Building a physics engine - part 5b: cars

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Basic linear dynamics

Basic linear dynamics

- Some easy assumptions for starters
- No gears, lateral forces, etc.
- Sports car with rear traction

Basic linear dynamics

Basic linear dynamics

- Longitudinal force

$$
\begin{aligned}
F_{\text{traction}} &= uF_{\text{engine}} && (1)\\
F_{\text{drag}} &= -C_{\text{drag}}v|v| && (2)\\
F_{\text{rr}} &= -C_{\text{rr}}v && (3)\\
F_{\text{long}} &= F_{\text{traction}} + F_{\text{drag}} + F_{\text{rr}} && (4)
\end{aligned}
$$

  - u is forward direction
  - $C_{\text{drag}} = 0.4257$
  - $C_{\text{rr}} = 12.8$

Basic linear dynamics

Basic linear dynamics

- Braking force

$$F_{\text{brake}} = -uC_{\text{brake}} \quad (5)$$

$$F_{\text{long}} = F_{\text{brake}} + F_{\text{drag}} + F_{\text{rr}} \quad (6)$$

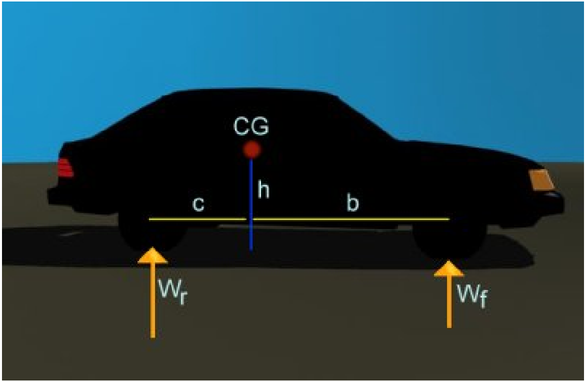
- C_{brake} is a constant that *just feels good*

Weight transfer

Weight transfer

- Acceleration causes a pitch of the car
- It shuffles weight between front and rear
- Tires with more or less friction, and thus capacity to support acceleration
- $F_{\max} = \mu W_w$ for a wheel carrying weight W_w
- $\mu \in [1 \dots 1.5]$

Weight transfer



Weight transfer

Weight transfer

- When the car is at rest

$$W_f = \frac{c}{l} W \quad (7)$$

$$W_r = \frac{b}{l} W \quad (8)$$

- When the car is accelerating

$$W_f = \frac{c}{l} W - \frac{h}{l} ma \quad (9)$$

$$W_r = \frac{b}{l} W + \frac{h}{l} ma \quad (10)$$

Weight transfer

Weight transfer

- When accelerating, pitch the car
- If the force applied by the engine to the wheels is bigger than $F_{\max}$, reduce μ and apply $F_{\max}$, and draw smoke/spinning wheels
- If the force applied by the engine to the wheels is less than $F_{\max}$, apply the engine force directly

Engine force

Engine force

- The engine is not directly connected to the wheels
- Gears apply the engine torque to different values of max wheel τ and max wheel ω
- Lower gears have higher τ
- Higher gears have higher ω

Engine force

Engine force

- $F_{\text{drive}} = u\frac{\tau_{\text{drive}}}{R_w}$ is the force applied to the rear axle
- $\tau_{\text{drive}} = \tau_{\text{engine}} x_g x_d n$ is the torque applied to the rear axle
- τ_{engine} is the torque coming from the engine given the current RPM
- x_g is the gear ratio, x_d is the differential ratio
- $m = 1500kg$ is the car mass
- $n = 0.7$ is the transmission efficiency
- $r_w = 0.34m$ is the wheels radius

Engine force

Gear ratios

- $x_g = 2.66\ 1.78\ 1.3\ 1.0\ 0.74\ 0.5$
- reverse gear $= 2.9$
- $x_d = 3.42$

Engine force

Torque and RPM

- *rpm* determines the current maximum torque
- torque accelerates the wheels
- wheels determine the next *rpm*
- *rpm* is capped; after a while (*red-line*) the engine breaks
- τ_{max} is capped as well; one cap per gear

Engine force

Torque and RPM

- Torque and RPM recurrences

$$\tau_{\text{max}} = \text{LookupCurve}(rpm) \quad (11)$$

$$\tau_{\text{engine}} = \tau_{\text{max}}\alpha_{\text{throttle}} \quad (12)$$

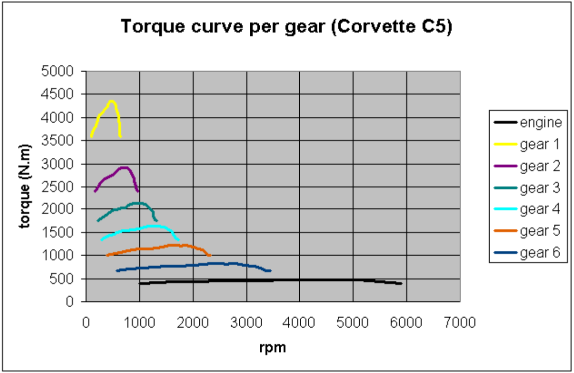
$$rpm = \max(1000, \frac{\omega_w x_g x_d}{2\pi}) \quad (13)$$

