALSE)
```

$\boxed{\texttt{(((}\lambda\texttt{p q}\rightarrow\texttt{((p p) q)) (}\lambda\texttt{t f}\rightarrow\texttt{t)) FALSE)}}$

Appendix


```
(((λp q→((p p) q)) (λt f→t)) FALSE)
```

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(((λp q→((p p) q)) (λt f→t)) FALSE)
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Appendix


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(((λp q→((p p) q)) (λt f→t)) FALSE)
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Appendix


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(((λp q→((p p) q)) (λt f→t)) FALSE)
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(((λp q→((p p) q)) TRUE) FALSE)
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(((λp q→((p p) q)) TRUE) FALSE)
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Appendix


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(((λp q→((p p) q)) TRUE) FALSE)
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(((λp q→((p p) q)) TRUE) FALSE)
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(((λp q→((p p) q)) TRUE) FALSE)
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Appendix


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(((λp q→((p p) q)) TRUE) FALSE)
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(((λp q→((p p) q)) TRUE) (λt f→f))
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Appendix


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(((λp q→((p p) q)) TRUE) (λt f→f))
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Appendix

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(((λp q→((p p) q)) TRUE) (λt f→f))
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(((λp q→((p p) q)) TRUE) (λt f→f))
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Appendix

```
(((λp q→((p p) q)) TRUE) (λt f→f) )
```

Appendix

$(((\lambda p\ q \rightarrow ((p\ p)\ q))\ \mathtt{TRUE})\ (\lambda t\ f \rightarrow f))$

$(((\lambda p\ q \rightarrow ((p\ p)\ q))\ \mathtt{TRUE})\ \mathtt{FALSE})$

Appendix


```
(((λp q→((p p) q)) TRUE) FALSE)
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Appendix


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(((λp q→((p p) q)) TRUE) FALSE)
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(((λp q→((p p) q)) TRUE) FALSE)
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(((λp q→((p p) q)) TRUE) FALSE)
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Appendix


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(((λp q→((p p) q)) TRUE) FALSE)
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(((λp q→((p p) q)) (λt f→t)) FALSE)
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(((λp q→((p p) q)) (λt f→t)) FALSE)
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(((λp q→((p p) q)) (λt f→t)) FALSE)
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( ((λp q→((p p) q)) (λt f→t)) FALSE)
```

Appendix


```
( ((λp q→((p p) q)) (λt f→t)) FALSE)
```

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```
( ((λp q→((p p) q)) (λt f→t)) FALSE)
```

```
((λq→(( (λt f→t) (λt f→t) ) q)) FALSE)
```

Appendix

$$\boxed{\texttt{((}\lambda\texttt{q}\rightarrow\texttt{(((}\lambda\texttt{t f}\rightarrow\texttt{t) (}\lambda\texttt{t f}\rightarrow\texttt{t)) q)) FALSE)}}$$

Appendix


```
((λq→((((λt f→t) (λt f→t)) q)) FALSE)
```

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((λq→((((λt f→t) (λt f→t)) q)) FALSE)
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((λq→(((λt f→t) (λt f→t)) q)) FALSE)
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Appendix


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((λq→(((λt f→t) (λt f→t)) q)) FALSE)
```

```
((λq→(((λt f→t) (λt f→t)) q)) (λt f→f))
```

Appendix

$$\boxed{\texttt{((}\lambda\texttt{q}\rightarrow\texttt{((((}\lambda\texttt{t f}\rightarrow\texttt{t) (}\lambda\texttt{t f}\rightarrow\texttt{t)) q)) (}\lambda\texttt{t f}\rightarrow\texttt{f))}}$$

Appendix


```
((λq→(((λt f→t) (λt f→t)) q)) (λt f→f))
```

```
((λq→(((λt f→t) (λt f→t)) q)) (λt f→f))
```

Appendix

$$((\lambda q \rightarrow (((\lambda t\ f \rightarrow t)\ (\lambda t\ f \rightarrow t))\ q))\ (\lambda t\ f \rightarrow f)\ )$$

Appendix


