Basic concepts from physics

Dr. Giuseppe Maggiore

NHTV University of Applied Sciences
Breda, Netherlands

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Introduction

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Particle kinematics

XY particle

- We start with a particle moving across the xy plane
- Position at time t is $r(t) = (x(t), y(t))$

Particle kinematics

XY particle

- Velocity at time t is $v(t) = \dot{r} = (\dot{x}, \dot{y})$ [a]
- Speed is $|v|$
- Acceleration is $a(t) = \dot{v} = \ddot{r} = (\ddot{x}, \ddot{y})$

[a] a dot above denotes derivation over time; this means that $\dot{x} = \frac{dx}{dt}$

Particle kinematics

XY particle

- Tangent is $T(t) = \frac{v}{|v|}$
- Normal is $N(t)$ and is perpendicular to T
- r, T, N is the *moving frame* of the particle (or body space, or local space)

Particle kinematics

XY particle motion with respect to frame

- $v = |v|T = \dot{s}T$
- $a = \dot{v} =$

Particle kinematics

XYZ particle

- We now consider a particle moving in space
- Position at time t is $r(t) = (x(t), y(t), z(t))$

Particle kinematics

XYZ particle

- Velocity at time t is $v(t) = \dot{r} = (\dot{x}, \dot{y}, \dot{z})$
- Speed is $|v|$
- Acceleration is $a(t) = \dot{v} = \ddot{r} = (\ddot{x}, \ddot{y}, \ddot{z})$

Particle kinematics

XYZ particle

- Tangent is $T(t) = \frac{v}{|v|}$
- We have an infinite set of possible vectors normal to T

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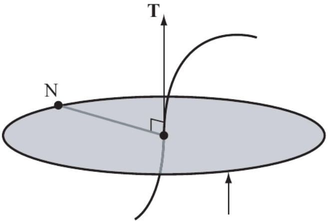


Circle of potential normals

Particle kinematics

XYZ particle

- Normal N is perpendicular to T
- We are missing an axis to have a complete frame
- Binormal is $B = T \times N$
- r, T, N, B is the *moving frame* of the particle (or body space, or local space)

Body kinematics

Rigid body

- $R = [T\ \ N\ \ B]$ put in matrix form (T, N, B are used as *columns* of the matrix) is the *rotation matrix* of the body
- $r(t) = R(t)r_0 + x(t)$ where $x(t)$ is the position of the *center* of the body
- $\omega(t)$, a vector, is the angular velocity of the body
  - Its direction $\frac{\omega(t)}{|\omega(t)|}$ is the rotation axis
  - Its magnitude $|\omega(t)|$ is in rad/s

Particle kinematics

Rigid body rotation

- We need to study $\dot{r}(t)$, in order to determine $\dot{R}$
- We decompose $r(t)$ into a, b where a is parallel to ω and b is perpendicular
  - linearly moving component
  - rotating component
- The instantaneous velocity of $r(t)$ is $\dot{r} = \omega(t) \times b = \omega(t) \times (a + b) = \omega(t) \times r(t)$

Particle kinematics

Rigid body rotation

- We now consider the inertial frames, which are the columns of the rotation matrix
- We compute $\dot{T} = \omega(t) \times T$, $\dot{N} = \omega(t) \times N$, and $\dot{B} = \omega(t) \times B$
- These are the velocities of the axes of the inertial frame
- Also known as the columns of $\dot{R}$

Newton's laws

Topic

- Inertia, the tendency of an object to remain in motion
- Force, the mechanism through which inertia is changed

Newton's laws

About the laws

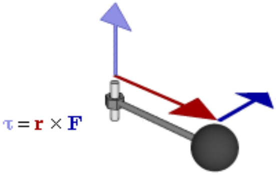
- The **second law** is the one we work the most with
- Mass is assumed to be always constant, so $F = \frac{d}{dt}(mv) = m\frac{d}{dt}v = ma$
- Each of the vector quantities of position, velocity, and acceleration is measured with respect to some arbitrary but fixed coordinate system, referred to as the *inertial frame*, or global space

Torque

From force to torque

- Removing log nuts with a wrench
- Exert a force on the end of the wrench, the nut turns
- The longer the wrench, the easier (but slower) the nut turns

SLIDE



Torque

Definition

- The *ease* of turning is proportional to the length of the wrench and the force applied
- This product is referred to as *torque* or *moment of force*
- Torque is defined as $\tau = r \times F$
  - Direction of torque is axis (and direction) of rotation
  - Length of torque is in rad/s

Torque

Multiple torques

- Multiple torques (just like forces) are simply added together
- $\tau = \sum_i r_i \times F_i$ (discrete body) or $\tau = \int r \times F dr$ (continuous body)