Engine force

Wheel angular velocity

- For LookupCurve, any reasonable bell-shaped curve (different for each gear) will do
- Or, copy from the sources of *Marco Monster's - Car Physics for Games* tutorial; they contain some data

Gear plot


Torque curve per gear (Corvette C5)
torque (N.m)
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0
0
1000
2000
3000
4000
5000
6000
7000
rpm
engine
gear 1
gear 2
gear 3
gear 4
gear 5
gear 6

Engine force

Shifting gears

- RPM changes suddenly when changing gear $rpm' = rpm\frac{x'_g}{x_g}$

Engine force

Wheel angular velocity

- Simple solution vs hard solution
- Simple solution: wheels rotating as car is moving

$$\omega_w \approx \frac{|v|}{r_w}$$

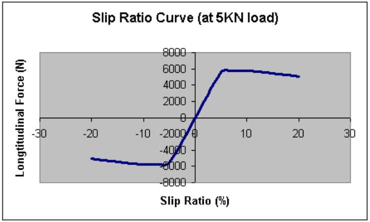
- Hard solution: track wheel angular velocities separately

Slip ratio

Slip ratio

- The amount of acceleration of the car depends on the friction between tires and road
- Rolling tires do not have friction; friction is given by tires rotating faster than they are moving
- Rear tires roll faster than front tires

Longitudinal force

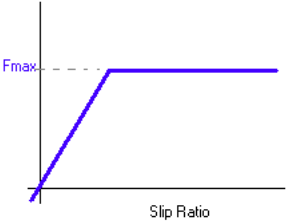


Slip ratio

Slip ratio

- Slip ratio determines the force given by the wheel to the car $\sigma = \frac{\omega_w r_w - v_{\text{long}}}{|v_{\text{long}}|}$
- The traction force given by the wheel at a certain slip ratio is $F_{\text{traction}} = \max(6000, C_t\sigma)$ $\tau_{\text{traction}} = F_{\text{traction}} \times R_w$

Longitudinal force simplified



Slip ratio

Slip ratio

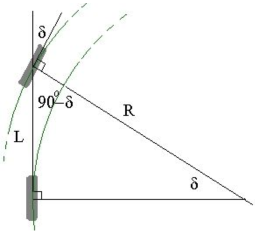
- We track ω_w for each wheel
- We compute the slip ratio and the corresponding torque on the axle $\tau_{\text{total}} = \tau_{\text{drive}} + \underbrace{\tau_{\text{traction}}}_{\text{two wheels}} + \tau_{\text{brake}}$
- We compute the angular acceleration of this force on the wheel $\alpha = \frac{\tau_{\text{total}}}{I_w}$
- The wheel rotating around its central axis has moment of inertia $I_w = \frac{mr_w^2}{2}$

Curves at a low speed

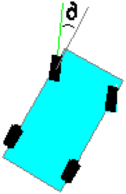
Curves at a low speed

- When travelling at low speed
- We just find the radius of the circle the car describes, depending on the wheel angle
- δ is the wheel turn angle
- $\sin\delta = \frac{L}{R}$
- From the radius we can determine the angular velocity and just rotate the car by that $\omega = \frac{v}{R} = \frac{v\sin\delta}{L}$

Rotation radius



Delta angle



Curves at a high speed

Curves at a high speed

- Turning the front wheels causes a change in their lateral forces
- We add new state information to our system
  - α is the side-slip angle of the wheel, which changes as we turn
  - C_a is the cornering stiffness, a pleasant, and utterly fake, constant

Curves at a high speed

Curves at a high speed

- We compute the lateral and longitudinal speed at a given side-slip angle, for each wheel

$$v_{\mathrm{lat}} = |v| \sin \alpha \tag{14}$$

$$v_{\mathrm{long}} = |v| \cos \alpha \tag{15}$$

Curves at a high speed

Curves at a high speed

- We also compute the side-slip angles given the current angular velocity (started up from low-speed turning) and lateral and longitudinal velocities

$$\alpha_{\text{front}} = \frac{v_{\text{lat}} + \omega b}{v_{\text{long}}} - \delta\,\text{sign}(v_{\text{long}}) \tag{16}$$

$$\alpha_{\text{rear}} = \frac{v_{\text{lat}} - \omega b}{v_{\text{long}}} \tag{17}$$

Curves at a high speed

Lateral forces

- Lateral force also depends on the current weight distribution
- $F_{\text{lateral}} = \max(6000, C_a\alpha)W_w$
- $\tau_{\text{lateral}} = F_{\text{lateral}} \times b$
- Each wheel has a different lateral force; we compute torque from lateral forces and use it as usual to further integrate ω

Assignment

Assignment

- Before the end of next week
- Group-work archive/video on Natschool or uploaded somewhere else and linked in your report
- Individual report by each of you on Natschool
- Add a personalized selection of forces to your simulator

That's it

Thank you!