```
((λq→(((λt f→t) (λt f→t)) q)) (λt f→f) )
```

```
((λq→(((λt f→t) (λt f→t)) q)) FALSE )
```

Appendix


```
((λq→(((λt f→t) (λt f→t)) q)) FALSE)
```

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```
((λq→(((λt f→t) (λt f→t)) q)) FALSE)
```

```
((λq→(((λt f→t) (λt f→t)) q)) FALSE)
```

```
((λq→(((λt f→t) (λt f→t)) q)) FALSE)
```

Appendix


```
((λq→(((λt f→t) (λt f→t)) q)) FALSE)
```

```
((λq→(((λt f→t) (λt f→t)) q)) (λt f→f))
```

Appendix

$$\boxed{\texttt{((}\lambda\texttt{q}\rightarrow\texttt{(((}\lambda\texttt{t f}\rightarrow\texttt{t) (}\lambda\texttt{t f}\rightarrow\texttt{t)) q)) (}\lambda\texttt{t f}\rightarrow\texttt{f))}}$$

Appendix

$((\lambda q \rightarrow (((\lambda t\ \ f \rightarrow t)\ \ (\lambda t\ \ f \rightarrow t))\ \ q))\ \ (\lambda t\ \ f \rightarrow f))$

$((\lambda q \rightarrow (((\lambda t\ \ f \rightarrow t)\ \ (\lambda t\ \ f \rightarrow t))\ \ q))\ \ (\lambda t\ \ f \rightarrow f))$

Appendix

$$((\lambda q \rightarrow (((\lambda t\ f \rightarrow t)\ (\lambda t\ f \rightarrow t))\ q))\ (\lambda t\ f \rightarrow f))$$

Appendix

`((λq→(((λt f→t) (λt f→t)) q)) (λt f→f))`

`(((λt f→t) (λt f→t)) (λt f→f))`

```
(((λt f→t) (λt f→t)) (λt f→f))
```

Appendix


```
(((λt f→t) (λt f→t)) (λt f→f))
```

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( ((λt f→t) (λt f→t)) (λt f→f))
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Appendix


```
( ((λt f→t) (λt f→t)) (λt f→f))
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Appendix


```
( ((λt f→t) (λt f→t)) (λt f→f))
```

```
((λf t f→ t) (λt f→f))
```

Appendix

```
((λf t f→t) (λt f→f))
```

Appendix

$((\lambda f\ \ t\ \ f \rightarrow t)\ \ (\lambda t\ \ f \rightarrow f))$

$((\lambda f\ \ t\ \ f \rightarrow t)\ \ (\lambda t\ \ f \rightarrow f))$

Appendix

$$((\lambda f\ \ t\ \ f \rightarrow t)\ \ (\lambda t\ \ f \rightarrow f)\ )$$

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$((\lambda f\ t\ f \to t)\ (\lambda t\ f \to f))$

$((\lambda f\ t\ f \to t)\ \text{FALSE})$

Appendix


```
((λf t f→t) FALSE)
```

Appendix


```
((λf t f→t) FALSE)
```

```
((λf t f→t) FALSE)
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Appendix


```
((λf t f→t) FALSE )
```

Appendix


```
((λf t f→t) FALSE)
```

```
((λf t f→t) (λt f→f))
```

Appendix

$$((\lambda \mathtt{f}\ \ \mathtt{t}\ \ \mathtt{f} \rightarrow \mathtt{t})\ \ (\lambda \mathtt{t}\ \ \mathtt{f} \rightarrow \mathtt{f}))$$

Appendix


```
((λf t f→t) (λt f→f))
```

```
((λf t f→t) (λt f→f))
```

Appendix

$((\lambda \mathtt{f\ t\ f} \rightarrow \mathtt{t})\ (\lambda \mathtt{t\ f} \rightarrow \mathtt{f}))$

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```
((λf t f→t) (λt f→f))
```

```
(λt f→t)
```

Appendix


```
(λt f→t)
```

Appendix


```
(λt  f→t)
```

```
(λt f→t)
```

Appendix

$(\lambda t\ f \rightarrow t)$

$(\lambda t\ f \rightarrow t)$

TRUE

Appendix

Remaining or derivations

Let us begin to with $\texttt{False} \vee \texttt{TRUE} \rightarrow_\beta \texttt{TRUE}$

$$\boxed{\texttt{(FALSE} \vee \texttt{TRUE)}}$$