Momenta

Momenta

- Quantification of Newton's **Second Law**
- How much *motion* does the body have?
  - **A lot** means that a lot of force is needed to change it
  - **Little** means that little force is needed to change it

Momenta

Linear momentum

- How much linear *motion* does the body have?
- $p = mv = \sum_i m_i v_i = \int_R v \; dm$
- Force integrates linear momentum directly
- $\frac{dp}{dt} = \frac{d(mv)}{dt} = m\frac{dv}{dt} = ma = F$

Momenta

Angular momentum

- How much rotational *motion* does the body have?
- $L = r \times p = mr \times v$
- Right-hand rule of cross-product:
  - Angular momentum refers to the tendency of the body to rotate around a given axis, L
  - The longer the axis, the harder it is to stop the rotation

Momenta

Angular momentum

- Just like the derivative of linear momentum is force...
- ...angular momentum derived yields torque (when the body does not change shape)
- $\frac{dL}{dp} = \tau$

Center of mass

Tracking particles?

- Do we really need to track all the particles of a rigid body?
- No!
  - Too slow
  - *Center* of mass
  - Properties of the motion *of the whole body*
  - Rigid body behaves as if all the mass were concentrated at a single point
- We compute the center of mass by a weighted average of the body particles relative positions and their respective masses

Center of mass

One dimension

- A wooden plank with two *weights* at the extremes
- Center of mass is $\bar{x} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = x_1 \frac{m_1}{m_1 + m_2} + x_2 \frac{m_2}{m_1 + m_2}$

SLIDE

Center of mass

Two dimensions

- Center of mass is $\bar{x} = \frac{\sum_i m_i x_i}{M}$, where x_i is a 2D vector [a]

[a] Component-wise, the result is $(\bar{x}, \bar{y}) = (\frac{\sum_i m_i x_i}{M}, \frac{\sum_i m_i y_i}{M})$

Center of mass

Three dimensions

- Center of mass is $\bar{x} = \frac{\sum_i m_i x_i}{M}$, where x_i is a 3D vector

Center of mass

Force projection

- When an external force F_{ext} is applied to a body from some position, r_f
- We use the center of mass to split the force between linear and torque
- $F = F_{ext} \cdot \frac{(r_f - \bar{x})}{|(r_f - \bar{x})|}$, $\tau = F_{ext} \times (r_f - \bar{x})$

Moments of inertia

Moments of inertia

- *How difficult is it to set an object into rotation around an axis?*
- Rotational equivalent to mass for linear movement

Moments of inertia

Moment of inertia in 2D

- A single number, because in 2D we can only rotate in one plane
- $I = \sum_i m_i |(x_i, y_i) - (\bar{x}, \bar{y})|^2 = \sum_i m_i (x_i^2 + y_i^2) - m(\bar{x}^2, \bar{y}^2)$

Moments of inertia

Moment of inertia in 3D

- Harder to express, because suddenly we can rotate along an infinite number of axes
- Let us engineer this from the angular momentum of a particle of the body
- Consider a particle
  - Located at relative vector r
  - Moving with linear velocity $v = \omega \times r$

Moments of inertia

Mass matrix in 3D

- $L_i = r_i \times m_i v_i = m_i r_i \times (\omega \times r_i) = J\omega$
- $J_i = m_i \begin{bmatrix} y_i^2 + z_i^2 & -x_i y_i & -x_i z_i \\ -x_i y_i & x_i^2 + z_i^2 & -y_i z_i \\ -x_i z_i & -y_i z_i & y_i^2 + z_i^2 \end{bmatrix}$
- $L_i = J_i \omega$, just like $P = mv$
- For the whole body, we sum all the J_i matrices of the particles
- $J = \sum_i J_i$, $L = J\omega$
- J must be recalculated from the rotated body, because $r_i = R r_0 + \bar{x}$

Whole kinematics

Whole kinematics

- Position, integrated from velocity $\dot{x} = v$
- Velocity, derived from linear momentum $v = \frac{P}{m}$
- Linear momentum, integrated from force $\dot{P} = F_{\text{ext}} \cdot \frac{(r_f - \bar{x})}{|(r_f - \bar{x})|}$

Whole kinematics

Whole kinematics

- Rotation, integrated from angular velocity
  $\dot{R} = [\omega \times T \ \omega \times N \ \omega \times B]$
- Angular velocity, derived from angular momentum $\omega = J^{-1} L$
- Angular momentum, integrated from torque
  $\dot{L} = \tau = \frac{(r_f - \bar{x})}{|(r_f - \bar{x})|} \times F_{\text{ext}}$

That's it

Thank you!