```
(FALSE ∨ TRUE)
```

```
((∨ FALSE) TRUE)
```

Appendix


```
((∨ FALSE) TRUE)
```

Appendix


```
(( ∨ FALSE) TRUE)
```

```
(( (λp q→((p p) q)) FALSE) TRUE)
```

Appendix


```
(((λp q→((p p) q)) FALSE) TRUE)
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Appendix


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(((λp q→((p p) q)) FALSE) TRUE)
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(((λp q→((p p) q)) FALSE) TRUE)
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Appendix


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(((λp q→((p p) q)) FALSE) TRUE)
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```
(((λp q→((p p) q)) FALSE) TRUE)
```

```
(((λp q→((p p) q)) (λt f→f)) TRUE)
```

Appendix


```
(((λp q→((p p) q)) (λt f→f)) TRUE)
```

Appendix

$$\texttt{(((}\lambda\texttt{p q}\rightarrow\texttt{((p p) q)) (}\lambda\texttt{t f}\rightarrow\texttt{f)) TRUE)}$$

$$\texttt{(((}\lambda\texttt{p q}\rightarrow\texttt{((p p) q)) }\boxed{\texttt{(}\lambda\texttt{t f}\rightarrow\texttt{f)}}\texttt{) TRUE)}$$

Appendix


```
(((λp q→((p p) q)) (λt f→f)) TRUE)
```

Appendix


```
(((λp q→((p p) q)) (λt f→f)) TRUE)
```

```
(((λp q→((p p) q)) FALSE) TRUE)
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Appendix


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(((λp q→((p p) q)) FALSE) TRUE)
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Appendix


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(((λp q→((p p) q)) FALSE) TRUE)
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(((λp q→((p p) q)) FALSE) TRUE)
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(((λp q→((p p) q)) FALSE) TRUE)
```

Appendix


```
(((λp q→((p p) q)) FALSE) TRUE)
```

```
(((λp q→((p p) q)) FALSE) (λt f→t))
```

Appendix

$$\boxed{\texttt{(((}\lambda\texttt{p q}\rightarrow\texttt{((p p) q)) FALSE) (}\lambda\texttt{t f}\rightarrow\texttt{t))}}$$

Appendix


```
(((λp q→((p p) q)) FALSE) (λt f→t))
```

```
(((λp q→((p p) q)) FALSE) (λt f→t))
```

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```
(((λp q→((p p) q)) FALSE) (λt f→t) )
```

```
(((λp q→((p p) q)) FALSE) (λt f→t))
```

```
(((λp q→((p p) q)) FALSE) TRUE)
```

Appendix

$$\texttt{(((}\lambda\texttt{p q}\rightarrow\texttt{((p p) q)) FALSE) TRUE)}$$

Appendix


```
(((λp q→((p p) q)) FALSE) TRUE)
```

```
(((λp q→((p p) q)) FALSE) TRUE)
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```
(((λp q→((p p) q)) FALSE) TRUE)
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Appendix


```
(((λp q→((p p) q)) FALSE) TRUE)
```

```
(((λp q→((p p) q)) (λt f→f)) TRUE)
```

Appendix


```
(((λp q→((p p) q)) (λt f→f)) TRUE)
```

Appendix


```
(((λp q→((p p) q)) (λt f→f)) TRUE)
```

```
( ((λp q→((p p) q)) (λt f→f)) TRUE)
```

Appendix


```
( ((λp q→((p p) q)) (λt f→f)) TRUE)
```

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```
( ((λp q→((p p) q)) (λt f→f)) TRUE)
```

```
((λq→(( (λt f→f) (λt f→f) ) q)) TRUE)
```

Appendix


```
((λq→(((λt f→f) (λt f→f)) q)) TRUE)
```

Appendix


```
((λq→(((λt f→f) (λt f→f)) q)) TRUE)
```

```
((λq→(((λt f→f) (λt f→f)) q)) TRUE)
```

Appendix


```
((λq→(((λt f→f) (λt f→f)) q)) TRUE )
```

```
((λq→(((λt f→f) (λt f→f)) q)) TRUE)
```

```
((λq→(((λt f→f) (λt f→f)) q)) (λt f→t))
```

Appendix


```
((λq→(((λt f→f) (λt f→f)) q)) (λt f→t))
```

Appendix

$$((\lambda q \rightarrow (((\lambda t\ \ f \rightarrow f)\ \ (\lambda t\ \ f \rightarrow f))\ \ q))\ \ (\lambda t\ \ f \rightarrow t))$$

$$((\lambda q \rightarrow (((\lambda t\ \ f \rightarrow f)\ \ (\lambda t\ \ f \rightarrow f))\ \ q))\ \ (\lambda t\ \ f \rightarrow t))$$

Appendix


```
((λq→(((λt f→f) (λt f→f)) q)) (λt f→t))
```

Appendix


```
((λq→(((λt f→f) (λt f→f)) q)) (λt f→t))
```

```
((λq→(((λt f→f) (λt f→f)) q)) TRUE)
```

Appendix


```
((λq→(((λt f→f) (λt f→f)) q)) TRUE)
```

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```
((λq→((((λt f→f) (λt f→f)) q)) TRUE)
```

```
((λq→((((λt f→f) (λt f→f)) q)) TRUE)
```

Appendix

$$((\lambda q \rightarrow (((\lambda t\ f \rightarrow f)\ (\lambda t\ f \rightarrow f))\ q))\ \texttt{TRUE})$$

Appendix


```
((λq→(((λt f→f) (λt f→f)) q)) TRUE)
```

```
((λq→(((λt f→f) (λt f→f)) q)) (λt f→t))
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Appendix


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((λq→(((λt f→f) (λt f→f)) q)) (λt f→t))
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((λq→(((λt f→f) (λt f→f)) q)) (λt f→t))
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((λq→(((λt f→f) (λt f→f)) q)) (λt f→t))
```

Appendix

$((\lambda q \rightarrow (((\lambda t\ f \rightarrow f)\ (\lambda t\ f \rightarrow f))\ q))\ (\lambda t\ f \rightarrow t))$

Appendix


```
((λq→(((λt f→f) (λt f→f)) q)) (λt f→t))
```

```
(((λt f→f) (λt f→f)) (λt f→t))
```

Appendix

$$\boxed{(((\lambda t\ f \rightarrow f)\ (\lambda t\ f \rightarrow f))\ (\lambda t\ f \rightarrow t))}$$

$(((\lambda t\ f \rightarrow f)\ (\lambda t\ f \rightarrow f))\ (\lambda t\ f \rightarrow t))$

$(\ ((\lambda t\ f \rightarrow f)\ (\lambda t\ f \rightarrow f))\ \ (\lambda t\ f \rightarrow t))$

Appendix

$\boxed{\texttt{(} \; \texttt{((}\lambda\texttt{t f}\rightarrow\texttt{f) (}\lambda\texttt{t f}\rightarrow\texttt{f))} \;\; \texttt{(}\lambda\texttt{t f}\rightarrow\texttt{t))}}$

Appendix


```
( ((λt f→f) (λt f→f)) (λt f→t))
```

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((λf→f) (λt f→t))
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Appendix


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((λf→f) (λt f→t))
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((λf→f) (λt f→t))
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((λf→f) (λt f→t))
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((λf→f) (λt f→t))
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((λf→f) (λt f→t))
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((λf→f) TRUE)
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((λf→f) TRUE)
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((λf→f) TRUE)
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((λf→f) TRUE)
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((λf→f) TRUE)
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((λf→f) TRUE)
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((λf→f) (λt f→t) )
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((λf→f) (λt f→t))
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((λf→f) (λt f→t))
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((λf→f) (λt f→t))
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((λf→f) (λt f→t))
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((λf→f) (λt f→t))
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(λt f→t)
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Appendix


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(λt f→t)
```

Appendix


```
(λt  f→t)
```

```
(λt  f→t)
```

Appendix

$(\lambda t\ f \rightarrow t)$

Appendix

`(λt f→t)`

`TRUE`

Remaining or derivations

Let us begin to with $\texttt{FALSE} \vee \texttt{FALSE} \rightarrow_\beta \texttt{FALSE}$

Appendix

$$\boxed{(\texttt{FALSE} \lor \texttt{FALSE})}$$

Appendix


```
(FALSE ∨ FALSE)
```

```
((∨ FALSE) FALSE)
```

Appendix


```
((∨ FALSE) FALSE)
```

Appendix

$((\lor$ FALSE) FALSE)

$((\ (\lambda p\ q \rightarrow ((p\ p)\ q))$ FALSE) FALSE)

Appendix


```
(((λp q→((p p) q)) FALSE) FALSE)
```

Appendix


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(((λp q→((p p) q)) FALSE) FALSE)
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(((λp q→((p p) q)) FALSE) FALSE)
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(((λp q→((p p) q)) FALSE) FALSE)
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Appendix


```
(((λp q→((p p) q)) FALSE) FALSE)
```

```
(((λp q→((p p) q)) (λt f→f)) FALSE)
```

Appendix


```
(((λp q→((p p) q)) (λt f→f)) FALSE)
```

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```
(((λp q→((p p) q)) (λt f→f)) FALSE)
```

```
(((λp q→((p p) q)) (λt f→f)) FALSE)
```

Appendix


```
(((λp q→((p p) q)) (λt f→f)) FALSE)
```

Appendix


```
(((λp q→((p p) q)) (λt f→f)) FALSE)
```

```
(((λp q→((p p) q)) FALSE) FALSE)
```

Appendix

$$(((\lambda \texttt{p q} \rightarrow ((\texttt{p p}) \texttt{ q})) \texttt{ FALSE}) \texttt{ FALSE})$$

Appendix


```
(((λp q→((p p) q)) FALSE) FALSE)
```

```
(((λp q→((p p) q)) FALSE) FALSE)
```

```
(((λp q→((p p) q)) FALSE) FALSE)
```

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```
(((λp q→((p p) q)) FALSE) FALSE)
```

```
(((λp q→((p p) q)) FALSE) (λt f→f))
```

Appendix


```
(((λp q→((p p) q)) FALSE) (λt f→f))
```

Appendix


```
(((λp q→((p p) q)) FALSE) (λt f→f))
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```
(((λp q→((p p) q)) FALSE) (λt f→f))
```

Appendix


```
(((λp q→((p p) q)) FALSE) (λt f→f) )
```

```
(((λp q→((p p) q)) FALSE) (λt f→f))
```

```
(((λp q→((p p) q)) FALSE) FALSE)
```

Appendix

$$\texttt{(((}\lambda\texttt{p q}\rightarrow\texttt{((p p) q)) FALSE) FALSE)}$$

Appendix


```
(((λp q→((p p) q)) FALSE) FALSE)
```

```
(((λp q→((p p) q)) FALSE) FALSE)
```

$$(((\lambda p\ q \rightarrow ((p\ p)\ q))\ \boxed{\mathtt{FALSE}})\ \mathtt{FALSE})$$

Appendix


```
(((λp q→((p p) q)) FALSE) FALSE)
```

```
(((λp q→((p p) q)) (λt f→f)) FALSE)
```

Appendix

$$\texttt{(((}\lambda\texttt{p q}\rightarrow\texttt{((p p) q)) (}\lambda\texttt{t f}\rightarrow\texttt{f)) FALSE)}$$

$$(((\lambda p\ q \rightarrow ((p\ p)\ q))\ (\lambda t\ f \rightarrow f))\ \mathtt{FALSE})$$

$$(\ ((\lambda p\ q \rightarrow ((p\ p)\ q))\ (\lambda t\ f \rightarrow f))\ \ \mathtt{FALSE})$$

Appendix

(((λp q$\rightarrow$((p p) q)) (λt f$\rightarrow$f)) FALSE)

Appendix


```
( ((λp q→((p p) q)) (λt f→f)) FALSE)
```

```
((λq→(( (λt f→f) (λt f→f) ) q)) FALSE)
```

Appendix


```
((λq→(((λt f→f) (λt f→f)) q)) FALSE)
```

Appendix


```
((λq→(((λt f→f) (λt f→f)) q)) FALSE)
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((λq→(((λt f→f) (λt f→f)) q)) FALSE)
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Appendix


```
((λq→(((λt f→f) (λt f→f)) q)) FALSE)
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Appendix

((λq→(((λt f→f) (λt f→f)) q)) FALSE)

((λq→(((λt f→f) (λt f→f)) q)) (λt f→f))

Appendix


```
((λq→(((λt f→f) (λt f→f)) q)) (λt f→f))
```

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```
((λq→(((λt f→f) (λt f→f)) q)) (λt f→f))
```

```
((λq→(((λt f→f) (λt f→f)) q)) (λt f→f))
```

Appendix

$$((\lambda q \rightarrow (((\lambda t\ f \rightarrow f)\ (\lambda t\ f \rightarrow f))\ q))\ (\lambda t\ f \rightarrow f))$$

Appendix


```
((λq→(((λt f→f) (λt f→f)) q)) (λt f→f))
```

```
((λq→(((λt f→f) (λt f→f)) q)) FALSE)
```

Appendix


```
((λq→(((λt f→f) (λt f→f)) q)) FALSE)
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Appendix


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((λq→(((λt f→f) (λt f→f)) q)) FALSE)
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((λq→(((λt f→f) (λt f→f)) q)) FALSE)
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Appendix


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((λq→(((λt f→f) (λt f→f)) q)) FALSE)
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Appendix


```
((λq→(((λt f→f) (λt f→f)) q)) FALSE)
```

```
((λq→(((λt f→f) (λt f→f)) q)) (λt f→f))
```

Appendix


```
((λq→(((λt f→f) (λt f→f)) q)) (λt f→f))
```

Appendix

`((λq→(((λt f→f) (λt f→f)) q)) (λt f→f))`

`((λq→(((λt f→f) (λt f→f)) q)) (λt f→f))`

Appendix

$((\lambda q \rightarrow (((\lambda t\ f \rightarrow f)\ (\lambda t\ f \rightarrow f))\ q))\ (\lambda t\ f \rightarrow f))$

Appendix

$((\lambda q \rightarrow (((\lambda t\ f \rightarrow f)\ (\lambda t\ f \rightarrow f))\ q))\ (\lambda t\ f \rightarrow f))$

$(((\lambda t\ f \rightarrow f)\ (\lambda t\ f \rightarrow f))\ (\lambda t\ f \rightarrow f))$

Appendix

$(((\lambda\texttt{t f}\rightarrow\texttt{f})\ (\lambda\texttt{t f}\rightarrow\texttt{f}))\ (\lambda\texttt{t f}\rightarrow\texttt{f}))$

Appendix

$(((\lambda \mathtt{t\ f} \rightarrow \mathtt{f})\ (\lambda \mathtt{t\ f} \rightarrow \mathtt{f}))\ (\lambda \mathtt{t\ f} \rightarrow \mathtt{f}))$

$(\ ((\lambda \mathtt{t\ f} \rightarrow \mathtt{f})\ (\lambda \mathtt{t\ f} \rightarrow \mathtt{f}))\ \ (\lambda \mathtt{t\ f} \rightarrow \mathtt{f}))$

Appendix

(((λt f→f) (λt f→f)) (λt f→f))

Appendix

`( ((λt f→f) (λt f→f)) (λt f→f))`

`((λf→f) (λt f→f))`

Appendix


```
((λf→f) (λt f→f))
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Appendix


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((λf→f) (λt f→f))
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((λf→f) (λt f→f))
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((λf→f) (λt f→f))
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```
((λf→f) (λt f→f))
```

```
((λf→f) FALSE)
```

Appendix


```
((λf→f) FALSE)
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Appendix


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((λf→f) FALSE)
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((λf→f) FALSE)
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Appendix


```
((λf→f) FALSE)
```

```
((λf→f) FALSE )
```

```
((λf→f) (λt f→f) )
```

Appendix


```
((λf→f) (λt f→f))
```

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```
((λf→f) (λt f→f))
```

```
((λf→f) (λt f→f))
```

Appendix

$((\lambda f \rightarrow f)\ (\lambda t\ f \rightarrow f))$

Appendix

$((\lambda f \rightarrow f)\ (\lambda t\ f \rightarrow f))$

$(\lambda t\ f \rightarrow f)$

Appendix

$(\lambda t\ \ f \rightarrow f)$

Appendix

$(\lambda t\ \ f \rightarrow f)$

$(\lambda t\ \ f \rightarrow f)$

Appendix

(λt f→f)

Appendix

$(\lambda$t f$\rightarrow$f)

FALSE

Remaining numeral derivations

Let us try out $0 \;=\; 0 \rightarrow_\beta \mathtt{TRUE}$

Appendix


```
(0 = 0)
```

Appendix


```
(0 = 0)
```

```
( 0?  0)
```

Appendix

```
( 0?  0)
```

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Appendix


```
( 0?  0)
```

```
( (λm n→((m (λx→FALSE)) TRUE))  0)
```

Appendix


```
((λm n→((m (λx→FALSE)) TRUE)) 0)
```

Appendix


```
((λm n→((m (λx→FALSE)) TRUE)) 0)
```

```
((λm n→((m (λx→FALSE)) TRUE)) 0)
```

Appendix


```
((λm n→((m (λx→FALSE)) TRUE)) 0)
```

Appendix

((λm n→((m (λx→FALSE)) TRUE)) 0)

((λm n→((m (λx→FALSE)) TRUE)) (λs z→z))

Appendix

$$\boxed{\texttt{((}\lambda\texttt{m n}\rightarrow\texttt{((m (}\lambda\texttt{x}\rightarrow\texttt{FALSE)) TRUE)) (}\lambda\texttt{s z}\rightarrow\texttt{z))}}$$

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$((\lambda m\ n \rightarrow ((m\ (\lambda x \rightarrow FALSE))\ TRUE))\ (\lambda s\ z \rightarrow z))$

$((\lambda m\ n \rightarrow ((m\ (\lambda x \rightarrow FALSE))\ TRUE))\ (\lambda s\ z \rightarrow z))$

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Appendix


```
((λm n→((m (λx→FALSE)) TRUE)) (λs z→z))
```

Appendix


```
((λm n→((m (λx→FALSE)) TRUE)) (λs z→z))
```

```
((λm n→((m (λx→FALSE)) TRUE)) 0)
```

Appendix


```
((λm n→((m (λx→FALSE)) TRUE)) 0)
```

Appendix

$$((\lambda m\ n \rightarrow ((m\ (\lambda x \rightarrow \mathtt{FALSE}))\ \mathtt{TRUE}))\ 0)$$

$$((\lambda m\ n \rightarrow ((m\ (\lambda x \rightarrow \mathtt{FALSE}))\ \mathtt{TRUE}))\ \boxed{0})$$

Appendix

$$((\lambda m\ n \rightarrow ((m\ (\lambda x \rightarrow \mathtt{FALSE}))\ \mathtt{TRUE}))\ \boxed{0})$$

Appendix

```
((λm n→((m (λx→FALSE)) TRUE)) 0 )
```

```
((λm n→((m (λx→FALSE)) TRUE)) (λs z→z) )
```

Appendix


```
((λm n→((m (λx→FALSE)) TRUE)) (λs z→z))
```

Appendix


```
((λm n→((m (λx→FALSE)) TRUE)) (λs z→z))
```

```
((λm n→((m (λx→FALSE)) TRUE)) (λs z→z))
```

Appendix


```
((λm n→((m (λx→FALSE)) TRUE)) (λs z→z))
```

Appendix

$((\lambda m\ n \rightarrow ((m\ (\lambda x \rightarrow FALSE))\ TRUE))\ (\lambda s\ z \rightarrow z))$

$(\lambda n \rightarrow ((\ (\lambda s\ z \rightarrow z)\ \ (\lambda x \rightarrow FALSE))\ \ TRUE))$

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$$\boxed{\mathtt{(\lambda n \to (((\lambda s\ \ z \to z)\ \ (\lambda x \to FALSE))\ \ TRUE))}}$$

Appendix


```
(λn→(((λs z→z) (λx→FALSE)) TRUE))
```

```
(λn→(((λs z→z) (λx→FALSE)) TRUE))
```

Appendix


```
(λn→(((λs z→z) (λx→FALSE)) TRUE))
```

Appendix


```
(λn→(((λs z→z) (λx→FALSE)) TRUE))
```

```
(λn→(((λs z→z) (λx→(λt f→f))) TRUE))
```

Appendix


```
(λn→(((λs z→z) (λx→ (λt f→f))) TRUE))
```

Appendix


```
(λn→(((λs z→z) (λx→ (λt f→f))) TRUE))
```

```
(λn→(((λs z→z) (λx t f→f)) TRUE))
```

Appendix

```
(λn→(((λs z→z) (λx t f→f)) TRUE))
```

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(λn→(((λs z→z) (λx t f→f)) TRUE))
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(λn→(((λs z→z) (λx t f→f)) (λt f→t)))
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This is it!

The best of luck, and thanks for the attention!